On admission to the examination room, you should acquaint yourself with the instructions below. You must listen carefully to all instructions given by the invigilators. You may read the question paper, but must not write anything until the invigilator informs you that you may start the examination.

You will be given five minutes at the end of the examination to complete the front of any answer books used.

DO NOT REMOVE THIS QUESTION PAPER FROM THE EXAM ROOM.

April 2016
MTMW20 2015/16  A001

Answer Book

Data Sheet
Any bilingual English language dictionary permitted
Any non-programmable calculator is permitted

UNIVERSITY OF READING

Global Circulation of the Atmosphere & Oceans (MTMW20)

Two hours

Answer Question 1 and either Question 2 or 3.

The marks for the individual components of each question are given in [ ] brackets. The total mark for the paper is 100
1. This question is intended to test your general knowledge of the global circulation of the oceans and atmosphere.

(a) Draw a sketch graph illustrating the zonal-average annual-mean distribution of absorbed solar radiation (ASR) and outgoing longwave radiation (OLR). Your sketch should indicate the location and approximate magnitude of the maxima and minima in each quantity (in \( \text{Wm}^{-2} \)).

[12 marks]

(b) Assume the mass flux in the atmospheric Hadley cell is \( 5 \times 10^{10} \text{ kg s}^{-1} \). If the lower branch at 1 km has temperature 20°C and specific humidity 12 g/kg, and the upper branch at 14 km has temperature -60°C and negligible specific humidity, evaluate the poleward moist static energy flux by the Hadley cell. Give your answer in Petawatts.

[12 marks]

(c) You are given a form of the horizontal momentum equations:

\[
\frac{Du}{Dt} - f v = - \frac{1}{\rho} \frac{\partial p}{\partial x} + F
\]

where \( F \) is a friction term. Using this equation or otherwise, write down a scale analysis for the Rossby number, \( Ro \). Using this, along with your knowledge of large-scale mid-latitude flow speeds, explain how you would expect the length-scales at which the motion is approximately geostrophic to differ between the atmosphere and ocean.

[10 marks]

(d) In geostropically balanced flow, the Coriolis force is exactly balanced by a pressure gradient force. Explain, with the aid of diagrams as necessary, why the introduction of surface drag leads to Ekman pumping in the atmosphere.

[10 marks]
(e) Imagine a world where the latent heat of water condensation in the atmosphere is reduced to zero. How might you expect this to impact on the mid-latitude storm tracks? [6 marks]
2. Consider a stably stratified rapidly rotating fluid with two layers in the vertical (i.e., a shallow low density layer over a deep high density layer with no mixing between the two layers).

(a) Imagine a wave propagating in the two-layer fluid. Sketch the structure of the height disturbances you would expect to see (i.e., the distortion of the interface and the free surface) if the wave’s vertical structure is described by:
   (i) the barotropic mode.
   (ii) the first baroclinic mode.

(b) The motion of the two-layer fluid can be described by the shallow water equations on a beta-plane with equivalent depth \( H_e \):

\[
\begin{align*}
\frac{\partial u}{\partial t} - f v + g \frac{\partial h}{\partial x} &= 0 \\
\frac{\partial v}{\partial t} + f u + g \frac{\partial h}{\partial y} &= 0 \\
\frac{\partial h}{\partial t} + H_e \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) &= 0
\end{align*}
\]

where \( h \) is the depth of the fluid. Show that, for geostrophic flow, \( h \) satisfies:

\[
\frac{\partial h}{\partial t} + H_e \left( -g \frac{\beta}{f^2} \frac{\partial h}{\partial x} \right) = 0
\]

[10 marks]
(c) For the situation described in part (c) (i.e., geostrophically balanced flow in the shallow water model described), sketch the horizontal circulation you would expect to find associated with a positive anomaly in depth $h$. [4 marks]

(d) Show that for plane wave-like solutions the dispersion relation for the system described in parts (c) and (d) can be written:

$$\omega = -\frac{\beta}{f^2} g H e k$$

[5 marks]
(e) If a Rossby wave is generated in the Atlantic Ocean at \(0^\circ W, 30^\circ N\), use the dispersion relationship given in part (e) to estimate how long it would take for it to reach \(50^\circ W, 30^\circ N\). Give your answer in years.

To do this, you will need to estimate the equivalent depth, \(H_e\), which is given by:

\[ H_e = \frac{\Delta \rho}{\rho} H \]

where \(\rho\) is the mean density, \(\Delta \rho\) is the difference in density between the two layers, and \(H\) is the depth of the shallow upper layer. Your estimates for these parameters should be based on Figure 1, below.

![Figure 1: Ocean potential density anomalies (with respect to a reference density). The x-axis is latitude, and the y-axis depth (in meters). For the purposes of this exam, you may assume the contour units are kg m\(^{-3}\) and the reference density is 1000 kg m\(^{-3}\).](image)

(f) Briefly explain the relevance of your result in part (f) in the context of climate variability.
3. (a) Draw a sketch of the zonal-mean, time-mean zonal wind speed in the troposphere. The sketch should clearly show the latitude-vs-height structure. In doing so, it should indicate the approximate location and magnitude of jet region, plus typical near-surface wind-speed in the tropics, the midlatitudes and near the poles. [11 marks]

(b) Briefly discuss orographic drag and surface drag as two processes by which angular momentum is transferred to the atmosphere. Identify the latitudinal regions where you would usually expect to find angular momentum sources and sinks at the surface for surface drag. [12 marks]

(c) Show that the poleward transport of a quantity $H$ by stationary eddies can be written in terms of its zonal-mean and zonal-anomaly components as follows:

$$[vH] = [v][H] + [v^*H^*]$$ [6 marks]
(d) The flux form of the zonal momentum equation may be written:

\[
\frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} + \frac{\partial vu}{\partial y} + \frac{\partial wu}{\partial z} = -F + f v - \frac{\partial p}{\partial x}
\]

Where F is a drag and all other terms have their usual meanings.

Consider now a steady flow in a zonally-periodic channel with no orography (i.e., the lower boundary of the atmosphere is completely flat: there are no mountains). By taking zonal and vertical averages (denoted by \([\cdot]\) and \(\langle\cdot\rangle\) respectively), show that this equation can be used to create a simple model of the Eddy-driven jet whereby the surface drag is balanced an eddy momentum flux convergence into the column. That is:

\[
\frac{\partial}{\partial y} \langle [v^* u^*] \rangle = -\langle [F] \rangle
\]

At each step, you should provide a brief physical interpretation of your work.

[12 marks]

(e) Use the simplified zonal-mean column-averaged form of the zonal-momentum equation given in part (d) to estimate the magnitude of the eddy momentum fluxes converging into the atmospheric column at 45°N. Give the units of your answer.

You will need to estimate the steady zonal-mean column-average midlatitude tropospheric wind-speeds and the meridional length scale over which the momentum flux is changing. You should briefly explain your choices for these values.

You may assume that \(\langle [F] \rangle\) takes the form of a bulk drag:

\[
\langle [F] \rangle = \rho C_D |\langle [u] \rangle| |\langle [u] \rangle|
\]

where \(C_D\) is \(10^{-3}\).
(End of Question Paper)