

**Candidates are admitted to the examination room ten minutes before the start of the examination. On admission to the examination room, you are permitted to acquaint yourself with the instructions below and to read the question paper.**

**Do not write anything until the invigilator informs you that you may start the examination. You will be given five minutes at the end of the examination to complete the front of any answer books used.**

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**April 2010**

Answer Book  
Data Sheet

Calculators and programmable calculators are permitted

**THE UNIVERSITY OF READING**

MSc/Diploma  
Course in Applied Meteorology  
Course in Atmosphere, Ocean and Climate

**PAPER MTMG38**

**Remote Sensing**

Two hours

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Answer ANY TWO questions

The marks for the individual components of each question are given in [ ] brackets.  
The total mark for the paper is 100

1 In the following you may use the following information: gravitational constant  $G = 6.67 \cdot 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$ ; Earth mass  $m_e = 5.97 \cdot 10^{24} \text{ kg}$ ; Earth polar radius  $r_e = 6357 \text{ km}$ ; Wien's displacement law:  $\lambda_{\text{max}} T = 2.897 \cdot 10^{-3} \text{ mK}$ , where  $\lambda$  is a wavelength and T a temperature.

(a) Describe the main advantages and disadvantages of remote sensing measurements with respect to in-situ observations.

[5 marks]

(b) Discuss the main advantages and disadvantages of remote sensing measurements of radiation emitted by the Earth with respect to those of scattered solar radiation.

[10 marks]

(c) The effective emission temperature of the Earth is about 250 K. Calculate the wavelength of peak emission for this temperature. Which spectral region does this correspond to?

[5 marks]

(d) Consider an atmosphere exclusively composed of molecules having a spectrum of absorption lines with central wavelengths  $\lambda_k = \lambda_0 / k^2$ , where k is an integer and  $\lambda_0$  is the 40  $\mu\text{m}$  wavelength. Assume that the width of the molecular absorption line at wavelength  $\lambda_k$  is  $\Delta\lambda_k = 0.1 \lambda_k$ . Considering only the spectral interval between 9 and 44  $\mu\text{m}$ , determine:

i) the spectral band that is most suited for the observation of the Earth surface.

[5 marks]

ii) a spectral interval that is suitable for measuring a vertical profile of some atmospheric parameter (e.g. temperature)

[5 marks]

Question 1 continues overleaf

Turn over

## Question 1 continued

- (e) The radiance emerging from the top of an atmosphere in local thermodynamic equilibrium for an optically thick atmosphere can be written as:  $L_{\lambda}(z_{TOA}) \approx \int_0^{z_{TOA}} B_{\lambda}(T(z))K_{\lambda}(z, z_{TOA})dz$  where  $T(z)$  is the temperature at the  $z$  height, and  $z_{TOA}$  is the top-of-the-atmosphere height. Describe the terms in this equation and discuss their meaning. Sketch a typical weighting function for temperature as a function of height, and identify the atmospheric region where the satellite sensor is most sensitive to atmospheric temperature variations.

[10 marks]

- (f) The revolution period  $T$  and orbital radius  $r$  of a satellite revolving around the Earth are related according to  $r^3 = \frac{Gm_e}{4\pi^2}T^2$  where  $G$  is the gravitational constant and  $m_e$  the Earth mass. The angular resolution of a circular aperture (e.g. telescope) is  $\theta = 1.22 \frac{\lambda}{D}$  where  $\lambda$  is the wavelength and  $D$  the diameter of the telescope. Determine the maximum orbital period of a polar orbiting satellite that allows an on-board instrument with a 0.5 m radius telescope to have a sub-satellite resolution of 10 m at a given wavelength in the visible part of the spectrum (e.g. 0.5  $\mu\text{m}$ )

[10 marks]

Turn over

2

In the following you may use the following results:  
the inverse of a non-singular 2 x 2 matrix  $\mathbf{A}$  is:

$$\mathbf{A}^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

The maximum *a posteriori* estimate can be written as

$\hat{\mathbf{x}} = \mathbf{x}_a + (\mathbf{K}^T \mathbf{S}_\varepsilon^{-1} \mathbf{K} + \mathbf{S}_a^{-1})^{-1} \mathbf{K}^T \mathbf{S}_\varepsilon^{-1} (\mathbf{y} - \mathbf{K} \mathbf{x}_a)$ , where  $\mathbf{x}_a$  is the prior estimate,  $\mathbf{y} = \mathbf{K} \mathbf{x} + \boldsymbol{\varepsilon}$  is the observation vector, with  $\mathbf{x}$  the true state and  $\boldsymbol{\varepsilon}$  the observation error, and  $\mathbf{S}_\varepsilon$  and  $\mathbf{S}_a$  are the observation error covariance and the prior error covariance.

