Reply

ANTHONY ILLINGWORTH

University of Reading, Reading, United Kingdom

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1. Introduction

I welcome this opportunity to resolve the apparent inconsistencies identified by Bringi et al. (2003, henceforth BCZU03) and hope that this reply will demonstrate that, once correct drop shapes are used, there is no major disagreement with the article by Illingworth and Blackman (2002, henceforth IB02) but that it is more a matter of interpretation; indeed BCZU03 provide further independent computations to confirm IB02. We are also grateful to BCZU03 for identifying the typographical error in Eq. (18). This consensus is reinforced by the discussions in the excellent book that was recently written by Bringi and Chandrasekar (2001), who are now using the correct drop shapes and normalized raindrop spectra as was first suggested by Illingworth and Blackman (1999) and Illingworth and Johnson (1999). In the title of the IB02 article, it was not possible to indicate that the article had two main thrusts: 1) to demonstrate the flawed logic of Ulbrich (1983) in using the 69 published reflectivity–rainfall rate (Z–R) relationships to derive the range of naturally occurring raindrop spectra that have been widely used to derive equations linking polarization parameters to rainfall rate and 2) to derive new relationships between rainfall rates and polarization parameters by using normalized gamma functions for raindrop spectra together with the ''new'' drop shapes. As BCZU03 correctly point out, the new drop shapes have the dominant role in changing these relationships.

In this response I follow the order of BCZU03. First, I address the Ulbrich range of drop size distributions (DSDs). Second, I discuss the role of drop shapes on differential reflectivity Z_{dr}, confirming that much published work does use relations based on ''old'' drop shapes that can lead to errors in R of up to 3 dB. Third, I address specific differential phase shift K_{dp} and resolve the paradox in BCZU03, who, while arguing from moments that the exponent in a forced K_{dp}–R relation cannot exceed 1.28 confirm that the exponent is 1.4 for Goddard et al. (1995) shapes, as calculated by IB02. Fourth, to support the suggestion that attenuation and differential phase are not linearly related once median volume diameter $D_o > 2.5$ mm, I appeal to recent work by Bringi and Chandrasekar (2001) that aims to resolve problems arising from this nonlinearity for $D_o > 2.5$ mm. Last, I question the conclusion of BCZU03 that recommends methods that are immune to the precise form of the drop shapes. A fuller discussion is provided in a chapter of a forthcoming book (Illingworth 2003; at the time of writing the chapter was available online at http://www.met.rdg.ac.uk/radar/publications.html).

2. Flawed logic and the Ulbrich range of DSD

Ulbrich (1983) identifies the values of coefficients a and b in $Z = aR^b$, obtained from integrating the nonnormalized gamma function [IB02, their Eq. (1)] and ''eliminating λ'' with the coefficients α and β in the 69 relationships of the form $Z = \alpha R^\beta$, and thus predicts the range of naturally occurring raindrop size spectra to be given by $\beta = (7 + \mu)/(4.67 + \mu)$ and

$$\alpha = \frac{10^6 N_o \Gamma(7 + \mu)}{[N_o a_r \Gamma(4.67 + \mu)]^\beta}, \qquad (1)$$

with (Ulbrich's units) $a_r = 331$ mm h^{-1} m^3 cm$^{-3.67}$ (Z: mm^6 m^{-3}). We should have emphasized the flawed logic more in the original paper and should have quoted Haddad et al. (1997) in full, who refer, rather undiplomatically, to this procedure of eliminating λ and equating coefficients by saying it ''is of course a nonsense.'' BCZU03 do not address this aspect in their comment, but it is reassuring to learn from BCZU03 that many of the papers using the ''Ulbrich range'' did, in fact, limit the maximum R to 100–300 mm h^{-1} and the maximum Z to 55 dBZ, even though this was ''not explicitly stated'' in all but one of the papers. Equation (1) is used to calculate N_o as a function of μ for $\alpha = 100$ and 500 in $Z = \alpha R^\beta$ as in Ulbrich (1983) to produce Fig. 1,

Corresponding author address: A. J. Illingworth, Dept. of Meteorology, University of Reading, Reading RG6 6BB, United Kingdom. E-mail: a.j.illingworth@reading.ac.uk

