You are allowed ten minutes before the start of the examination to acquaint yourself with the instructions below and to read the question paper.

Do not write anything until the invigilator informs you that you may start the examination. You will be given five minutes at the end of the examination to complete the front of any answer books used.

April 2011

Answer Book
General Data Sheet
Any bilingual English language dictionary permitted
Only Casio-fx83 calculators are permitted

UNIVERSITY OF READING

MSc/Diploma
Course in Applied Meteorology
Course in Atmosphere, Ocean and Climate

MSc in Mathematics and Numerical Modelling of the Atmosphere and Oceans

PAPER MTMG49

Boundary Layer Meteorology and Micrometeorology

One and a half hours

Answer ANY TWO questions

The marks for the individual components of each question are given in [ ] brackets. The total mark for the paper is 100
1

(a) Sketch typical profiles of horizontal wind-speed $U$, potential temperature, $\theta$, and specific humidity, $q$, in the atmospheric boundary layer for day-time and night-time, marking in typical depths of each characteristic layer. [12 marks]

(b) Sketch the day-time potential temperature profile when there is a layer of stratocumulus present. Explain the thermodynamic processes which act at different heights to give the temperature profile its shape. Explain what happens to the stratocumulus layer as it is exposed to a diurnal cycle in the sun’s radiation. How might the potential temperature profile change as a result? [17 marks]

(c) The dynamic stability can be quantified using the Monin-Obukhov stability parameter $\zeta = z/L$, where the Obukhov length $L$ is given by

$$L = \frac{\frac{u^*}{k}}{\frac{g}{\theta p c_p}}$$

What does $\zeta$ tell us about the production of turbulence, and how does it vary under different atmospheric conditions? What is the physical interpretation of $L$? [10 marks]

(d) In Monin-Obukhov theory, the potential temperature profile in the surface layer may be described by

$$\frac{d\theta}{dz} = \frac{\theta_s}{kz} \phi_h(\zeta)$$

where $\theta_s$ is the turbulent temperature scale. In stable conditions the stability function is

$$\phi_h = (1 + 5\zeta).$$

Derive an expression for the potential temperature gradient during stable conditions using these equations. [11 marks]
2

(a) The evolution of turbulent kinetic energy per unit mass, $e$, is given by

$$\frac{De}{Dt} = -\overline{u w} \frac{d\overline{u}}{dz} + \frac{g}{\overline{\theta}} \frac{w}{\overline{\theta}} - d \frac{w e}{dz} - \frac{1}{\rho} \frac{d}{dz} \frac{w p'}{p} - e.$$

State briefly the physical interpretation of each of the five terms on the right hand side of the above equation. [10 marks]

(b) Under what conditions does the turbulent kinetic energy equation assume the form

$$e = -\overline{u w} \frac{d\overline{u}}{dz} + \frac{g}{\overline{\theta}} \frac{w}{\overline{\theta}}$$

(1)

Stating any assumptions that you make, show that an equivalent form for the surface layer is given by

$$e = \frac{u_*^3}{kz} - \frac{g}{\overline{\theta}} u_* T_*.$$  \hspace{1cm} (2)

[8 marks]

(c) Some students carry out turbulence measurements in order to test out the prediction given by equation (2). They obtain the following values: $u_* = 0.5 \text{ms}^{-1}$, $\overline{u} = 5 \text{ms}^{-1}$, surface heat flux $H_0 = 200 \text{ Wm}^{-2}$, and surface temperature $\overline{\theta} = 293 \text{K}$. What is the value of $e$ at a height of $z = 10 \text{m}$?

Conditions become overcast but the wind-speed remains the same. What value of $e$ would be obtained under these conditions? State your assumptions clearly. [8 marks]

(d) The students try a different method for obtaining $e$ using spectral analysis. Figure 2.1 shows the spectral energy density $S(f)$ multiplied by frequency $f$ as a function of frequency, calculated for the measured velocity component $u$. Note that $fS(f) = \kappa E(\kappa)$, where $E(\kappa)$ is the spectral energy density as a function of wavenumber.

Question 2 continued overleaf
Question 2 continues

Describe the processes affecting turbulence at low, middle and high frequencies in the spectrum, quoting typical length-scales where appropriate.

[10 marks]

(e) Table 2.1 shows the data-points which are plotted in Figure 2.1.

The form of the spectrum in the inertial sub-range is

\[ S_u(f) = \alpha \varepsilon^{2/3} \left( \frac{u}{2\pi} \right)^{-2/3} f^{-5/3} \]  

(3)

where \( \alpha = 0.5 \). Explain how the students calculate a value of \( \varepsilon \) from this data, stating any assumptions they make. What value do they get?

[14 marks]

<table>
<thead>
<tr>
<th>Data-point</th>
<th>Frequency f [Hz]</th>
<th>Spectral energy density ( S(f) ) [m² s⁻¹]</th>
<th>Frequency.Spectral energy density ( fS(f) ) [m² s⁻²]</th>
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</thead>
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<td>0.0058</td>
</tr>
</tbody>
</table>

Table 2.1

Question 2 (e) continued overleaf
Question 2 (e) continues

Figure 2.1

Turn over
3

(a) Explain the processes leading to the Urban Heat Island phenomenon, and how it might be quantified. Include references to the evolution of the phenomenon, the surface energy balance, and differences between the urban and surrounding rural surface. When is the effect at its maximum during a diurnal cycle?

[12 marks]

(b) The Carson model of the convective boundary layer gives the following expression for the height $h(t)$ of the boundary layer as a function of time $t$:

$$h(t) = \sqrt{\frac{2(1+2E)}{\rho c_p \gamma}} \int_0^t H_0 dt$$

This model describes growth of the convective boundary layer by entrainment. Describe which heat fluxes are responsible for controlling the boundary layer depth, and how they are related. In doing so you should explain the meaning of the terms $E$, $\gamma$ and $H_0$.

[8 marks]

(c) Assuming that $H_0$ has a sinusoidal form

$$H_0 = H_{0\text{MAX}} \sin(\omega t),$$

derive an equation for the evolution of $h(t)$ given that $t=0$ at dawn. Given that $H_{0\text{MAX}}$ in a rural area is 100 W m$^{-2}$ and that the urban value of $H_{0\text{MAX}}$ is 50% higher, what is the depth of the rural boundary layer at dusk, and how much deeper is the nearby urban boundary layer? Assume that $\gamma = 3.3 \times 10^{-3}$ K m$^{-1}$, $E = 0.21$, and that dusk occurs 12 hours after dawn.

[15 marks]

(d) Ozone is a serious pollutant which can be found in the boundary layer. Describe the physical and chemical processes affecting the evolution of ozone concentrations throughout a daily cycle and compare resulting concentrations in the city centre with those at a rural site downstream of a city. Make reference to the Urban Heat Island in your answer.

[15 marks]

(End of Question Paper)