The Need to Represent Raindrop Size Spectra as Normalized Gamma Distributions for the Interpretation of Polarization Radar Observations

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ABSTRACT

Polarization radar techniques essentially rely on detecting the oblateness of raindrops to provide a measure of mean raindrop size and then using this information to give a better estimate of rainfall rate R than is available from radar reflectivity Z alone. To derive rainfall rates from these new parameters such as differential reflectivity Z_{DR} and specific differential phase shift K_{DP} and to gauge their performance, it is necessary to know the range of naturally occurring raindrop size spectra. A three parameter gamma function is in widespread use, with the three variables N_o, D_o, and μ providing a measure of drop concentration, mean size, and spectral shape, respectively. It has become standard practice to derive the range of these three variables in rain by comparing the 69 published values of the constants a and b in the empirical relationships $Z = aR^b$ with the values of a and b obtained when R and Z are derived by integrating the appropriately weighted gamma function. The relationships in common use both for inferring R from Z, Z_{DR}, and K_{DP}, and for developing attenuation correction routines have been derived from a best fit through the values obtained by cycling over these predicted ranges of N_o, D_o, and μ. It is pointed out that this derivation of the predicted range of N_o, D_o, and μ arises using a flawed logic for a particular nonnormalized form of the gamma function, and it is shown that the predicted ranges give rise to some very unrealistic drop spectra, including many with high rainfall and very small drop sizes. It is suggested that attenuation correction routines relying on differential phase may be suspect and the commonly used relationships between rainfall rate and Z, Z_{DR}, and K_{DP} need to be reexamined. When more realistic drop shapes are also used, it may be that published relationships for deriving R from Z and Z_{DR} are in error by over a factor of 2; a new equation is proposed that, in the absence of hail and attenuation, should yield values of R accurate to 25%, provided that Z_{DR} can be estimated to 0.2 dB and Z is calibrated to 1 dB. Relationships of the form $R = aK_{DP}^b$, with $b = 1.15$, are in widespread use, but more realistic drop spectra and drop shapes yield a value of b closer to 1.4, similar to the exponent in Z–R relationships. In accord, although K_{DP} has the advantage of insensitivity to hail, it may have the same sensitivity to variations in drop spectra as Z does. In addition, the higher value of the exponent b implies that the proposed use of the total phase shift to give the path-integrated total rainfall is also questionable. However, the consistency of Z, Z_{DR}, and K_{DP} in rain can be used to provide absolute calibration of Z to 0.5 dB (12%), and when it fails it indicates that hail is present, in which case a relationship of the form $K_{DP} = aR^{1.4}$ should be used. The technique should work at S, C, and X band, but, in all cases, paths should be chosen so that the total phase shift is not large enough to introduce significant attenuation of Z and Z_{DR}.

1. Introduction

Polarization radar techniques provide additional parameters such as differential reflectivity Z_{DR} and specific differential phase shift K_{DP}, which essentially depend upon the shape and fall mode of the hydrometeors. Use of such additional information should provide better estimates of rainfall rate than is available from the reflectivity Z alone. If the radar beam is dwelling wholly within the rain, then part of the error when using conventional Z–R relationships arises because of variations in raindrop size spectra. The differential reflectivity (Seliga and Bringi 1976) Z_{DR} [$=10 \log(Z_H/Z_V)$, where Z_H and Z_V are the reflectivities measured with horizontal and vertical polarization, respectively] provides an estimate of mean raindrop size and hence, because larger raindrops become increasingly oblate, their size. The specific propagation differential phase shift (Sachidananda and Zrnić 1987) K_{DP} relies on the horizontally polarized beam traveling more slowly than the vertically polarized one when traversing a region of heavy rain with large oblate raindrops. As a result, the phase of the horizontally polarized return ϕ_H lags progressively more and more behind the phase of the vertically polarized return ϕ_V and the differential phase $\phi_{DP} = \phi_V - \phi_H$ increases with increasing range. The parameter K_{DP} (° km^{-1}) is the rate of change of ϕ_{DP} with range

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and has been proposed as a new measure of rain rate. One advantage of K_{DP} is that it should be unaffected by hail. Because of their large size, hailstones can lead to very large values of Z, and the application of a conventional Z–R relationship will predict spuriously high rainfall rates; however, hail tumbles as it falls and should not contribute to K_{DP}, which should respond only to the contribution of the oblate raindrops.

The use of the Z_{DR} and K_{DP} parameters has great potential to improve the radar estimates of rainfall by providing information on mean raindrop size and eliminating the hail contribution. However, field campaigns have shown mixed results (e.g., Petersen et al. 1999; Brandes et al. 2001; May et al. 1999) and do not demonstrate the expected improvement when compared with rainfall estimated from Z alone. The equations used to derive the improved rainfall rate from Z_{DR} and K_{DP} in these studies have been derived by representing naturally occurring raindrop size spectra as a gamma function:

$$N(D) = N_o D^{\mu} \exp(-\lambda D) \qquad (0 \leq D \leq D_{max}), \quad (1)$$

where $\lambda = (3.67 + \mu)/D_o$ and, for $D_{max} = \infty$, D_o is the median volume diameter, N_o has the units: $m^{-3}\ mm^{-1-\mu}$, and the three parameters N_o, D_o, and μ provide a measure of raindrop concentration, mean size, and shape of the spectrum, respectively. The value of μ governs the shape of the distribution, with $\mu = 0$ being an exponential, in which case $\lambda = 3.67/D_o$, and, if N_o is constant and equal to 8000 $m^{-3}\ mm^{-1}$, then we have the original Marshall–Palmer (1948) expression. Ulbrich and Atlas (1998) have drawn attention to the sensitivity of truncating spectra at a maximum drop size, but once μ is greater than 2 then the gamma function introduces this truncation naturally.

Ulbrich (1983) derived the range of these three variables, N_o, D_o, and μ by comparing the values of the constants a and b in the 69 published relationships (Battan 1973) of the form $Z = aR^b$ with the values derived when the appropriately weighted gamma function is integrated to give an expression for Z and R. This leads to the widely quoted range of values of μ from -1 to 4 and D_o from 0 to 2.5 mm. Ulbrich derived the following relationship between N_o and μ:

$$N_o = 60\,000 \exp(3.2\mu) \quad m^{-3}\ cm^{-1-\mu}, \qquad (2)$$

with a range from $10^{5.5} \exp(2.8\mu)$ to $10^{4.2} \exp(3.57\mu)$ $m^{-3}\ cm^{-1-\mu}$. If μ is in the range 1–4, this implies that N_o varies over 15 orders of magnitude. We shall refer to the spectra within these limits as the ''Ulbrich'' spectra.

