Some simple models of baroclinic instability

hhh: 12/11/2012

QG dynamics

$$(u, v, \theta) = (-\psi_y, \psi_x, \psi_z)$$

QGPV: $Q = f + \nabla^2 \psi$

In the interior: $\left(\frac{\partial}{\partial t} + \boldsymbol{u} \cdot \nabla\right) Q = 0$

At horizontal boundaries: $\left(\frac{\partial}{\partial t} + \boldsymbol{u} \cdot \nabla\right) \theta = 0$

Scaling:

$z \to (N/f)z \qquad \theta \to \frac{g}{\theta_0 f}\theta$

Eady (1949) Tellus

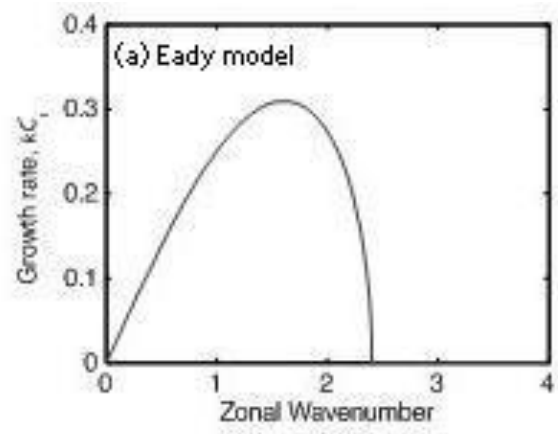
Long Waves and Cyclone Waves

By E. T. EADY, Imperial College of Science, London

(Manuscript received 28 Febr. 1949)

Abstract

By obtaining complete solutions, satisfying all the relevant simultaneous differential equations and boundary conditions, representing small disturbances of simple states of steady baroclinic large-scale atmospheric motion it is shown that these simple states of motion are almost invariably unstable. An arbitrary disturbance (corresponding to some inhomogeneity of an actual system) may be regarded as analysed into "components" of a certain simple type, some of which grow exponentially with time. In all the cases examined there exists one particular component which grows faster than any other. It is shown how, by a process analogous to "natural selection", this component becomes dominant in that almost any disturbance tends eventually to a definite size, structure and growth-rate (and to a characteristic life-history after the disturbance has ceased to be "small"), which depends only on the broad characteristics of the initial (unperturbed) system. The characteristic disturbances (forms of breakdown) of certain types of initial system (approximating to those observed in practice) are identified as the ideal forms of the observed cyclone waves and long waves of middle and high latitudes. The implications regarding the ultimate limitations of weather forecasting are discussed.



Long Waves and Cyclone Waves

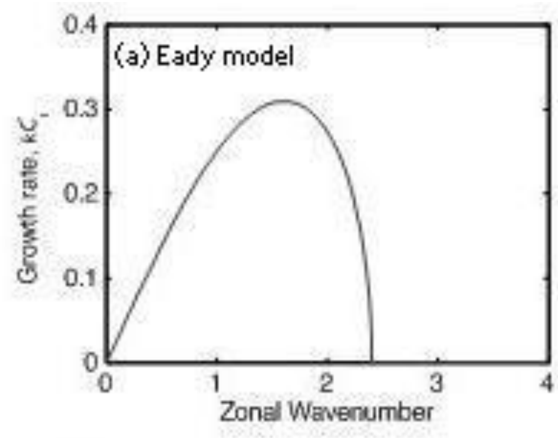
By E. T. EADY, Imperial College of Science, London

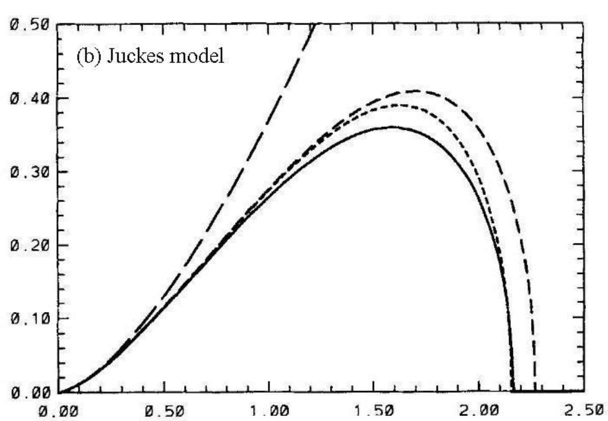
(Manuscript received 28 Febr. 1949)

Abstract

By obtaining complete solutions, satisfying all the relevant simultaneous differential equations and boundary conditions, representing small disturbances of simple states of steady baroclinic large-scale atmospheric motion it is shown that these simple states of motion are almost invariably unstable. An arbitrary disturbance (corresponding to some inhomogeneity of an actual system) may be regarded as analysed into "components" of a certain simple type, some of which grow exponentially with time. In all the cases examined there exists one particular component which grows faster than any other. It is shown how, by a process analogous to "natural selection", this component becomes dominant in that almost any disturbance tends eventually to a definite size, structure and growth-rate (and to a characteristic life-history after the disturbance has ceased to be "small"), which depends only on the broad characteristics of the initial (unperturbed) system. The characteristic disturbances (forms of breakdown) of certain types of initial system (approximating to those observed in practice) are identified as the ideal forms of the observed cyclone waves and long waves of middle and high latitudes. The implications regarding the ultimate limitations of weather forecasting are discussed.

Eady (1949) Tellus





Juckes (1998) QJRMS

Q. J. R. Meteorol. Soc. (1998), **124**, pp. 2227–2257

Baroclinic instability of semi-geostrophic fronts with uniform potential vorticity. I: An analytic solution

By M. N. JUCKES*

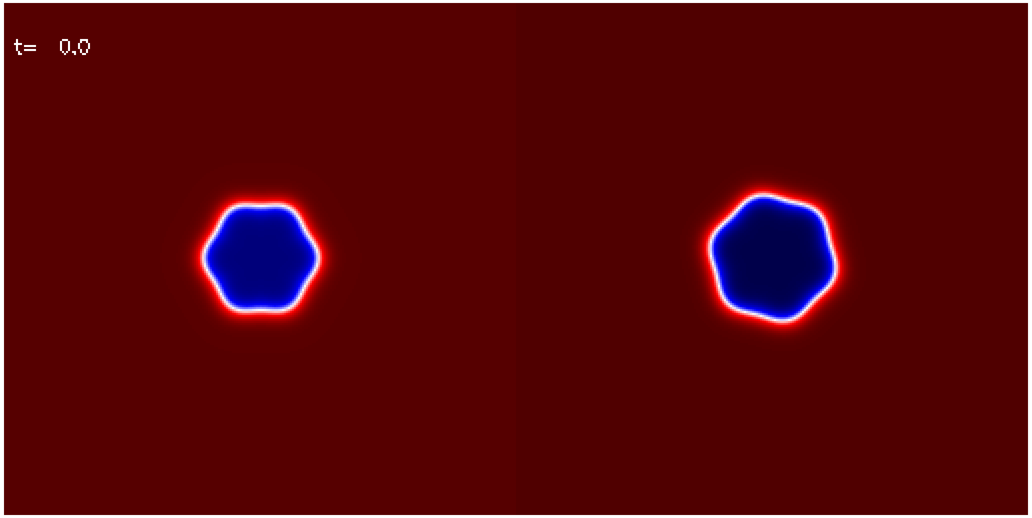
University of Munich, Germany

(Received 5 August 1997, revised 16 April 1998)

