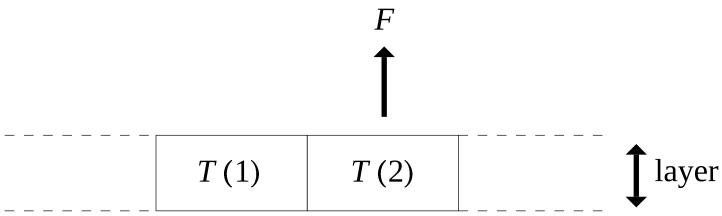
MSc Exam Question - 3d-Var

The total flux, F, of thermal radiation emitted from a layer of the atmosphere is being monitored by a special instrument (a bolometer). The flux is related to the temperature, T, of the layer via the Stefan-Boltzmann law,

$$F = \kappa T^4,$$

where κ is the Stefan-Boltzmann constant.

A model of the layer represents its temperature in two grid boxes by the column vector $\vec{x} = (T(1), T(2))^T$ (see figure). It has forecast that the temperature is $\vec{x}_B = (T_B(1), T_B(2))^T$ at the time that the measurement is made. The measurement is to be used to adjust the model state using three-dimensional variational data assimilation.



(a) Write down a suitable cost function that uses $\vec{x}$ as the control variable. Define the symbols that you use. **[3 marks]**

(b) The instrument has made a single observation of grid box 2 (see figure). What is the innovation on the first iteration of the 3d-Var. scheme? **[2 marks]**

(c) Let $\mathbf{H}$ be the Jacobean of the forward model evaluated at the forecast. Determine the number of rows and columns of $\mathbf{H}$, and write down each of its matrix elements. **[2 marks]**

(d) Write down the gradient of the cost function with respect to $\vec{x}$ given that the variance of the measurement F is σ and that the inverse of the background error covariance matrix is,

$$\mathbf{B}^{-1} = \begin{pmatrix} \alpha & \gamma \\ \gamma & \beta \end{pmatrix}.$$

Explain the role of the adjoint operator, $\mathbf{H}^T$, which appears in the gradient. **[4 marks]**

(e) Write down the Hessian of the cost function. **[4 marks]**

(f) By applying the results used above, write an expression that gives the adjustment to $\vec{x}_B$ after one Newton iteration (do not perform the matrix inversion). **[8 marks]**

(g) By referring to the nature of the forward model, κT^4, would you expect that the minimum of J is reached exactly after just one Newton iteration? **[2 marks]**