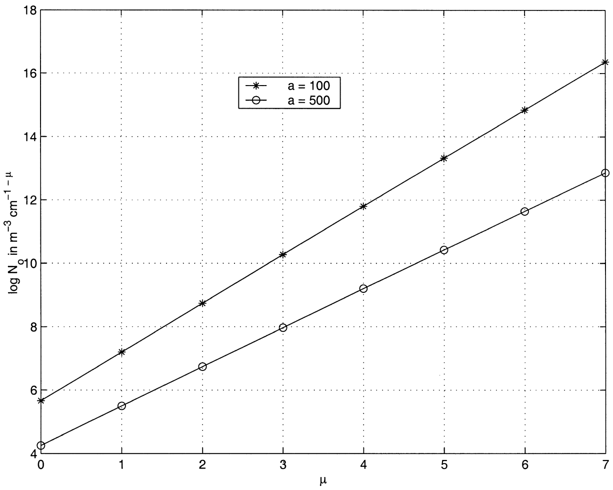

FIG. 1. Values of the nonnormalized N_o as a function of μ derived by Ulbrich (1983) using the 69 $Z = \alpha R^{\beta}$ relationships for $\alpha = 100$ and 500. The straight lines are the range of N_o–μ used by BCZU03.

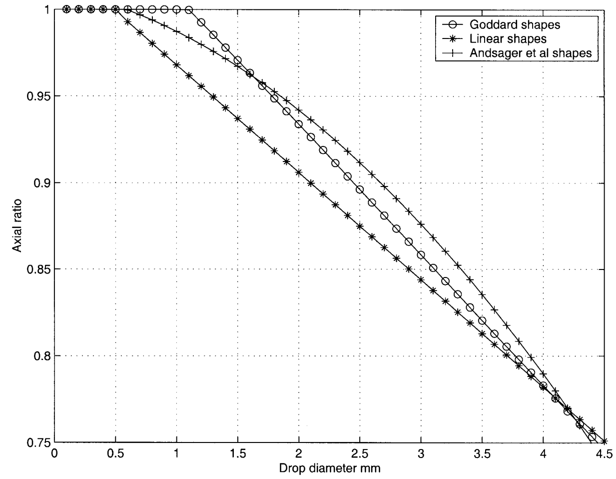

FIG. 2. Drop shapes for the three models: Goddard et al. (1995), Andsager et al. (1999), and the linear shapes of Pruppacher and Beard (1970).

which is identical to Ulbrich's Fig. 7, with the values of the two straight lines given by $N_o = 10^{4.2} \exp(2.8\mu)$ for $\alpha = 500$ and $N_o = 10^{5.5} \exp(3.57\mu)$ for $\alpha = 100$ (m^{-3} cm$^{-1-\mu}$). With a change of units, these straight lines are identical to the N_o–μ range quoted by BCZU03. It seems difficult to reconcile this point with the repeated statement in BCZU03 that they used disdrometers and ''it is reemphasized that we did not use Ulbrich's analysis of Z–R relations to arrive at our N_o–μ bounds.''

3. Drop shapes and implications for differential reflectivity

The crucial importance of the use of ''new drop shapes'' rather than the ''linear'' shapes of Pruppacher and Beard (1970) was repeatedly emphasized throughout IB02. It is mentioned twice in the abstract, and the phrase ''and more realistic drop shapes'' is specifically included in the ''provocative conclusions'' 1 and 2 quoted by BCZU03 but with the drop shape remarks omitted. IB02 used the empirical adjustment of the drop shapes suggested by Goddard et al. (1982, 1995) for small drops to force agreement to match radar-measured and disdrometer-inferred Z_{dr}. BCZU03 criticize this ''overreliance'' on the ''empirical'' Goddard et al. shapes, which only provide an '''effective' axis ratio . . . , not the 'true' axis ratio'' and which were ''not intended . . . to be representative of all rainfall conditions'' and are ''not the true relation that exists in natural rainfall.'' Instead, they have adopted the recent drop shapes of Andsager et al. (1999). Bringi and Chandrasekar (2001, 393–394) describe the Goddard et al. (1982, 1995) empirical adjustments as ''remarkable'' but suggest their applicability may be limited because they were ''obtained in very light stratiform rain and the measured Z_{dr} was generally less than 1–1.5 dB.'' I believe that this adjustment is realistic because it was derived using Z values of up to 40 dBZ and so the rain was not really ''very light'' and because observations of Z_{dr} in the range of 1–1.5 dB are similar to most values observed in the Colorado flash flood of 1997 (Petersen et al. 1999).