Following Ulbrich's work, this relationship between N_o and μ has been widely quoted, and the range of N_o, D_o, and μ implied by the scatter of the 69 relationships has been used for calculating the expected range of naturally occurring raindrop size spectra. These spectra have been used by Chandrasekar and Bringi (1987) to examine Z–R relationships and by Bringi et al. (1990) and Chandrasekar and Bringi (1988b) to predict attenuation relationships to be expected in rainfall. Bringi et al. (1991), Chandrasekar and Bringi (1988a), Chandrasekar et al. (1990), Gorgucci et al. (1994, 1999, 2000), Jameson (1994), Ryzhkov and Zrnić (1995, 1996), Sachidananda and Zrnić (1987), Scarchilli et al. (1993, 1996), and Matrosov et al. (1999) have used this range of Ulbrich spectra to examine the characteristics of Z_{DR} and K_{DP} and to propose relationships to derive improved rainfall estimates from them. Ryzhkov and Zrnić (1995) restricted the range to a maximum rainfall rate of 250 mm h^{-1}. These relationships have been used in nearly all polarization radar rainfall studies reported over the past decade. Perhaps the most widespread is from Sachidananda and Zrnić (1987) used for predicting rainfall rate from the one-way differential phase at S band:

$$R = 37.1 K_{DP}^{0.866}, \quad \text{or} \quad K_{DP} = 0.0154 R^{1.155}. \qquad (3)$$

Kozu and Nakamura (1991) and Tokay and Short (1996) have confirmed that Eq. (1) is an excellent representation of the higher moments of naturally occurring raindrop spectra: they fitted observed spectra to the equation by forcing the third, fourth, and sixth moments of the spectrum to be equal to the integral of Eq. (1) with appropriate weighting and found that R calculated from the fitted values of λ, μ, and N_o agreed to within 0.02 dB (0.5%) of the value of R derived from the raw spectra. The values of μ were in the range 0–20, much larger than those predicted by Ulbrich. The values of N_o, however, covered 15 orders of magnitude in agreement with Ulbrich. Sempere Torres et al. (1994) demonstrated that the gamma function is a special case of a more general formulation of raindrop size distributions. As discussed above, the gamma function has the advantage that it captures accurately the relationship between the higher moments used in radar–rainfall work.

The purpose of this paper is to show that the Ulbrich range of N_o, D_o, and μ quoted above and the relationship in Eq. (2) arise because of the particular form of the gamma function in Eq. (1). This is illustrated in Fig. 1a, in which drop size spectra for Eq. (1) are plotted out for a constant values of N_o = 8000 $m^{-3}\ mm^{-1-\mu}$ and D_o = 1 mm, but for various values of μ. As μ increases, the concentration of all sizes of raindrops falls dramatically, so that, for example, the rainfall rates for values of μ = 0, 2, 5, and 10 are 2, 0.22, 0.009, and 0.00005 mm h^{-1}, respectively. This latter rainfall is unrealistic for a D_o of 1 mm; so, to compensate for this when μ = 10, Ulbrich uses a much higher value of N_o to increase the rain rate. As noted by Feingold and Levin (1986), the apparent correlation in Eq. (2) reflects the observation that, in natural rainfall for a constant D_o, the absolute raindrop concentration does not fall markedly when μ increases.

We seek a form of Eq. (1) with three *independent* parameters to describe the distribution, each of which represents something physically meaningful about the distribution: D_o should be a measure of the median drop

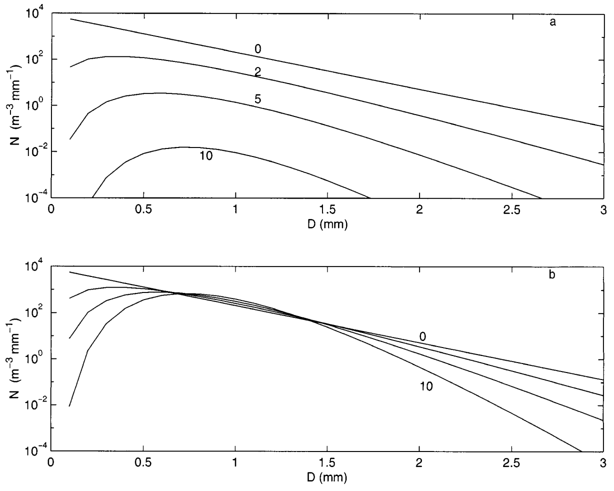

FIG. 1. Raindrop size spectra for $D_o = 1$ mm. (a) Nonnormalized gamma functions with $N_o = 8000$ m^{-3} mm$^{-1-\mu}$ and (top) $\mu = 0$, 2, 5, and (bottom) 10. (b) Gamma functions normalized with respect to liquid water content for $N_L = 8000$ m^{-3} mm^{-1} and the same range μ as in (a).

size, μ should be the shape of the distribution, and N_o should be the absolute concentration. It is clear that if N_o varies over 15 orders of magnitude when one of the other parameters is changing, it is neither an independent measure of concentration nor is it physically telling us anything useful about the distribution. Last, we note that for any other value of μ apart from 0, the dimensions of Eq. (1) become physically meaningless as was also observed by Testud et al. (2000).

Equation (1) could be normalized so that either total drop concentration, liquid water content, or rainfall rate remain constant when μ is changed. In section 2 we show that, once any of these normalizations are carried out, the 69 $Z = aR^b$ relationships lead to a constant value of b, and a is a function of both μ and N_o, so no useful information on the range of raindrop particle spectra can be inferred. Furthermore, it has been shown (Haddad et al. 1997) that it is mathematically invalid to equate the values of a and b in the expression $Z = aR^b$ obtained by integration of Eq. (1) with the values of a and b in the 69 empirical Z–R relationships. In addition, Ulbrich's approach assumes that the Z–R relationships reflect the variability of individual spectra, whereas in reality each Z–R relationship is itself an average of many spectra.

We follow Willis (1984) and suggest normalization of Eq. (2) so that liquid water content is kept constant if μ is changed—in this case the three parameters become independent, with N_o scaling the absolute values of the concentration of all sizes of drops, μ scaling the shape of the distribution, and D_o scaling the median volume diameter. In section 3 we suggest a more realistic range for the variables N_o, D_o, and μ, and in section 4 we review the status of the equations predicting the expected attenuation and the equations relating R to Z, Z_{DR}, and K_{DP} to improve rainfall estimates, which have been based upon the range of drop size distributions derived from N_o and μ from the 69 Z–R relationships.

2. The Z–R relationships and gamma functions

In his 1983 paper, Ulbrich integrated the gamma function in Eq. (1) with various weighting functions D^p:

$$P = a_p \int D^p N(D)\, dD, \tag{4}$$

where $p = 0$ for total drop number N_T; $p = 3$ for liquid water content; $p = 3.67$ for rainfall rate R, assuming the terminal velocity varies as $D^{0.67}$; $p = 6$ for radar reflectivity Z and a_p is, for example, unity for Z but $\pi/6$ for liquid water content. Using Eq. (1) gave

$$P = a_p N_o \frac{\Gamma(p + \mu + 1)}{\lambda^{p+\mu+1}}, \tag{5}$$

so that, for example, the total number N_T is given by

$$N_T = N_o \frac{\Gamma(\mu + 1)}{\lambda^{\mu+1}}. \tag{6}$$

Ulbrich then derived expressions for Z and R from Eq. (5):

$$Z = N_o \frac{\Gamma(7 + \mu)}{\lambda^{7+\mu}}, \quad \text{and} \quad R = N_o a_r \frac{\Gamma(4.67 + \mu)}{\lambda^{4.67+\mu}}. \tag{7}$$

Eliminating λ, we have

$$Z = \frac{N_o \Gamma(7 + \mu)}{[N_o a_r \Gamma(4.67 + \mu)]^b} R^b, \tag{8}$$

where $b = (7 + \mu)/(4.67 + \mu)$. Ulbrich then compared Eq. (8) with $Z = aR^b$ and found that the value of b uniquely defined μ and that, once μ was known, the value of a could be used to find N_o. Examination of the 69 proposed values of a and b then leads to a range of values of μ and N_o believed to occur in natural rainfall. Nearly all values of μ ranged from -2 to $+6$, and N_o ranged from 1 to 10^{16} m^{-3} cm$^{-1-\mu}$, but $\log N_o$ and μ had a correlation coefficient of 0.98, which arises from degrees of freedom consideration, and a best fit given by Eq. (2). The variation of the constant a in $Z = aR^b$ from 100 to 500 leads to a spread in the values of N_o and μ; for N_o the upper and lower bound are

$$N_o = 10^{5.5} \exp(2.8\mu) \quad \text{and}$$

$$N_o = 10^{4.2} \exp(3.57\mu)\ \text{m}^{-3}\ \text{cm}^{-1-\mu}, \tag{9}$$

respectively.

