Surface effects in QG dynamics

University of Reading
NATURAL ENVIRONMENT RESEARCH COUNCIL

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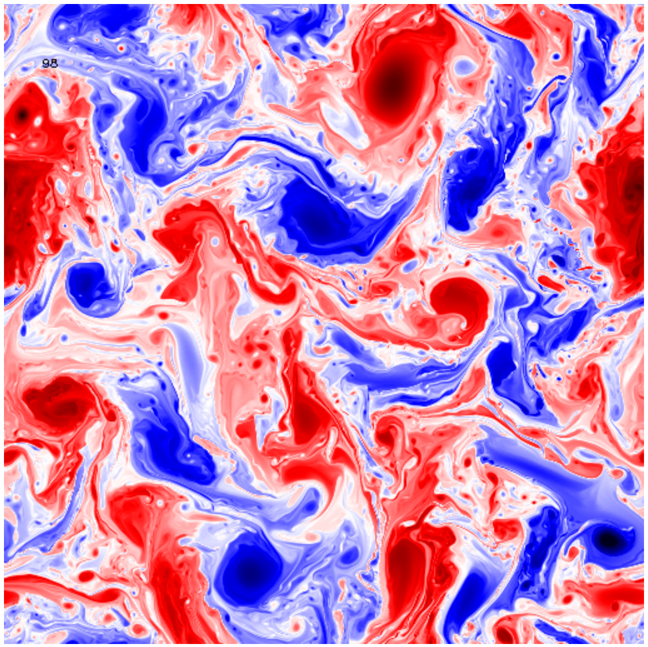
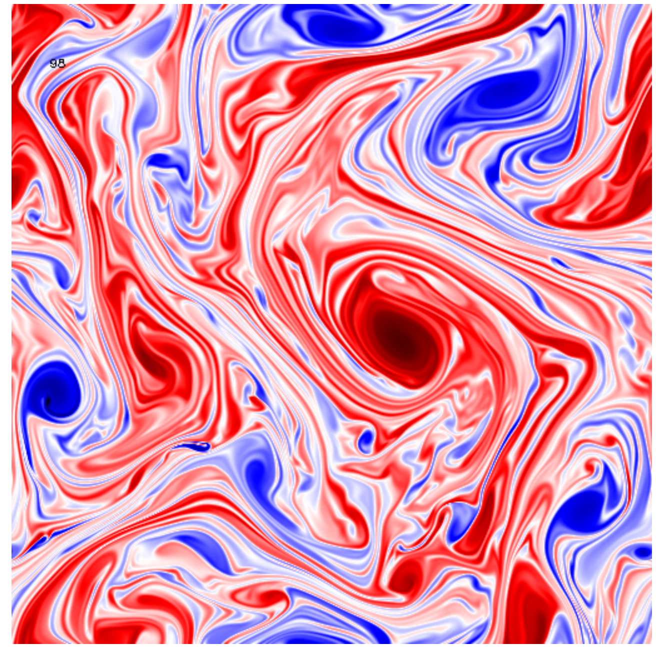
Surface QG dynamics

Surface QG (temperature)



2-d Euler (vorticity)



Outline of talk

Introduction:	PV basics Quasi-geostrophic theory The surface QG equations
Filament instability:	
Baroclinic instability:	

Potential vorticity: a (very short) introduction

Vorticity ξ is a measure of the local rotation rate of a fluid

It's conserved in 2-d incompressible and frictionless flows:
(*2-d Euler equations*)

$$\frac{D\xi}{Dt} = 0, \quad \xi = \nabla^2 \psi$$

This represents the conservation of angular momentum

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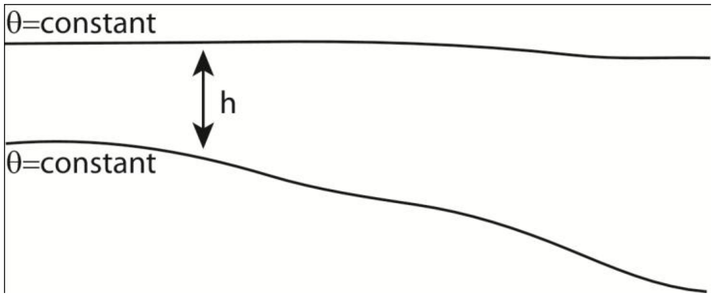

$$P = \frac{f + \xi}{h}$$

Two important properties:

(i) P (and θ) is conserved by *adiabatic* and *frictionless* flows

(ii) It can be \`inverted' under suitable balance conditions

Quasi-geostrophic theory

The simplest such balance

Based on the fact that the atmosphere is close to geostrophic and hydrostatic balance (at large scales)...thermal wind balance

Or, equivalently, that the Rossby number is small ($Ro=U/fL<<1$) and the Richardson number is large ($Ri=N^2H^2/U^2>>1/Ro$)

Quasi-geostrophic theory

Linearise the PV equation for small h variations (write h = h_0+h'):

$$P = \frac{f+\xi}{h} = \frac{f+\xi}{h_0}\left(1 - \frac{h'}{h_0} + \ldots\right) \approx \frac{f}{h_0} + \left(\frac{\xi}{h_0} - \frac{fh'}{h_0{}^2}\right)$$

rotation stratification

Quasi-geostrophic theory

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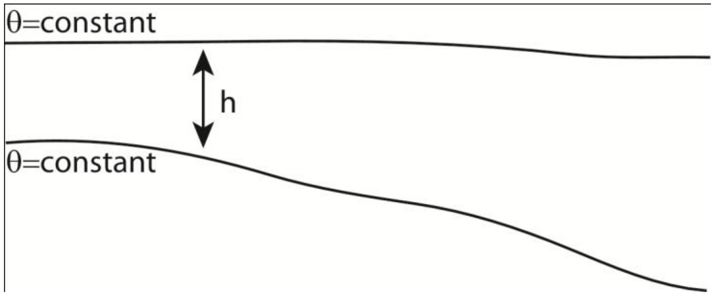
rotation stratification

The continuously stratified (Boussinesq, constant N) version takes the form:

$$q = h_0 P = f + \xi + \frac{fg}{N^2\theta_0}\frac{\partial\theta'}{\partial z}$$



Quasi-geostrophic theory

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The continuously stratified (Boussinesq, constant N) version takes the form:

$$q = h_0 P = f + \xi + \frac{fg}{N^2\theta_0}\frac{\partial \theta'}{\partial z}$$

Finally, hydrostatic balance gives: $\theta' = \frac{\theta_0 f}{g}\frac{\partial \psi}{\partial z}$

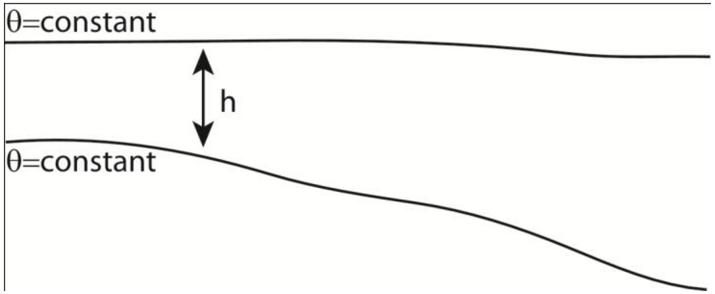
So that

$$q = f + \nabla_h^{\,2}\psi + \frac{f_0^{\,2}}{N^2}\frac{\partial^2 \psi}{\partial^2 z^2}$$

(typically N/f=100)

Quasi-geostrophic theory

But what happens at the surface?



Quasi-geostrophic theory

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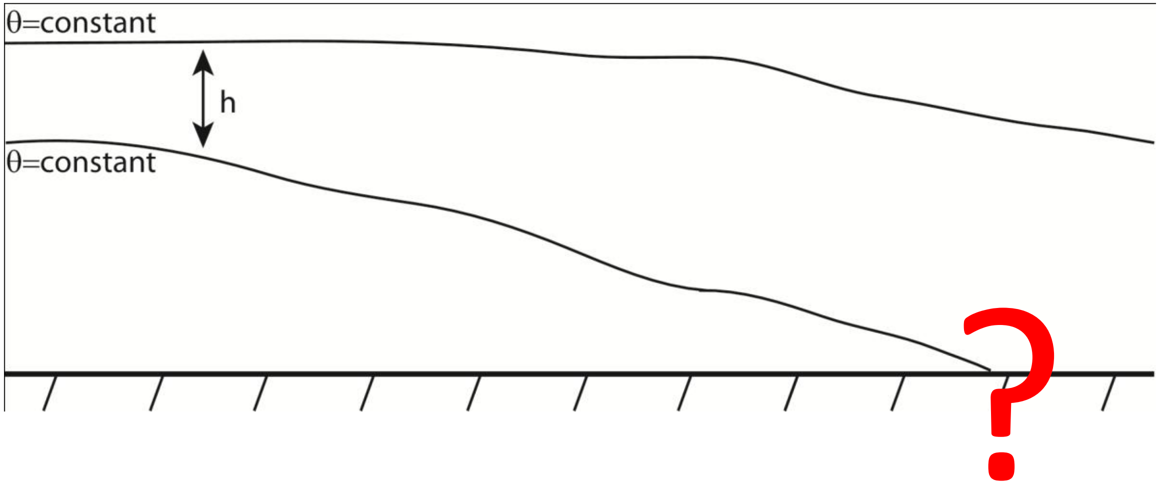

