The propagation of Rossby waves on a smooth potential vorticity front

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1. Motivation

The isentropic gradient of potential vorticity (PV) at the tropopause is typically weaker in global numerical weather forecasts than in corresponding analyses (Gray et al., 2014).

The average PV gradient declines with lead time and tends towards a steady value dependent on resolution (Figure 1).

Key Question: **Does this systematic forecast error influence the propagation and evolution of large-scale Rossby waves?**

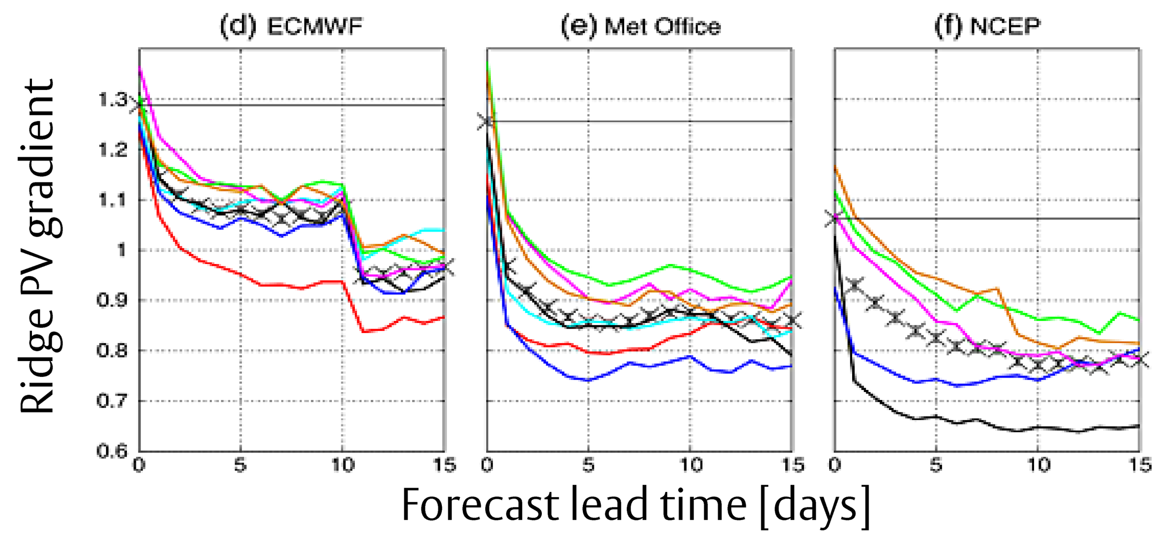

Figure 1 The average PV gradient (units: PVU per 100km) at the northern edge of northern hemisphere wintertime ridges as a function of lead time for ECMWF, Met Office and NCEP operational global forecasts. Each line represents a single winter between 2006-2012 and black markers indicate the average over all winters. Adapted from Gray et al. (2014).

2. Illustration

We use the quasi-geostrophic shallow water model (QGSW) with a slightly-smoothed PV front representing the transition between tropospheric and tropospheric air at the tropopause.

Figure 2 highlights two potential issues, identified in a simple numerical simulation, caused by the PV front being too smooth:

- The **phase speed** of the Rossby wave is **too slow**
- The **amplitude** of the Rossby wave **decays with time**

This poster explores the first of these.