We now demonstrate that the empirical adjustment made by Goddard et al. (1982) was indeed remarkably prescient and that the values of Z_{dr} and K_{dp} predicted by such drop shapes are virtually identical to those using the Andsager et al. (1999) shapes. Figure 2 shows a comparison of the three drop-shape models. The empirical new drop shapes of Goddard et al. (1982, 1995) are close to the recent laboratory measurements of Andsager et al. (1999) and, as shown in Fig. 7.10 of Bringi and Chandrasekar (2001), the two curves lie within the observational errors. They are very different from the linear shapes of Pruppacher and Beard (1970), which have, until very recently, been used for analyzing most polarization radar observations. The small differences between the drop shapes of Andsager et al. (1999) and Goddard et al. are shown in an exaggerated form in BCZU03's Fig. 4, which plots the logarithm of (1 − axial ratio) against drop size. This representation means that the difference between an axial ratio of 0.999 and 0.99 appears to be very serious, whereas in fact they are both so close to spheres that they will contribute little to the observed polarization parameters. In a similar way, BCZU03's Fig. 5 implies a massive relative difference in the forward-scattering amplitude between horizontal and vertical polarization for the two drop-shape models when drops are almost spherical, but the actual difference in the contributions to the polarization parameters for real raindrop size spectra is almost negligible.

The computed values of 10 log(Z/R) as a function of Z_{dr}, using the various drop-shape models at S band for normalized gamma distributions with $\mu = 5$, are plotted in Fig. 3, which is similar to Fig. 3 of BCZU03. The

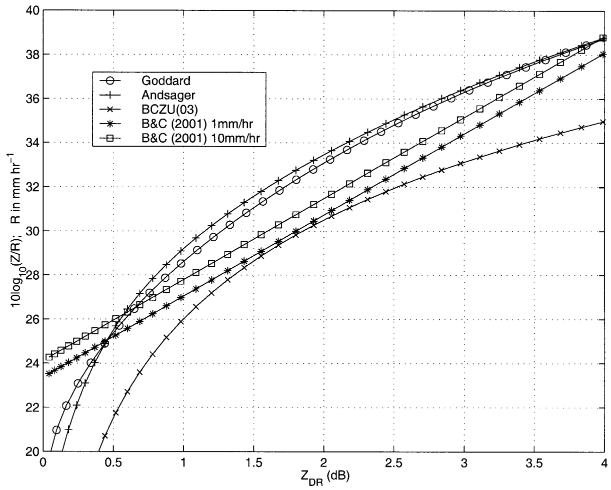

FIG. 3. Values of Z/R for a normalized gamma function as a function of Z_{dr} with $\mu = 5$ at S band. The values using Andsager et al. (1999) drop shapes and those of Goddard et al. are almost identical. The power-law relationship proposed by Bringi and Chandrasekar (2001), for the new drop shapes for $R = 1$ (lower line) and 10 mm h^{-1} (upper line), overestimates rain by up to about 60%, whereas the curve based upon Green's shapes from BCZU03 overestimates R by at least a factor of 2.

computed values of 10 log(Z/R) for the Andsager et al. (1999) drop shapes differ from those of Goddard et al by less than 0.5 dB, and the predicted rainfall rates from the two drop shapes will consequently change by less than 10%. Also plotted in Fig. 3 is the relationship quoted by Bringi and Chandrasekar (2001, p. 538):

$$R = 0.067Z^{0.93}10^{-0.343Z_{dr}}, \tag{2}$$

where Z_{dr} is in decibels. The Z exponent of 0.93 means that, for a given Z_{dr}, Z and R do not scale linearly, and so the upper line for $R = 10$ mm h^{-1} is 0.7 dB above the lower 1 mm h^{-1} line. As discussed at length in IB02, this nonlinearity is puzzling; if the normalized drop spectra do capture the natural variation, then the only explanation must be that the value of μ for heavy rain must be about 10, and that for light rain the value is close to zero; but there is no independent evidence of this explanation. Equation (2) was obtained by using Andsager et al. (1999) drop shapes and independently varying N_w (the normalized drop concentration), D_o, and μ over a reasonable range; for a given Z and Z_{dr}, Fig. 3 suggests that it would lead to overestimates of R by up to 2 dB (60%). The reason for this overestimation is not clear, but forcing a simple straight-line fit for the nonlinear regression rather than a smooth curve may be responsible.