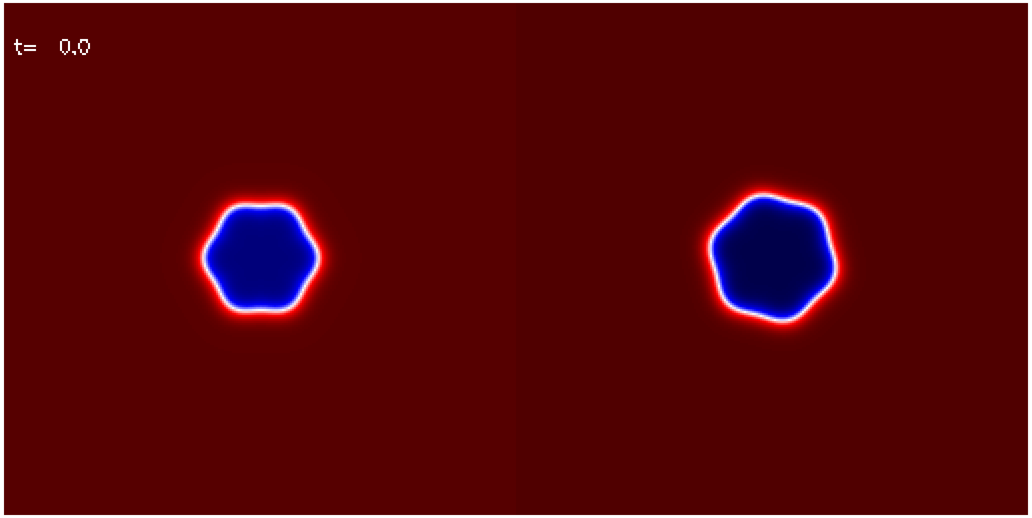
SUMMARY

Baroclinic instability of uniform potential-vorticity flow between solid upper and lower boundaries is analysed. The instability is driven by meridional temperature gradients on the boundaries. The classic Eady model of baroclinic instability uses just this system with the further idealization that the temperature gradients in the basic state are uniform. Here, this last idealization will be replaced with a complementary one in which the basic-state temperature gradients are taken to be concentrated in a front. The analysis then takes advantage of the fact that the boundary temperature anomalies created by the growing baroclinic wave are localized at the front. The dependence of the growth rates and phase structure on wave number are remarkably similar to those of the Eady model. The wave number of maximum instability and the short-wave cut-off differ from those of the Eady model by less than 2% and 10% respectively. The solution is asymptotic in the limit of zero frontal width in geostrophic coordinates. For a physical flow this limit can never be achieved, but comparison with direct numerical solutions shows that the analytic solution is still accurate at physically relevant frontal widths. Part I develops the solution based on an equation for the evolution of the displacement of the surface and upper fronts. Part II will look at the three-dimensional structure of the disturbance in more detail.

KEYWORDS: Baroclinic Instability Eady model Frontal dynamics



Bottom Top



Bottom Top

Outline

Some simple baroclinic instability solutions:

- Eady, Juckes

Some simple Rossby wave solutions:

- 2-d Euler, surface QG

An axisymmetric model of baroclinic instability

2-d Euler dynamics

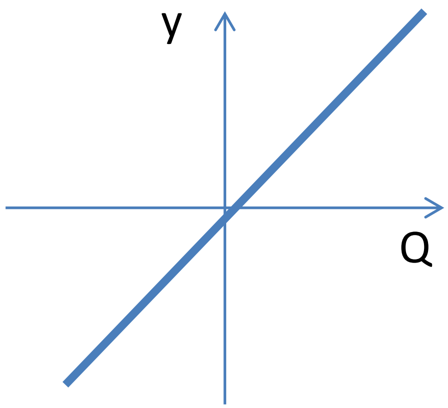
(or 2-d incompressible flow, or barotropic vorticity dynamics, or ...)

$$(u, v) = (-\psi_y, \psi_x)$$

$$Q = \nabla_h^2 \psi$$

$$\left(\frac{\partial}{\partial t} + \boldsymbol{u} \cdot \nabla\right) Q = 0$$

Rossby waves I: Uniform gradient

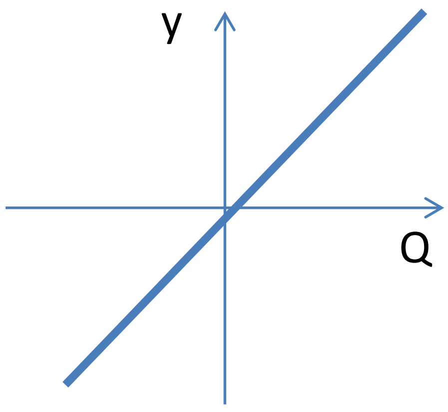


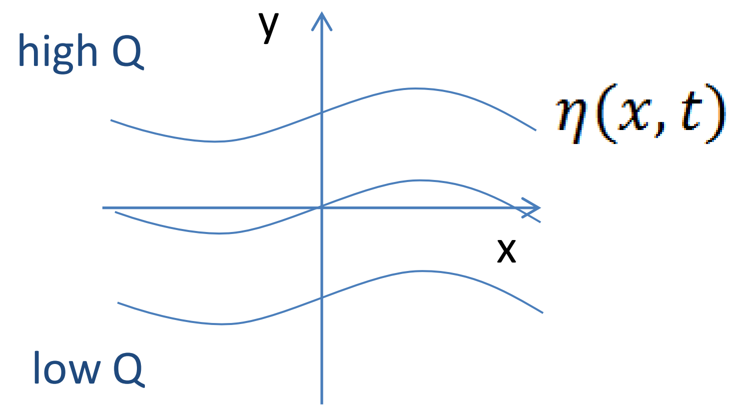
$$Q = \beta y + q$$

2-d Euler

$$(u, v) = (-\psi_y, \psi_x)$$
$$Q = \nabla_h^2 \psi$$
$$\left(\frac{\partial}{\partial t} + \boldsymbol{u} \cdot \nabla\right) Q = 0$$

Rossby waves I: Uniform gradient





$$Q = \beta y + q \qquad \Rightarrow \frac{\partial \eta}{\partial t} + U_0 \frac{\partial \eta}{\partial x} = v$$

$$\eta = \eta_0 e^{ik(x-ct)}$$

$$\Rightarrow q = -\beta\eta \text{ and } \psi = \frac{\beta}{k^2}\eta$$

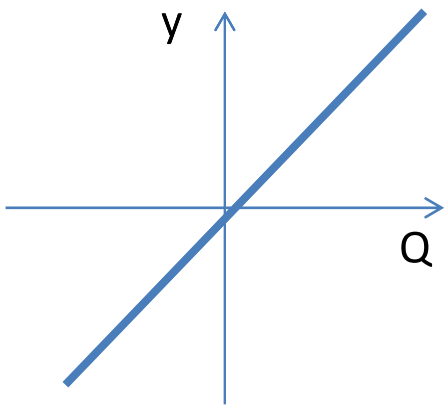
2-d Euler

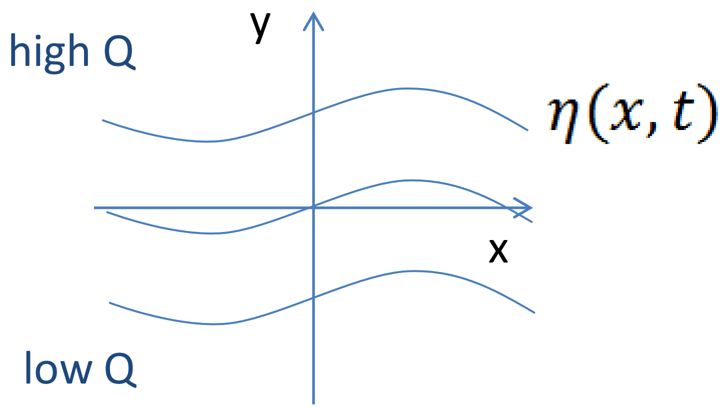
$$(u, v) = (-\psi_y, \psi_x)$$

$$Q = \nabla_h^2 \psi$$

$$\left(\frac{\partial}{\partial t} + \boldsymbol{u} \cdot \nabla\right) Q = 0$$

Rossby waves I: Uniform gradient





$$Q = \beta y + q \qquad \Rightarrow \frac{\partial \eta}{\partial t} + U_0 \frac{\partial \eta}{\partial x} = v$$

