

# A method for estimating the turbulent kinetic energy dissipation rate from a vertically-pointing Doppler lidar, and independent evaluation from balloon-borne in-situ measurements

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## ABSTRACT

A method of estimating dissipation rates from a vertically-pointing Doppler lidar with high temporal and spatial resolution has been evaluated by comparison with independent measurements derived from a balloon-borne sonic anemometer. This method utilises the variance of the mean Doppler velocity from a number of sequential samples and requires an estimate of the horizontal wind speed. The noise contribution to the variance can be estimated from the observed signal-to-noise-ratio and removed where appropriate. The relative size of the noise variance to the observed variance provides a measure of the confidence in the retrieval. Comparison with in-situ dissipation rates derived from the balloon-borne sonic anemometer reveal that this particular Doppler lidar is capable of retrieving dissipation rates over a range of at least three orders of magnitude.

This method is most suitable for retrieval of dissipation rates within the convective well-mixed boundary-layer where the scales of motion that the Doppler lidar probes remain well within the inertial sub-range. Caution must be applied when estimating dissipation rates in more quiescent conditions. For the particular Doppler lidar described here, the selection of suitably short integration times will permit this method to be applicable in such situations but at the expense of accuracy in the Doppler velocity estimates. The two case studies presented here suggest that, with profiles every four seconds, reliable estimates of  $\epsilon$  can be derived to within at least an order of magnitude throughout almost all of the lowest 2 km and, in the convective boundary layer, to within 50%. Increasing the integration time for individual profiles to 30 s can improve the accuracy substantially but potentially confines retrievals to within the convective boundary layer. Therefore, optimisation of certain instrument parameters may be required for specific implementations.

## 1. Introduction

Turbulent properties of the boundary layer can be measured by aircraft (e.g. Fairall *et al.*, 1980) and vertical profiles of the turbulent kinetic energy dissipation rate have been obtained from balloon-borne turbulence probes in the convective boundary layer (Caughey and Palmer, 1979), nocturnal boundary layer (Caughey *et al.*, 1979), and cloudy boundary layers (Hignett, 1991; Siebert *et al.*, 2003). Such in-situ observations, however, are necessarily restricted both spatially and temporally.

Applications such as investigating the role of turbulence in new aerosol particle formation (Wehner *et al.*, 2010), or cloud microphysics (Pinsky *et al.*, 2008), place the emphasis on high resolution, especially in the vertical, with a measurement accuracy of within an order of magnitude probably sufficient for these purposes. Long time-series are necessary for evaluating and improving turbulence schemes in

Numerical Weather Prediction models, and the preference here is for robust statistics with low bias.

An active remote sensing approach is required to achieve routine, continuous coverage with simultaneous measurement at all altitudes across a significant part of the lower atmosphere, including the full depth of the boundary layer. Doppler radars and lidars can provide the necessary high resolution velocities and there are a number of methods currently available for estimating the dissipation rate. These broadly fall into 3 categories; Doppler spectral width, temporal spectra or structure function methods, and conical scanning. An example of a method from each of these categories, as applied to Doppler lidar, is given by Banakh *et al.* (1999). The methods in these categories may be applicable to both Doppler radar (Brewster and Zrnić, 1986; Cohn, 1995; Doviak and Zrnić, 1993; Chapman and Browning, 2001) and Doppler lidar (Gal-Chen *et al.*, 1992; Banakh and Smalikho, 1997; Davies *et al.*, 2004). Other possibilities include dual-Doppler lidar (Davies *et al.*, 2005) and radars sensitive to clear-air echoes, which can use the returns arising from turbulent mixing across atmospheric re-

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fractive index gradients to estimate dissipation rates (Cohn, 1995).

Evaluation of the various methods for estimating dissipation rate from a Doppler lidar or radar is usually performed by comparison with ground-based or tower-based sonic anemometers and sodars (Drobinski *et al.*, 2004). Comparisons with other instruments have also been carried out, such as a lightweight three-dimensional magnetometer carried on a radiosonde (Harrison *et al.*, 2009).

The method of estimating the dissipation rate,  $\epsilon$ , from the Doppler spectral width assumes that turbulence is entirely responsible for the spectral broadening; in practice there are additional sources of spectral broadening that must be accounted for (Doviak and Zrnić, 1993), such as wind-shear. Not all Doppler lidars provide the full Doppler spectrum, so in this paper we utilise a method which requires only the mean Doppler velocity. We make the assumption that the variations of the mean Doppler velocity over a short sampling time are entirely due to turbulence. Using the variance of a number of samples of the mean Doppler velocity sidesteps most of the issues involved in correcting the various sources of additional spectral broadening associated with the Doppler spectral width method, but, since a longer integration time is required, care must be taken that the scales of turbulent motion now encompassed still remain within the inertial sub-range (Frehlich and Cornman, 2002).

In this paper we outline a simple method for estimating dissipation rate from unattended continuously operating Doppler lidars. In section 2 we present the method for estimating dissipation rate from the variance of the mean Doppler velocity, with corrections for the expected uncertainty in the observed Doppler velocities for a heterodyne system. An error analysis is given in section 3 and validation of the method by comparison with balloon-borne in-situ data is presented in section 4.

## 2. Estimating dissipation rate

### a. Standard method from velocity power spectra

Standard methods for estimating dissipation rate from high-frequency measurements of turbulent velocities typically involve the transformation of the velocity spectra into the frequency domain (e.g. by the use of Fast Fourier Transforms, FFTs). In theory, with sufficient resolution, these vertical velocity spectra are presumed to have a form similar to that shown in Fig. 1, when plotted versus frequency. Production of turbulent kinetic energy, TKE, is dominated by large eddies (length scales of a hundred meters or more), which then decay into smaller and smaller eddies (the inertial sub-range) until the length scales

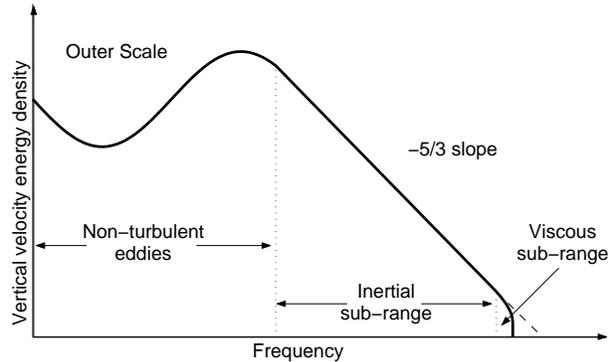


FIG. 1: Schematic of vertical velocity energy density spectra versus frequency conforming to Kolmogorov's hypothesis.

are small enough for the kinetic energy to be dissipated into heat by molecular diffusion in the viscous sub-range (scales on the order of centimeters or less).