In Fig. 1a, we noted that when the drop size spectra of Eq. (1) are plotted out for a constant values of $N_o =$ 8000 m^{-3} mm$^{-1-\mu}$ and $D_o = 1$ mm, then as μ increases the concentration of all sizes of raindrops falls dramatically. Chandrasekar and Bringi (1987) noted that the apparent correlation between N_o and μ in Eq. (2) is to compensate for this effect and suggested modifying the gamma function so that the total drop concentration N_T is conserved. The equation

$$N(D) = \frac{N_T\lambda(D\lambda)^{\mu}}{\Gamma(\mu + 1)}\exp(-\lambda D) \qquad (10)$$

has the required property that integrating over all drop sizes leads to the total drop concentration being N_T and independent of μ.

In this case the expressions in Eq. (7) become

$$Z = N_T\frac{\Gamma(7 + \mu)}{\lambda^6\Gamma(\mu + 1)} \quad \text{and}$$

$$R = N_T a_r \frac{\Gamma(4.67 + \mu)}{\lambda^{3.67}\Gamma(\mu + 1)}. \qquad (11)$$

Eliminating λ as before and then expressing Z as a function of R gives

$$Z = \frac{N_T\Gamma(7 + \mu)}{\Gamma(\mu + 1)}\left[\frac{\Gamma(\mu + 1)}{N_T a_r\Gamma(4.67 + \mu)}\right]^{1.63} R^{1.63} = aR^b. \qquad (12)$$

In this case $b = 1.63$ is a constant and so, in contrast to Eq. (8), provides no information on μ; a is a function of both N_T and μ, and so a whole family of N_T and μ values is possible for a given a. Willis (1984) suggested normalization with respect to liquid water content; this idea was revived by Illingworth and Blackman (1999) and Testud et al. (2001). Illingworth and Johnson (1999) have shown (see section 3b) that if drop spectra are fitted to such a normalized gamma function, then the correlation between concentration and μ vanishes. In this case the appropriate distribution function is

$$N(D) = \frac{N_L 0.033(3.67 + \mu)^{\mu+4}}{\Gamma(\mu + 4)}\left(\frac{D}{D_o}\right)^{\mu} \times \exp\left[-(3.67 + \mu)\frac{D}{D_o}\right], \qquad (13)$$

which has the required property that integration over all drop sizes leads to a constant liquid water content that is independent of μ and is obtained by multiplying Eq. (1) by the normalization factor

$$\frac{0.0033D_o^4\lambda^{\mu+4}}{\Gamma(\mu + 4)} = \frac{0.0033(3.67 + \mu)^{\mu+4}}{\Gamma(\mu + 4)D_o^{\mu}}, \qquad (14)$$

where N_L has the units: m^{-3} mm^{-1}, so that when $\mu = 0$, Eq. (13) reduces to the simple exponential with $N_L = N_o$. Because the normalization factor expressed in Eq. (14) involves both λ and D_o, it is convenient to express Eq. (13) as a function of N_L, D_o, and μ rather than the concentration λ and μ used in Eq. (10). Testud et al. (2000) point out that the mathematical definition is $N_L = 3.67^4\text{LWC}/(\pi\rho_w D_o^4)$, where LWC is liquid water content and ρ_w is density of the drops. Drop spectra from Eq. (13) are plotted out in Fig. 1b, with $N_L = N_o = 8000$ m^{-3} mm^{-1}, and $D_o = 1$ mm for different values of μ. In this case, as μ changes from 0, 2, 5, to 10, even though normalization is with respect to LWC, the rainfall rate only changes from 2.01 to 1.99 mm h^{-1}, confirming that the representation in Eq. (13) has the desired properties that the three free variables N_L, D_o, and μ are independent measures of the absolute concentration of the drops, their median volume diameter, and the shape of the spectrum. This is preferable to the normalization with respect to N_T [Eq. (10)] in which case, as μ takes the values 0, 2, 5, and 10, then the rainfall rate increases from 2.0, 5.4, 8.5, to 11.0 mm h^{-1}, respectively.

When values of Z and R are calculated by integrating Eq. (13) and the value of D_o is eliminated so that we obtain an expression of the form $Z = aR^b$, we find that b is again a constant as in Eq. (12) but is now equal to 1.5 and a is a function of both N_o and μ. This result is puzzling. Why should b vary with the normalization? Haddad et al. (1997) have identified the ''all too common logically faulty step'' that involves equating the value of a and b in $Z = aR^b$, obtained from integrating the spectra after ''eliminating λ,'' with the α and β in another expression $Z = \alpha R^{\beta}$. They point out that it is only possible to conclude that $a = \alpha$ and $b = \beta$ provided that a and b are mutually independent of R. Thus one would need to know that N_o and R are independent; given that this is not the case, there is an infinite variety of possible choices of α and β. We conclude that, from the 69 Z–R relationships, we can draw no conclusions with respect to the range of values of N_o, λ, and μ occurring in natural rainfall. However, Testud et al. (2000) argue further that because Z and R both scale linearly with concentration then a should also scale linearly with (normalized) drop concentration.

3. The range of naturally occurring raindrop spectra

In this section we appeal to specific measurements that indicate the range of natural drop spectra rather than rely on satisfying arguments based on statistical drop size distribution constraints.

a. Representativity of the Ulbrich spectra

It is now relevant to ask how representative are the Ulbrich range of drop spectra based on these 69 Z–R relationships and to see if a bias is introduced by averaging over all these possible spectra to provide a mean relationship between rainfall rate and the radar observables, Z, Z_{DR}, and K_{DP}. To give an indication of the range of spectra implied in the Ulbrich range, in Fig. 2 we have plotted R against median volume diameter D_o for the maximum and minimum values of N_o in Eq. (9) for values of $\mu = 0$ and 4; in addition, the solid curve is

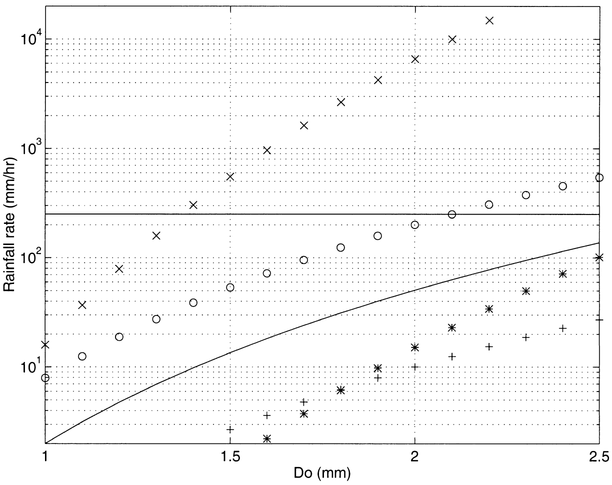

FIG. 2. Rainfall rates against D_o for raindrop spectra in the Ulbrich range [Eq. (9)]. Here, $\mu = 4$ is the maximum concentration ×, minimum concentration *; and $\mu = 0$ is the maximum concentration ○, minimum concentration +. The solid curve is the Marshall–Palmer. The solid horizontal line is the 250 mm h^{-1} limit used by Ryzhkov and Zrnić (1995).