$$\eta = \eta_0 e^{ik(x-ct)}$$

$$\Rightarrow q = -\beta\eta \text{ and } \psi = \frac{\beta}{k^2}\eta$$

$$c = U_0 - \frac{\beta}{k^2} \text{ and } c_g = U_0 + \frac{\beta}{k^2}$$

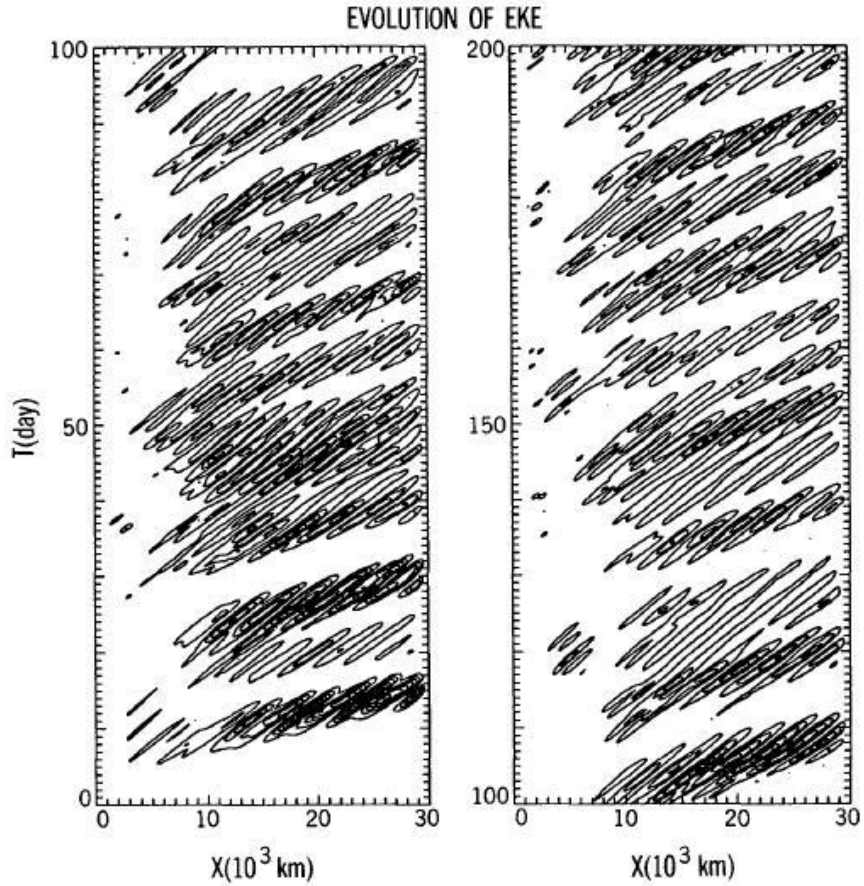
2-d Euler

$$(u, v) = (-\psi_y, \psi_x)$$

$$Q = \nabla_h^2 \psi$$

$$\left(\frac{\partial}{\partial t} + \boldsymbol{u} \cdot \nabla\right) Q = 0$$

Rossby waves I: Uniform gradient



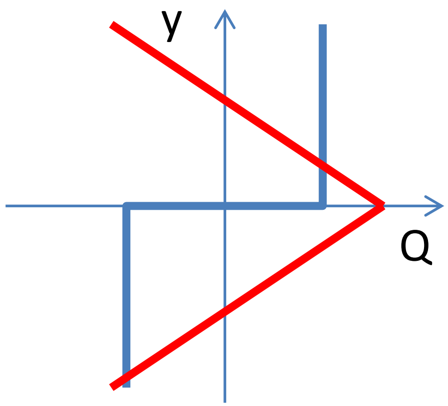
Chang & Orlanski (1993)

$$c = U_0 - \frac{\beta}{k^2} \text{ and } c_g = U_0 + \frac{\beta}{k^2}$$

2-d Euler

$$(u, v) = (-\psi_y, \psi_x)$$
$$Q = \nabla_h^2 \psi$$
$$\left(\frac{\partial}{\partial t} + \boldsymbol{u} \cdot \nabla\right) Q = 0$$

Rossby waves II: Front



$$Q = \begin{cases} \frac{Q_0}{2} & \text{for } y > 0 \\ -\frac{Q_0}{2} & \text{for } y < 0 \end{cases}$$

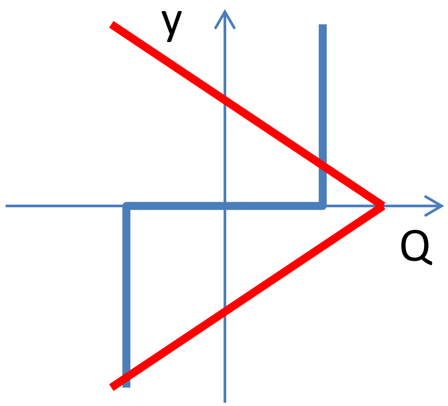
Basic state:

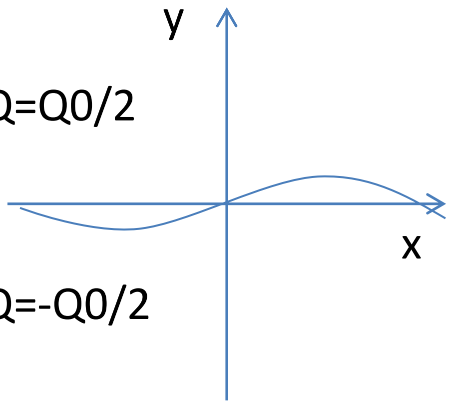
$$U(y) = U_0 - Q_o|y|$$

2-d Euler

$$(u, v) = (-\psi_y, \psi_x)$$
$$Q = \nabla_h^2 \psi$$
$$\left(\frac{\partial}{\partial t} + \boldsymbol{u} \cdot \nabla\right) Q = 0$$

Rossby waves II: Front





$$Q = \begin{cases} \dfrac{Q_0}{2} & \text{for } y > 0 \\ -\dfrac{Q_0}{2} & \text{for } y < 0 \end{cases} \Rightarrow \frac{\partial \eta}{\partial t} + U\frac{\partial \eta}{\partial x} = v$$

$$\eta = \eta_0 e^{ik(x-ct)}$$

$$\Rightarrow q \approx -Q_0 \delta(y)\eta \text{ and } \psi \approx e^{-|ky|}\frac{Q_0}{|k|}\eta$$

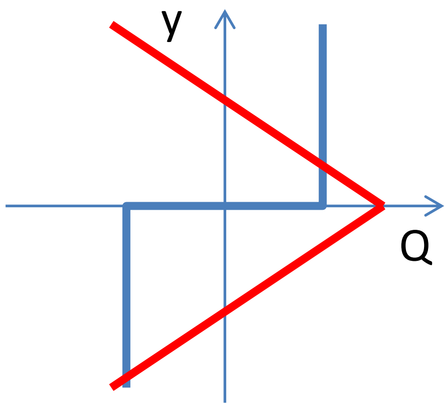
2-d Euler

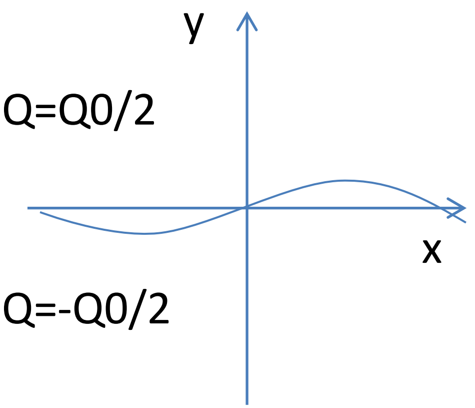
$$(u, v) = (-\psi_y, \psi_x)$$

$$Q = \nabla_h^2 \psi$$

$$\left(\frac{\partial}{\partial t} + \boldsymbol{u} \cdot \nabla\right) Q = 0$$

Rossby waves II: Front





$$Q = \begin{cases} \frac{Q_0}{2} & \text{for } y > 0 \\ -\frac{Q_0}{2} & \text{for } y < 0 \end{cases} \Rightarrow \frac{\partial \eta}{\partial t} + U\frac{\partial \eta}{\partial x} = v$$

