What is the cut-off total wavenumber for each vertical mode of unbalanced pressure?

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1. Background and equations

The PV scheme [1] permits the existence of an unbalanced rotational wind in VAR. This wind is diagnosed from the unbalanced pressure via an anti-balance relation, Eqs. (26)-(29) of [1]. The anti-balance relation has a form that greatly amplifies those structures of unbalanced pressure that have small vertical scales and large horizontal scales. This is damaging to the data assimilation as such modes that are present in unbalanced pressure translate to unrealistically large unbalanced horizontal winds. It is the role of this document to determine a way of cutting-off the spectrum of unbalanced pressure so that such modes are not present.

In terms of control variables, unbalanced pressure is stored as a function of horizontal wavenumber (below, referred to as just "wavenumber") and vertical mode (referred to as just "mode"). Part of the role of the horizontal **U**-transform is to multiply the unbalanced pressure by a factor that depends upon wavenumber and mode. This document considers how a relationship between wavenumber and mode can be determined such that structures of unbalanced pressure only of small horizontal and large vertical scales are allowed to pass through the horizontal transform (and hence through the anti-balance relation). In effect, the horizontal transform multiplies prohibited structures by zero.

The Rossby radius definition, L_R is defined as

$$L_R = \frac{NH}{f},$$

where N is the Brunt-Vaisala frequency, H is the vertical scale and f is the Coriolis parameter. It is desirable to cut-off horizontal unbalanced pressure scales, L, that are greater than some multiple of L_R

$$L_{\text{crit}} > \varepsilon L_R, \qquad \varepsilon > 1.$$

In terms of wavenumber, this cut-off of unbalanced pressure corresponds to global wavenumbers that are smaller than n_{crit}

$$n < n_{\text{crit}},$$

$$n_{\text{crit}} = \frac{2\pi R}{L_{\text{crit}}} = \frac{2\pi R}{\varepsilon L_R} = \frac{2\pi Rf}{\varepsilon NH}, \qquad (1)$$

where R is the radius of the Earth

$$R = 6371 \text{ km},$$

a typical value of N is

$$N \sim 1.5 \times 10^{-2} \text{ sec}^{-1},$$

and a midlatitude value for f is

$$f \sim 2 \times 7.292 \times 10^{-5} \times \sin 45° \text{ sec}^{-1}$$

$$\sim 1.03124 \times 10^{-4} \text{ sec}^{-1}.$$

Next, it is necessary to obtain a relationship between vertical scales of unbalanced pressure, H, and the unbalanced pressure vertical mode index, ν. For the models considered this relationship can empirically be found to be of the form

$$H \sim A\nu^{-B} \text{ km}, \qquad (2)$$

(see Sec. 2 for details of how A and B are found). Using (1) and (2), the cut-off global wavenumber for mode ν is

$$n_{\text{crit}} = \frac{2\pi Rf}{\varepsilon NA}\nu^{B}. \qquad (3)$$

How n_{crit} varies with mode is plotted in Fig. 1, for the results of a 50-level model studied and with the choice of $\varepsilon = 2$, showing which parts of the unbalanced pressure are expelled from the data assimilation problem.
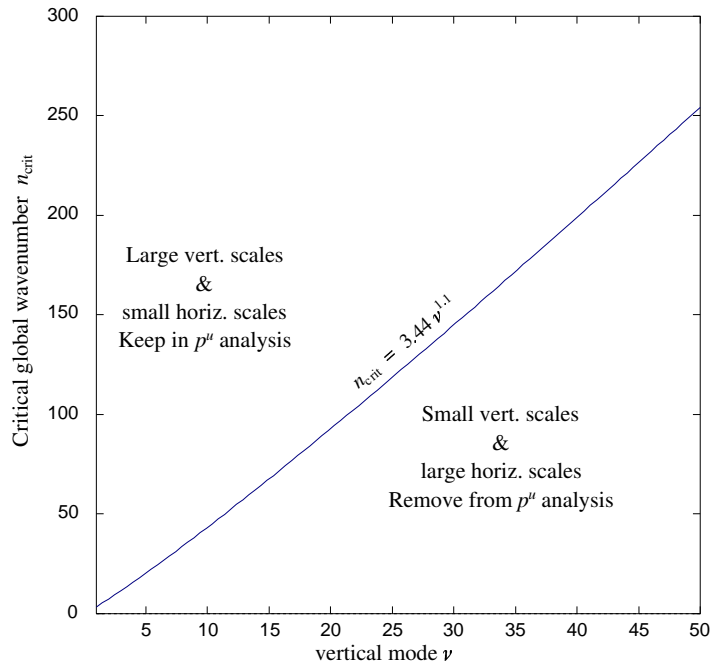
Figure 1: Spectrum of wavenumber verses mode showing how the cut-off wavenumber varies with mode number for a 50-level model tested and with $\varepsilon = 2$.