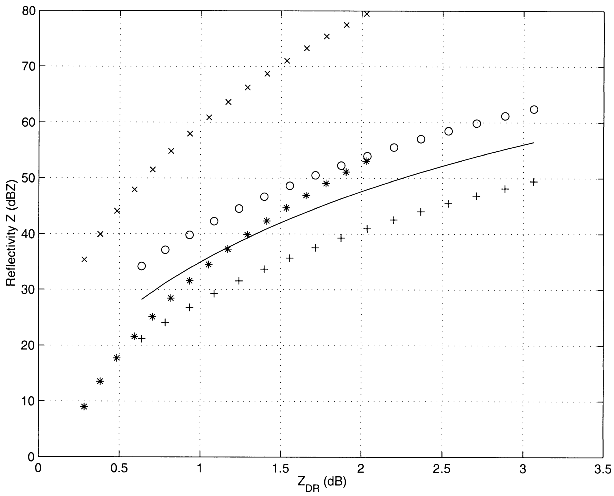

FIG. 3. Variation of Z against Z_{DR} at S band for spectra in the Ulbrich range with values of D_o increasing in steps of 0.1 mm from $D_o = 1$ up to a maximum of 2.5 mm. Symbols and solid curve are as in Fig. 2.

for $\mu = 0$ and the Marshall–Palmer value of 8000 m^{-3} mm^{-1}. The higher values of μ lead to some enormous values of R; for example, if $D_o = 2.5$ mm and $\mu = 4$, then R is 40 000 mm h^{-1}. Ryzhkov and Zrnić (1995) did limit the range of spectra in their averaging to those with R below 250 mm h^{-1} (the solid horizontal line in Fig. 2), but from Fig. 2 it is evident that cycling over the Ulbrich range still leads to many more spectra having much smaller drop size and higher concentrations than the Marshall–Palmer values.

It is possible to gain some idea of the values of D_o for various rainfall rates in natural rainfall from an analysis of observations of reflectivity Z and differential reflectivity Z_{DR}. Unless stated otherwise, the calculations in this paper are for a temperature of 0°C, a wavelength of 9.75 cm, and a raindrop axial ratio r for drops of diameter D (mm) given by

$$r = 1.075 - 0.065D - 0.0036D^2 + 0.0004D^3, \quad (15)$$

as proposed by Goddard et al. (1995) for drops larger than 1 mm, based on careful distrometer comparisons of Goddard et al. (1982), and subsequently essentially confirmed by laboratory experiments (Andsager et al. 1999) rather than by the more widespread simple linear relationship between r and D in common use (Pruppacher and Pitter 1971).

In Fig. 3 we have plotted the values of Z and Z_{DR} predicted at S band for the Ulbrich range of D_o, N_o, and μ. The figure predicts that the maximum value of Z_{DR} in rain with $\mu = 0$ should be 3.1 dB; for $\mu = 4$ it should not exceed 2.1 dB; and in heavy rain having Z values of 50–52 dBZ, Z_{DR} should be in the range 0.7–1.9 dB. Illingworth and Caylor (1989) carried out an extensive analysis of observations of Z and Z_{DR} in rain over two summers in the United Kingdom, and a histogram of the values of Z_{DR} occurring for all their observations of Z in the range 50–52 dBZ displayed in Fig. 4 reveals a very different behavior. There are virtually no occasions with Z_{DR} below 1.4 dB, but there is a broad maximum in the range 1.6–3.2 dB and a significant number of observations above 3.2 dB that are completely outside the Ulbrich range. For the range 40–42 dBZ, the mean observed value of Z_{DR} is 1.5 dB with individual values of greater than 2 dB, but Fig. 3 predicts lower values. The observations are more consistent with concentrations closer to the Marshall–Palmer curve. The observed values of Z_{DR} are much higher than those predicted from the Ulbrich range as would be expected if this range includes so many spectra with unrealistically small drop sizes. We now accordingly consider other

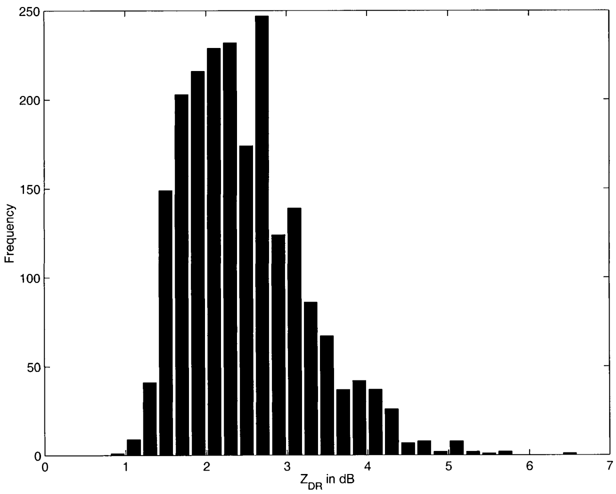

FIG. 4. A histogram of the S-band Z_{DR} values in rain for Z in the range 50–52 dBZ [from Illingworth and Caylor (1989)].

evidence for the range of N_o, D_o, and μ occurring in natural rainfall.

b. *Reported values of μ in rainfall*

Kozu and Nakamura (1991) analyzed 1000 3-min samples of rainfall spectra recorded over a 2-yr period in Japan and fitted the spectra using the method of moments to calculate N_o, D_o, and μ in the nonnormalized gamma function. Virtually all the values of μ lay in the range 0–20, with the peak of the histogram having a value of 5. Tokay and Short (1996) fitted 127 h of tropical rainfall spectra recorded at Kapingamarangi Atoll in the Pacific Ocean with 1-min resolution using the same technique and for the 7000 spectra found a similar range of μ. The nonnormalized values of N_o ranged over 9 orders of magnitude, as might be expected for a nonnormalized gamma function. Illingworth and Johnson (1999) fitted 1260 spectra recorded every 30 s during July of 1988 in the United Kingdom and found the same range of values of μ, with a mean value of 6 and a standard deviation of 5. Using the nonnormalized gamma function of Eq. (1), the variation of N_o with μ was over 25 orders of magnitude, but once the normalized form of Eq. (13) was employed then the value of N_L was no longer a function of μ and its standard deviation was only a factor of 3.6. These ranges of μ are consistent with the mean value of 5 derived by Goddard and Cherry (1984), who calculated Z, Z_{DR}, and R from naturally occurring values of raindrop spectra recorded with a ground-based distrometer. They then compared the values of R derived from Z and Z_{DR} with R from the raw spectra and found that the minimum bias of only 1% occurred for a value of μ of 5, with a standard deviation of 14%.