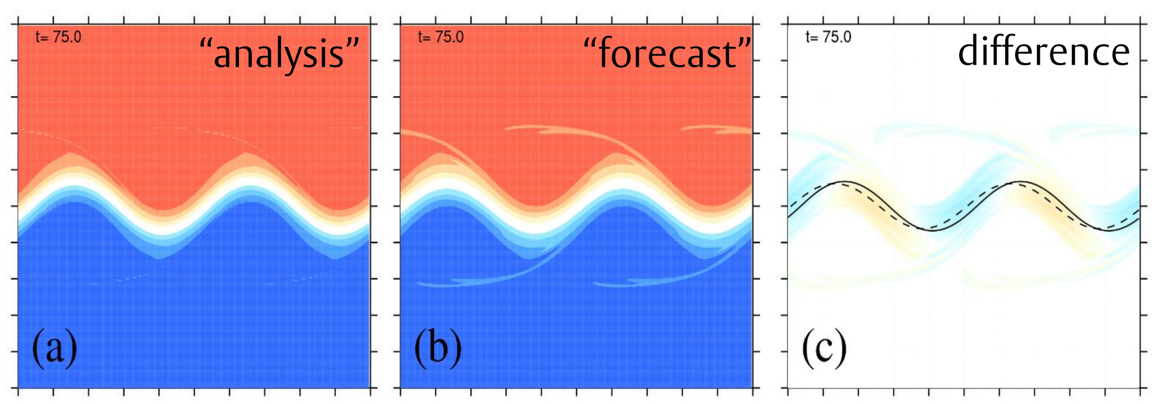

Figure 2 Simulations of large-amplitude Rossby waves on a smooth PV front. Panels (a) and (b) show PV (warm colours = high PV) for frontal widths corresponding approximately to 1.1PVU/100km and 1.3PVU/100km respectively, and panel (c) shows their difference (colours) together with the position of the central PV contour from (a; solid) and (b; dashed).

3. General considerations

The phase speed of a zonally-propagating Rossby wave can be split into the sum of advection by the basic state jet and an intrinsic upstream propagation term:

$$c(k) = U_{adv} - c_{int}(k) \qquad (1)$$

Both of these terms will be modified by smoothing the front.

Question: **Is the net effect of smoothing an increase or a decrease of the phase speed, and how large is the change?**

To answer this we extend the analytic theory of waves on a sharp PV front to the case of a slightly-smoothed PV front.

4. Waves on a sharp PV front

The derivation of the dispersion relation for Rossby waves on a sharp PV front is standard theory. Here we present a brief outline derivation.

Basic state (Fig 3, red lines):

$$Q(y) = f_0 + \frac{Q_0}{2}\mathrm{sgn}(y) \qquad U(y) = \frac{Q_0 L_R}{2} e^{-|y|/L_R} \qquad (2)$$

Dispersion relation (Fig 4b, red lines):

1. perturb the contour $y = 0$ to $y = \eta(x,t) = \hat{\eta}_0 e^{ik(x-ct)}$
2. substitute into the linear continuity equation to find:

$$c(k) = U(0) - \phi(0) = \frac{Q_0 L_R}{2}\left(1 - \frac{1}{\kappa}\right) \qquad (3)$$

where $\phi(y) = \frac{Q_0 L_R}{2} \frac{e^{-\kappa|y|/L_R}}{\kappa}$ represents the meridional structure of the perturbation and $\kappa^2 = 1 + k^2 L_R^2$ is the effective wavenumber. Equation (3) says that waves on a sharp PV front are advected at the speed of the jet maximum and propagate upstream at speed proportional to κ^{-1}.