In the case of homogeneous and isotropic turbulence, the Kolmogorov hypothesis (1941) states that within the inertial sub-range the statistical representation of the turbulent energy spectrum  $S(k)$  is given by

$$S(k) = a\epsilon^{2/3}k^{-5/3}, \quad (1)$$

where  $a = 0.55$  is the Kolmogorov constant for 1-dimensional wind spectra (Paquin and Pond, 1971),  $\epsilon$  is the dissipation rate and  $k$  is the wavenumber, which can be related to a length scale,  $L$  ( $k = 2\pi/L$ ) by invoking Taylor's hypothesis of frozen turbulence (Taylor, 1935). If observed spectra fit the form shown in figure 1, a  $-5/3$  power-law can be fitted to the portion of the spectrum that lies within the inertial sub-range and thus,  $\epsilon$  can be estimated (e.g. Lothon *et al.*, 2009).

### b. Variance of mean Doppler velocity

We now introduce a new parameter  $\sigma_v^2$ , which is the variance of the observed mean Doppler velocity over a defined number of sequential samples,  $N$  (O'Connor *et al.*, 2005). Initially, we consider the case where the observed variance is dominated by the turbulent processes in the vertical and there are no significant contributions from other sources. The velocity variance is then equivalent (Bouniol *et al.*, 2003) to integrating (1) so that

$$\sigma_v^2 = \int_k^{k_1} S(k) dk, \quad (2)$$

$$= -\frac{3}{2}a\epsilon^{2/3} (k_1^{-2/3} - k^{-2/3}), \quad (3)$$

$$= \frac{3a}{2} \left(\frac{\epsilon}{2\pi}\right)^{2/3} (L^{2/3} - L_1^{2/3}), \quad (4)$$

where the wavenumber  $k_1 = 2\pi/L_1$  corresponds to the length scale describing the scattering volume di-

mension for the dwell time of the lidar for a single sample, and  $k = 2\pi/L$  relates to the length scale of the large eddies travelling through the lidar beam during the  $N$  sampling intervals.

The length scale for an individual sample is given by

$$L_1 = Ut + 2z \sin\left(\frac{\theta}{2}\right), \quad (5)$$

where  $\theta$  is the half-angle divergence of the lidar beam,  $U$  is the horizontal wind,  $t$  is the dwell time and  $z$  is the height in m. Usually, the second term in (5) is negligible as Doppler lidars typically have a very small divergence,  $< 0.1$  mrad. Over  $N$  sampling intervals, the length scale is  $L = NUt$ . If the lidar instrument is set to acquire one profile of velocity measurements every 4 seconds, the length scales for a typical wind speed of  $U = 10 \text{ m s}^{-1}$  in the boundary layer are  $L_1 = 40 \text{ m}$  and, if 10 samples are used to calculate  $\sigma_{\bar{v}}^2$ ,  $L = 400 \text{ m}$ . The length scales for horizontal wind speeds as low as  $0.25 \text{ m s}^{-1}$ , with an integration time of 4 seconds or greater, should still be much larger than the expected cut-off in the viscous sub-range. Assuming both length scales lie within the inertial sub-range, we can now write

$$\epsilon = 2\pi \left(\frac{2}{3a}\right)^{3/2} \sigma_{\bar{v}}^3 (L^{2/3} - L_1^{2/3})^{-3/2} \quad (6)$$

and hence estimate  $\epsilon$  directly from  $\sigma_{\bar{v}}^2$  without the need to calculate FFTs. It should be noted, however, that it is not as easy to determine if the length scales are appropriate in the absence of spectra.

### c. Noise contribution to variance

So far we have assumed that turbulence is the only source of variance. We now consider the influence of noise on the Doppler velocity measurement. The error in an individual Doppler lidar velocity estimate is dependent on the signal to noise ratio (SNR), of the measurement. For a heterodyne Doppler lidar, Pearson *et al.* (2009) have shown that when many pulses have been averaged, the theoretical standard deviation of the Doppler velocity estimate,  $\sigma_e$ , for weak signals can be reliably approximated by (Rye and Hardesty, 1993)

$$\sigma_e^2 = \frac{\Delta v^2 \sqrt{8}}{\alpha N_p} \left(1 + \frac{\alpha}{\sqrt{2\pi}}\right)^2, \quad (7)$$

$$\alpha = \frac{\text{SNR } B}{\sqrt{2\pi} \Delta v}, \quad (8)$$

where  $\alpha$  is the ratio of the lidar detector photon count to the speckle count (Rye, 1979),  $\Delta v$  is the signal spectral width and  $B$  is the receiver bandwidth (both expressed here in  $\text{m s}^{-1}$  so that  $B$  corresponds

TABLE 1: Doppler lidar specifications. The instrument deployed during the 2nd REPARTEE experiment in London is essentially the same as the one deployed at Chilbolton, but with certain parameters adjusted to maximise the measurement capabilities within the boundary layer. Where particular parameters differ, the REPARTEE instrument parameters are given in brackets. Both instruments were built by Halo Photonics.

Wavelength	1.5 $\mu\text{m}$
Pulse repetition rate	15 kHz (20 kHz)
Nyquist velocity	10 $\text{m s}^{-1}$ (14 $\text{m s}^{-1}$ )
Sampling frequency	50 MHz (30 MHz)
Points per range gate	12 (6)
Pulses averaged	20000
Raw profiles averaged	5 (1)
Range resolution	36 m (30 m)
Integration time	30 s (4 s)
Pulse duration	0.2 $\mu\text{s}$
Lens diameter	6 cm (8 cm)
Divergence	33 $\mu\text{rad}$
Focus	$\infty$ (801 m)
Telescope	monostatic optic-fibre coupled

to twice the Nyquist velocity), and  $N_p$  is the accumulated photon count. Both  $N_p$  and  $\alpha$  are determined from the instrument characteristics and the wideband SNR of the target return for a single point sample:

$$N_p = \text{SNR } n M, \quad (9)$$

where  $n$  is the number of pulses averaged per profile and  $M$  is the number of points sampled within a specified range gate to obtain a raw velocity. The term wideband SNR refers to the ratio of the average total signal power to the average noise power over the full bandwidth. Note that, due to oversampling and subsequent averaging, the final range gate length does not necessarily coincide with the pulse length.

