

**Candidates are admitted to the examination room ten minutes before the start of the examination. On admission to the examination room, you are permitted to acquaint yourself with the instructions below and to read the question paper.**

**Do not write anything until the invigilator informs you that you may start the examination. You will be given five minutes at the end of the examination to complete the front of any answer books used.**

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**April 2012**

**Answer Book**

**Data Sheet**

**Any bilingual English language dictionary permitted**

**Only Casio-fx83 calculators are permitted**

**Final Examination for MSc**

**Course in Applied Meteorology**

**Course in Atmosphere, Oceans and Climate**

**Course in Data Assimilation and Inverse Modelling in Geosciences**

**Course in Applied Meteorology and Climate with Management**

**MTMW20**

**Global Circulation of the atmosphere and oceans**

**Two hours**

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**Answer ANY TWO questions**

The marks for the individual components of each question are given in [ ] brackets. The total mark for the paper is 100.

1. Effects of global warming on sea level rise and thermohaline circulation.

Global warming is expected to increase sea level through two main effects: 1) Thermal expansion of a warming ocean; 2) Addition of melted land ice to the ocean. The following seeks to examine which process is the most efficient at using the global warming excess energy to increase sea level.

- (a) By assuming that global warming results in an annual excess of incoming solar energy of  $1 \text{ W/m}^2$  on average over the whole planet, estimate the global excess energy  $Q$  (in Joules) that this represents when integrated over the whole Earth surface area and over one year.

[6 marks]

- (b) First examine the case where the excess energy  $Q$  computed in (a) is used to warm up uniformly the first  $h=1000$  metres of the oceans.

(i) By recalling that the heat capacity of sea water is about  $C_p=4000 \text{ J/kg/K}$ , and that the oceans cover  $2/3$  of the total surface, compute the resulting average increase in temperature  $\Delta T$  caused by the excess energy  $Q$ .

[8 marks]

(ii) By using  $\alpha = 10^{-4} \text{ K}^{-1}$  as the effective thermal expansion coefficient of the oceans, estimate the increase in sea level associated with the increase in temperature computed in (b)(i) in one year.

[12 marks]

- (c) Second, examine the case where the excess energy  $Q$  is used to melt land ice of temperature  $T_{\text{ice}} = -10^\circ\text{C}$  instead.

(i) Derive an expression linking the mass of ice  $M_{\text{ice}}$  that can be melted by the energy excess  $Q$ , assuming that the heat capacity of ice is  $C_{\text{ice}} = 2000 \text{ J/kg/K}$  and the latent heat of fusion at atmosphere pressure and  $T=0^\circ\text{C}$  is  $L_{\text{fusion}} = 330 \text{ kJ/kg}$ .

[12 marks]

(ii) Assume that the mass of ice  $M_{\text{ice}}$  computed in (i) is now added to the oceans. Compute the increase in sea level resulting from the addition. Conclude as to what process is the most efficient use of the global warming excess energy  $Q$  to increase sea level.

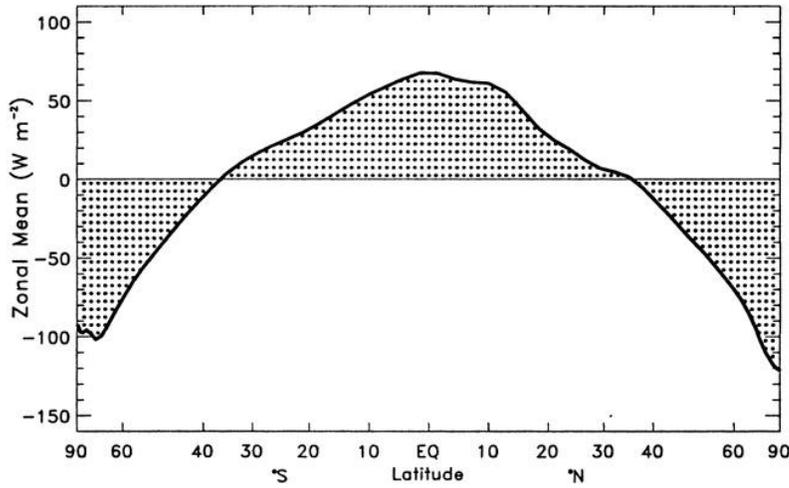
[8 marks]

- (d) Discuss what will be the respective impacts of a warming of the upper ocean and land ice melting on the thermohaline circulation?