The difficulty with the above techniques, which use distrometers, is their poor sampling of the larger drops, which are important for the higher moments. It is more convincing to rely on μ derived from radar measurements, because the radar sampling volume contains an enormously larger number of the larger drops. One approach adopted by Illingworth and Caylor (1991) relied on the lowering of the copolar correlation by the mixture of drop shapes present in rainfall to provide an estimate of the value μ. The correlation changes are small (especially when μ is above 5) and are difficult to measure, but they deduced that the value of μ was probably between 0 and 5. Better evidence comes from the values of μ derived by Wilson et al. (1997) from the ''differential Doppler velocity'' (DDV). They showed that DDV, the difference in the Doppler velocity of rain for horizontally and vertically polarized radiation with the radar beam dwelling at a finite elevation, when expressed as a function of Z_{DR} has a well-defined dependence on μ, and they found that the average value of μ was about 5 but that actual values varied between 2 and 10.

The value of μ depends upon the moments chosen for the fit. Higher moments are appropriate for relationships between Z and R and lead to higher values of μ than do fits to lower moments. Testud et al. (2001) infer μ values closer to unity; this may be because they first derive N_o and D_o in an exponential spectrum by fitting the Z and R moments but then choose μ to minimize a least squares fit to the observed spectrum. This procedure for fixing μ assigns equal weight to drops of all sizes and may lead to values of μ that are inappropriate for the higher moments involved in radar studies.

c. *Reported range of values of N_L in rainfall*

There are many report of large variations of drop concentration in natural rainfall. If we first consider exponential fits, then $N_L \equiv N_o$ and, for example, Waldvogel (1974) observed ranges of N_o of up to a factor of 10 around the Marshall–Palmer level for an exponential fit. Sauvageot and Lacaux (1995) found that when 53 531 spectra with 1-min resolution at three different geographical locations were sorted into different rain rates then even the average value of N_o varied from 1612 to 65 343 $\text{m}^{-3}\ \text{mm}^{-1}$, a factor of up to 5 lower to 8 higher than the Marshall–Palmer value. Illingworth and Johnson (1999) fitted 1 month's rainfall spectra in the United Kingdom to a normalized gamma function and found the mean value of $\log_{10}(N_L)$ was 3.93 with a standard deviation of 0.56, equivalent to 8511 $\text{m}^{-3}\ \text{mm}^{-1}$ (very close to the Marshall–Palmer value), with a standard deviation of a factor of 3.6. The 25 orders of magnitude change for the value of N_o for the nonnormalized gamma function reported by Illingworth and Johnson (1999) is purely an artifact of this inconsistent distribution function. Last, in the Tropics, Marecal et al. (1997) derived a mean value of N_o in the exponential to be close to 32 000 $\text{m}^{-3}\ \text{mm}^{-1}$. Tokay and Short (1996) also reported larger values of N_o in the Tropics.

d. *Reported range of D_o in rainfall*

The Ulbrich parameterizations assumed a maximum value of D_o of 2.5 mm, implying that Z_{DR} in rainfall at S band should never exceed 3.1 dB (Fig. 3). However, higher values of Z_{DR} are frequently observed in regions of rain in heavy convective storms, as shown, for example, in the histogram in Fig. 4, suggesting that the maximum value of D_o can reach 4 or 5 mm.

In summary, evidence from both the Tropics and midlatitudes suggests that in natural rainfall the average value of μ is about 5 or 6, with a standard deviation of about 5; values of D_o can exceed 2.5 mm and reach 4 or 5 mm, and the mean value of N_o can vary by a factor of 4 or so from the Marshall–Palmer value. If $Z = 300R^{1.4}$ for the Next-Generation Weather Radar (NEXRAD; e.g., Peterson et al. 1999) is correct for the average, then variations of N_o of a factor of 4 would lead to rainfall errors of about ±50%, or 2 dB.

4. Implications for polarization radar techniques

a. Differential reflectivity

To derive relationships between R, Z, and Z_{DR}, Chandrasekar and Bringi (1988a) cycled over the Ulbrich ensemble of drop spectra and obtained the best fit,

$$R = 0.002\,397Z^{0.94}Z_{DR}^{-1.08}, \quad (16)$$

and Chandrasekar et al. (1990) derived

$$R = 0.001\,98Z^{0.97}Z_{DR}^{-1.05}. \quad (17)$$

The above approach is different from the original formulation of the Z_{DR} technique by Seliga and Bringi (1976), who stressed that the advantage of Z_{DR} was that it provided a measure of mean drop size that was independent of the concentration of raindrops and that, if an exponential raindrop size distribution is assumed, D_o is uniquely defined as a simple monotonic function of Z_{DR}, providing the shape of the raindrops as a function of size is known. For a given D_o, the value of Z scales with N_o, so, once D_o is known, the value of N_o can be derived from the absolute value of Z. Rainfall rates are derived in a similar manner: D_o is derived from Z_{DR}, and then, for a given D_o (and thus Z_{DR}), Z scales with R because both are linearly related to N_o. Thus, R should be linearly related to Z (provided Z_{DR} remains constant and the ratio of the moments of the drop spectrum do not change with drop concentration), and so, in Eqs. (16) and (17), the Z power should be unity rather than 0.96 and 0.97. Of course, if Z_{DR} increases as Z increases (as it will on the average) then Z and R are not linearly related and we have a relationship of the form $Z = aR^b$, and b is not unity. The use of gamma functions leads to a $\pm 25\%$ change in rain rates (see below) but does not affect the linearity argument.

Computed values of Z/R, the values of Z (dBZ) at S band for a rainfall rate of 1 mm h^{-1}, are plotted in Fig. 5 over the range $D_o = 1$–5 mm (Z_{DR} of 0.25–5.4 dB) for the newly proposed normalized gamma function in Eq. (13) with $\mu = 5$ using the drop shapes in Eq. (15). A third-order polynomial fit of Z/R as a function of Z_{DR} gives the following expression:

$$Z/R\ [\text{dB}Z\ (\text{mm h}^{-1})^{-1}] = f(Z_{DR}) = 21.48 + 8.14Z_{DR} - 1.385(Z_{DR})^2 + 0.010\,39(Z_{DR})^3, \quad (18)$$

which is accurate to better than 0.5 dB. Because the rainfall rate R is proportional to Z, then, for a given Z_{DR}, the actual rainfall rate for an observed value of Z is given by $R = Z/f(Z_{DR})$.

Figure 5 also displays the values of Z/R, the value of Z for a rainfall rate of 1 mm h^{-1}, as a function of Z_{DR} for μ values of 0, 2, and 10. The effect of truncating the drop spectrum at 8 and 10 mm is also plotted and is important only for $\mu = 0$ and 2 and for values of Z_{DR} above 2.5 dB. Note that, in the calculations of Fig. 5, the normalization factor is unimportant because the concentration of drops of all sizes is scaled to give a rainfall rate of 1 mm h^{-1} and then Z is calculated. The curves in Fig. 5 indicate that if the range of values of μ from 0 to 10 encompasses those occurring in natural rainfall, then uncertainty in the value of μ will introduce an error of only ± 1 dB or $\pm 25\%$ in rainfall rates derived from Z and Z_{DR}. Note also from the slope of the curves in Fig. 5, that Z_{DR} must be estimated to an accuracy of 0.2 dB to achieve this accuracy; Z, of course, must be accurately calibrated to 25%.