Quasi-geostrophic theory

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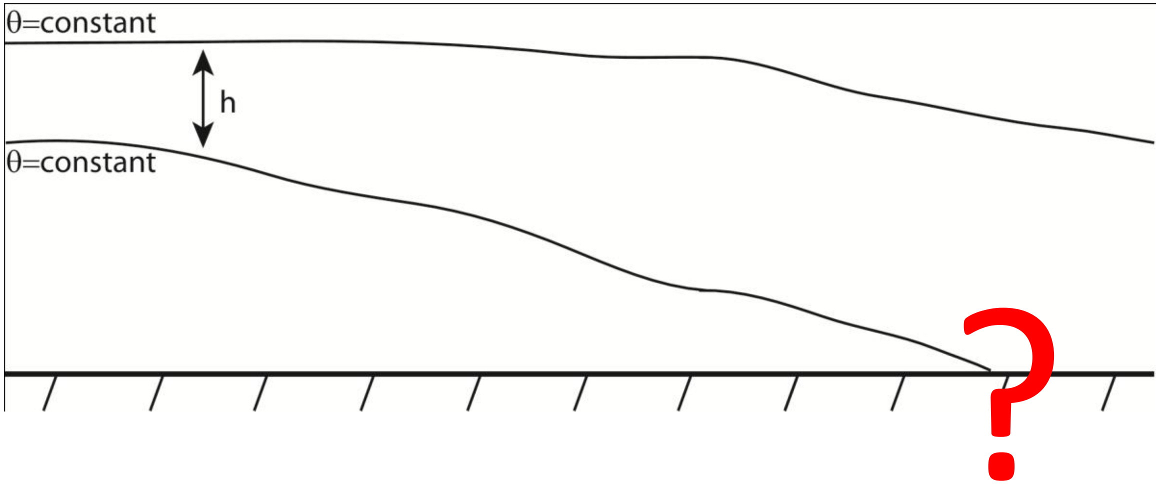

Use that θ is conserved

The result is a two component system:

Interior potential vorticity: $q = f + \nabla_h^2 \psi + \frac{f_0^2}{N^2} \frac{\partial^2 \psi}{\partial^2 z^2}$

Surface temperature: $\theta' = \frac{\theta_0 f}{g} \frac{\partial \psi}{\partial z}$

The surface QG equations

Focus on the surface component by setting q=0

Then, at the surface,

$$\frac{D\theta}{Dt} = 0$$

With PV inversion given by

$$\nabla_h{}^2\psi + \frac{f_0{}^2}{N^2}\frac{\partial^2\psi}{\partial^2 z^2} = 0 \qquad \theta = \frac{\theta_0 f}{g}\frac{\partial\psi}{\partial z}$$

Key ingredients:

A horizontal boundary
QG f-plane dynamics
Negligible interior PV

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QG f-plane dynamics
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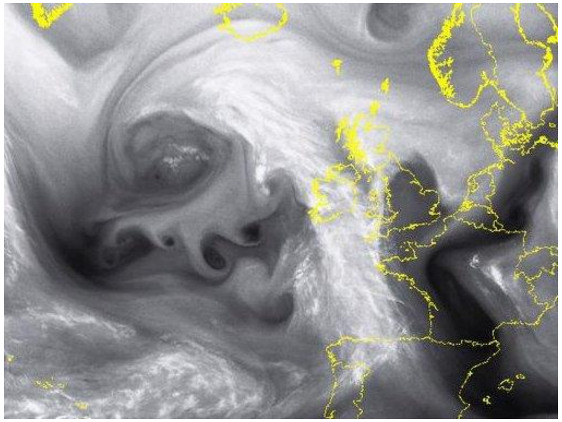
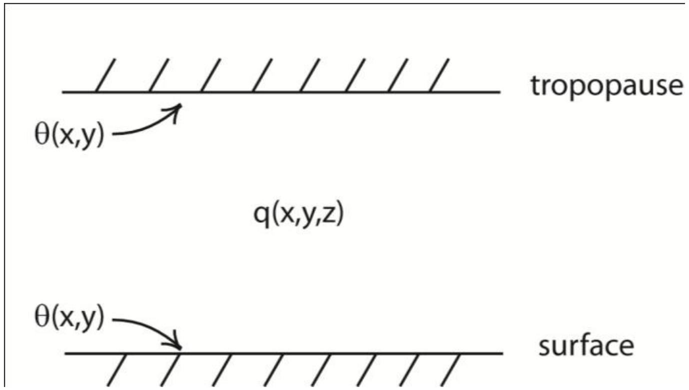
Other applications:

Tropopause perturbations

Juckes (1994) showed that small scale tropopause perturbations can be approximated by surface QG





The surface QG equations

Key ingredients:

A horizontal boundary
QG dynamics
Negligible interior PV

Other applications:

Tropopause perturbations

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Upper level ocean eddies

Lapeyre and Klein (2006) apply surface QG dynamics to upper level ocean dynamics

How to model numerically?

$$\frac{D\theta}{Dt} = 0 \qquad \nabla_h^{\ 2}\psi + \frac{f_0^{\ 2}}{N^2}\frac{\partial^2\psi}{\partial^2 z^2} = 0 \qquad \theta = \frac{\theta_0 f}{g}\frac{\partial\psi}{\partial z}$$

This is effectively a two-dimensional system

Surface QG

$$\theta^n(x,y)$$
$$\downarrow$$
$$\hat{\theta}^n(\mathbf{k}) = -\frac{\hat{\psi}^n(\mathbf{k})}{|\mathbf{k}|}$$
$$\downarrow$$
$$u^n = -\psi^n{}_y$$
$$v^n = \psi^n{}_x$$
$$\downarrow$$
$$\theta^{n+1}(x,y)$$

2-d Euler

$$\xi^n(x,y)$$
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$$\hat{\xi}^n(\mathbf{k}) = -\frac{\hat{\psi}^n(\mathbf{k})}{|\mathbf{k}|^2}$$
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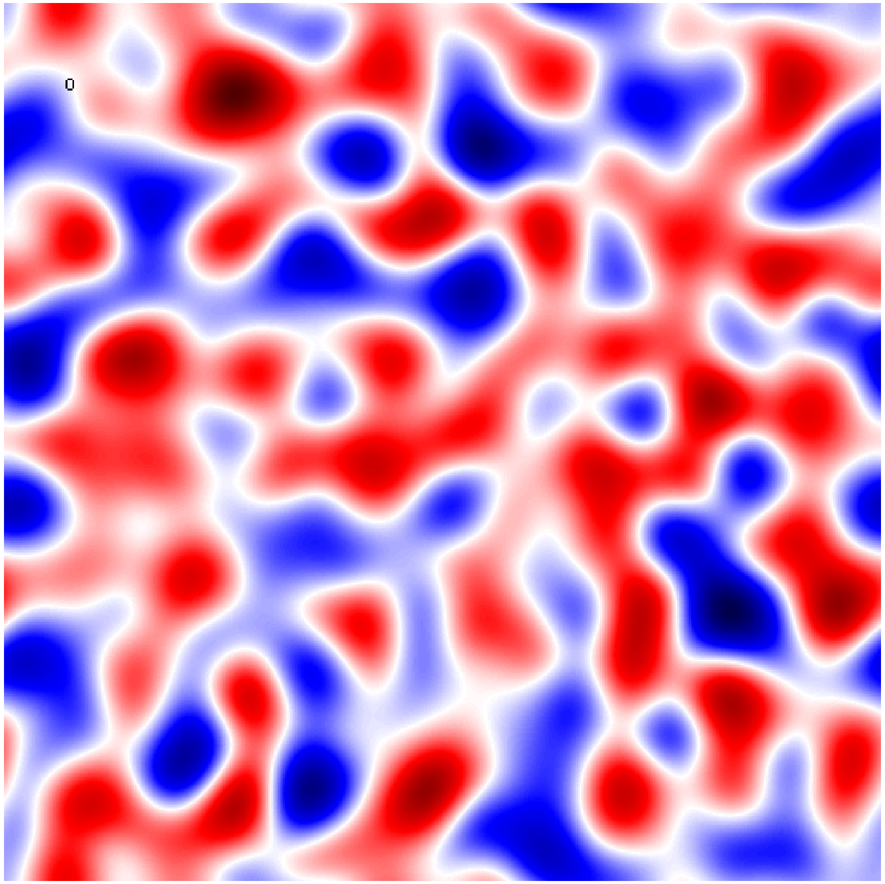
Surface QG dynamics

A turbulence simulation:

(animation)