[4 marks]

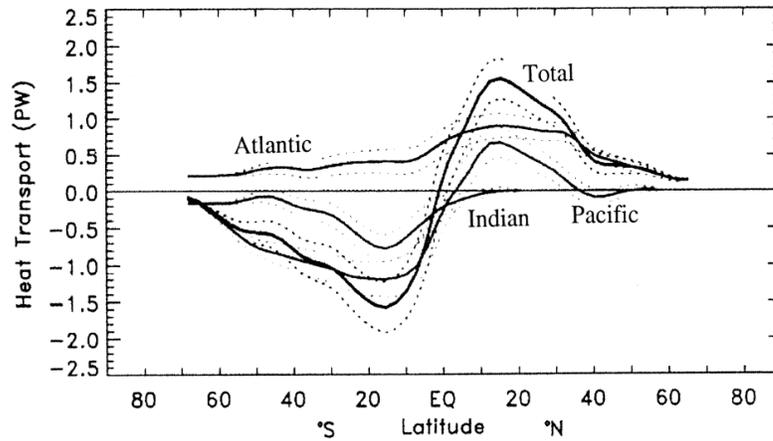
2. General questions about global oceanic and atmospheric circulations

- (a) This figure depicts the excess of energy received by the Earth surface at the top of the atmosphere as a function of latitude. Based on this figure, estimate the heat transport at 30S and 30N. Explain how you arrive at your result.



[12 marks]

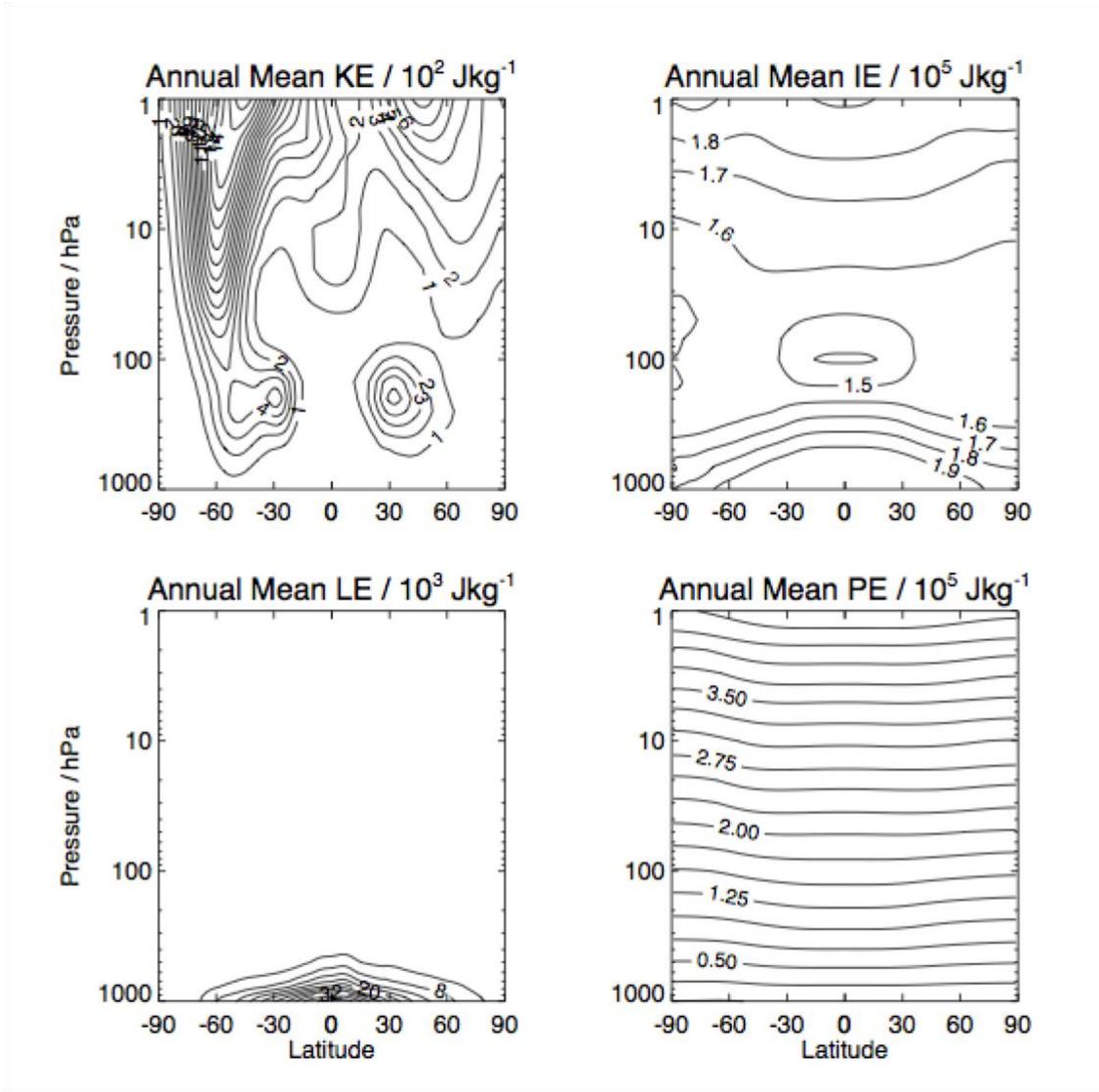
- (b) The following figure shows estimates of the northward heat transports by the global oceans and its major ocean basins.