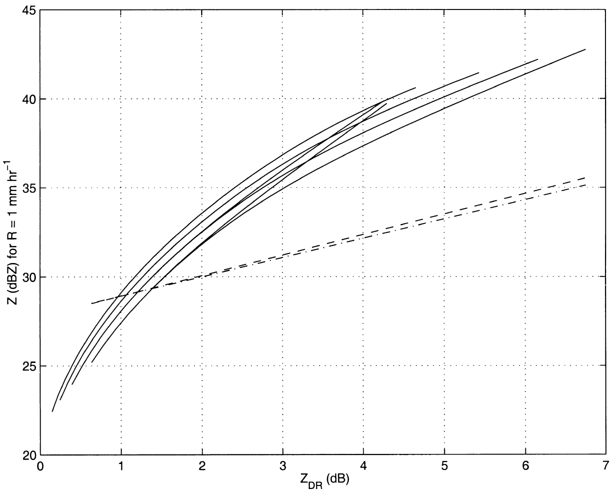

FIG. 5. Theoretical values of Z/R (the value of Z for an R of 1 mm h^{-1}) as a function of Z_{DR} at S band. For a given Z_{DR}, values of Z scale with rainfall rate. Solid lines are for normalized gamma functions with $\mu = 0$ (lower line), 2, 5, and 10 (upper line), with variable D_o to a maximum of 5 mm. The upper bifurcation of the lower two curves when Z_{DR} is greater than 3.5 dB shows the effect of truncating the spectrum at 8 mm. The dashed curve is Eq. (16) and the dot–dashed curve is Eq. (17) predicted from the Ulbrich range of spectra.

Also included in Fig. 5 are the curves proposed by Chandrasekar and Bringi (1988a) and Chandrasekar et al. (1990), which have been determined by setting $R = 1$ mm h^{-1} in Eqs. (16) and (17). Equations (16) and (17) were derived from averaging over all the Ulbrich ensemble of drop spectra distributions for linear drop shapes, and, as expected, this averaging leads to an overestimate of the rainfall rate for a given Z_{DR} at higher rainfall rates. The use of linear drop shapes in Fig. 5 only changes the curves for Z_{DR} of less than 2 dB; with the Z values lowered by about 2 dB when $Z_{DR} = 1$ dB. From Fig. 5, it appears that the use of these empirical relationships should introduce an underestimate of derived rainfall rates of about 3 dB for a Z_{DR} of 0.5 dB and overestimates of about 5 dB for a Z_{DR} of 3 dB and 7 dB for a Z_{DR} of 5 dB when compared with the predictions of Eq. (18) based on normalized gamma functions and the observed range of raindrop spectra. However, the value of the Z exponent of 0.94 does compensate somewhat for the underestimation in heavy high-Z rainfall; for a rate of 100 mm h^{-1} it leads to a scaling of Z by a factor of 76 rather than 100, which, for a Z_{DR} of 3 and 5 dB, would reduce the overestimates

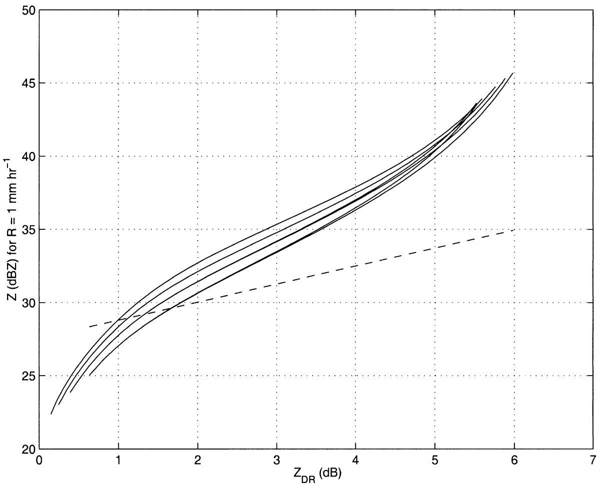

FIG. 6. Solid curves as for Fig. 5, but at C band. Again, for a given Z_{DR}, values of Z scale with rainfall rate. The dashed curve is Eq. (20) from Aydin and Giridhar (1992).

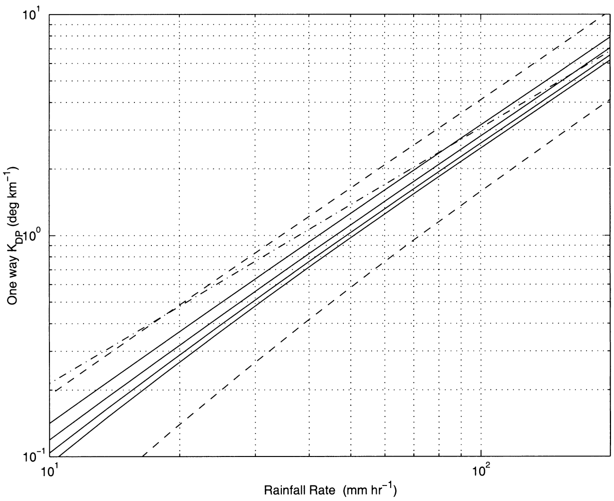

FIG. 7. Predicted values of one-way K_{DP} as a function of rainfall rate. Solid lines are for a normalized gamma function with (top) N_L = 8000 m^{-3} mm^{-1} and μ = 0, 2, 5, and (bottom) 10. Dashed lines are for (top) μ = 6 and N_T = 2000 and (bottom) 32 000 m^{-3} mm^{-1}. Dash–dot line is Eq. (3).

by 1 dB, to 4 and 6 dB, respectively. These errors are still a factor of 2.5–4. For C band (5.6 cm), the new equation equivalent to Eq. (18) is computed to be

$$Z/R(\mathrm{dB}Z) = f(Z_{DR}) = 21.50 + 8.35Z_{DR} - 1.89(Z_{DR})^2 + 0.1976(Z_{DR})^3, \quad (19)$$

which is plotted in Fig. 6, together with the formula of Aydin and Giridhar (1992):

$$R = 0.002\,37Z^{0.95}Z_{DR}^{-1.17}, \quad (20)$$

for the range Z_{DR} of 0.1–3 dB. Equation (20) underestimates rainfall by 4 dB at the lower range of Z_{DR} and overestimates it at the upper range when compared with Eq. (19). These errors at both C and S band are much larger than the oft-quoted errors of 2–3 dB associated with simple Z–R relationships, so it seems that the use of Eqs. (16), (17), or (20) will not lead to the expected improvement in accuracy of rainfall estimates. It would be interesting to see if the performance of Eqs. (18) and (19) is better.