Surface QG dynamics

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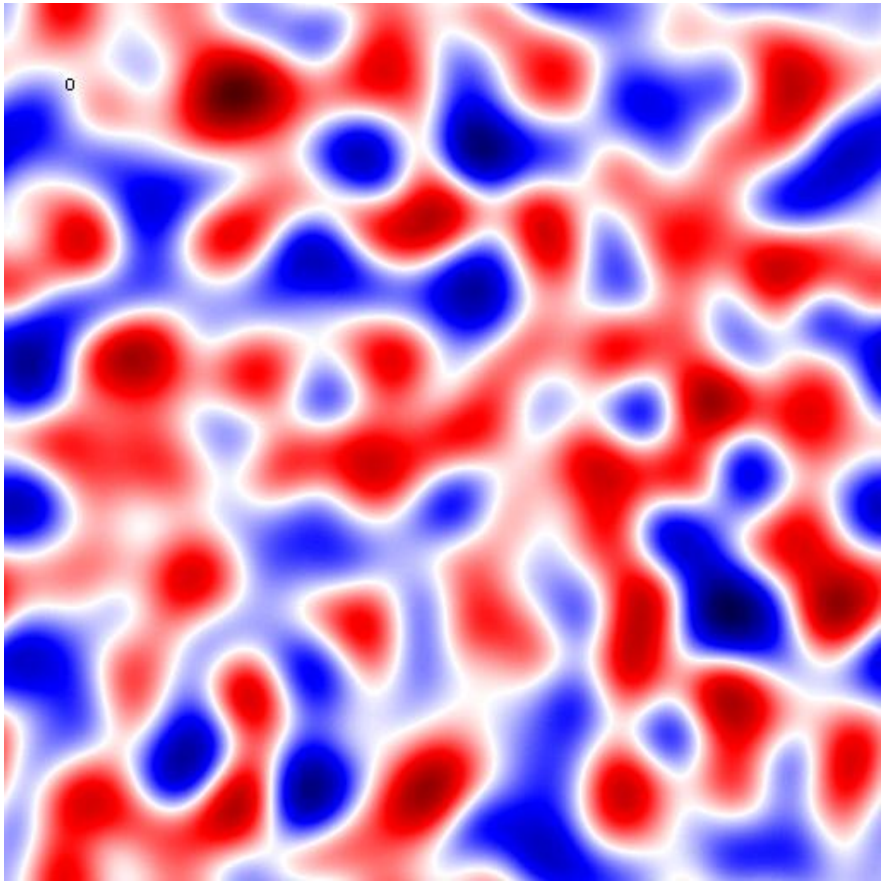
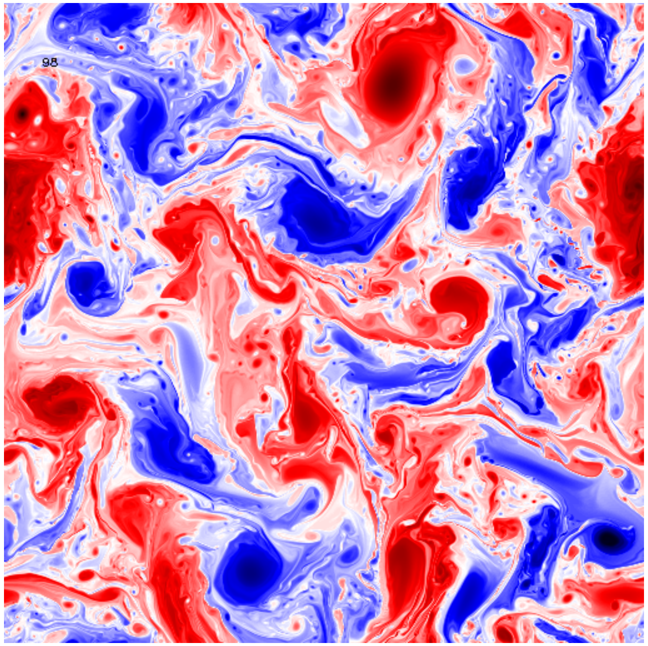
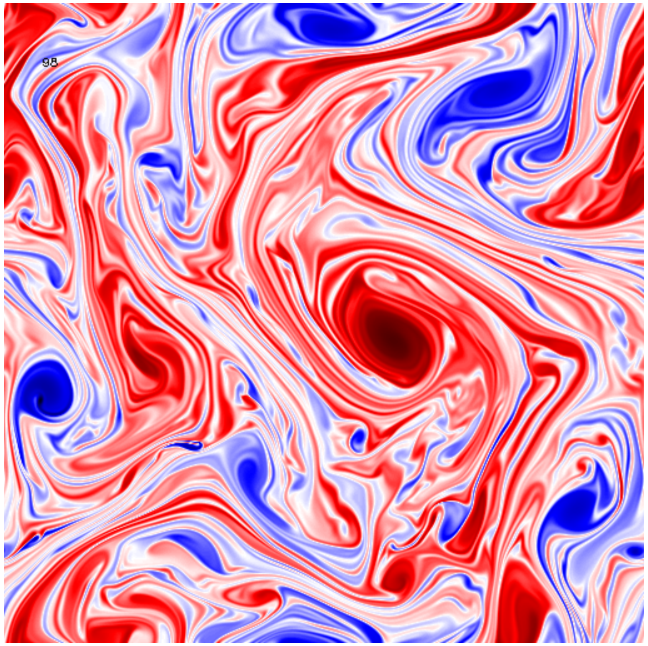


Surface QG dynamics

Surface QG (temperature)



2-d Euler (vorticity)



Both have: large coherent vortices plus complicated small scale structure

But: SQG vortices are less tightly bound and small scale structure is more ‘messy’

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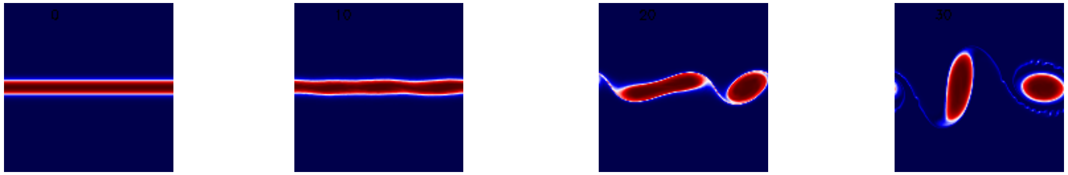
Filament instability:	Motivation The effects of straining
Baroclinic instability:	

Filament instability

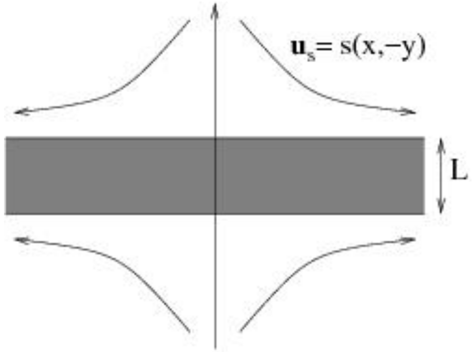
Motivation:

Both 2-d Euler vorticity filaments and surface QG temperature filaments are *unstable* in isolation (barotropic instability, the Rayleigh problem,...):



However...the presence of external flows often act to stabilise vorticity filaments (Dritschel et al 1991)

e.g., straining:



The filament remains coherent if the strain rate is large enough:

$$s > 0.25\xi_0$$

The question: What happens for surface QG dynamics?

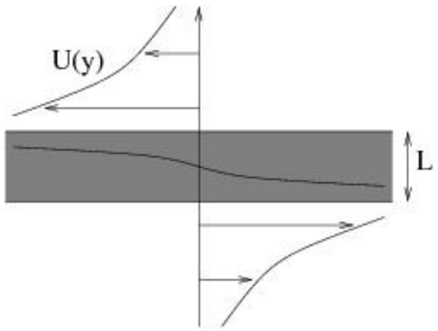
Filament instability

Calculated by Juckes (1995).

Basic state:

$$\theta(y) = \begin{cases} \theta_0 & |y| < L/2 \\ 0 & |y| > L/2 \end{cases}$$

$$U(y) = \frac{\theta_0}{\pi} \log \left| \frac{y - L/2}{y + L/2} \right|$$



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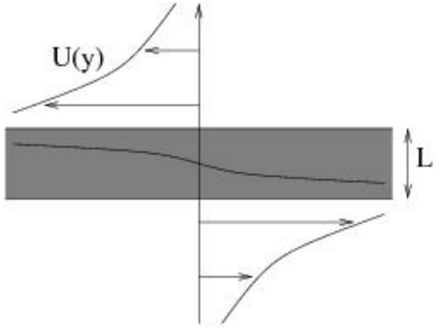

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Perturbation:

$$(\eta_1, \eta_2) = \hat{\boldsymbol{\eta}}(t) e^{ikx}$$

$$i\frac{d\hat{\boldsymbol{\eta}}}{dt} = \frac{\theta_0}{L} \begin{pmatrix} P(\kappa) & I(\kappa) \\ -I(\kappa) & -P(\kappa) \end{pmatrix} \hat{\boldsymbol{\eta}}$$

where $\kappa = kL$

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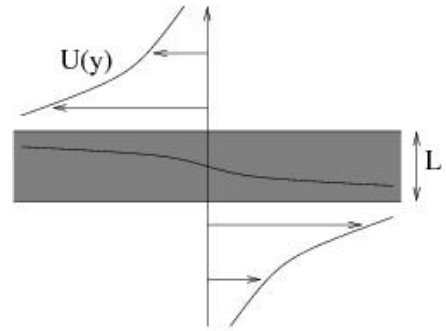

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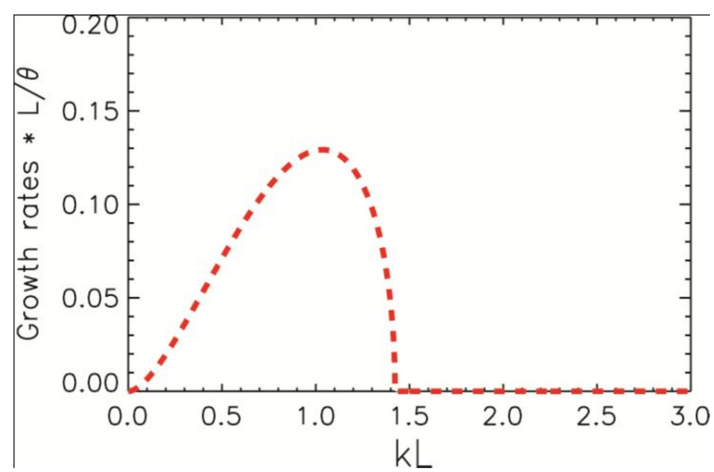
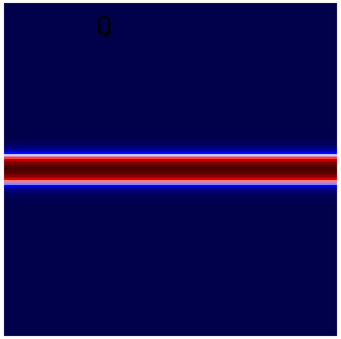
Growth rates proportional to $\dfrac{\theta_0}{L}$



Filament instability

A numerical simulation:



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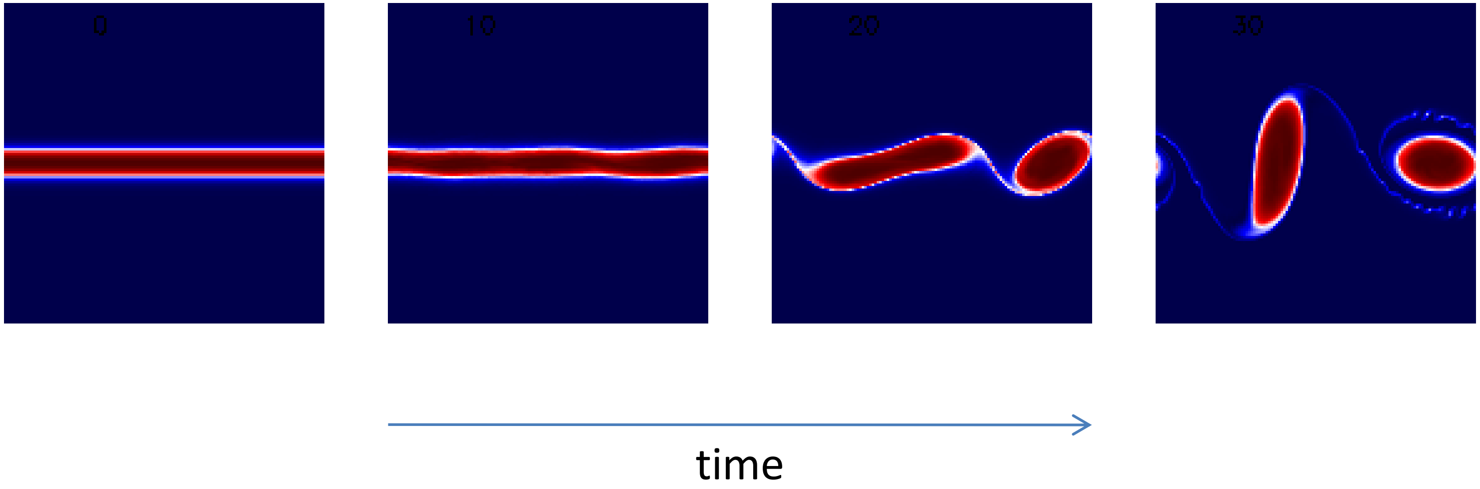
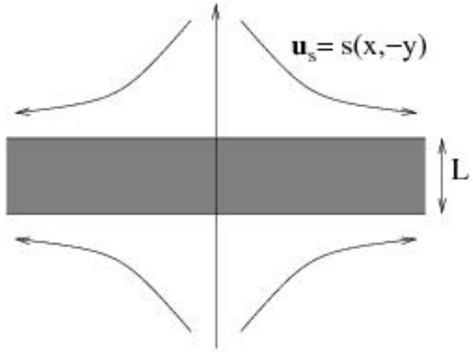


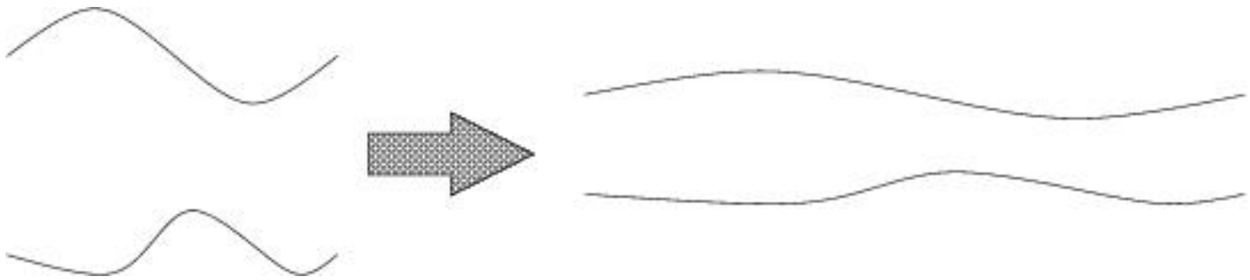
Filament instability

Now add a straining flow:

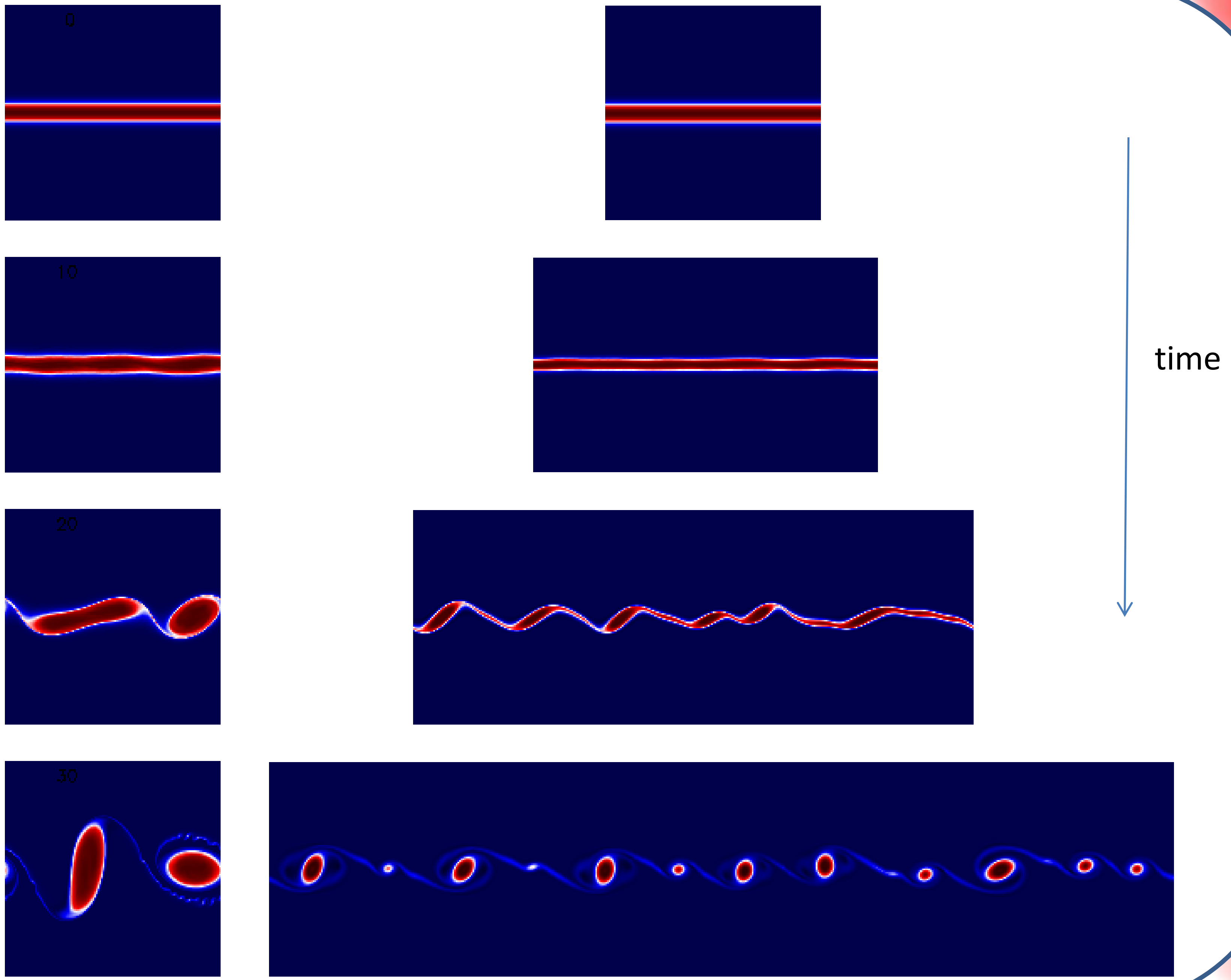
(*s* is the strain rate)



The filament will be stretched and squashed exponentially:



Which normally would act to stabilise any irregularities along the filament.


0
10
20
30
time

Filament instability

The equation for linear perturbations takes the same form:

$$i\frac{d\hat{\boldsymbol{\eta}}}{dt} = \frac{\theta_0}{L}\begin{pmatrix} P(\kappa) & I(\kappa) \\ -I(\kappa) & -P(\kappa) \end{pmatrix}\hat{\boldsymbol{\eta}}$$

Except that now the wavenumber and filament width are functions of time:

$$L = L_0 e^{-st} \qquad \kappa = \kappa_0 e^{-2st}$$

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There are three competing effects:

1. Kinematic decay: $A \propto e^{-2st}$
2. Growth rate increase: $\sigma \propto \theta_0 / L$

(1 and 2:) Stable until $L = L_c \propto \theta_0 / s$

3. Wave-number decrease

Filament instability

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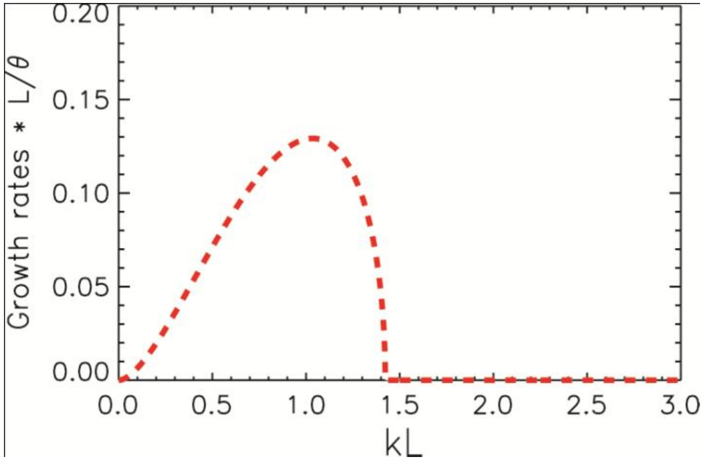