The situation for Green's drop shapes is very different [Eq. (1) from BCZU03]:

$$R = 0.003\,47Z^{0.945}Z_{dr}^{-1.457}, \tag{3}$$

leading to rainfall rates that are always too large by a factor of 2. Because these linear drop shapes have been used to interpret many field observations in the past, we stand by our second ''provocative conclusion'' that many rain rates derived using Z and Z_{dr} that are reported in the literature may well be in error by 3 dB and that reanalysis using new drop shapes from either Goddard et al. or Andsager et al. (1999) would be a worthwhile exercise.

4. Specific differential phase and forced K_{dp}–R relations

BCZU03 confirm the computations of IB02 that use of Goddard et al. drop shapes for a gamma function with $\mu = 5$ and $N_w = 8000$ m^{-3} mm^{-1} leads to $b = 1.4$, where b is the exponent in $R = a\,K_{dp}^b$. BCZU03 argue paradoxically, using moments, that the exponent in a forced K_{dp}–R relation must always be between 1.07 and 1.28. This paradox arises because BCZU03 have used the incorrect expression for the Andsager et al. (1999) drop shapes in their Eq. (4). The correct expression is

$$1 - r = k + \beta D + \alpha D^2, \tag{4}$$

where r is the axial ratio, $k = -0.012$, $\beta = 0.014\,45$ mm^{-1}, $\alpha = 0.010\,28$ mm^{-2}, and, for small drop diameter D, the r is not allowed to exceed unity. The k term, which expresses the fact that drops only become aspherical when they attain a certain size, has been omitted by BCZU03—the same error was made in Fig. 1 of Gorgucci et al. (2000), which we discuss later. With the assumptions made by BCZU03, we have

$$K_{dp} = E[D^3(1 - r)], \tag{5}$$

where E [ ] is the expectation over $N(D)$. The correct form of Eq. (4) in BCZU03 should include k and leads to

$$K_{dp} = kE[D^3] + \beta E[D^4] + \alpha E[D^5]. \tag{6}$$

Following BCZU03 and integrating the normalized gamma function, we have

$$\frac{K_{dp}}{N_w} = \frac{6}{3.67^4}\left[kD_o^4 + \frac{\mu + 4}{3.67 + \mu}\beta D_o^5 + \frac{(\mu + 5)(\mu + 4)}{(3.67 + \mu)^2}\alpha D_o^6\right]. \tag{7}$$

By contrast, R/N_w depends upon $D_o^{4.67}$ and Z/N_w depends upon D_o^7. If the k term is omitted, then the exponent of a forced K_{dp}–R relationship must be between 5/4.67 and 6/4.67. We can identify several difficulties in this treatment. First, when a finite k is introduced that has opposite sign to α and β, the terms in Eq. (7) can cancel and K_{dp} can depend upon higher moments of D_o. Second, for small values of D, the drops should be spherical (e.g., Fig. 1) and should make no contribution to K_{dp}; but in Eq. (5) the value of r can exceed unity, and the small drops can contribute a spurious negative component to the total K_{dp}. Third, even for $r < 1$, Eq. (5) is an approximation. Figure 4 illustrates the effects of