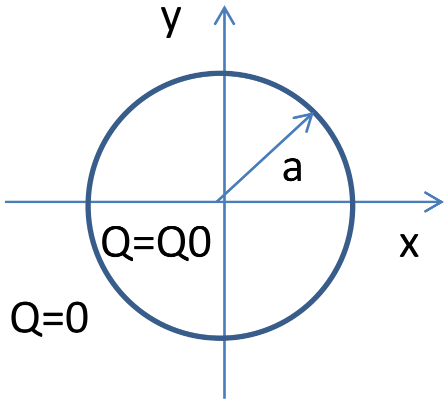
$$\eta = \eta_0 e^{ik(x-ct)}$$
$$\Rightarrow q \approx -Q_0 \delta(y)\eta \text{ and } \psi \approx e^{-|ky|}\frac{Q_0}{|k|}\eta$$

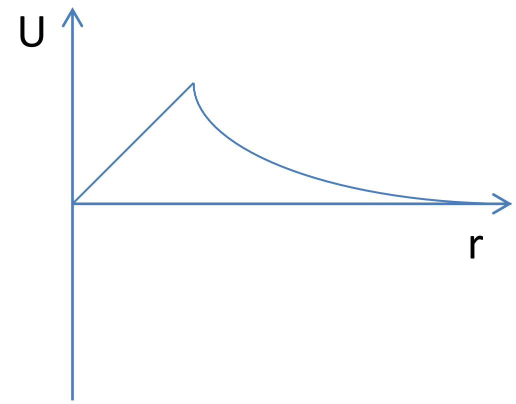
$$c = U_0 - \frac{Q_0}{|k|} \text{ and } c_g = U_0 + 0$$

2-d Euler

$$(u, v) = (-\psi_y, \psi_x)$$
$$Q = \nabla_h^2 \psi$$
$$\left(\frac{\partial}{\partial t} + \boldsymbol{u} \cdot \nabla\right) Q = 0$$

Rossby waves III: Rankine vortex





$$\eta = a + \eta_0 e^{in(\theta - ct/a)}$$

$$c = \frac{aQ_0}{2}\left(1 - \frac{1}{n}\right) \text{ and } c_g = \frac{aQ_0}{2}$$

2-d Euler

$$(u, v) = (-\psi_y, \psi_x)$$

$$Q = \nabla_h^2 \psi$$

$$\left(\frac{\partial}{\partial t} + \boldsymbol{u} \cdot \nabla\right) Q = 0$$

	Uniform gradient waves	Frontal waves	Rankine vortex
2-d Euler dynamics	$c = U_0 - \frac{\beta}{k^2}$	$c = U_0 - \frac{Q_0}{\lvert k \rvert}$	$c = \frac{aQ_0}{2}\left(1 - \frac{1}{n}\right)$

	Uniform gradient waves	Frontal waves	Rankine vortex
2-d Euler dynamics	$c = U_0 - \frac{\beta}{k^2}$	$c = U_0 - \frac{Q_0}{\lvert k \rvert}$	$c = \frac{aQ_0}{2}\left(1 - \frac{1}{n}\right)$
Surface QG dynamics			

QG dynamics

$$(u, v, \theta) = (-\psi_y, \psi_x, \psi_z)$$

QGPV: $Q = f + \nabla^2 \psi$

In the interior: $\left(\frac{\partial}{\partial t} + \boldsymbol{u} \cdot \nabla\right) Q = 0$

At horizontal boundaries: $\left(\frac{\partial}{\partial t} + \boldsymbol{u} \cdot \nabla\right) \theta = 0$

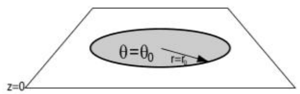
Two 2-d limits

No z variations
(and theta constant on boundaries)

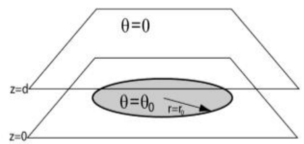
2-d Euler dynamics

No interior PV (Q=0)

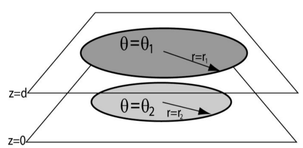
Surface QG dynamics



Capped surface QG dynamics



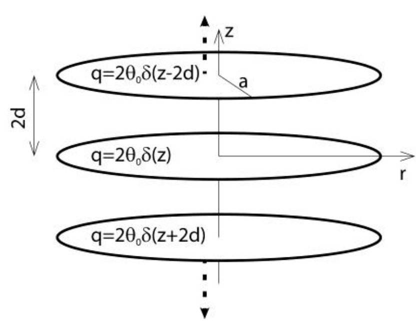
Uniform PV dynamics



Two 2-d limits

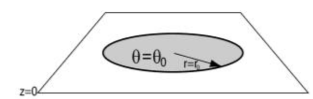
No z variations
(and theta constant on boundaries)

2-d Euler dynamics

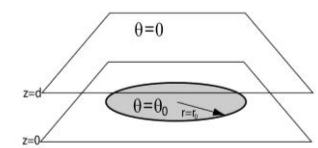


No interior PV (Q=0)

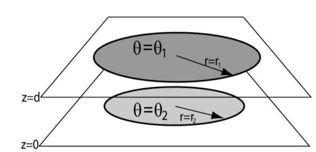
Surface QG dynamics



Capped surface QG dynamics



Uniform PV dynamics



Surface QG dynamics

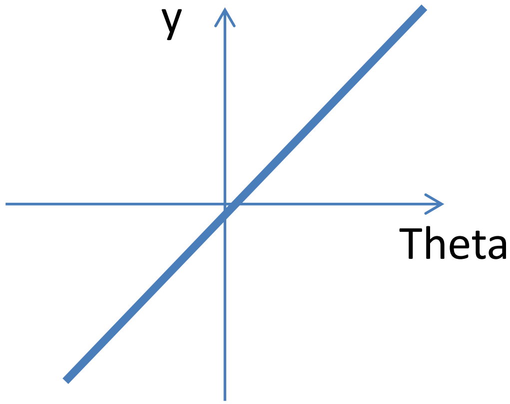
$$(u, v, \theta) = (-\psi_y, \psi_x, \psi_z)$$

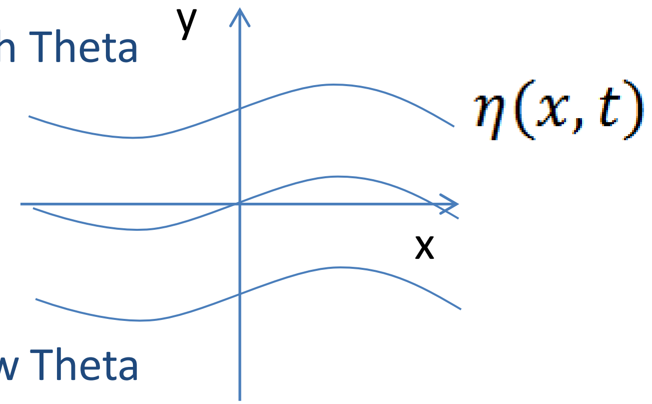
$$\nabla^2 \psi = 0$$

$$\theta \rightarrow 0 \text{ at } z \rightarrow \infty$$

$$\left(\frac{\partial}{\partial t} + \boldsymbol{u} \cdot \nabla\right)\theta = 0$$