For a direct detection system, the theoretical minimum for the standard deviation of the Doppler velocity estimate is given by  $\sigma_e = \Delta v / (N_p^{0.5})$ , although in practice there are additional factors to consider (McKay, 1998).

Data from two coherent heterodyne Doppler lidars are presented here. The instruments are very similar in design but have had certain parameters optimised for different objectives. The instrument at Chilbolton has a longer integration time to improve sensitivity as it is configured for a primary function of observing liquid and ice cloud at all heights up to 10 km. The instrument deployed for the 2nd REPARTEE campaign in central London (Martin *et al.*, 2009) is optimised for boundary-layer studies and has achieved the required sensitivity with a

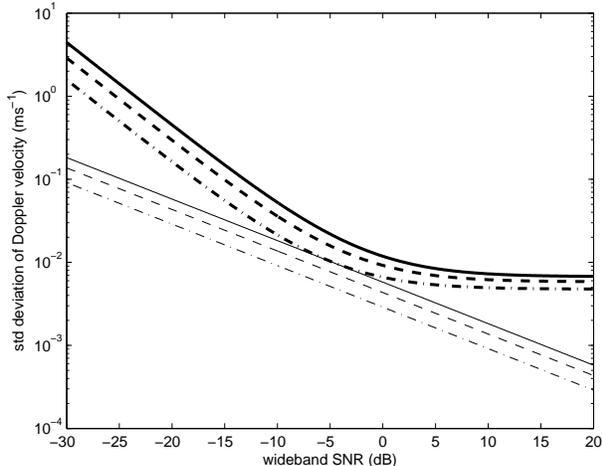


FIG. 2: Theoretical standard deviation of Doppler velocity estimate for the REPARTEE heterodyne Doppler lidar (thick lines) for three signal spectral widths, equivalent to 1 (dot-dashed), 1.5 (dashed) and 2 (solid)  $\text{m s}^{-1}$ , computed using Eqs. (7) and (8). Also shown are the theoretical standard deviation of Doppler velocity estimates for direct detection lidar systems (thin lines). The wideband SNR refers to the ratio of the average total signal power to the average noise power over the full bandwidth for the target return from a single pulse.

shorter integration time by having the telescope focus set to approximately 800 m (note that this reduces the instrument sensitivity dramatically above 2 km). The specifications of the two Doppler lidar instruments are given in Table 1. The pulse length for both instruments is the same, 30 m, but the signals are oversampled (by a factor of 10 for the instrument at Chilbolton and by 6 for the REPARTEE instrument). These high-resolution samples, or points, are then averaged up to yield the raw velocity estimates at the selected range gate length; the number averaged is given by the number of points per range gate parameter in Table 1. The acquisition time for a single profile obtained from 20,000 pulses is dependent on the pulse repetition rate of the instrument and additional time is then required for real-time computation of the velocities; Chilbolton requires approximately 1.33 s for acquisition and 4 s for computation per profile, REPARTEE requires 1 s for acquisition plus 3 s for computation per profile. The Chilbolton instrument then performs additional averaging of 5 profiles to give a total integration time of about 30 s.

We first consider the REPARTEE instrument as it is potentially more suitable for estimating dissipation rate because of its much shorter integration time. The theoretical standard deviation of the Doppler velocity estimate as a function of wideband SNR is given in figure 2 for the REPARTEE instrument. It is immediately obvious that the relationship between  $\sigma_e$  and wideband SNR for a heterodyne system is not

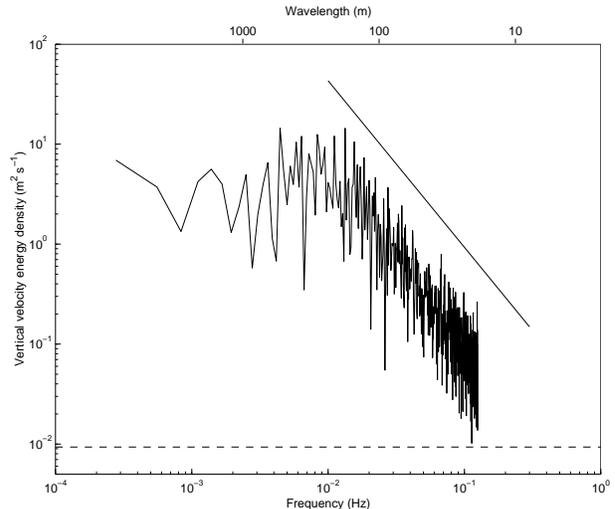


FIG. 3: Power spectra of 60 minutes of data with a time resolution 4 s, centred on 0800 UTC, 29 October 2007, at a height of 135 m (average taken from 3 adjacent range gates at 105-165 m). Also shown is the expected noise contribution (dashed line) calculated from the mean SNR of  $-7$  dB, and a reference  $-5/3$  power law (solid line). Data is from the REPARTEE instrument.

the same as that for a direct detection system. In fact, at  $-25$  dB,  $\sigma_e$  is an order of magnitude higher. It is also apparent that, for a heterodyne system, once the wideband SNR has reached 0 dB, increasing the SNR further does not greatly improve  $\sigma_e$  (see Rye and Hardesty (1993) for a comprehensive explanation). The choice of signal spectral width,  $\Delta v$ , also has some influence on estimating  $\sigma_e$ . Pearson *et al.* (2009) suggested a value of  $1.5 \text{ m s}^{-1}$  for  $\Delta v$ , our results indicate that  $2 \text{ m s}^{-1}$  is more suitable and we select this as a typical value for the rest of the paper.