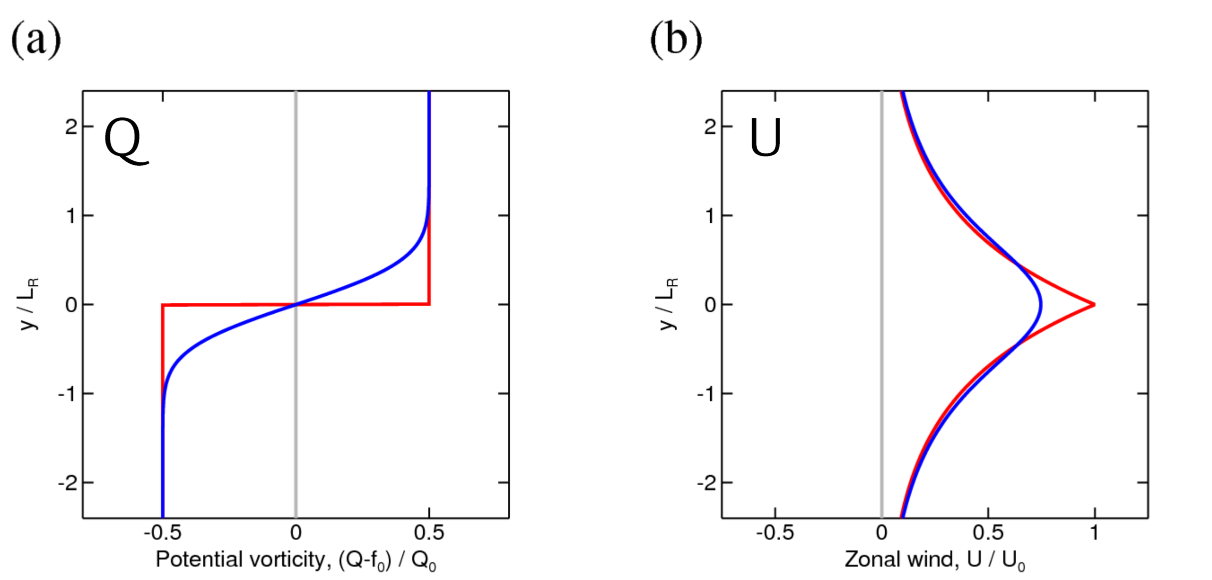

Figure 3 Basic state PV (a) and wind (b) profiles for the sharp PV front (red) and for the smooth front with a Gaussian weighting function (blue). PV is scaled by Q_0 and wind speeds by $U_0 \equiv Q_0 L_R/2$.

5. Waves on a smooth PV front

To generalise (3) to a slightly-smoothed PV front we first generate a suitable basic state by convolving (2) with a weighting function of width r_0, e.g. using a Gaussian:

$$w(y) = W(y/r_0)/r_0 \quad \text{with} \quad W(Y) = \frac{e^{-Y^2/2}}{\sqrt{2\pi}}$$

Basic state (Fig 3, blue lines):

$$Q_s(y) = \int Q(y')w(y-y')dy' \qquad U_s(y) = \int U(y')w(y-y')dy'$$

Dispersion relation (Fig 4, blue lines):

Unlike for the sharp PV front, normal modes are not readily calculated for general $w(y)$: in shear flows with single-signed PV gradients the phase between perturbations at different y cannot be maintained indefinitely. However, when $\epsilon \equiv kr_0 \ll 1$ coherent meandering disturbances are observed to exist for finite time (e.g. as in Fig 2), and we derive an approximate expression for their phase speed as follows:

1. perturb all contours $y = \tilde{y}$ to $y = \tilde{y} + \hat{\eta}(\tilde{y},t)e^{ikx}$
2. decompose the perturbation into meandering and internal components: $\hat{\eta}(y,t) = \hat{\eta}_m(t) + \hat{\eta}_i(y,t)$

   with $\hat{\eta}_m(t) = \int \hat{\eta}(y,t)w(y)dy$
3. substitute into the linear continuity equation to find:

$$i\frac{d\hat{\eta}_m}{dt} = kc_m\hat{\eta}_m + O(\epsilon^2 k\hat{\eta}_i) \qquad (4)$$

where $c_m = \int (U_s(y) - \phi_s(y))w(y)dy$

and $\phi_s(y) = \int \phi(y')w(y-y')dy'$.

Equation (4) says that the meandering component propagates approximately at speed c_m when ϵ is small, and this is verified numerically in Fig 4. The phase speed c_m consists of advection at the average of the jet speed weighted by the PV gradient and the upstream propagation similarly a smooth version of the sharp PV front case.