Briefly discuss the extent to which these distributions can be understood in terms of wind-driven and thermohaline circulations in each basin.

[12 marks]

This figure provides a global distribution of the zonally and annually averaged forms of energy in the atmosphere. Explain the physical basis (using the definitions of each kind of energy) for the 4 different panels.



[14 marks]

- (d) Write the expression for the total angular momentum as the sum of two terms, and explain the physical differences between the two terms. What are the main sources and sinks of angular momentum on Earth?

[12 marks]

3. Propagation of Atmospheric and Oceanic Rossby waves

- (a) Observed oceanic Rossby waves are commonly assumed to have a normal mode structure of the form  $w(x,z,t) = F(z)e^{i(kx-\omega t)}$ , where  $F(z)$  satisfies the following normal mode equation:

$$\frac{d^2 F}{dz^2} + \frac{N^2}{c^2} F = 0$$

with the boundary conditions  $F(0)=F(-H)=0$ , where  $N$  is the buoyancy frequency, and  $c$  is the gravity wave speed that determines the speed of long

Rossby waves  $c_R = \frac{\beta c^2}{f^2}$ . Assuming  $N$  constant, show that the eigenmodes

can be written as  $F(z) = \sin[N(z+H)/c]$ . What is the general solution for  $c$ ?

What is the fastest solution?

[8 marks]

- (b) Observations usually show faster propagation than that estimated in 3(a). A possible explanation is that topographic effects modify the bottom boundary condition  $F'(-H)=0$  instead of  $F(-H)=0$ . Show that the eigenmode then becomes  $F(z) = \cos[N(z+H)/c]$ . Compute the change in Rossby phase speed on the fastest mode. How much faster is the fastest mode as compared to the previous one?

[8 marks]

- (c) Atmospheric Rossby waves are assumed to satisfy the following vorticity equation:

$$\left( \frac{\partial}{\partial t} + \bar{U} \frac{\partial}{\partial x} \right) \nabla^2 \Psi + \beta \frac{\partial \Psi}{\partial x} = 0$$

- (i) By assuming a wave solution of the form:  $\Psi = \Psi_0 e^{i(kx+ly-\omega t)}$ , derive the dispersion relationship linking the frequency  $\omega$  to the wavenumbers  $k$  and  $l$ .

[8 marks]

- (ii) Show that stationary waves, i.e., waves satisfying  $\omega=0$ , can only exist for certain types of flow. State what these flows are.

[6 marks]

(iii) What is the zonal wavelength of wavenumber 1 and 2 Rossby waves at the equator? State your results in terms of the diameter of the Earth along the equator.

[6 marks]

(iv) Rossby wave energy propagates along preferred directions called rays, which are aligned with the group velocity vector:

$$\vec{c}_g = \left( \frac{\partial \omega}{\partial k}, \frac{\partial \omega}{\partial l} \right)$$

Compute the two components of the group velocity. Cite two sources of Rossby waves in the atmosphere.

[8 marks]

(v) As said above, stationary waves are such that  $\omega=0$ . What is their meridional and zonal phase speeds? Contrast the phase speeds with the group velocity for such stationary waves.

[6 marks]

[End of Question paper]