MSc exam question (Data Assimilation, 3d Var.) Ross Bannister, 2005

Observations made in a time interval are to be assimilated for operational weather forecasting. The interval is short enough for the motion of the atmosphere to be ignored. A 3d-Var. cost function can be constructed as follows (ignoring the control variable transform step for simplicity),

$$J = \tfrac{1}{2}(\mathbf{x}-\mathbf{x}_B)^T \mathbf{B}^{-1}(\mathbf{x}-\mathbf{x}_B) + \tfrac{1}{2}(\mathbf{y}-\mathbf{y}_m)^T \mathbf{R}^{-1}(\mathbf{y}-\mathbf{y}_m),$$

where $\mathbf{y}_m = \mathbf{h}[\mathbf{x}]$, $\mathbf{h}$ is the observation operator and other notation is standard.

(a) Explain the meaning of each of the symbols: $\mathbf{x}, \mathbf{x}_B, \mathbf{y}, \mathbf{y}_m, \mathbf{B}$ and $\mathbf{R}$. [6 marks]

(b) A model's domain has a longitude, latitude and height grid that covers the globe of N_λ, N_ϕ and N_z elements respectively. The assimilation system is designed to work with this model. There are p observations.

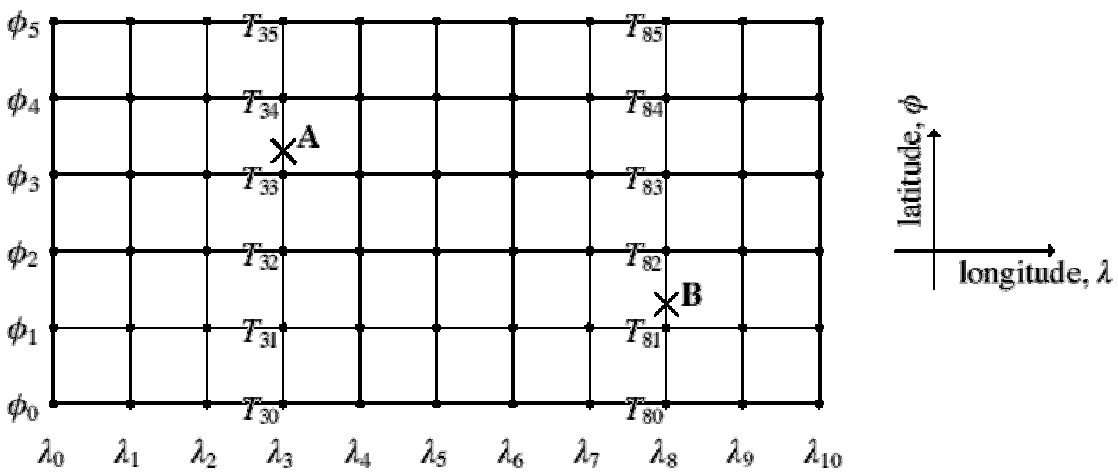
(i) How many elements do the vectors $\mathbf{x}$ and $\mathbf{y}$ have? [2 marks]

(ii) What are the sizes of the matrices $\mathbf{B}$ and $\mathbf{R}$? [2 marks]

(c) During a particular data assimilation cycle, only two observations are recorded. One is of temperature, T_A, and is made at position 'A' (λ_A, ϕ_A) in the Fig. The other is of potential temperature, θ_B, and is made at position 'B' (λ_B, ϕ_B). [In meteorology potential temperature, θ, is related to temperature, T, and pressure, p, by,

$$\theta = \left(\frac{p}{p_0}\right)^{-\kappa} T,$$

where p_0 and κ are constants.] The observations are made at exactly the same height plane shown in the Fig. The state $\mathbf{x}$ includes the variables temperature, T_{ij}, pressure, p_{ij}, plus wind and moisture variables, where ij labels grid position (temperatures for some grid points are shown explicitly in the Fig.). The grid points are at positions (λ_i, ϕ_j).



(i) Write down a simple observation operator for observation A (use linear interpolation where appropriate). [3 marks]

(ii) Similarly, write down a suitable observation operator for observation B. [3 marks]

(iii) By ignoring the effect of the background error part of J for now, list the model variables that will be influenced in the assimilation by these observations. [2 marks]

(iv) Now by including the effect of the background error term, explain briefly how other variables and grid points would be influenced in the assimilation. [3 marks]

(v) The variational method has widely replaced sequential methods - such as optimal interpolation (OI) – in operational data assimilation. Comment on how 3d-Var. is superior to OI. Could observations like those considered above be dealt with just as well in OI? [4 marks]