We first investigate observed vertical-velocity energy density spectra to confirm that they have the same shape as the idealised form given in Fig. 1, to examine the noise contribution, and to note whether the spectra contain a sufficient portion within the inertial sub-range for (6) to be valid. Data from the REPARTEE instrument has been selected because the shorter integration time allows the spectra to encompass smaller scales. Figures 3 - 5 display vertical-velocity energy density spectra at three different heights (135, 825 and 1275 m) calculated from 60 minutes of data over 3 adjacent gates (approximately 2700 individual velocity estimates). The targets in all cases are aerosol particles in the boundary layer. If velocity measurements and their random estimation error are uncorrelated, then Frehlich (2001) states that the temporally uncorrelated estimation error will appear as a constant-amplitude high-frequency region in the velocity spectra. The noise contribution is computed as the ensemble mean,

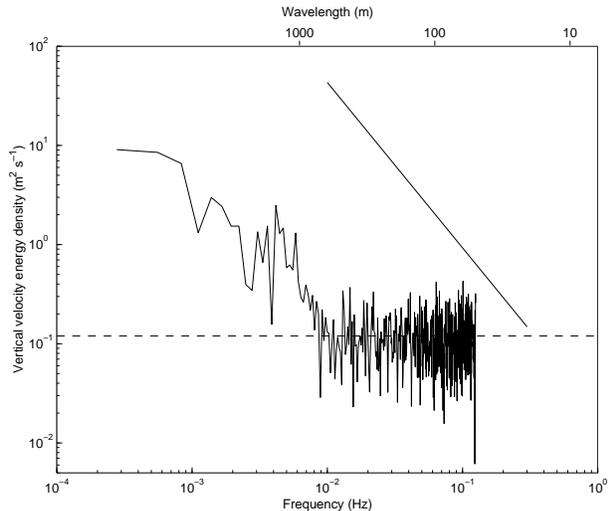


FIG. 4: Same as Figure 3 except at a height of 825 m. The mean SNR is  $-14$  dB.

$\langle \sigma_e \rangle$ , of the individual estimates of  $\sigma_e$  calculated for each individual measurement using (7). In terms of vertical-velocity energy density spectra, the levels of the theoretical noise contribution  $\langle \sigma_e \rangle^2$  are then scaled by dividing by the frequency span of the spectrum (approximately 0.125 Hz) to obtain the noise variance displayed as dashed lines in figures 3 - 5.

The vertical-velocity energy density spectrum in figure 3 was obtained from data with a mean SNR close to  $-7$  dB and there is no indication of noise at the high end of the frequency spectrum, which is consistent with a very low theoretical noise level of  $0.009 \text{ m}^2 \text{ s}^{-1}$ . This spectrum has the same form as the idealised version given in Fig. 1 and a  $-5/3$  power-law can be fitted to the high frequency end of the spectrum (from approximately  $1 \times 10^{-2}$  Hz to the Nyquist frequency, 0.125 Hz). The horizontal wind speeds given by the Doppler lidar in scanning mode, and the Met Office North Atlantic and European (NAE) operational numerical weather prediction model, were both close to  $2 \text{ m s}^{-1}$  at 135 m and 0800 UTC. The frequency range therefore corresponds to a spatial range of approximately 200 - 16 m and implies that the transition from the inertial sub-range to the outer scale for non-turbulent eddies occurs at length scales of 200 m. From a  $-5/3$  power-law fit to this frequency range we calculate  $\epsilon = 3 \times 10^{-3} \text{ m}^2 \text{ s}^{-3}$ .

The energy density spectrum in figure 4 was obtained from data with a mean SNR of  $-14$  dB, for which the theoretical noise level is  $0.12 \text{ m}^2 \text{ s}^{-1}$ . This agrees very well with the observed spectrum, which abruptly flattens out above  $10^{-2}$  Hz, and is consistent with the explanation given by Frehlich (2001). A  $-5/3$  power-law can still be fitted to the portion

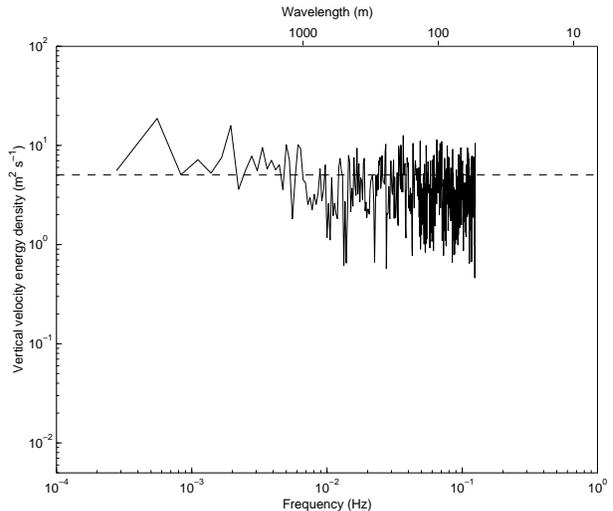


FIG. 5: Same as Figure 3 except at a height of 1275 m. The mean SNR is  $-20$  dB.

of the spectrum from  $4 \times 10^{-3}$  to  $1 \times 10^{-2}$  Hz, which, since the horizontal wind speed at 825 m is about  $6 \text{ m s}^{-1}$ , corresponds to a spatial range of approximately 1500 - 600 m. The transition from the inertial sub-range to outer-scale at this height is at length scales of about 1500 m and, from the  $-5/3$  power-law fit,  $\epsilon = 2 \times 10^{-6} \text{ m}^2 \text{ s}^{-3}$ .

In figure 5, the energy density spectrum was derived from data with a mean SNR of about  $-20$  dB. The noise dominates the spectrum in this case, and is consistent with a theoretical noise level of  $5 \text{ m}^2 \text{ s}^{-1}$ . Again, the observed spectrum has a constant amplitude characteristic of temporally uncorrelated estimation noise. It is not reasonable to attempt to fit a  $-5/3$  power-law to any portion of this particular spectrum.

If it is assumed that the sources of variance have a Gaussian distribution and are independent of one another, the observed variance,  $\sigma_v^2$ , is the sum of the variances from each source (Doviak and Zrnić, 1993; Frehlich *et al.*, 1998) such that

$$\sigma_v^2 = \sigma_w^2 + \sigma_e^2 + \sigma_d^2, \quad (10)$$

where  $\sigma_w^2$  is the contribution from air turbulence that we are interested in, the contribution from noise is  $\sigma_e^2 \approx \langle \sigma_e \rangle^2$ , and  $\sigma_d^2$  is the contribution from the variation in still-air terminal fall speeds of particulates within the measurement volume from one sample to the next. Since aerosol particles and liquid cloud droplets have terminal fall speeds  $< 1 \text{ cm s}^{-1}$ , the variance  $\sigma_d^2$  can be safely ignored for returns from these targets. Figures 3 - 5 show that the variance arising from the uncertainty in the Doppler velocity measurements can be estimated reliably and that it is valid to assume that the two sources of vari-

ance, turbulence and estimator noise, are independent. Thus, given an observed total variance and a calculated noise variance,  $\epsilon$  can be derived using (6) by replacing the theoretical  $\sigma_v^2$  with  $\sigma_w^2 = \sigma_v^2 - \sigma_e^2$ . Targets such as rain or ice particles will have significant terminal fall speeds; therefore it may be necessary to quantify  $\sigma_d^2$  when attempting to calculate  $\epsilon$  in such situations.