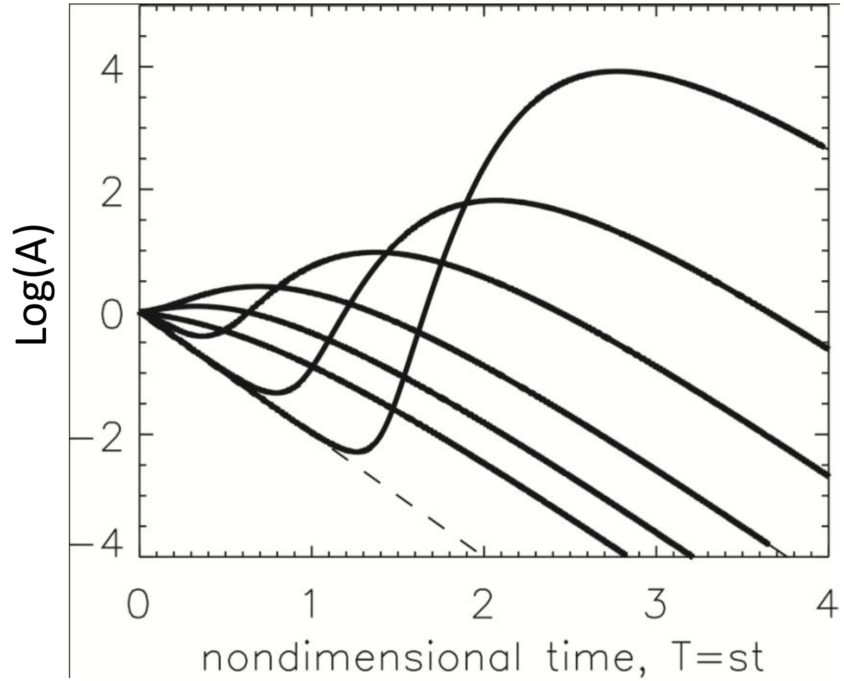

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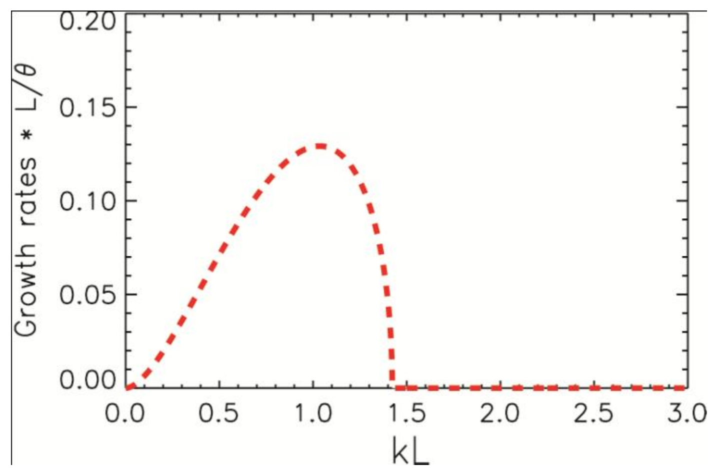

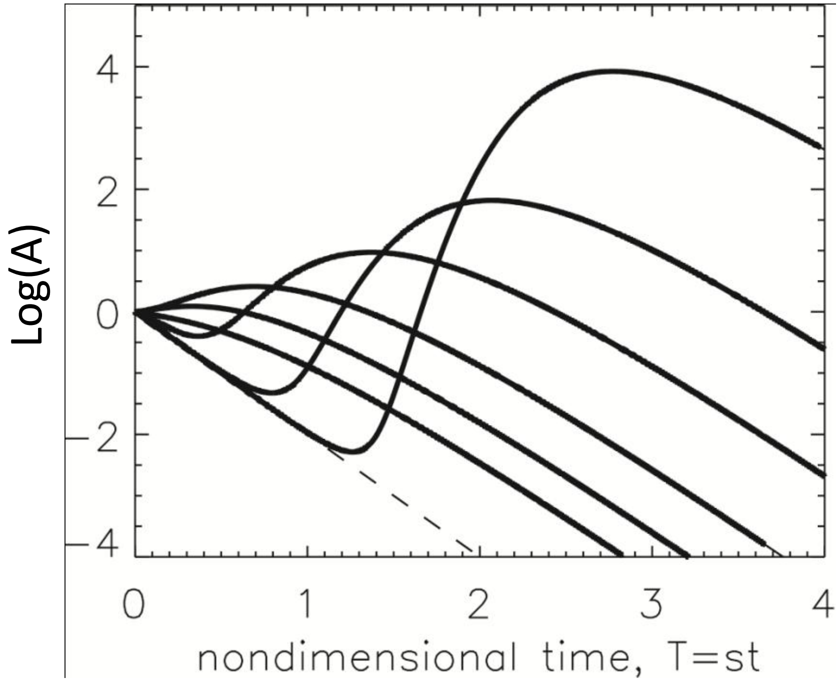

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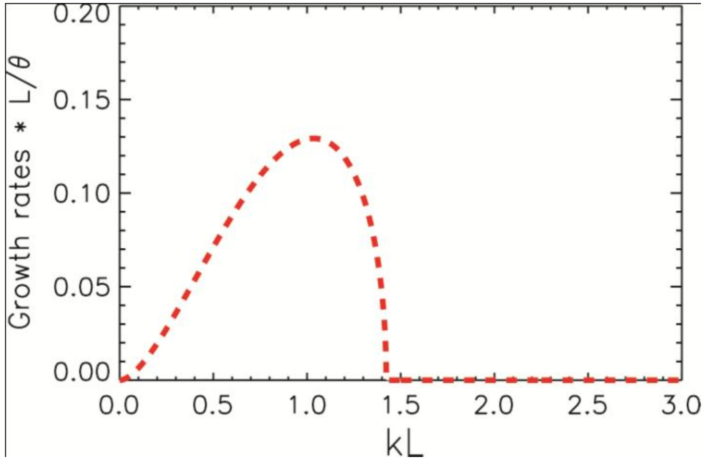

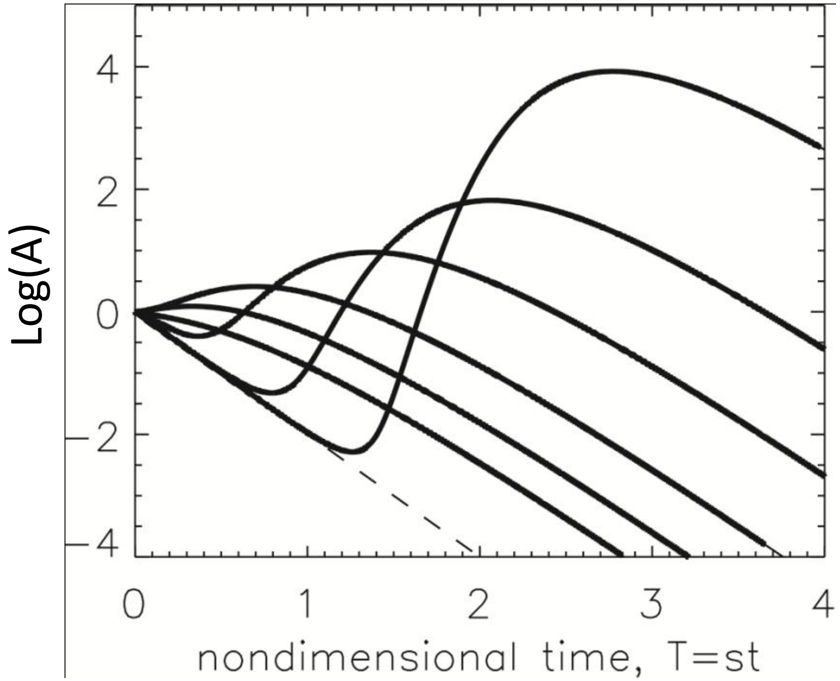

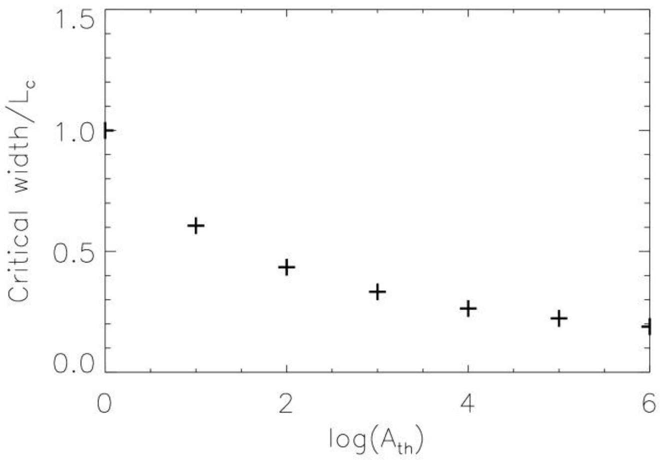

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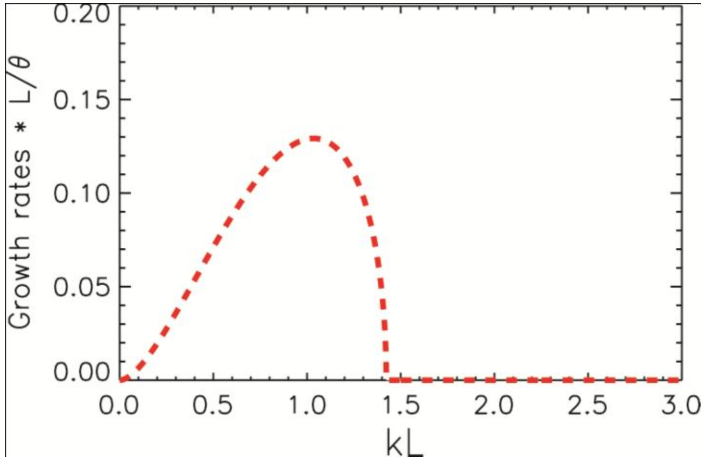

Filament instability

The conceptual picture:

In both models filaments are formed by the presence of straining, then...

In 2-d Euler dynamics:

the straining keeps the filaments stable. Instability only occurs if the straining stops or the filament moves away from it.

In surface QG:

the straining keeps the filaments stable, but only for a short time. Once they reach a critical width perturbation growth dominates.