Surface QG waves I: Uniform gradient





$$\Theta = \Lambda y + \theta \qquad \Rightarrow \frac{\partial \eta}{\partial t} + U_0 \frac{\partial \eta}{\partial x} = v$$

$$\eta = \eta_0 e^{ik(x-ct)}$$

$$\Rightarrow \theta = -\Lambda \eta \text{ and } \psi = e^{-|kz|} \frac{\Lambda}{|k|} \eta$$

$$c = U_0 - \frac{\Lambda}{|k|} \text{ and } c_g = U_0 + 0$$

Surface QG

$$(u, v, \theta) = (-\psi_y, \psi_x, \psi_z)$$

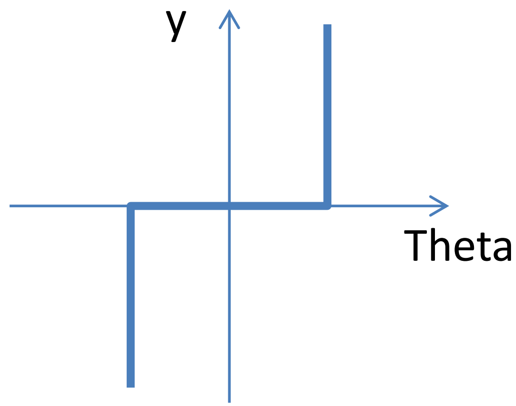
$$\nabla^2 \psi = 0$$

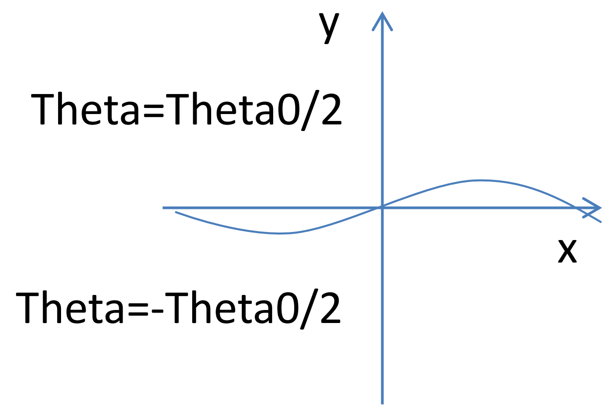
$$\theta \to 0 \text{ at } z \to \infty$$

$$\left(\frac{\partial}{\partial t} + \boldsymbol{u} \cdot \nabla\right) \theta = 0$$

	Uniform gradient waves	Frontal waves	Rankine vortex
2-d Euler dynamics	$c = U_0 - \frac{\beta}{k^2}$	$c = U_0 - \frac{Q_0}{\lvert k \rvert}$	$c = \frac{aQ_0}{2}\left(1 - \frac{1}{n}\right)$
Surface QG dynamics	$c = U_0 - \frac{\Lambda}{\lvert k \rvert}$		

Surface QG waves II: Front





$$\Theta = \begin{cases} \frac{\Theta_0}{2} \text{ for } y > 0 \\ -\frac{\Theta_0}{2} \text{ for } y < 0 \end{cases}$$

Surface QG

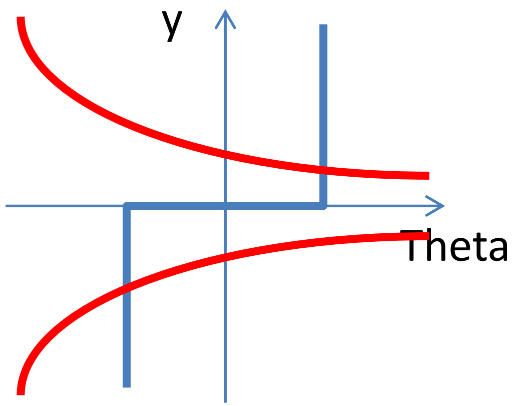
$$(u, v, \theta) = (-\psi_y, \psi_x, \psi_z)$$

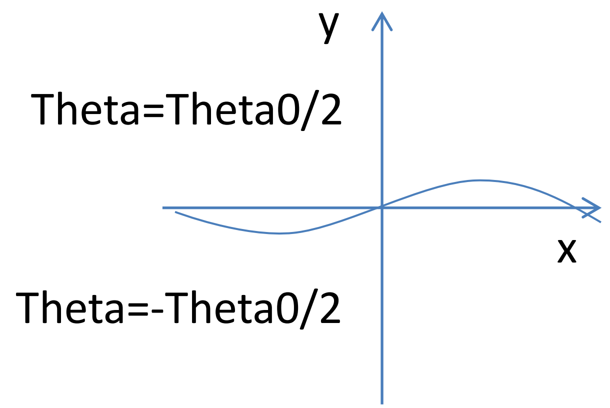
$$\nabla^2 \psi = 0$$

$$\theta \to 0 \text{ at } z \to \infty$$

$$\left(\frac{\partial}{\partial t} + \boldsymbol{u} \cdot \nabla\right) \theta = 0$$

Surface QG waves II: Front





$$\Theta = \begin{cases} \frac{\Theta_0}{2} \text{ for } y > 0 \\ -\frac{\Theta_0}{2} \text{ for } y < 0 \end{cases}$$

Basic state (3-d fields):

$$U(y,z) = U_0 - \frac{\Theta_0}{2} \log \sqrt{y^2 + z^2}$$

$$\Theta(\mathrm{y}, \mathrm{z}) = \frac{\Theta_0}{2} \tan^{-1}(\mathrm{y}/\mathrm{z})$$

Surface QG

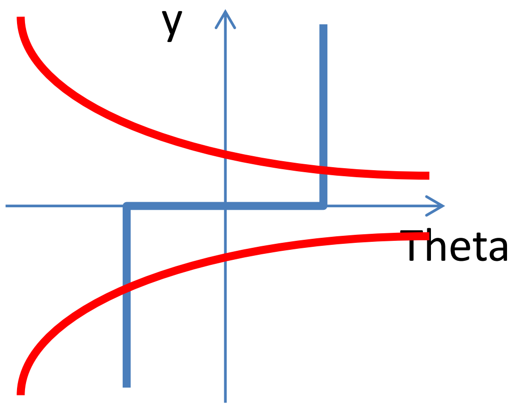
$$(u, v, \theta) = (-\psi_y, \psi_x, \psi_z)$$

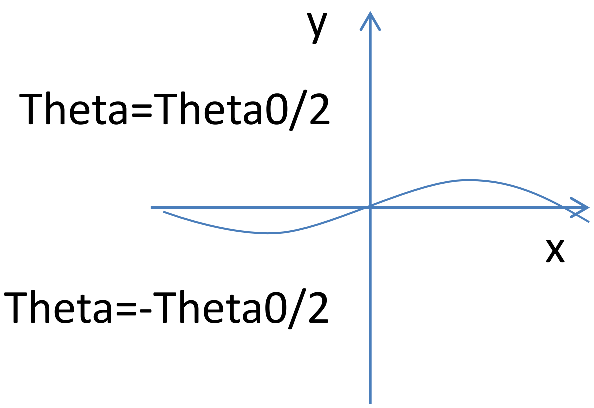
$$\nabla^2 \psi = 0$$

$$\theta \to 0 \text{ at } z \to \infty$$

$$\left(\frac{\partial}{\partial t} + \boldsymbol{u} \cdot \nabla\right) \theta = 0$$

Surface QG waves II: Front





$$\Theta = \begin{cases} \dfrac{\Theta_0}{2} & \text{for } y > 0 \\ -\dfrac{\Theta_0}{2} & \text{for } y < 0 \end{cases} \Rightarrow \frac{\partial \eta}{\partial t} + U\frac{\partial \eta}{\partial x} = v$$

$$\eta = \eta_0 e^{ik(x-ct)}$$

$$\Rightarrow \theta \approx -\Theta_0 \delta(y)\eta \text{ and } \psi \approx \Theta_0 K_0(k\sqrt{y^2+z^2})\eta$$

$$c = U_0 + \Theta_0 \log k \text{ and } c_g = c + \Theta_0$$

Surface QG

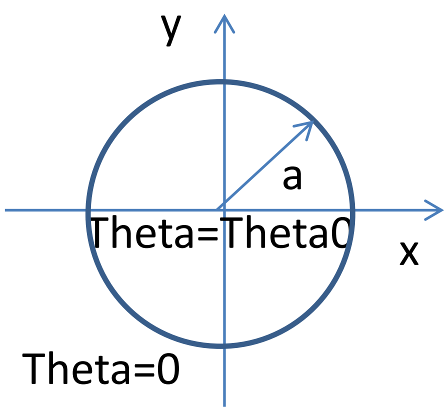
$$(u, v, \theta) = (-\psi_y, \psi_x, \psi_z)$$