### 3. Error in derived dissipation rates

To estimate the error in  $\epsilon$  we first assume that  $L_1 \ll L$  in (6), so that  $\epsilon \propto \sigma_w^3/L$ , and through the propagation of errors, the fractional error in  $\epsilon$  is

$$\frac{\Delta\epsilon}{\epsilon} = \frac{3\Delta\sigma_w}{\sigma_w} + \frac{\Delta L}{L}. \quad (11)$$

Radiosonde, tower measurements or wind profilers can be used to estimate the horizontal length scale, as can the output from an operational forecast model. In this study, we use model winds from an operational forecast model to derive  $L$  and, since the horizontal winds from the Met Office mesoscale model are generally accurate to 1-2 m s<sup>-1</sup> (Panagi *et al.*, 2001), we estimate the fractional error in  $L$  to be about 10% for a typical horizontal wind speed of 10 m s<sup>-1</sup>. The extremely small divergence of the Doppler lidar in this study (0.033 mrad) means that the second term in (5) can be ignored even at very low  $U$  and short observation times.

Following Lenschow *et al.* (2000) we can estimate the measurement error in a variance as follows,

$$\Delta\sigma_w^2 \approx \sigma_w^2 \sqrt{\frac{4}{N} \frac{\sigma_e^2}{\sigma_w^2}}, \quad (12)$$

and therefore provide the fractional error for each individual estimate of  $\epsilon$ . It should be noted that (12) assumes that each velocity sample used to calculate the variance has a similar error to the ensemble mean error,  $\sigma_e \approx \langle \sigma_e \rangle$ . This assumption can be tested and those variances for which this is no longer approximately true should be flagged as unreliable.

Estimates of  $\epsilon$  derived from the REPARTEE Doppler lidar data are shown in Fig. 6, along with the derived  $\langle \sigma_e \rangle$  and fractional error in dissipation rate, for the same day as in Figs. 3 - 5. The REPARTEE instrument has a maximum range of about 2 km and, as Fig. 6a indicates, is sufficiently sensitive to detect aerosol (or clouds) at almost all ranges, potentially providing an estimate of dissipation rate throughout most of the lower atmosphere.

The convective boundary layer is clearly visible in Fig. 6c, by noting where  $\epsilon$  is high, in this case  $> 10^{-4}$  m<sup>2</sup> s<sup>-3</sup>. From midnight (local time is UTC) to 0730 UTC, the well-mixed layer reaches from the

surface to about 250 m; it then begins to grow from 0730 UTC and the convective boundary layer top reaches 1.5 km at 1400 UTC. Maximum values of  $\epsilon$  approach  $5 \times 10^{-3}$  m<sup>2</sup> s<sup>-3</sup>. The convective boundary layer then decays after 1600 UTC and returns to a shallow well-mixed layer that again reaches from the surface to about 250 m.

Within the convective boundary layer, the limiting factor in providing accurate estimates of  $\epsilon$  is the uncertainty in the horizontal winds used to estimate the length scales. The wind speed was 2 m s<sup>-1</sup> in Fig. 3 and 6 m s<sup>-1</sup> in Fig. 4; in the absence of lidar-derived winds, the resulting relative errors in  $L$  should be increased from 10% to 75 and 25%, respectively. Values of the ensemble mean of the theoretical error in the observed velocities are  $< 0.1$  m s<sup>-1</sup> throughout much of the lowest 1 km (Fig. 6b). However, the resulting error variance,  $\sigma_e^2$ , although small, may still be a substantial fraction of the observed  $\sigma_w^2$ . As suggested by the expression (12), this can be mitigated by increasing the number of samples used in calculating  $\epsilon$  until the fractional error,  $\Delta\sigma_w/\sigma_w$  is reduced to an acceptable level. The number of samples used in calculating Fig. 6 was 45 (equivalent to 3 minutes), which resulted in a fractional error in  $\sigma_w$  of about 10% within the convective boundary layer.

Outside the convective boundary layer, uncertainty in the velocity variance estimates is much more likely to be the dominant source of error; a smaller velocity variance due to less turbulent conditions is compounded by more uncertainty in the variance estimate due to low SNR. Figures 6b and 6d corroborate this. Within the convective boundary layer, where  $\epsilon$  is high, the fractional error in  $\epsilon$  is estimated to be as low as 30%. For quiescent conditions in early morning or late evening with similar mean errors in velocity, the derived fractional error in  $\epsilon$  is at least 100%. Fractional errors in  $\sigma_w > 100\%$  indicate where noise is the dominant source of variance; from (11) this translates to fractional errors in  $\epsilon$  of over 300% and provides a quality flag for identifying unreliable  $\epsilon$  estimates. Such values are found, for example, for locations within the time-height period (0730-0830 UTC and 1230-1320 m) used for generating Fig. 5, whose noisy spectrum does not show any sign of an inertial sub-range.

Thus, by limiting retrievals to estimates with fractional errors in  $\epsilon < 300\%$ , reliable estimates of  $\epsilon$  can be derived to within at least an order of magnitude throughout almost all of the lowest 2 km and, in the convective boundary layer, to within 50% or better for this instrument.

In providing these error estimates, we have also implicitly assumed that the sampling time is sufficiently short to ensure that we remain in the inertial