2. Estimating the empirical relationship between vertical lengthscale and mode number

This is a multi-stage process. A covariance file for the model of interest is first required.

- The first task is to generate a text file of covariance statistics for this file. This is done via a Var_CovAccStats print job (see e.g. *vbwi.x*). This produces a text file of covariance statistics called *stats.html*, which can be found in the output directory (specified in the varui job) on the linux system. In the case of *vbwi.x*, this is *~frxb/var/out/ VertModes50.Ob*.

- Run the IDL script *PlotModes.pro*, which can be found on the web page specified in [2]. The number of levels and the location of the *stats.html* file needs to be specified by editing the IDL script. To run this script, start IDL on the linux box (type *idl*), compile the code with *.compile PlotModes.pro*, and run the code with *PlotModes*. This script extracts vertical mode data for unbalanced pressure from the *stats.html* file and generates plots of vertical modes of unbalanced pressure. Plots are found in the postscript files *ApModesx.ps*, where *x* is the page number (there are 40 vertical modes plotted per page). Example plots are available [3,4,5] (renamed and converted to pdf files).

- Use the plots to measure the vertical scales of each mode, $H(\nu)$ where possible (e.g., by estimating each mode's peak-to-peak distance in the middle troposphere.

- Plot these data and fit to a curve. The PV code assumes that the data can be reasonably modelled with form (2). Example data are shown in Figs. 2-4 for various models tested.
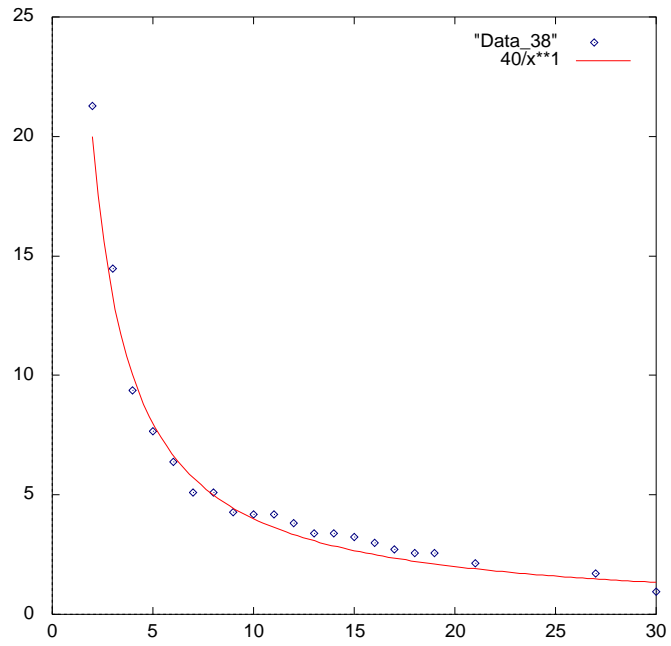
Figure 2: Variation of the vertical lengthscales of unbalanced pressure, H, as a function of the vertical mode index, ν. This plot is for a 38-level model ($A = 40$ and $B = 1$).
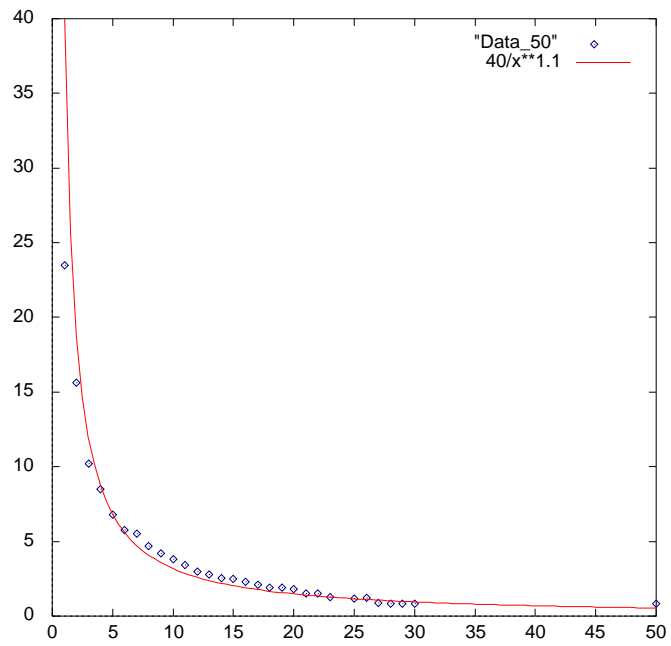