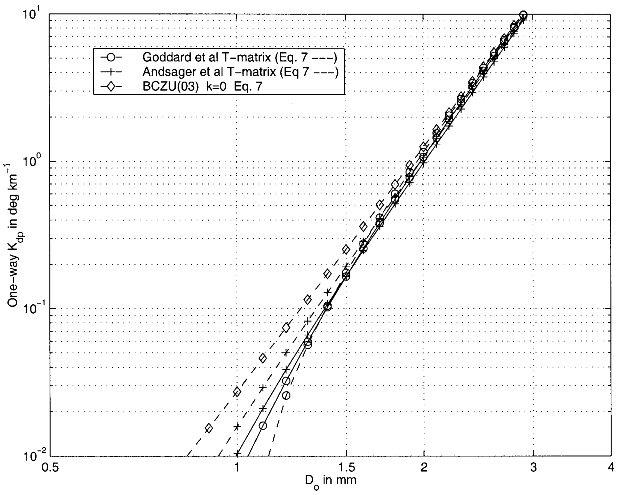

FIG. 4. Values of one-way K_{dp} at S band (3.076 GHz) as a function of D_o for $N_w = 8000$ m^{-3} mm^{-1} and $\mu = 5$ derived from Eq. (7) and for the full T-matrix solutions. For $D_o > 1.4$ mm, the values of K_{dp} for the Andsager et al. (1999) shapes and Goddard et al. shapes differ by less than 10%, and the forced exponents in a K_{dp}–R relation are 1.34 and 1.4, respectively. The BCZU03 assumption of $k = 0$ yields an exponent of 1.17 and values of K_{dp} that are too high by up to a factor of 2.

these three assumptions. The two solid lines are the values of K_{dp} as a function of D_o for $N_w = 8000$ m^{-3} mm^{-1} and $\mu = 5$; using the full T-matrix solution for the Andsager et al. (1999) shapes and the Goddard et al. shapes, whereas the dotted lines are derived using Eq. (7).

From Fig. 4, first we note that, for the approximation $k = 0$ made by BCZU03, the smallest drops are aspherical, leading to values of K_{dp} for small D_o that are much too high. Over the range $D_o = 1$–2 mm ($R = 2$–51 mm h^{-1}), the $k = 0$ line has a slope of 5.5 in Fig. 4, leading to an R–K_{dp} exponent of 1.17, which, as argued by BCZU03, is indeed below 1.28. Using the correct value of k for the Andsager et al. (1999) shapes, Eq. (7) leads to a slope of 6.1 in Fig. 4 and a forced exponent of 1.3. For the Goddard et al. drops, a D^6 term is added in Eq. (7), but the unrealistic assumption that the small drops can have $r > 1$ leads to a negative K_{dp} for small D_o. As BCZU03 point out, the smallest observable value of K_{dp} is about 0.1° km^{-1} and even then the spatial resolution is a very poor 20 km, and so for observable $K_{dp} > 0.1°$ km^{-1} it is more realistic to consider a range of D_o from 1.4 to 2.3 mm, or rain rates of 9.8–97 mm h^{-1}. For this range, the correct full T-matrix solution for the Andsager et al. (1999) and Goddard et al. shapes, the values of K_{dp} differ by less than 10% and are not of practical significance; the slopes in Fig. 4 are 6.16 and 6.46, respectively, and forced K_{dp}–R exponents are 1.34 and 1.40. The exponent of 1.4 for the Goddard et al. shapes, confirmed by BCZU03, is the same as the exponent in the Next-Generation Weather Radar (NEXRAD) Z–R relation. In accord, our first and fourth "provocative conclusions" quoted by BCZU03 are upheld: In the absence of hail, "K_{DP} is no better than Z for estimating rain rate," particularly when the noisy nature of K_{dp} is considered, and "the use of total differential phase shift along a long path to derive an integrated rainfall rate . . . is unlikely to outperform an equivalent algorithm based on integrating the value of Z."

5. Attenuation correction

BCZU03 question our comments concerning the difficulty of correcting for attenuation using phase once $D_o > 2.5$ mm and suggest that the linearity only breaks down once "$D_o > 3.5$ mm but . . . DSDs with $D_o > 3.5$ mm are not associated with any significant rain rates and attenuation." The problems associated with this nonlinearity for $D_o > 2.5$ mm are handled very succinctly in Bringi and Chandrasekar (2001, 495–497): "The coefficient α (in $A_h = \alpha K_{dp}$) will increase with D_o. This dependence of α on D_o will only occur for DSDs with large D_o ($\geq$2.5 mm). Similarly, β (in $A_{dp} = \beta K_{dp}$) will increase as D_o increases beyond 2.5 mm. If large D_o-values occur along the propagation path, then the attenuation-correction procedure is more complicated since α and β are no longer constant but depend on D_o." This difficulty may well have been resolved by Bringi et al. (2001), who have combined the phase technique with the constraint that Z_{dr} on the far side of a storm be zero (Smyth and Illingworth 1998) so that the uncertain values of α and β are no longer prescribed but are derived along with values of D_o; the attenuation-corrected results are very encouraging. Thus, recent work by Bringi et al. supports our third "provocative conclusion" (quoted by BCZU03) that there is not a constant linear relationship between K_{dp} and attenuation once D_o is above 2.5 mm.