b. Specific differential phase shift

Specific differential phase shift K_{DP}, unlike Z_{DR}, is directly proportional to the absolute drop concentration. It has often been claimed (e.g., Ryzhkov and Zrnić 1995, 1996) that one of the advantages of the K_{DP} technique is that it is much less sensitive to changes in the drop size distribution than is Z_{DR} and that K_{DP} is more linearly related to rainfall rate, as indicated in Eq. (3) in which the exponent of R is only 1.15, as opposed to an exponent of 1.4 or 1.6 for Z–R relationships. In addition, Ryzhkov et al. (2000) have assumed that K_{DP} and R are linearly related and suggested that the total differential phase shift along a 30-km path can yield the mean integrated rain rate along the path to an accuracy of 0.3 mm h^{-1}. The exponent of 1.15 is based upon cycling over the Ulbrich range of spectra and using a linear drop shape variation with size. The predicted variation of K_{DP} with R at S band (9.75 cm) for μ in the range 0–10 with an N_L of 8000 m^{-3} m^{-1} in the normalized gamma function and the drop shapes of Eq. (15) is plotted as the solid lines in Fig. 7. The change in R for a given K_{DP} when μ changes from 0 to 10 is less than ±10% for a rainfall rate of 10 mm h^{-1} and falls to only about ±8% for 100 mm h^{-1}, whereas for Z_{DR} the changes were ±25%, suggesting that K_{DP} is indeed insensitive to changes in the shape of the drop spectra. However, the best fit for one-way K_{DP} with μ = 5 and N_L = 8000 m^{-3} m^{-1} over the range 10–100 mm h^{-1} is given by

$$R = 47.5K_{DP}^{-0.71} \quad \text{or} \quad K_{DP} = 0.004\,35R^{1.40}. \quad (21)$$

The exponent of 1.4 is close to the value of 1.37 deduced by Blackman and Illingworth (1995) for a Marshall–Palmer distribution. The widely used Eq. (3) with exponent 1.155 derived from the Ulbrich spectra and linear drop shapes is plotted in Fig. 7 as the dash–dot line and shows a more linear dependence with a larger K_{DP} for the lower rainfall rates. The new drops shapes of Eq. (15) are having an important effect: using linear shapes yields a power of 1.2 in Eq. (21) rather than 1.4. The equivalent figures for C band (5.6 cm) are

$$R = 31.2K_{DP}^{-0.71} \quad \text{or} \quad K_{DP} = 0.007\,87R^{1.41}, \quad (22)$$

showing a virtually linear dependence with frequency. Testud et al. (2000) have extended this argument further by noting that K_{DP}/N_L and R/N_L are functionally related and independent of the value of N_L.

The proposed value of 1.4 for the exponent in Eqs. (21) and (22) as opposed to the widely used 1.15 in Eq. (3) has important implications. First, it means that the

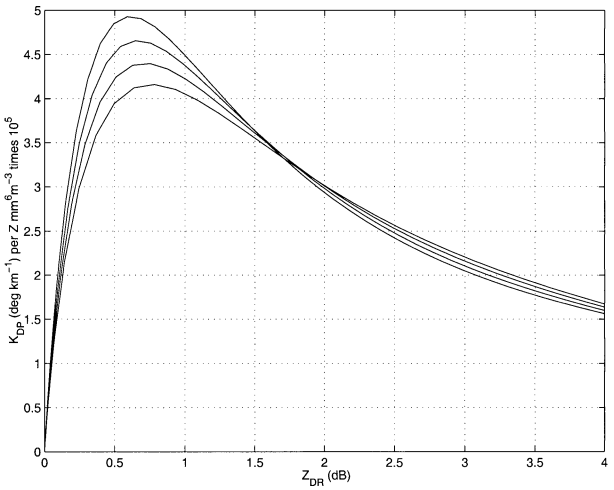

FIG. 8. Calibration technique (see text): the relationship between two-way K_{DP} per unit linear Z as a function of Z_{DR} for $\mu = 0$ (lower curve), 2, 5, and 10 (upper curve).

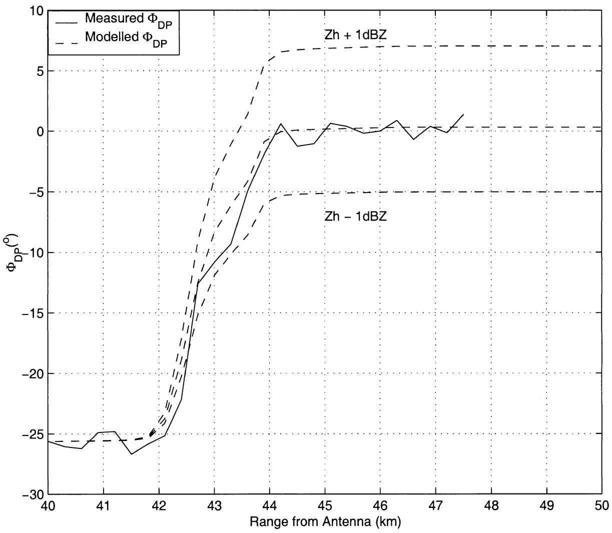

FIG. 9. Example of autocalibration. Different calibrations of Z lead to three different traces for differential phase shift. Comparison of the observed differential phase shift in rain fixes the calibration of Z to 0.5 dB (10%).

R–K_{DP} relationship is sensitive to the absolute value of the drop concentrations to the same degree as the recommended NEXRAD Z–R relationship: $Z = 300R^{1.4}$ (Peterson et al. 1999). Second, as initially pointed out by Blackman and Illingworth (1995), it means there are large errors in deriving the path-integrated rainfall over a river catchment from the observed total differential phase shift along the path, because such a method assumes that K_{DP} and R are linearly related.

The dashed curves in Fig. 7 demonstrate the sensitivity of the R–K_{DP} relationship [Eq. (21)] at S band to changes in N_L to values 4 times higher and lower than the Marshall–Palmer value of 8000 $\mathrm{m^{-3}\ mm^{-1}}$ while keeping $\mu = 5$. For a given K_{DP}, the rain rates change by a factor of about 50%, as would be expected from the 1.4 exponent in Eq. (21); if Z is proportional to $R^{1.4}$, then these changes in N_L would also give 50% changes in R for a given value of Z. This leads us to the unexpected conclusion that K_{DP} is no better than Z for estimating rainfall rates when pure rain is falling. In addition, the K_{DP} is noisy and tends to have poor range resolution, whereas Z and Z_{DR} are available at each gate. The advantage of the nonresponse of K_{DP} to hail remains, as does the ease of calibration of a phase measurement when compared with problems of calibrating Z.

c. Combination of Z_{DR} and K_{DP} for absolute calibration of Z

Ryzkhov and Zrnić (1995, 1996) have suggested that rainfall rate can be derived from a combination of K_{DP} and Z_{DR} based on a regression analysis of the values derived from the Ulbrich range of drop spectra. At S band they derived

$$R = 26.7K_{DP}^{0.96}Z_{DR}^{-0.447}. \quad (23)$$

Such relations involving both K_{DP} and Z_{DR} were used by Peterson et al. (1999) in their analysis of the Fort Collins flash flood of July of 1997. Again, as for Eqs. (16) and (17), because Z_{DR} is independent of concentration but both R and K_{DP} scale linearly with drop concentration, physical arguments would suggest an exponent of unity for K_{DP} in Eq. (23). Hail was not reported in the flash flood, so the values of Z_{DR} should be reliable once they have been corrected for differential attenuation and, as argued in section 4a, both K_{DP} and Z should scale with concentration, so this equation reduces to a form similar to Eqs. (16) or (17).