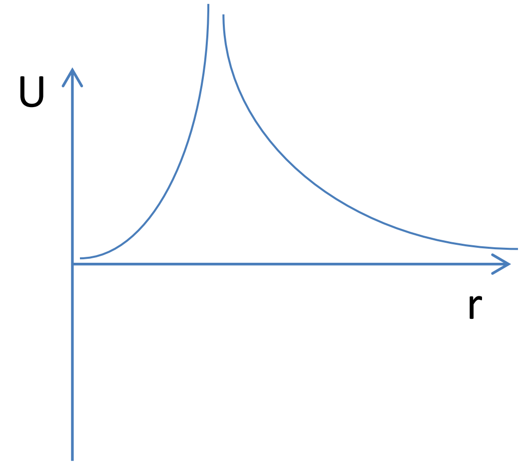
$$\nabla^2 \psi = 0$$

$$\theta \to 0 \text{ at } z \to \infty$$

$$\left(\frac{\partial}{\partial t} + \boldsymbol{u}\cdot\nabla\right)\theta = 0$$

Surface QG waves III: Rankine vortex





$$\eta = a + \eta_0 e^{in(\theta - ct/a)}$$

Surface QG

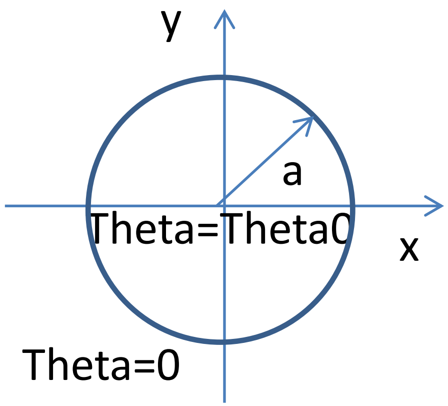
$$(u, v, \theta) = (-\psi_y, \psi_x, \psi_z)$$

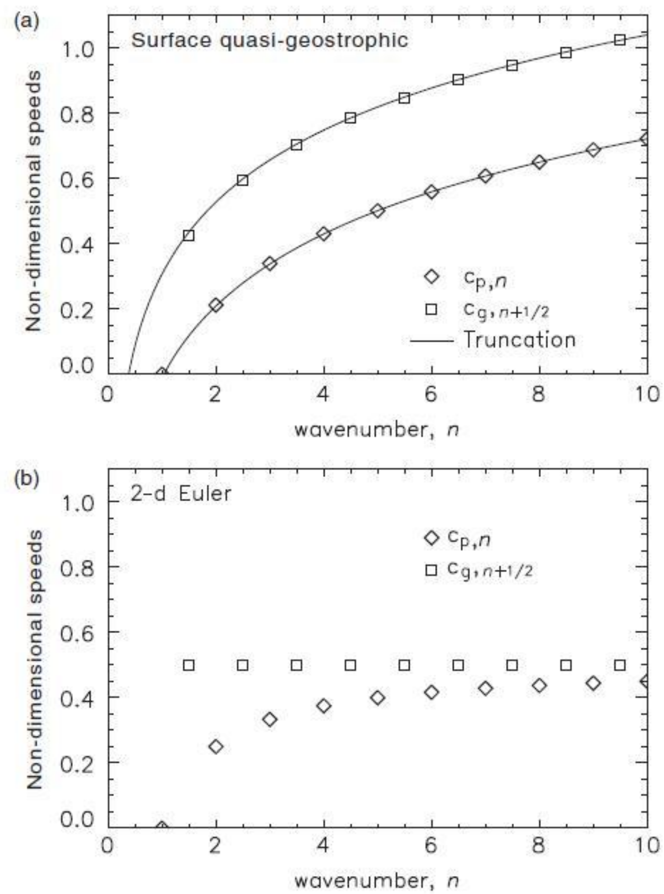
$$\nabla^2 \psi = 0$$

$$\theta \to 0 \text{ at } z \to \infty$$

$$\left(\frac{\partial}{\partial t} + \boldsymbol{u} \cdot \nabla\right) \theta = 0$$

Surface QG waves III: Rankine vortex





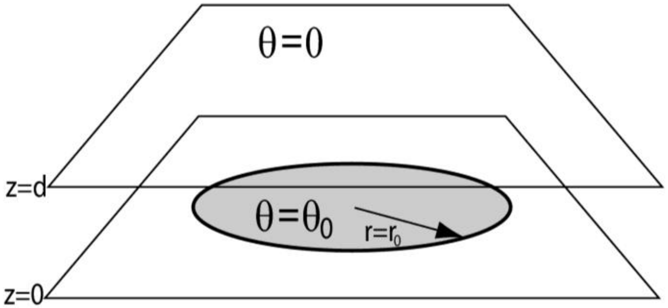
$$\eta = a + \eta_0 e^{in(\theta - ct/a)}$$

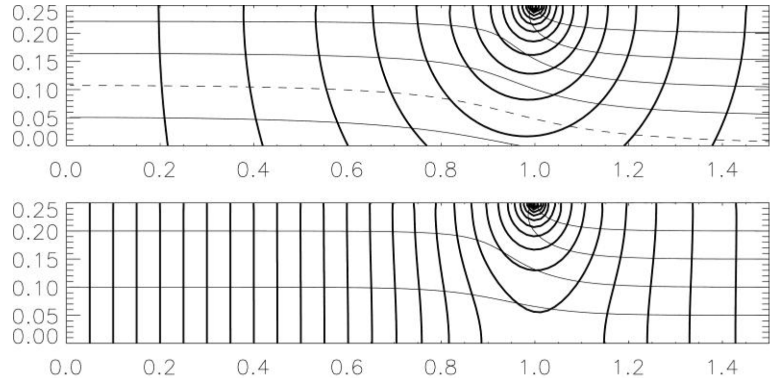
$$c = \Theta_0 \sum_{j=2}^{n} \frac{1}{j - 1/2} \text{ and } c_g = c + \Theta_0$$

	Uniform gradient waves	Frontal waves	Rankine vortex
2-d Euler dynamics	$c = U_0 - \frac{\beta}{k^2}$	$c = U_0 - \frac{Q_0}{\lvert k \rvert}$	$c = \frac{aQ_0}{2}\left(1 - \frac{1}{n}\right)$
Surface QG dynamics	$c = U_0 - \frac{\Lambda}{\lvert k \rvert}$	$c = U_0 + \Theta_0 \log k$	$c = \Theta_0 \sum_{j=2}^{n} \frac{1}{j - 1/2}$

	Uniform gradient waves	Frontal waves	Rankine vortex
2-d Euler dynamics	$c = U_0 - \frac{\beta}{k^2}$	$c = U_0 - \frac{Q_0}{\lvert k \rvert}$	$c = \frac{aQ_0}{2}\left(1 - \frac{1}{n}\right)$
Surface QG dynamics	$c = U_0 - \frac{\Lambda}{\lvert k \rvert}$	$c = U_0 + \Theta_0 \log k$	$c = \Theta_0 \sum_{j=2}^{n} \frac{1}{j - 1/2}$
Capped surface QG dynamics	$c = U_0 - \frac{\Lambda}{\lvert k \rvert \tanh(\lvert k \rvert d)}$	?	
	Eady model (Davies & Bishop (1994))	Juckes model	New model