6. Use of variable drop shapes

In their final paragraph, BCZU03 recommend the combined use of K_{dp} and Z_{dr} to make the rain-rate estimate relatively less sensitive to the precise form of the assumed axis-ratio relation. Indeed, Gorgucci et al. (2000) allow β, the slope of the axis ratio with raindrop size, to be a free variable whose value can be derived from the observed values of Z, Z_{dr}, and K_{dp} in rain. They suggest that, for Z in the range of 40–45 dBZ, the average value of β is 0.061 mm^{-1}, which is very close to the Pruppacher and Beard (1970) value, but β falls to 0.053 mm^{-1} when Z is above 53 dBZ. They ascribe this change to collision-induced raindrop oscillations in the heavier rain. Illingworth (2003) discusses some of the difficulties that may be encountered by this approach. In brief:

1) The linear drop-shape model has two unknowns: the intercept and the slope (see Fig. 1). Only allowing

the slope to change may not capture the real drop-shape variations, and so computations of Z_{dr} and K_{dp} are insufficiently accurate to improve R estimates. Any changes in the intercept value [k in Eq. (4)] are significant.
2) IB02 recommend using the redundancy between K_{dp}/Z and Z_{dr} to autocalibrate Z to better than 10% (Goddard et al. 1994), assuming that the drop shape is known.
3) The Gorgucci et al. (2000) variable β approach uses the observed and very noisy value of K_{dp}, whereas the autocalibration uses the much more accurate integrated total differential phase shift.
4) If the redundancy is being used to fix β, then it is assumed that Z is somehow perfectly calibrated. A 1-dB error in calibration will change β by 10%, rendering the retrieved values useless.
5) The apparent fall in β in heavy high Z_{dr} can be explained simply from the fall in the K_{dp}/Z ratio with increasing Z_{dr} in the Goddard et al. (1994) calibration curve, thus avoiding the need to introduce arbitrarily another degree of freedom to the system when it is doubtful that one exists.

7. Summary

To recapitulate, we agree on the following fundamental issues: 1) the normalized gamma functions should be used for computing the polarization parameters in rain and 2) the Pruppacher and Beard (1970) drop shapes are inappropriate and lead to unacceptable errors in rain rates derived from Z_{dr} and K_{dp}. Because most of the published literature has used the Pruppacher and Beard (1970) drop shapes, I repeat the last sentence of IB02: ‘‘It would be interesting to reanalyze existing data to see if the rainfall estimates are improved when the suggestions above are incorporated into the algorithms.’’ In addition, I argue the following:

1) The values of Z_{dr} and K_{dp} predicted using the Goddard et al. (1995) shapes are virtually identical to those predicted using the Andsager et al. (1999) shapes.
2) BCZU03 confirm that if raindrop size distributions can be represented by a normalized gamma function with $\mu = 5$, then we have relationships of the form $R = aK_{dp}^{1.4}$ for the Goddard et al. drop shapes. Using Andsager et al. (1999) shapes yields a very similar index of 1.34, but a very different index of 1.17 is derived from the linear drop shapes that have been widely used until recently [see papers quoted in IB02]. The index of 1.4 or 1.34 is similar to that in the NEXRAD $Z = 300R^{1.4}$ relationship, and so K_{dp} may not outperform Z. In addition, the use of total differential phase shift along a path to derive integrated rainfall rate is unlikely to outperform an equivalent algorithm based on integrating the value of Z.
3) I agree with the discussion in Bringi and Chandrasekar (2001, p. 495) that the linear relationship between differential phase shift and attenuation breaks down once $D_o > 2.5$ mm rather than with the opposing view expressed in BCZU03.

Last, I argue that because observed values of K_{dp} are obtained by differentiating an already noisy phase profile, they are far too noisy to be used for most practical applications. It is much better to use the path-integrated phase shift, which can be measured accurately, as a constraint. Examples of the use of this constraint are the autocalibration of Z to 10% (Goddard et al. 1994) and the ‘‘ZPHI’’ technique of Testud et al. (2000).

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