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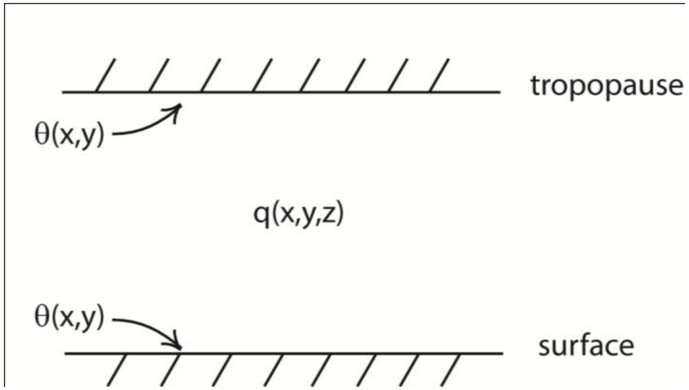
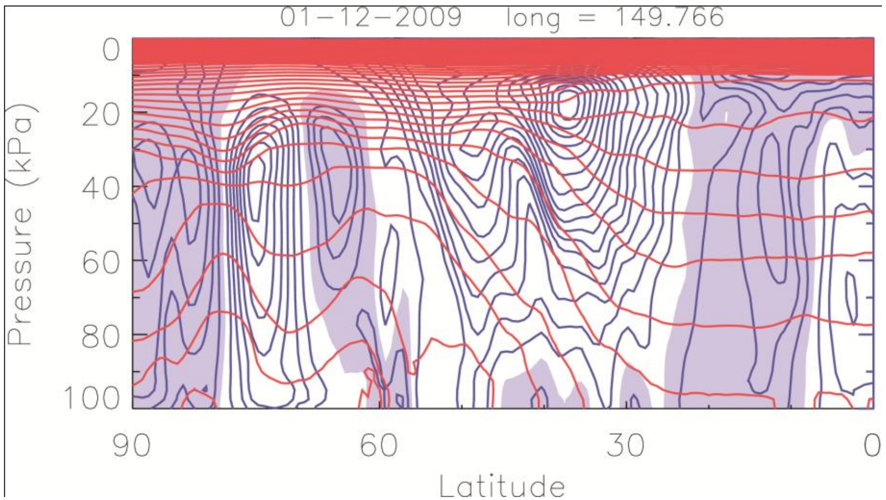
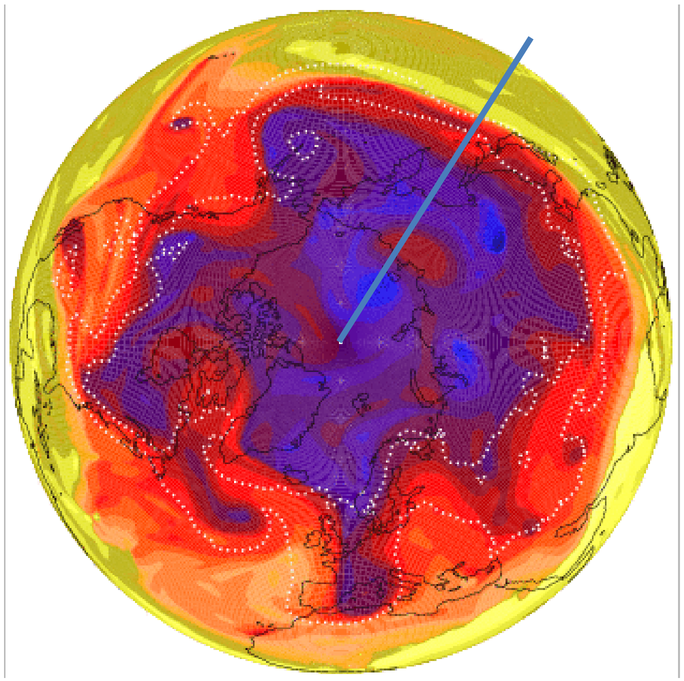
Filament instability:	Motivation The effects of straining
Baroclinic instability:	Uniform PV models A new model



Baroclinic instability

Motivation:





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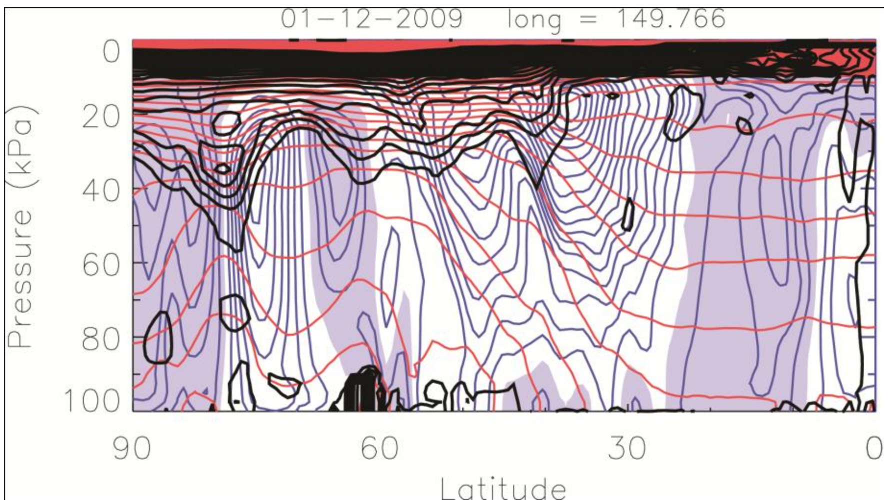

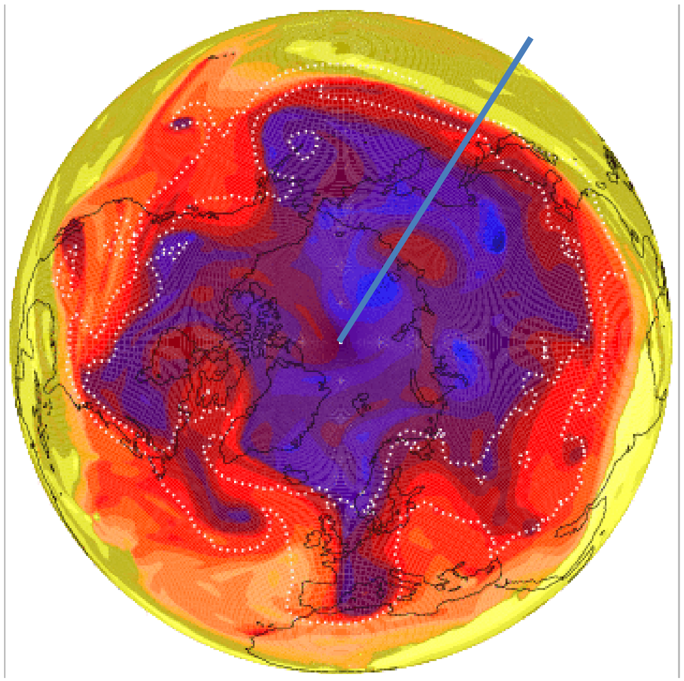

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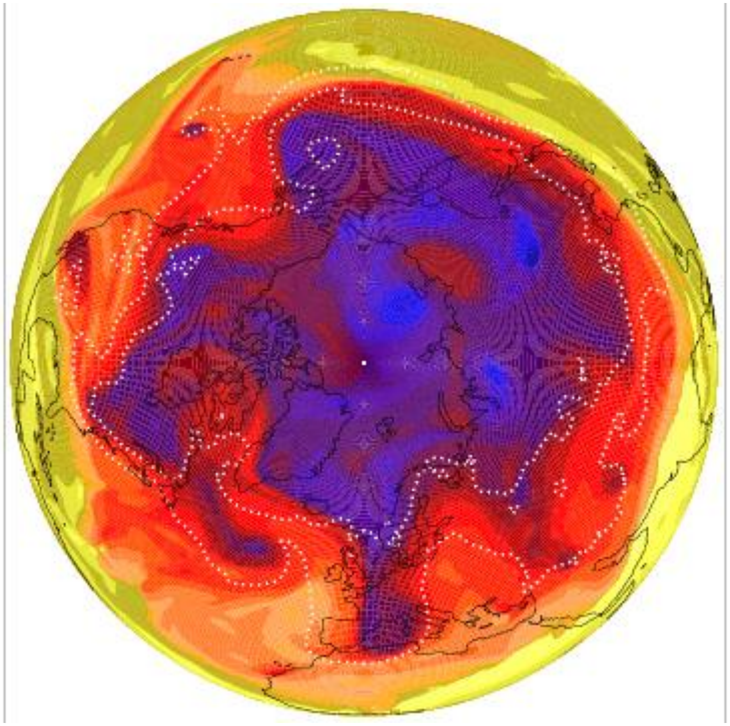
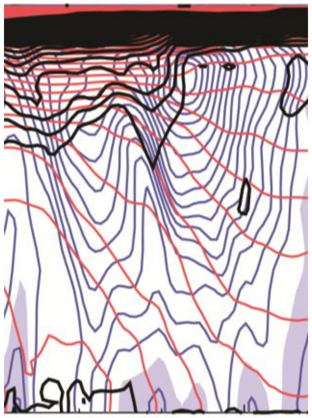
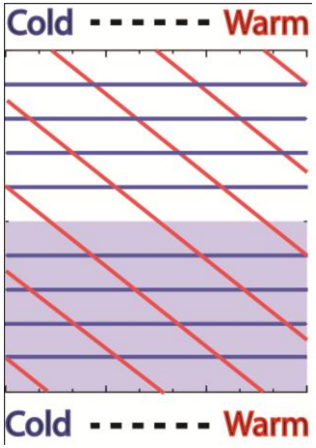


Baroclinic instability: Uniform PV models

Example



Eady (1949)



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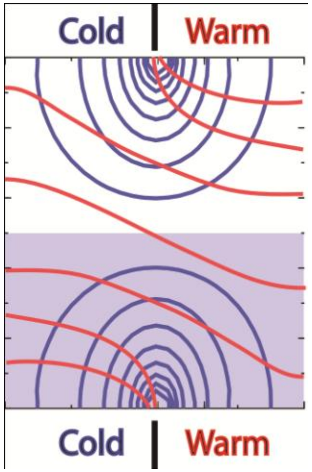