Figure 3: As Fig. 2 but for a 50-level model ($A = 40$ and $B = 1.1$).
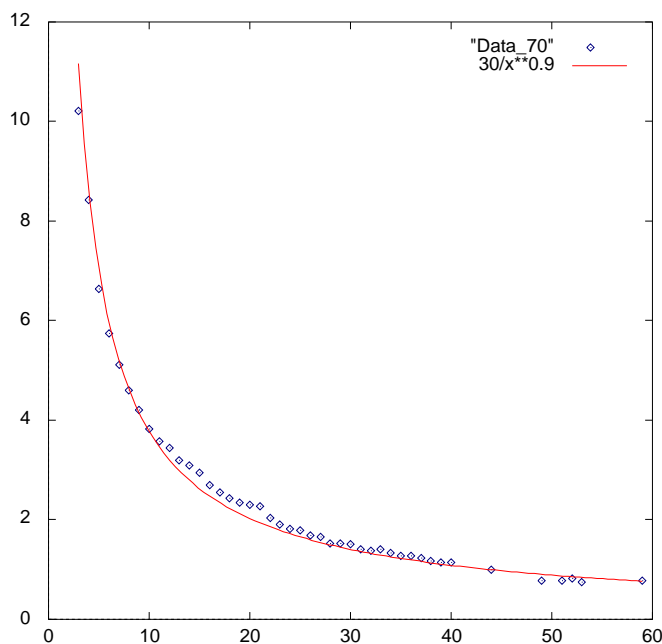
Figure 3: As Fig. 2 but for a 70-level model ($A = 30$ and $B = 0.9$).

These results for A and B are used in VAR routine `Var_UhGlobalSpectral.f90` with help from (3). The following 'Transform namelist' parameters need to be set.

- `PVcutoffApmodes` (.true. to do the above spectral cut-off, .false. to leave it unmodified).
- `PVcutoff_fac1` is $2\pi Rf / \varepsilon NA$, as appearing in (3).
- `PVcutoff_fac2` is B, as appearing in (3).

Possible values of `PVcutoff_fac1` and `PVcutoff_fac2`, derived from the plots shown in this document, are summarised in the table below.

Parameter	L38			L50			L70		
	$\varepsilon = 1$	$\varepsilon = 2$	$\varepsilon = 3$	$\varepsilon = 1$	$\varepsilon = 2$	$\varepsilon = 3$	$\varepsilon = 1$	$\varepsilon = 2$	$\varepsilon = 3$
`PVcutoff_fac1`	16.88	23.44	32.29	16.88	23.44	32.29	19.17	24.59	33.06
`PVcutoff_fac2`	1.0	1.0	1.0	1.1	1.1	1.1	0.9	0.9	0.9

References

[1] Bannister R.N., Cullen M.J.P., 2008, *http://www.met.rdg.ac.uk/~ross/DARC/PVcv/PVcv.pdf*

[2] Bannister R.N., 2009, *http://www.met.rdg.ac.uk/~ross/Computing/IDL/PV/PlotModes.pro*

[3] Vertical modes for 38-level model *http://www.met.rdg.ac.uk/~ross/DARC/PVcv/VertModes/ ApModes_L38_p1.pdf* and *ApModes_L38_p2.pdf*

[4] Vertical modes for 50-level model *http://www.met.rdg.ac.uk/~ross/DARC/PVcv/VertModes/ ApModes_L50_p1.pdf* and *ApModes_L50_p2.pdf*

[5] Vertical modes for 70-level model *http://www.met.rdg.ac.uk/~ross/DARC/PVcv/VertModes/ ApModes_L70_p1.pdf* and *ApModes_L70_p2.pdf*