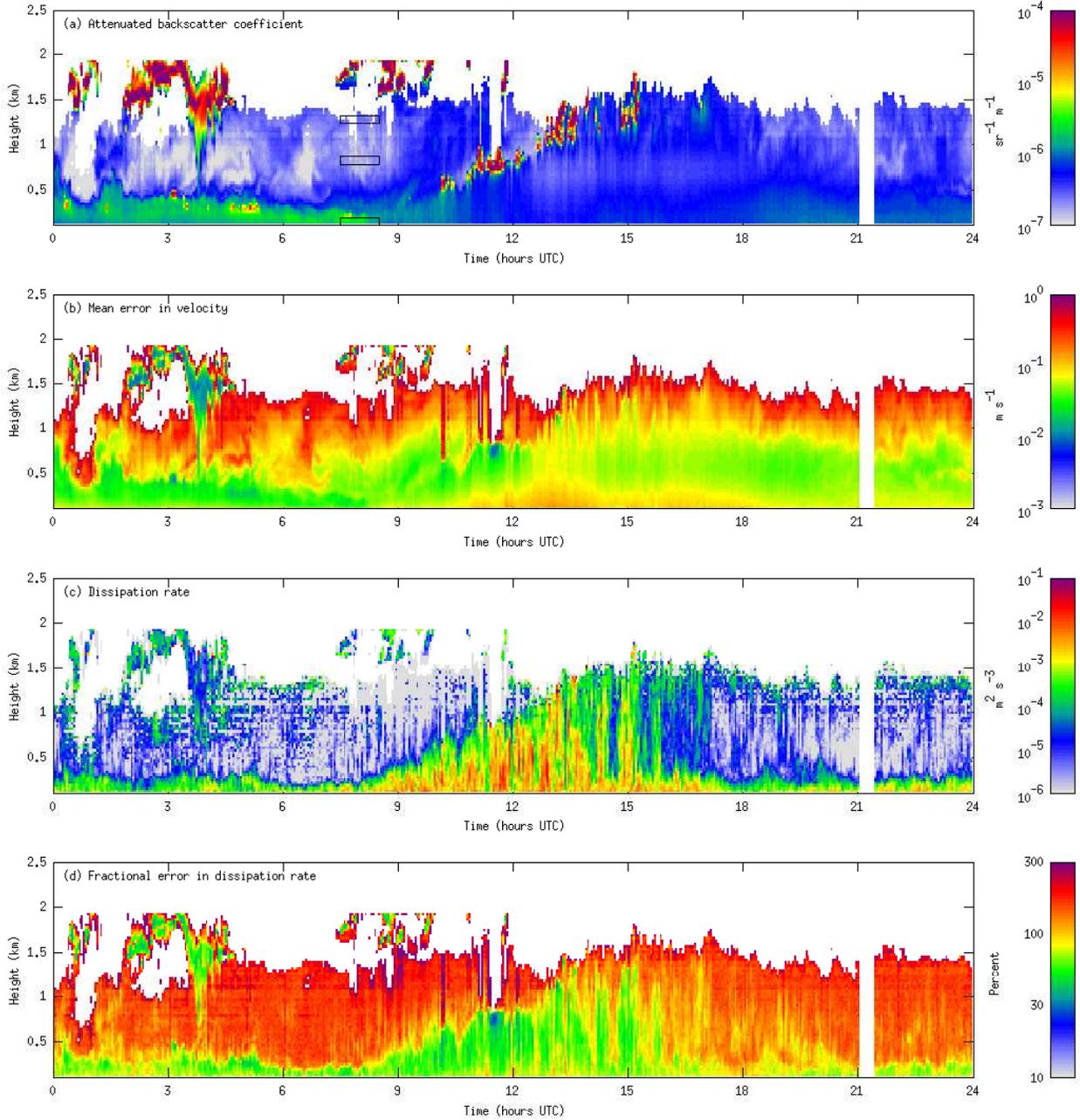


FIG. 6: Top panel (a); attenuated backscatter coefficient from the Doppler lidar during the REPAREE campaign in London on 29 October 2007. The three boxes centred on 0800 UTC indicate the locations for the data presented in Figs. 3-5. Panel (b) displays the ensemble mean of the theoretical uncertainty in observed velocities calculated using Eq. 6 for each sample used to derive (c) the dissipation rate. The estimated fractional error in dissipation rate, calculated using Eq. 11, is given in panel (d).

sub-range, where (1) applies. If the sampling time is too long then the observed velocity variance will include contributions from the outer scales of turbulence, where (1) no longer applies, and  $\epsilon$  will be underestimated. For this case, the horizontal winds used to derive  $\epsilon$  were taken from the Met Office NAE model and ranged from 2-6 m s<sup>-1</sup>. With  $N = 45$

consecutive 4-second samples used to calculate  $\sigma_v^2$ , this corresponds to length scales for  $L$  of 360-1080 m, which, according to Figs. 3 and 4, suggests that the assumption is reasonable at ranges near to the surface (although possibly not when very close to the surface) and is valid at greater ranges. The length scales for  $L_1$ , 36-108 m, are substantially greater than

the transition from the inertial sub-range to the viscous sub-range. At high levels of turbulence within the well-mixed boundary-layer the sampling time of this Doppler lidar is sufficiently fast to acquire enough samples while remaining in the inertial sub-range, but this may no longer be true in very quiescent conditions above the well-mixed boundary-layer. In these cases the value of  $\epsilon$  can be severely underestimated.

#### 4. Balloon-borne in-situ evaluation

We now present estimates of the dissipation rate for data taken from the Doppler lidar at Chilbolton on 22 April 2008. Appropriate parameters for this day are given in Fig. 7. Low cloud or fog at 300 m in height is present from 0400 to 0800 UTC and completely attenuates the lidar signal (Fig. 7a) but otherwise there is potential coverage throughout most of the lowest 2 km. A convective boundary layer is again evident in Fig. 7c, from about 1000 to 1600 UTC, which grows to over 1.5 km with  $\epsilon$  values reaching  $5 \times 10^{-2} \text{ m}^2 \text{ s}^{-3}$ . Similar to the REPARTEE case, the horizontal winds taken from the Met Office NAE model again ranged from  $2\text{-}6 \text{ m s}^{-1}$ , but, since the Chilbolton instrument has a longer integration time to improve the sensitivity, while  $N = 10$  consecutive 30-second samples were used to calculate  $\sigma_v^2$ , the length scales for L are larger, at 600-1800 m. The advantage of this longer integration time is clearly visible in Fig. 7b, where the increase in accumulated photocount,  $N_p$ , leads to lower theoretical errors in velocity for a similar SNR.

Since these length scales may now incorporate unwanted contributions from the outer scale as well as the inertial sub-range, especially when close to the surface, we also performed calculations using  $N = 12$  samples taken from 4 consecutive rays in time and from 3 adjacent gates in height. The length scales for L using this approach are potentially more applicable, at 240-720 m, although again probably too large close to the surface. The absolute values of  $\epsilon$  are not exactly the same as those in Fig. 7c but the pertinent features of such a figure are very similar and so not included here. Within the convective boundary layer, the histogram of  $\epsilon$  has approximately the same shape and mean, indicating that the length scales for both sampling regimes do remain within the inertial sub-range. Examining Fig. 1 reminds us that we would expect  $\epsilon$  estimates in the sampling regime with length scales remaining within the inertial sub-range to be larger than those with length scales extending into the outer range. This explains why values of  $\epsilon$  outside of the convective boundary layer are often larger than those in Fig. 7c, sometimes by as much as factor of 5.