The axisymmetric model



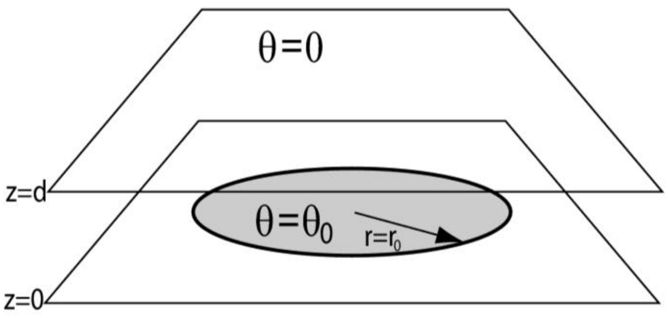


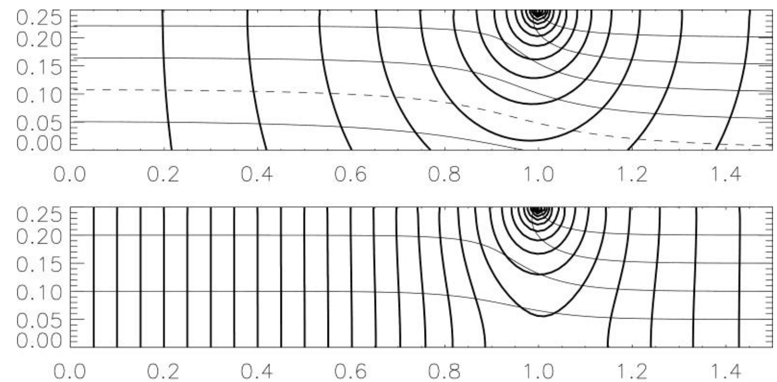
Surface QG dynamics

Capped surface QG dynamics

Use a=4d

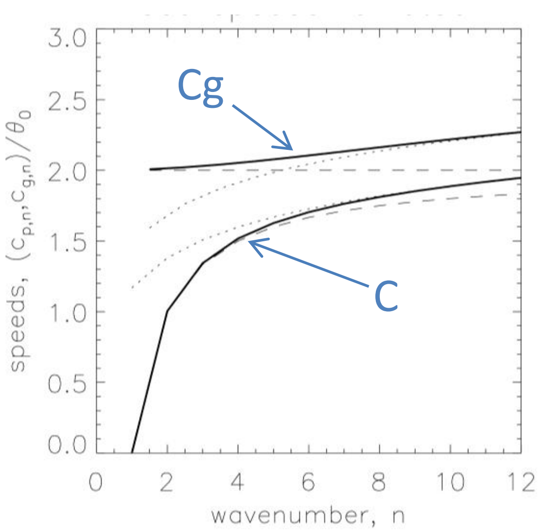
The axisymmetric model



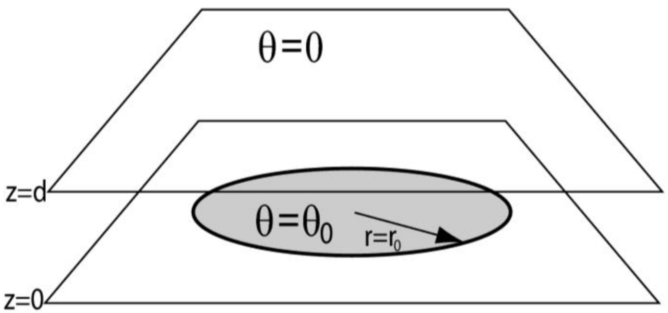


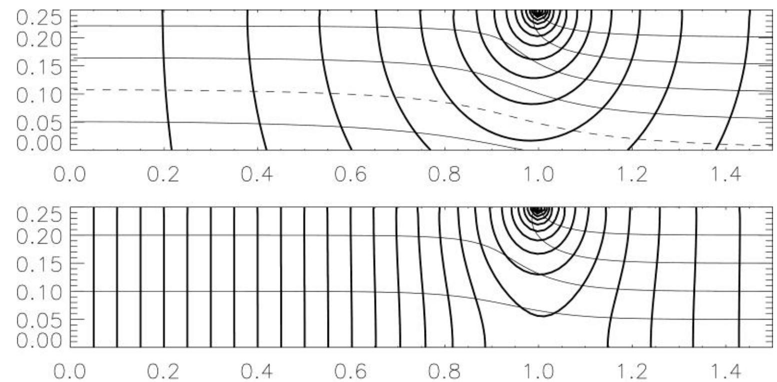
Surface QG dynamics

Capped surface QG dynamics



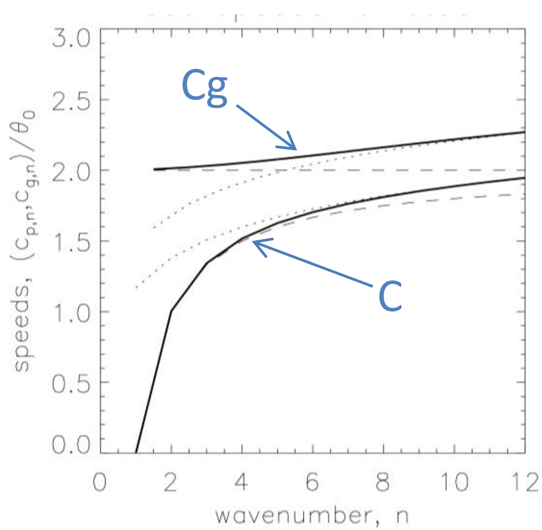
The axisymmetric model

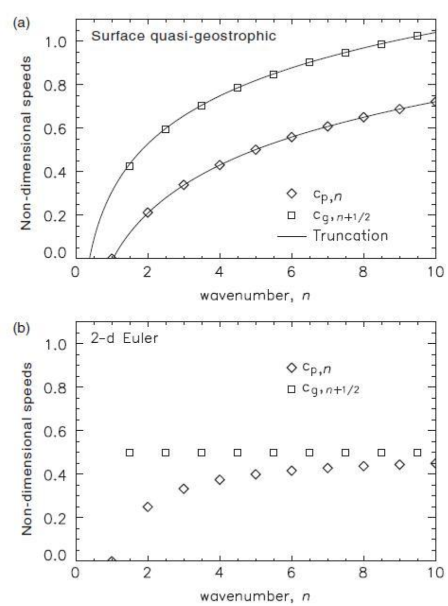




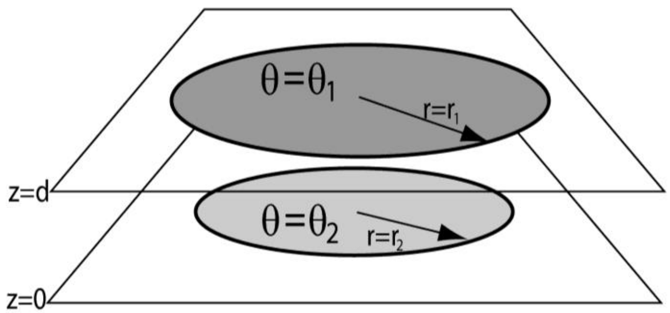
Surface QG dynamics

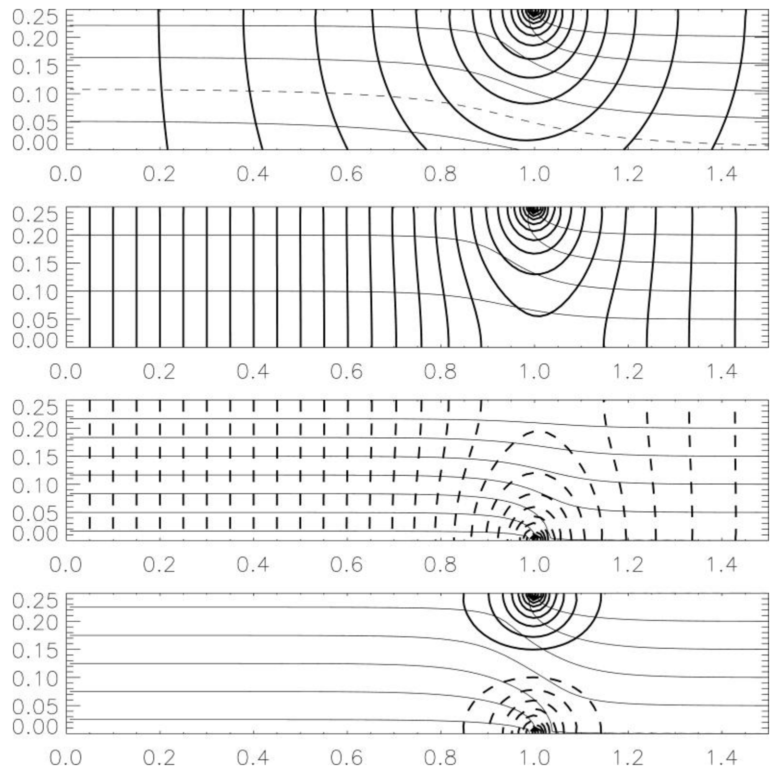
Capped surface QG dynamics





The axisymmetric model





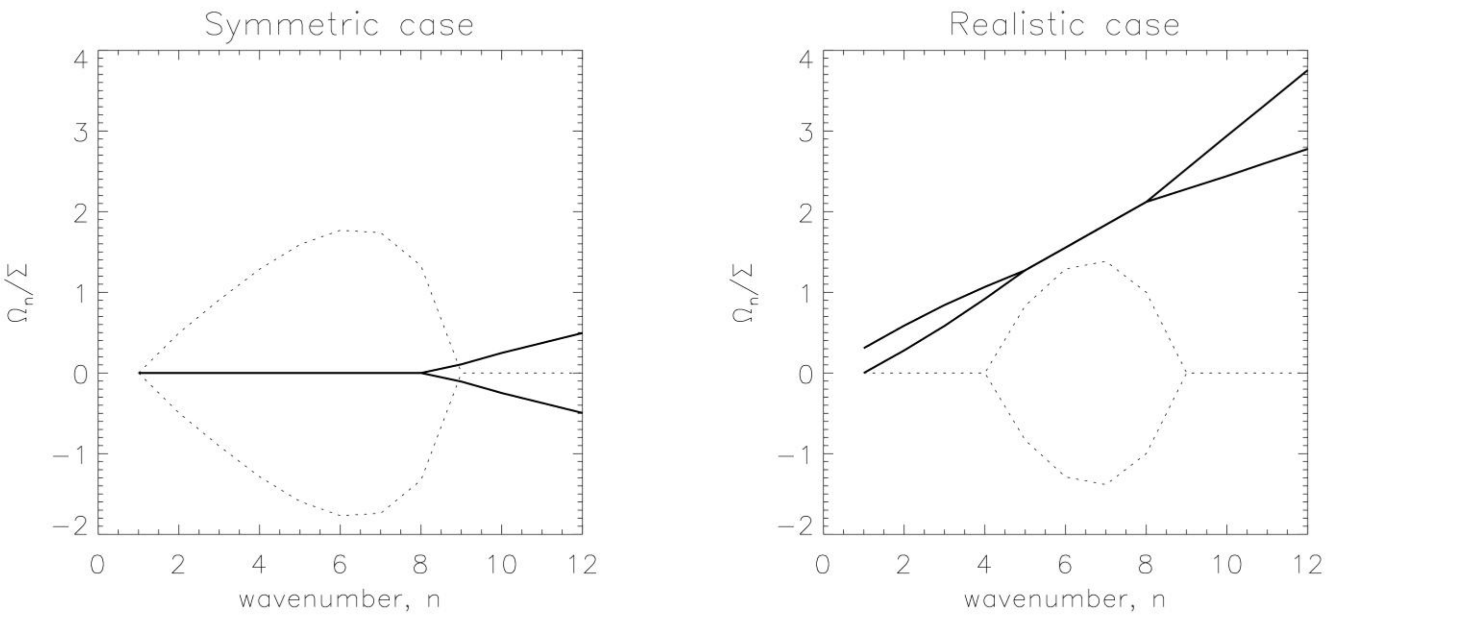
Surface QG dynamics