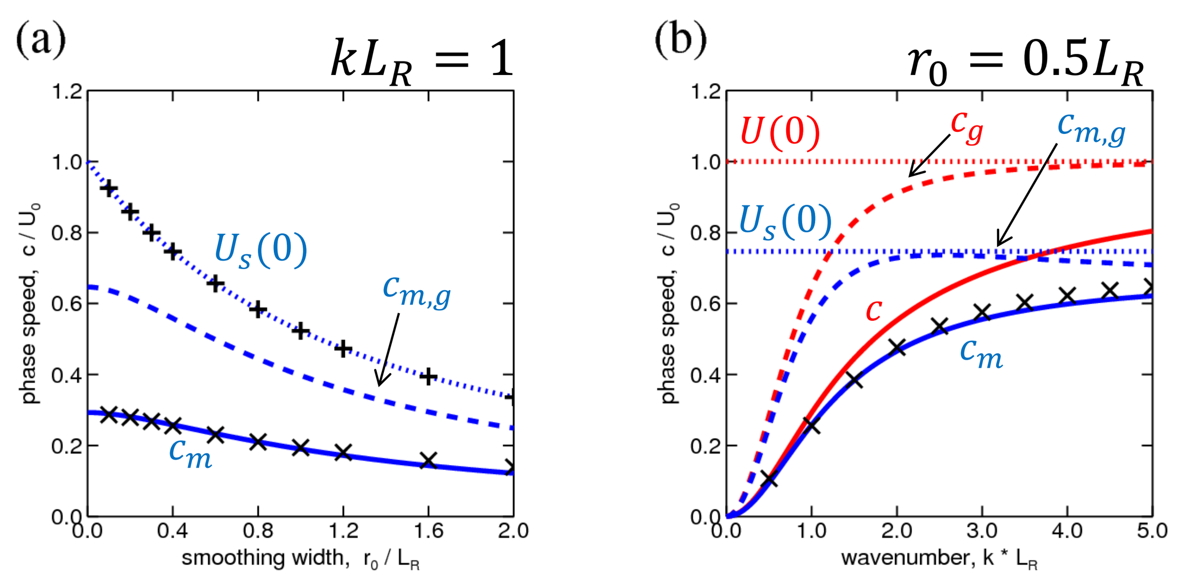

Figure 4 The smooth front dispersion relation evaluated for the Gaussian weighting function (blue). Both panels show phase (solid) and group (dashed) speeds and the maximum of the basic state jet (dotted). Black markers show results from numerical simulations and the red lines in panel (b) are the sharp PV front result of (2). All speeds are scaled by $U_0 \equiv Q_0 L_R/2$.

5. Properties (illustrated in Fig 4a)

1. c_m decreases with the smoothing width r_0
2. to first order the changes in U_{adv} and c_{int} cancel
3. however, the decrease in U_{adv} dominates at higher order.

This holds for any weighting function w and more generally for a wide range of PV inversion operators (Harvey et al, 2015).

6. Implications

Typical dimensional values for the QGSW PV front model are $L_R = 700$ km and $Q_0 = 2 \times 10^{-4}\mathrm{s}^{-1}$. Frontal widths of the troposphere-stratosphere boundary are estimated from the PV gradients in Fig 1a. Assuming a transition of 4 PVU gives widths of $r_0 = 308$ km and $r_0 = 381$ km for a typical analysis and forecast respectively.

Table 1 shows the resulting values of the jet maximum and the phase speed estimate c_m. There is a decrease of $1\mathrm{ms}^{-1}$ between the analysis and forecast values, which is around 5% and corresponds to a 400km displacement over a 5 day forecast.

	r_0 [km]	r_0/L_R	$U_s(0)$ [ms^{-1}]	$c_m(r_0)$ [ms^{-1}]	$d(kc_m)/dk(r_0)$ [ms^{-1}]
Sharp PV front	0	0	70.0	20.5	45.3
Typical analysis values	308	0.440	50.9	17.7	38.2
Typical 5-day forecast values	381	0.544	47.6	16.8	36.0

Table 1 Typical values for the sharp PV front (top) and smooth PV fronts with width based on the analysis (centre) and forecast (bottom) values in Fig 1. All values are for wavenumber $kL_R = 1$.

Key Results

- Smoothing a PV front to width r_0:
  - reduces the **jet maximum** by $O(r_0)$
  - reduces the Rossby wave **phase speeds** by $O(r_0^2)$
- There is a **leading-order compensation** in (1) between the change in **jet speed** and the change in the ability of waves to **propagate upstream**, rendering the phase speeds **relatively insensitive** to the PV smoothing
- However, the **decrease in jet speed dominates** at higher order: typically the difference between analysis and forecast values of r_0 may cause a phase speed error of order **1m/s, or around 5%,** for synoptic scale Rossby waves.

Gray, S. L, Dunning, C. M., Methven, J., Masato, G. and Chagnon, J. (2014) Systematic model forecast error in Rossby wave structure *Geophys. Res. Lett.* **41**, 2979-1987

For further details of this work please see:
Harvey, B. J., Methven, J. and Ambaum, M. H. P. (in prep for *J. Fluid Mech.*) The propagation of Rossby waves on a slightly-smoothed PV front