During April 2008 the University of Leeds UFAM

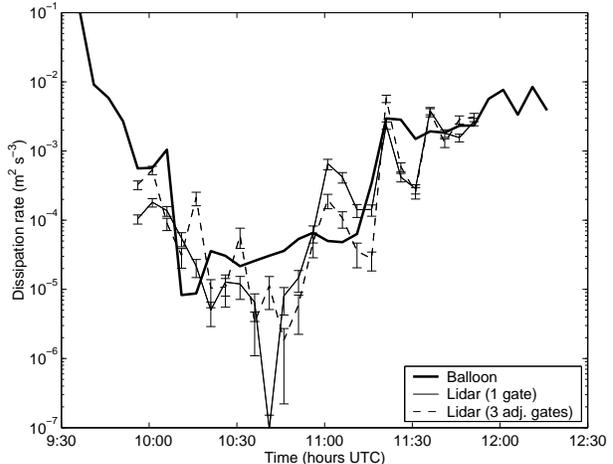


FIG. 8: Observed rate of dissipation of TKE,  $\epsilon$ , in the boundary layer from the lidar and in-situ balloon measurements between 100 and 600 m on 22 April 2008.

SkyDoc balloon was flown at Chilbolton in close proximity to the Doppler lidar carrying a turbulence sonde and mean meteorology instrumentation. The turbulence sonde is a prototype instrument incorporating the sensing head and control electronics of a Gill Windmaster 3-axis sonic anemometer in a compact aerofoil housing. The sonic anemometer measures the 3 components of the turbulent wind at 40 Hz and internally averages the measurements to 10 Hz before outputting the data via a serial interface. The data stream was logged via a compact embedded linux computer installed in the housing along with a 12V battery pack. A separate enclosure housed a mean meteorology package to measure air pressure, mean temperature and relative humidity, and mean wind speed; a compact aerosol probe (CLASP, Hill et al. 2008) was also included to measure aerosol size spectra for a related study. These instruments were suspended approximately 20 m below the balloon, which is about 3.5 m in diameter when inflated.

For calculation of the dissipation rate we select the portion of the power spectrum at frequencies greater than 2 Hz. This limit is chosen to avoid that part of the spectra contaminated by the motion of the tether-sonde; this spans a frequency range of approximately 0.08-0.2 Hz. It should be noted that, for a  $4 \text{ m s}^{-1}$  horizontal wind speed, frequencies from 2 to 10 Hz correspond to length scales from 2 m down to 40 cm, which, although above the viscous sub-range, are significantly smaller than the length scales probed by the lidar.

The flight track of the balloon is superimposed on the plots in Fig. 7 and the weight of the turbulence sonde limited the maximum height to about 600 m. Lidar estimates of  $\epsilon$  closest to the location of the bal-

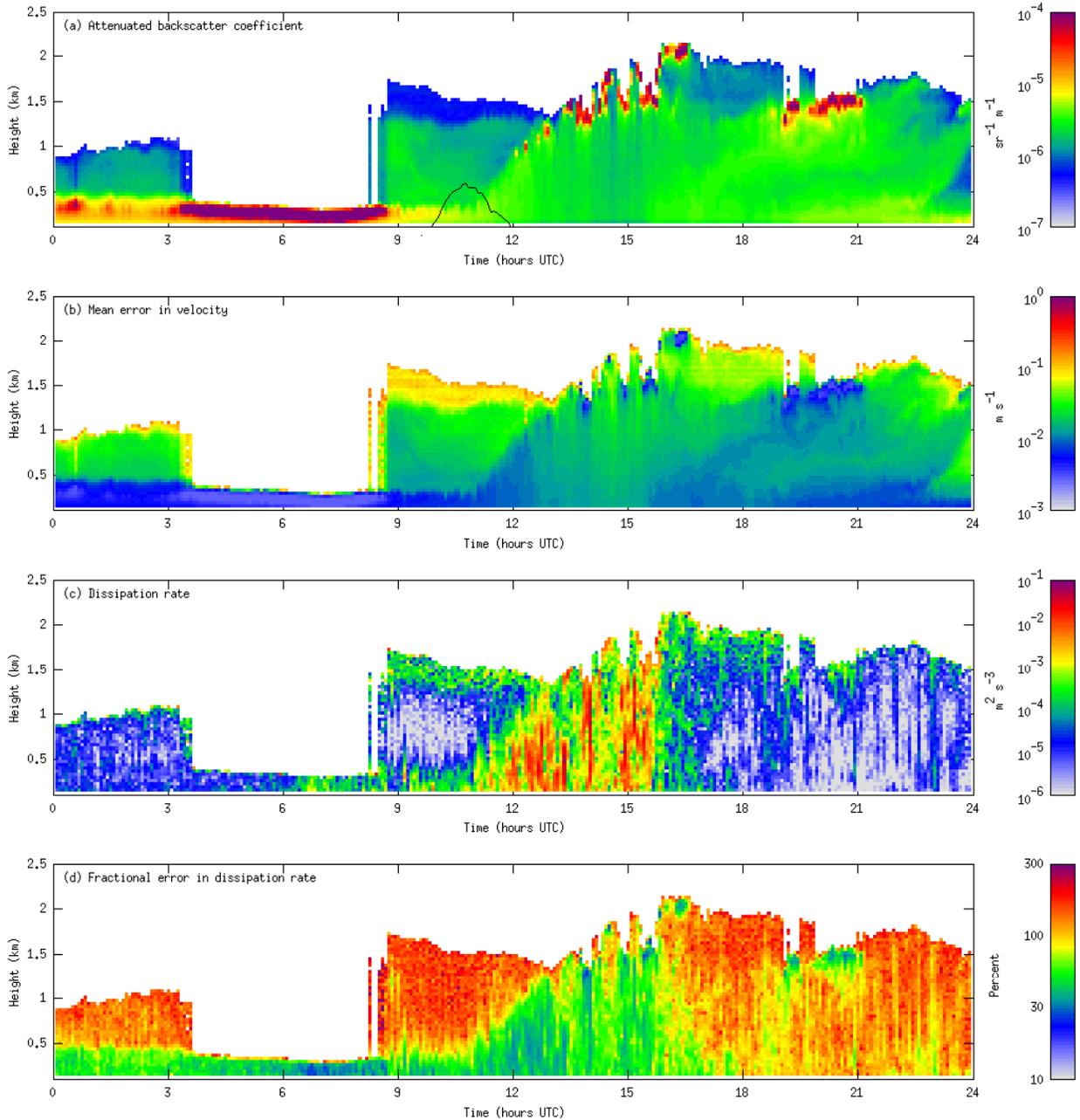


FIG. 7: Top panel (a); attenuated backscatter coefficient from the Doppler lidar at Chilbolton for 22 April 2008. Panel (b) displays the ensemble mean of the theoretical uncertainty in observed velocities calculated using Eq. 11 for each sample used to derive (c) the dissipation rate. The estimated fractional error in dissipation rate, calculated using Eq. 11, is given in panel (d). The black line in each panel (from about 1000 to 1200 UTC) denotes the balloon flight track.