- (a) We want to estimate the atmospheric temperature  $T_A$  and  $T_B$  at a given location over two horizontal layers (denoted with the letters A and B). Assume we have two remote sounding measurements  $y_1$  and  $y_2$  for the location of interest, such that we can write

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 2a & a \\ b & 2b \end{pmatrix} \begin{pmatrix} T_A \\ T_B \end{pmatrix}$$

where  $a$  and  $b$  are two constants.

- (i) Describe the relative sensitivity of each remote sounding measurement to the temperatures in layers A and B.  
[3 marks]
- (ii) Calculate  $T_A$  and  $T_B$  when  $y_1 = 220$  K,  $y_2 = 278$  K, in the case when the measurements are accurate enough so that their error does not affect the precision of the atmospheric temperature profile that is required at this time.

[10 marks]

Question 2 continued overleaf

Turn over

Question 2 continued

- (iii) Now assume that we need to have a better estimate of  $T_A$  and  $T_B$  so that this time we need to have an estimate of the measurement error. The instrument manufacturer tells us that the errors on  $y_1$  and  $y_2$  both have error standard deviation of 1 K and that they are mutually uncorrelated. We also realize that in the past other investigators had measured the temperature over the same location for layer A ( $x_{a1}$ ) and layer B ( $x_{a2}$ ), finding values of  $x_{a1} = 55 \pm 2$ ,  $x_{a2} = 110 \pm 3$ , where the two measurements have a correlation of 0.5. Combine your measurements  $\mathbf{y} = (y_1, y_2)$  and past measurements  $\mathbf{x}_a = (x_{a1}, x_{a2})$  to get a new estimate of  $T_A$  and  $T_B$  when  $a = b = 1$ . [12 marks]
- (b) What scattering mechanism is involved when a meteorological radar detects an echo from a raindrop? Why does this lead to the radar reflectivity,  $Z$ , having units of  $\text{mm}^6 \text{m}^{-3}$ ? Why is the value of  $Z$ , observed when the beam of an operational rain radar is filled with rain, not a unique function of the rainfall rate,  $R$  (in  $\text{mm hr}^{-1}$ )? Empirical relationships of the form  $Z = a R^b$  are often used. Why does 'b' have the value of about 1.5? [8 marks]
- (c) Explain why melting snow appears as a 'bright band' on a radar display with a value of  $Z$  about a factor of ten higher than the  $Z$  in the rain below. [7 marks]
- (d) On a day of widespread frontal rain the zero degree isotherm is at 2km height. For an operational rain radar with a one degree beamwidth and scanning at an elevation of one degree, explain how the presence of the bright band may lead to errors in the rainfall rate,  $R$ , derived from the observed  $Z$  for the following horizontal ranges:  
 i) up to 50km, ii) 50-100km, and iii) beyond 100km? [10 marks]

Turn over

- 3 (a) Explain how a meteorological radar is able to estimate the velocity of target and how the concept of the maximum unambiguous velocity,  $V_{\max}$ , arises. If  $V_{\max}$  is given by:

$$V_{\max} = \lambda (\text{prf}) / 4,$$

where  $\lambda$ , is the wavelength and ‘prf’ is the pulse repetition frequency. What is the value of  $V_{\max}$  for a vertically pointing ground-based Dopplerised cloud radar operating with  $\lambda = 3.2\text{mm}$  and a prf of 10kHz?

[8 marks]

- (b) At one particular gate the radar in part a) detects drizzle droplets which are all falling with a velocity of  $2 \text{ m s}^{-1}$ . What phase shift does this correspond to? Sketch out a plot of the steady phase progression of the received backscattered signal from this gate for ten successive transmitted pulses.

[8 marks]

- (c) If the beamwidth of the cloud radar is  $\theta$  and the horizontal wind is  $V_{\text{horiz}}$ , derive a simple expression for the apparent spread of the Doppler velocities of the drizzle droplets in part b). What will this Doppler width be if the cloud radar has an antenna of diameter 0.8m and the horizontal wind is  $50 \text{ m s}^{-1}$ ?

[12 marks]

- (d) How much random phase noise will the increased Doppler width of the drizzle target introduce into the phase progression plotted in part b)? Modify your sketch to include this effect.

[10 marks]

- (e) How much will the Doppler width of the drizzle target increase if the same radar is on a satellite in low earth orbit with a  $V_{\text{horiz}}$  of  $7 \text{ km s}^{-1}$ . Modify your sketch of the phase progression in part d) to account for this increased Doppler width. Will it still be possible to derive the velocity of the drizzle from the satellite?

[12 marks]

(End of Question Paper)