Baroclinic instability: Uniform PV models

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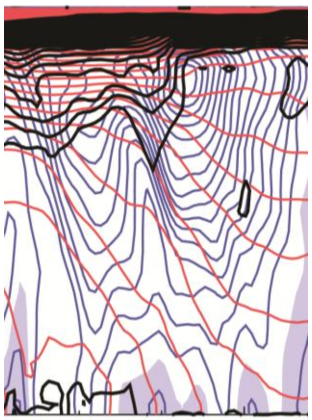

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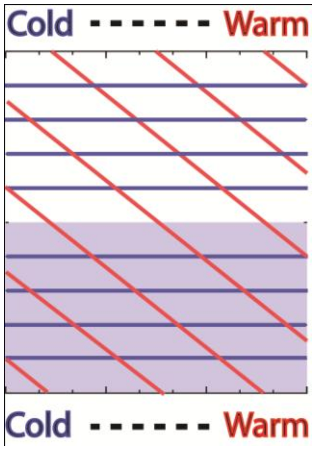

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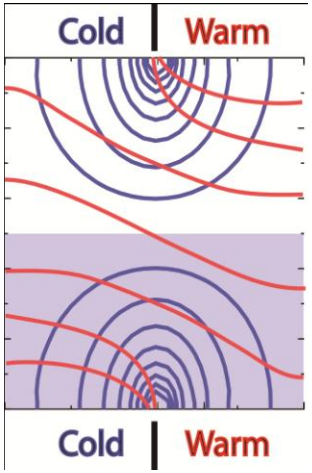
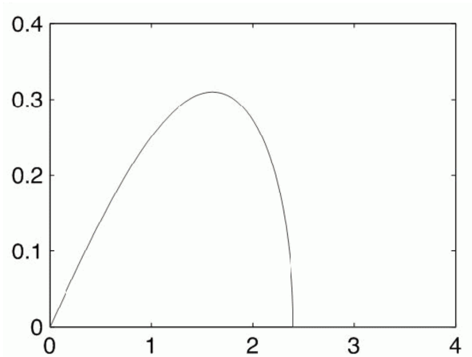
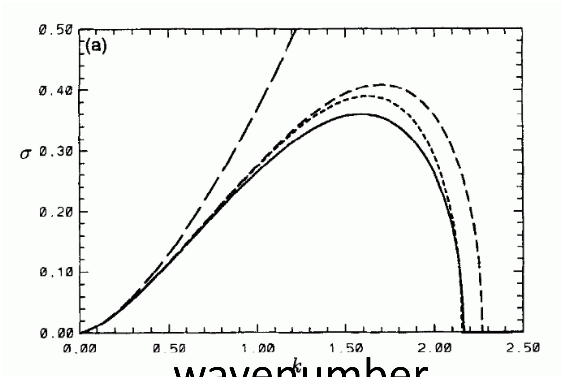


Growth rates



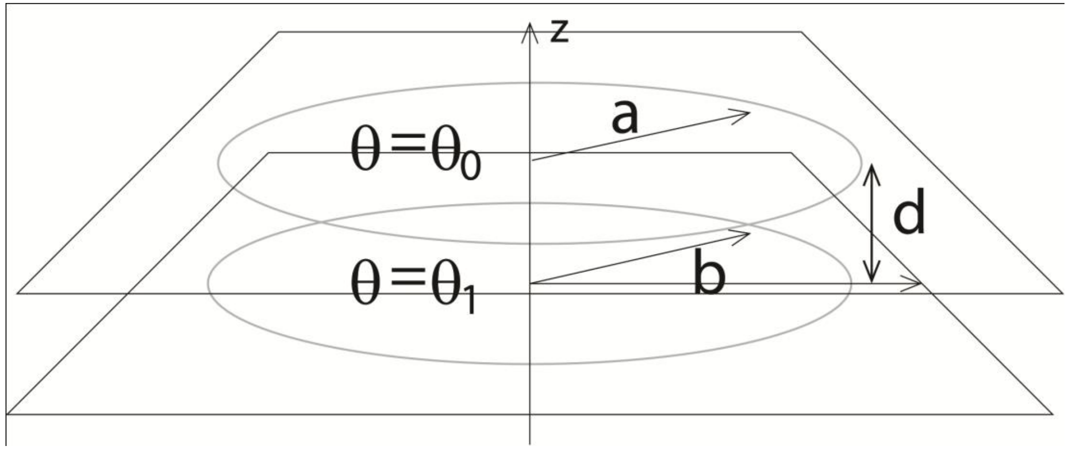
wavenumber



wavenumber

Baroclinic instability: Uniform PV models

Yet another model:



Interesting features:

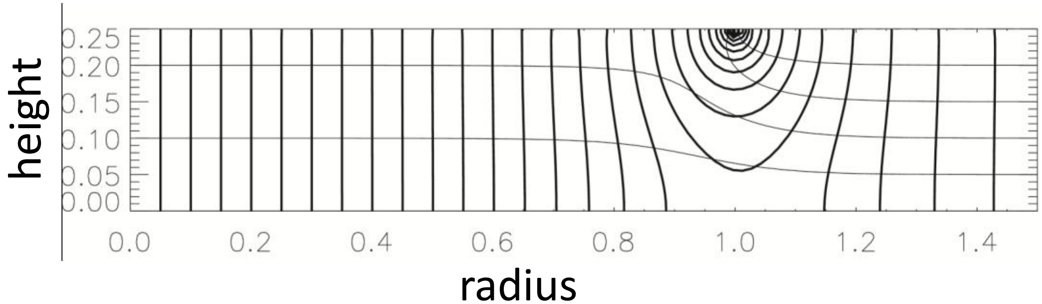
No longer restricted to tropopause-surface symmetry

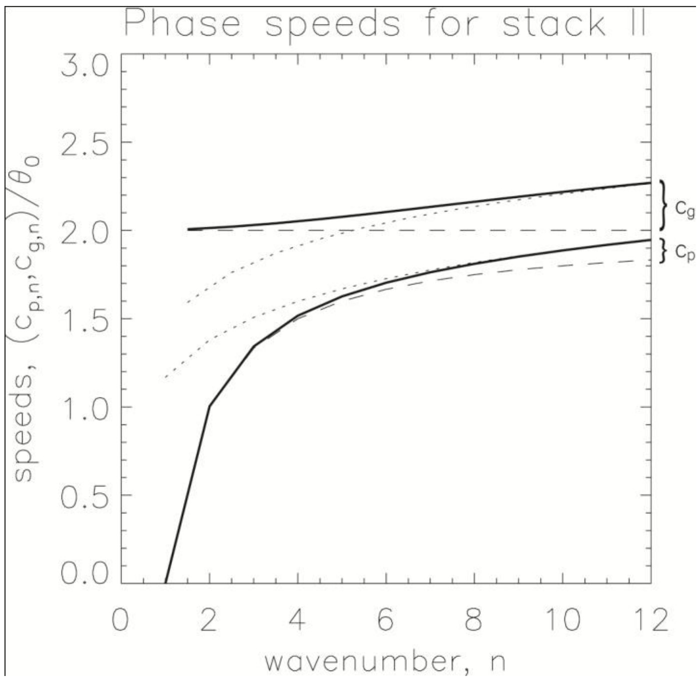
Nonlinear evolution may be more realistic due to circular geometry
- is the direction of wavebreaking simply related to the basic state?

Baroclinic instability: Uniform PV models

Linear dynamics:

First consider a single temperature patch on the tropopause

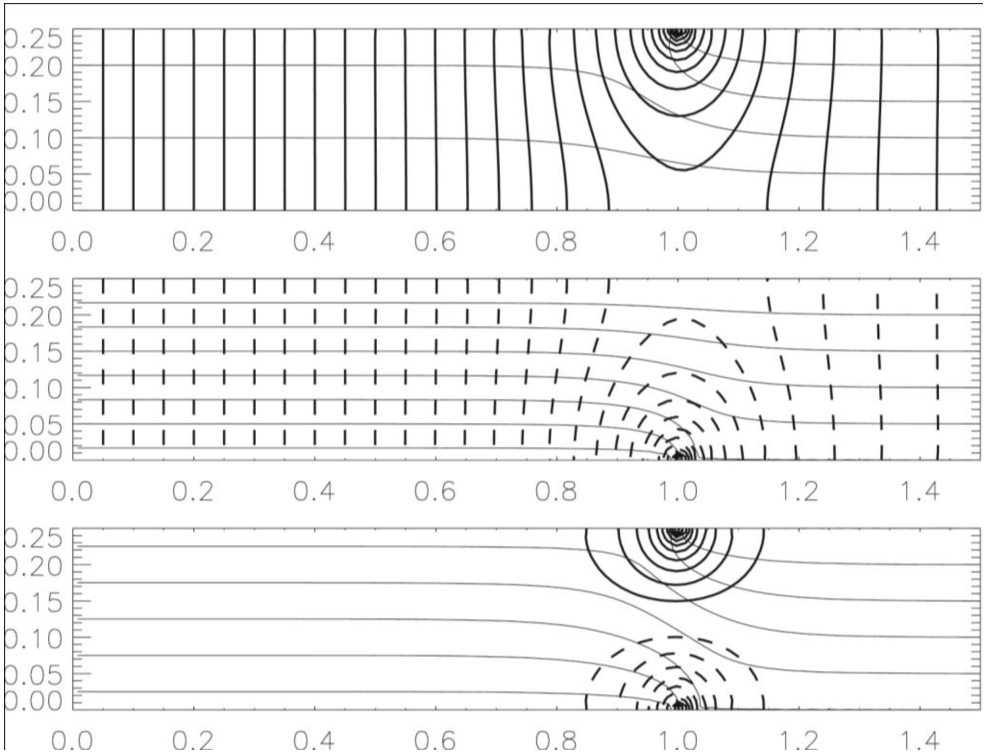


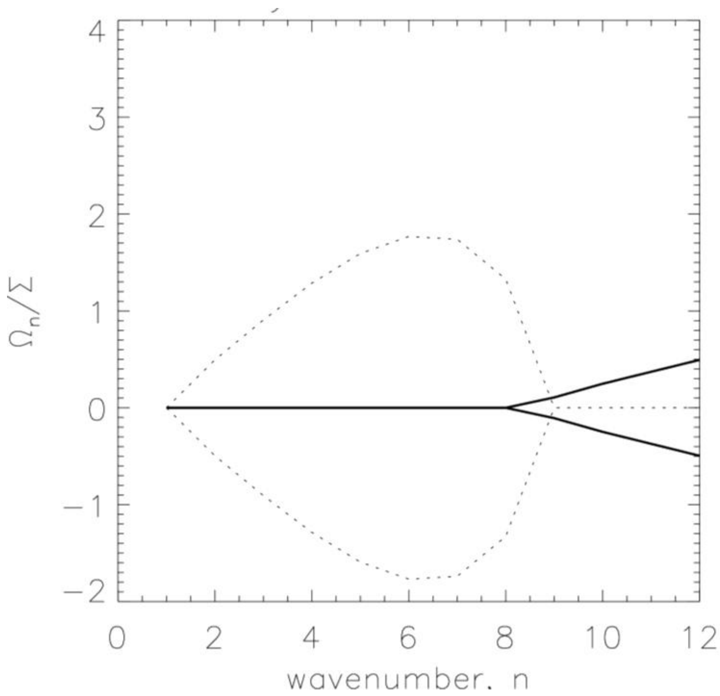


Baroclinic instability:
Uniform PV models

Linear dynamics:

Next add an opposing patch at the surface

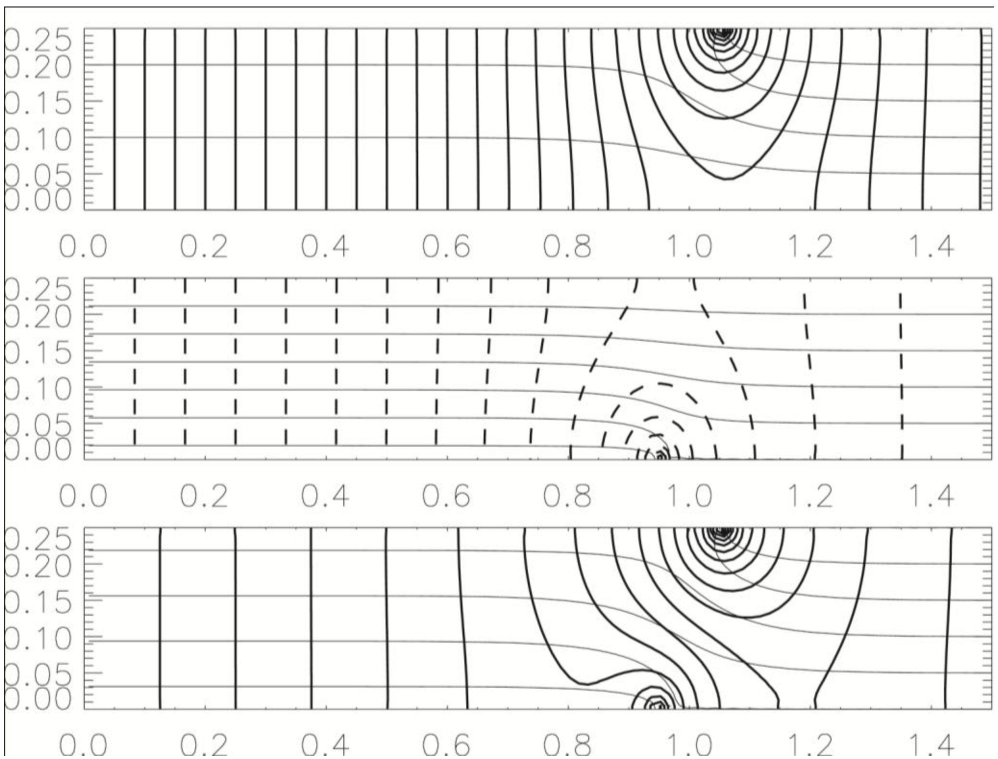


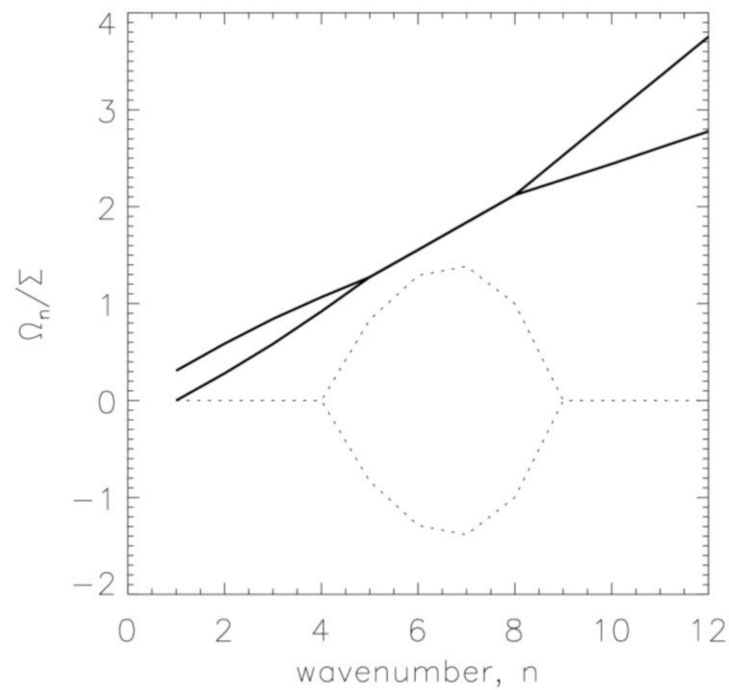


Baroclinic instability: Uniform PV models

Linear dynamics:

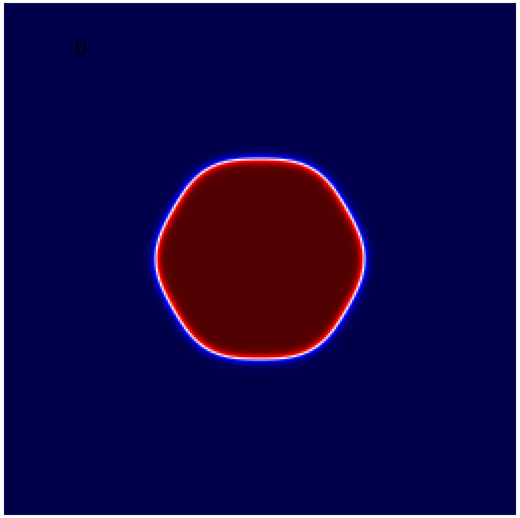
A non-symmetric example

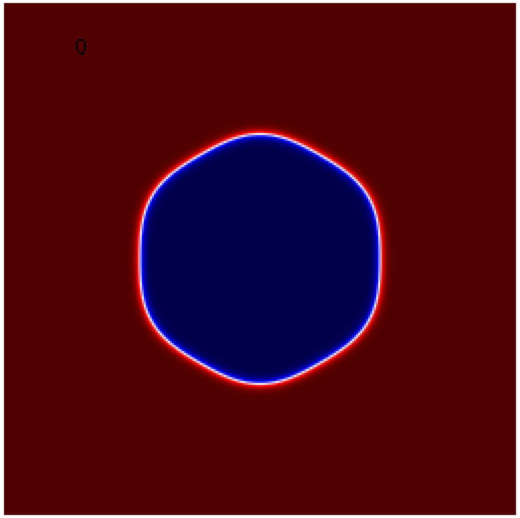




Baroclinic instability:
Uniform PV models

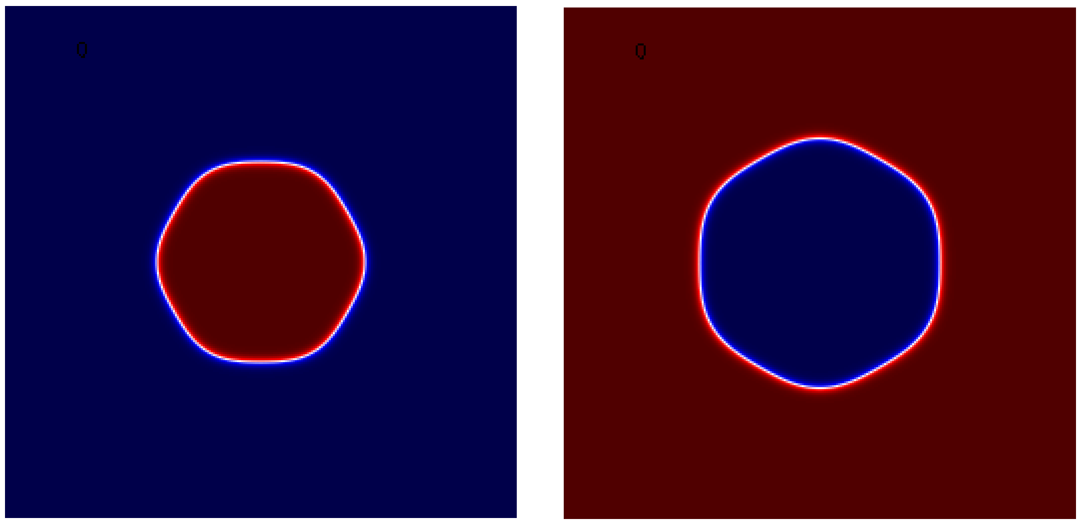
Nonlinear simulations:





Baroclinic instability: Uniform PV models

Nonlinear simulations:



Conclusions

•Small scale stratospheric intrusions can be modelled as filaments in the surface QG equations

•Filaments in the surface QG model behave very differently to those of the more familiar 2-d Euler system, because...

•...straining does not stabilise them

•In fact, straining just constrains when the instability occurs.

•Uniform PV quasi-geostrophic models provide analytically tractable examples of baroclinic instability

•We've formulated a new circular model as an extension to the `polar-front' model of Juckes (1998)

•The linear theory gives realistic growth rates for atmospheric parameter values, and the nonlinear evolutions may be insightful for understanding wavebreaking