Goddard et al. (1994) have shown more fundamentally that Z, Z_{DR}, and K_{DP} in rain are not independent. Both K_{DP} and Z scale with concentration, and so their ratio will be independent of concentration, as is Z_{DR}. The relationship among the three variables is illustrated in Fig. 8, which shows that the value of K_{DP} (two way) per unit (linear) Z is a well-defined function of Z_{DR} and is virtually independent of μ. Goddard et al. (1994) showed that this redundancy, which arises by chance because of the natural form of drop spectra and drop shapes, can be exploited to provide an automatic means of calibrating the Z measurement to 0.5 dB (12%). The technique, as demonstrated in Fig. 9, is to take a path through rain over which there is appreciable differential phase shift mostly caused by rain with Z_{DR} above 1.5 dB and to use Fig. 8 to derive the value of K_{DP} at each gate using the observed Z and Z_{DR} at that gate; the calibration of Z is then adjusted so that the theoretical total phase shift from adding up the predicted K_{DP} at each gate agrees with the total observed differential phase shift along the whole path. Figure 9 shows clearly that a calibration to better than 0.5 dB (12%) can be achieved. The technique is used routinely at the Rutherford Appleton Laboratory (RAL) radar at Chilbolton in southern England. The advantages are that, first, the predicted phase shift is compared with an observed total phase shift integrated along a path, which can be estimated very accurately, rather than introducing further noise by differentiating an already noisy differential phase estimate and, second, many adjacent rays can be averaged to reduce the standard deviation of the calibration error. Curves similar to those in Fig. 7 are found at C band and X band, so the calibration procedure should operate at these frequencies. In all cases, the path should be chosen so that total differential phase shift is limited to values that imply negligible attenuation and the observed values of Z and Z_{DR} along the path are the true ones.

The very lack of independence that enables the consistency of Z, Z_{DR}, and K_{DP} to be used to calibrate the reflectivity of the radar precludes their use, in combination with Z, to provide independent of estimates of N_o, D_o, and μ in the drop size distribution and so to improve the rainfall estimate. Smyth et al. (1999) suggested using the consistency of the three variables at S band to monitor that the integrated phase along the path agrees with the predicted phase shift to confirm that rain is present along the beam. When rain is present, then Z can be accurately calibrated and the Z–Z_{DR} technique (Eq. 18) should provide a rainfall rate accurate to 25%. When the consistency fails this indicates that hail is present, in which case the Z–Z_{DR} technique fails; the value of Z_{DR} is no longer related to the mean size of the raindrops because hail has the effect of depressing Z_{DR} towards zero, and the value of Z is inflated by the large hailstones, which contribute little to the rainfall rate. Where hail is indicated, they suggested that an R–K_{DP} relation, such as in Eq. (21), provides the best rainfall estimate.

d. Attenuation correction using differential phase shift

We note finally that the linearity between differential phase shift and both total and differential attenuation derived by Bringi et al. (1990) has been widely used (especially at C band) for correcting Z and Z_{DR} for attenuation affects. The linearity was derived from a statistical fit using the Ulbrich range of drop spectra with a maximum value of D_o of 2.5 mm (Z_{DR} = 3 dB), but Smyth and Illingworth (1998) showed that once D_o exceeds 2.5 mm then even at S band this linearity unfortunately breaks down. Most attenuation events involve values of Z_{DR} above 3 dB, and so the correction technique may have difficulties. To overcome this problem at C band, Carey et al. (2000) propose the use of different relationships between phase shift and attenuation depending upon the value of Z_{DR}.

5. Conclusions

1) The use of nonnormalized gamma functions to describe raindrop size distributions and the derivation of the Ulbrich range of values of N_o, D_o, and μ in rain based on interpretation of the 69 Z–R relationships of Battan (1973) leads to a range of drop spectra which is inconsistent with observations. Once normalized functions are used, μ is no longer a function of the b in $Z = aR^b$, and N_o is no longer correlated with μ, but N_o, D_o, and μ are independent parameters describing the concentration, mean size, and breadth of the drop spectra.
2) The linearity between differential phase and both total and differential attenuation proposed by Bringi et al. (1990) is in widespread use for correction of differential and total attenuation, but, even at S band, it breaks down when the Ulbrich maximum value of D_o of 2.5 mm (Z_{DR} = 3 dB) is exceeded. Occasions on which there are high values of attenuation (Smyth and Illingworth 1998) are often associated with values of Z_{DR} of more than 3 dB, so the use of differential phase to correct for attenuation is questionable. The situation at C band is less clear.
3) Most experiments to derive better rainfall rates using Z and Z_{DR} have been based on equations [e.g., Eqs. (16), (17), and (20)] derived from the unrealistic Ulbrich range of spectra and appear often to be in error by 3 dB—a larger error than the use of the conventional Z–R relationship. A new relationship is proposed that, in the absence of hail, should yield a rainfall rate accurate to $\pm$1 dB (25%), providing Z_{DR} can be estimated to 0.2 dB and Z is calibrated to 1 dB. This relationship is based on a mean value of μ in natural rainfall of 5 and more realistic drop shapes.
4) The advantages of K_{DP} technique for improving the accuracy of rainfall rates may not be as powerful as previously claimed. Based on the Ulbrich range of drop spectra, equations of the form K_{DP} proportional to $R^{1.15}$ have been widely used, but if raindrop spectra are based on a normalized gamma function with a mean value of μ of 5 and more realistic drop shapes, then the exponent at both C and S band is 1.4 and is similar to the Z–R value for NEXRAD. Thus, rainfall rates from K_{DP} will have a sensitivity to changes in raindrop spectra that is similar to those based on Z. The important advantage of K_{DP} is its immunity to hail.
5) The larger exponent in the R–K_{DP} expression means that the use of total differential phase shift along a long path to derive an integrated rainfall rate along that path is unlikely to outperform an equivalent algorithm based on integrating the value of Z.
6) In rainfall, Z, Z_{DR}, and K_{DP} are not independent. This redundancy can be exploited to provide an absolute calibration of Z to within 0.5 dB (12%). The value of phase shift is computed at each gate using the theoretical value for K_{DP} derived from Z and Z_{DR} observed at that gate, and the calibration of Z adjusted so that the computed total phase shift along the path agrees with that observed. The advantage

of the method is that the total integrated phase shift can be accurately estimated, rather than differentiating an observed noisy differential phase shift to obtain an even noisier value of K_{DP}. The technique should also work at C and X band. In all cases, a path should be chosen over which the total phase shift is not too large and so indicates negligible attenuation of Z or Z_{DR}.

7) The redundancy of Z, Z_{DR}, and K_{DP} in rain means that the three parameters cannot be used to derive values of N_o, D_o, and μ in rain. However, an inconsistency of the three variables can be used at S band when attenuation is insignificant to flag the presence of hail. When hail is absent, then R may be derived from Z and Z_{DR} to an accuracy of 25%, but, when hail is indicated, then an R–K_{DP} relationship for $N_L = 8000\ \mathrm{m^{-3}\ m^{-1}}$ [e.g., Eq. (21)] should provide the most accurate rainfall estimate.

The conclusions above indicate that polarization techniques should be very powerful for improving rainfall estimates in severe storms provided that the correct assumptions are made. It would be interesting to reanalyze existing data to see if the rainfall estimates are improved when the suggestions above are incorporated into the algorithms.

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