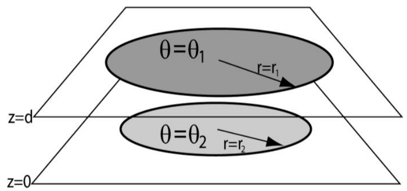
Capped surface QG dynamics

Capped surface QG dynamics

Uniform PV dynamics

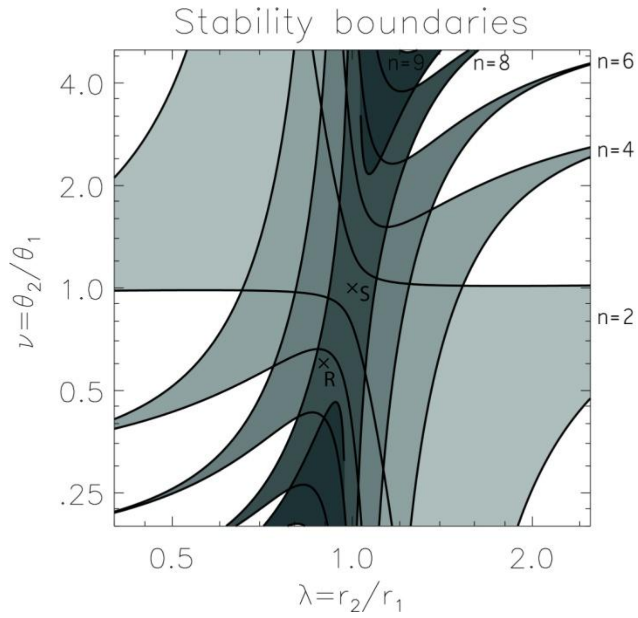
The axisymmetric model

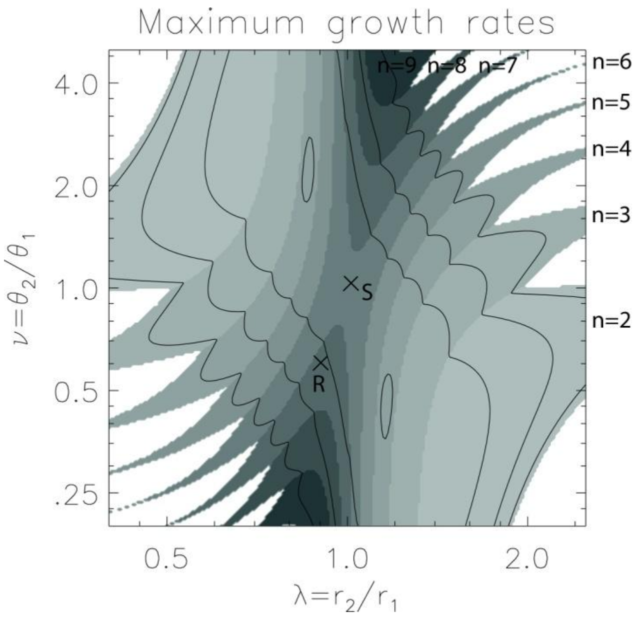


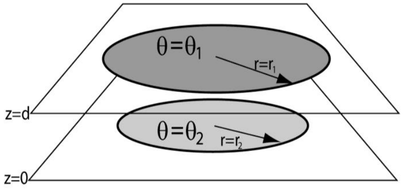


$$\Sigma = \frac{\theta_1 + \theta_2}{2d} \approx 0.7 day^{-1}$$

The axisymmetric model







The axisymmetric model – summary

Analytic model, similar to Eady/Juckes models

Growth rates of similar order of magnitude to Eady

Wide range of basic states can be explored

Questions:

Is there a simple 'Eady growth rate' diagnostic for this model?
What does the nonlinear wave breaking look like?
What is the zonal mean state after wave breaking?