loon in height and time were selected for comparison. The balloon is obviously not co-located exactly with the lidar beam, and depending on wind conditions, may be as much as 400 m away in the horizontal. We consciously used model forecast horizontal winds to estimate the Doppler lidar  $\epsilon$  values, whereas the balloon values were calculated from in-situ measurements of the horizontal wind, so that we could ex-

amine how the lidar technique would perform in an operational context.

A comparison of the in-situ measurements of  $\epsilon$  with those inferred from the Doppler lidar shows good general agreement both in time, Fig. 8, and in height, Fig. 9. The balloon observes a significant decrease in  $\epsilon$  with height of over 3 orders of magnitude; the lidar method captures this decrease, and is also able

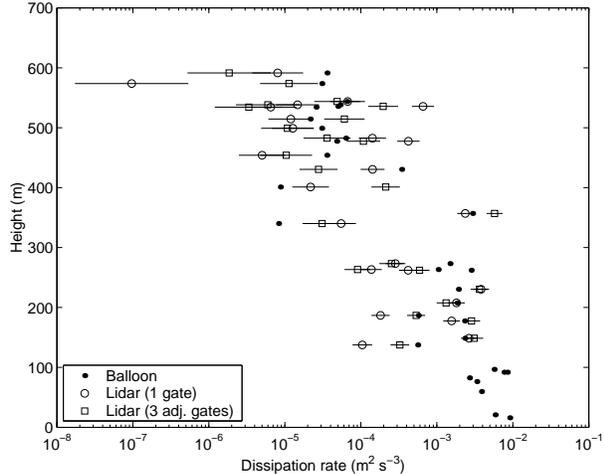


FIG. 9: Observed rate of dissipation of TKE,  $\epsilon$ , in the boundary layer from the lidar and in-situ balloon measurements between 0930 and 1230 UTC on 22 April 2008.

to cover the wide variation in magnitude. For values close to the ground, between 100 and 250 m, better agreement is found between the balloon and lidar estimates derived from 4 consecutive samples and 3 adjacent gates. Concerns noted earlier about the length scales involved near the surface being too large when 10 consecutive samples at one gate are used to derive the lidar estimates are most likely responsible for this discrepancy. The values of lidar  $\epsilon$  below  $10^{-6} \text{ m}^2 \text{ s}^{-3}$  are those which display the greatest disagreement with the balloon measurements and are again due to the limitations in using 10 consecutive samples at one gate. As shown in Fig. 9, this occurs at a height of almost 600 m, which, at 10.40 UTC, is not yet encompassed by the growing convective boundary layer. Two factors limit the ability of the lidar to estimate  $\epsilon$  in this particular case; not only is the lidar SNR low above the convective boundary layer, the length scales over which the velocity variance is calculated are again unlikely to be wholly contained within the inertial sub-range. Using 4 consecutive samples and 3 adjacent gates to estimate  $\epsilon$  at this height does show considerable improvement but it is still not certain that the shorter length scales involved will remain within the inertial sub-range outside the convective boundary layer.

One method of diagnosing whether the length scales probed by the lidar only encompass the inertial sub-range is to vary the number of samples,  $N$ , used to calculate  $\epsilon$ . If the derived values of  $\epsilon$  are no longer consistent with each other then the probability is high that the observed variance,  $\sigma_{\bar{v}}^2$ , contains contributions from the outer scales of turbulence.

## 5. Conclusion

We have demonstrated the potential for estimating  $\epsilon$  from a Doppler lidar by using the standard deviation of successive samples of the mean Doppler velocity. We have shown that the noise contribution to the velocity variance can be estimated reliably and that there is sufficient SNR throughout most of the well-mixed boundary layer for good Doppler observations. The range of values found for  $\epsilon$  agree well with the wide range of dissipation rates measured by Chen (1975); Siebert *et al.* (2005). The agreement with the in-situ balloon-borne measurements is very encouraging; however, it should be noted that this comparison was mainly performed within the well-mixed boundary-layer, where the lidar signals are generally strong and the scales of motion contained within the inertial sub-range are large enough to encompass the entire  $N$  sampling times required to derive  $\sigma_{\bar{v}}$ .

In principle the retrieval of  $\epsilon$  in low SNR conditions can be improved by discarding the individual Doppler velocity measurements with large errors before computing  $\sigma_{\bar{v}}$ , as discussed by Frehlich (2001). For low-power Doppler lidars, which require averaging of many pulses to achieve a reasonable sensitivity, there are a limited number of individual samples available within the required timeframe for keeping length scales within the inertial sub-range and it is highly likely that removal of the noisy samples will bias the calculation of  $\epsilon$ . A threshold on the relative frequency of noisy samples within a variance measurement provides a simple quality flag for the  $\epsilon$  estimates.

It is clear that a shorter integration time is preferable for ensuring that the length scales probed are always within the inertial sub-range to ensure that (1) is applicable. The boundary between the outer scale and the inertial sub-range may well lie at much smaller scales in some regions of the atmosphere (and in some types of boundary layers). However, for the particular instruments considered here, there is a tradeoff between the sensitivity of the instrument and the applicability of the method. This method can still be applied where longer integration times have been used to improve the instrument sensitivity but there will be more situations when the length scales are no longer within the inertial sub range. From the measurements discussed here it seems that an instrument temporal resolution of 4 s, resulting in a  $\sigma_{\bar{v}}$  estimate over  $< 120$  s, is required to remain within the inertial sub-range throughout the boundary layer, whereas an instrument with a temporal resolution of 30 s may be limited to the length scales found in convective boundary layers. In contrast, the ability of the instrument to measure very low velocity variances with sufficient accuracy is much improved as

the accumulated photon count increases, necessitating the extended integration time. With 30-second samples there is potential for reducing length scales by taking samples from adjacent gates so that the number of consecutive samples in time can be reduced accordingly, while still providing estimates of dissipation rates with reasonable accuracy.

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