Estimating the sensitivity of urban surface drag to building morphology

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Declaration

I confirm that this is my own work and the use of all material from other sources has been properly and fully acknowledged.

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ABSTRACT

The near-surface wind flow velocity and turbulence profiles in urban areas are very different to those observed in rural areas and these differences are important considerations for pollution dispersion modelling and numerical weather prediction.

In urban areas, buildings increase the roughness of the surface which modifies the near-surface flow field and promotes the vertical exchange of momentum between the surface and the atmosphere. The roughness of the surface is often quantified in terms of the roughness length or the bulk drag coefficient and these values are strongly related to the size, shape and layout (morphology) of buildings in a neighbourhood. The morphology of the surface can be described quantitatively in terms of the building plan area index ($\lambda_P$) and frontal area index ($\lambda_F$). Existing algorithms which relate these two indices to the surface roughness often do so for idealised cube-shaped obstacles for which $\lambda_P = \lambda_F$.

In the first part of this study, a GIS analysis of buildings in the Greater London area was conducted and it showed that in typical neighbourhoods, the plan and frontal area indices are not equal and therefore on average buildings are non-cubic in shape.

In the second part of this study, the typical range of $\lambda_P$ and $\lambda_F$ values for London were used to design a series of non-cubic idealised obstacle arrays as part of a wind tunnel experiment. The integral momentum method was used to estimate the sensitivity of the bulk surface drag coefficient to the plan and frontal area index separately. The results prompted the modification of an existing algorithm capable of estimating the normalised roughness length for uniform height, non-cubic obstacle arrays.

In the final part of the study, the variation of the roughness length due to varying wind directions was considered. Four flow processes which change with wind direction were identified, of which the surface roughness was found to be most sensitive to the frontal area index.
“The higher you climb the mountain, the harder the wind blows”

Samuel Cummings

(1927 – 1998)
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Glossary of Terms

Standard terms

\( t \) time(s)
\( s = (x, y, z) \) displacement vector, decomposed into standard orthogonal co-ordinate system with \( x \) aligned with the mean wind
\( u = (u, v, w) \) velocity vector, decomposed into orthogonal co-ordinates \( u \) aligned with the mean wind
\( u', v', w' \) fluctuating velocity components for the three orthogonal dimensions (ms\(^{-1}\))
\( c_v \) coefficient of variation
\( C_R \) wind tunnel contraction ratio
\( \pi \) pi (3.142)
\( \varepsilon_U \) standard error of the measured velocity
\( \theta_i \) incident wind angle (deg)

Averaging operators

\( \bar{x} \) temporal average property of \( x \)
\( \langle x \rangle \) spatial average property of \( x \)

Subscripts

\( i, j, k \) general subscripts
\( \infty \) free-stream
Boundary layer variables

$z^*$ roughness sublayer depth (m)
$z^{**}$ inertial sublayer depth (m)
$\delta$ boundary layer depth (m)
$\delta^*$ displacement thickness (m)
$\theta$ momentum thickness (m)
$\kappa$ von Karman constant (0.4)
$\Phi$ momentum flux deficit (Nm$^{-2}$)

Fluid properties

$p$ air pressure (Pa)
$R$ specific gas constant (287 JK$^{-1}$K$^{-1}$)
$T$ temperature of air (K)
$\rho$ density of air (1.2 kgm$^{-3}$, at mean sea level and 293K)
$\nu$ kinematic viscosity of air (1.5 x 10$^{-5}$ m$^2$s$^{-1}$, at mean sea level and 293K)

Hot-wire anemometry variables

$E$ wire voltage (v)
$E_{os}$ dc off-set voltage (v)
$G$ voltage Gain
$f_{LP}$ low pass filter cut off frequency (Hz)
$OH$ overheat ratio
$R_o$ ambient wire resistance (Ω)
$R_L$ shorting probe resistance (Ω)
$R_W$ mean operating wire resistance (Ω)
$T_W$ mean operating wire temperature (K)
$\phi_{yaw}$ wire yaw angle (deg)
Obstacle array dimensions

\[ z_H \] obstacle height (m)
\[ z_{H_{\text{Max}}} \] maximum obstacle height (m)
\[ A_F \] obstacle frontal area (m²)
\[ A_P \] obstacle plan area (m²)
\[ A_T \] lot area (m²)
\[ D_X \] lot area length (m)
\[ D_Y \] lot area width (m)
\[ G_X \] obstacle wake length (m)
\[ L_X \] obstacle length (m)
\[ L_Y \] obstacle width (m)
\[ P_0 \] canopy porosity factor
\[ W_C \] free-flowing channel width (m)
\[ W_X \] inter-obstacle spacing length (m)
\[ W_Y \] inter-obstacle spacing width (m)
\[ \lambda_B \] obstacle surface area-to-plan area ratio
\[ \lambda_C \] channelling ratio
\[ \lambda_F \] frontal area density ratio
\[ \lambda_P \] plan area density ratio
\[ \lambda_S \] obstacle height-to-width ratio
\[ \sigma_H \] standard deviation of obstacle height
\[ \sigma_{\lambda_P} \] standard deviation of the plan area density of obstacles
\[ \rho_N \] density of obstacles (per unit area)
Surface variables

\( k_s \quad \) drag coefficient correction for the effects of the shear flow velocity profile
\( k_i \quad \) drag coefficient correction due to the effects of varying turbulence length scales
\( k_i \quad \) drag coefficient correction due to the effects of varying turbulence intensity
\( k_\phi \quad \) drag coefficient correction due to the effects of incident wind direction
\( k_r \quad \) drag coefficient correction due to the effects of the curvature of wall edges
\( u_* \quad \) friction velocity (ms\(^{-1}\))
\( x_0 \quad \) flow adjustment distance (m)
\( z_D \quad \) displacement height (m)
\( z_0 \quad \) surface roughness length (m)
\( C_D \quad \) bulk drag coefficient
\( C_{D(1)} \quad \) drag coefficient of an isolated roughness element
\( C_{D(S)} \quad \) sectional drag coefficient
\( C_{D(SUB)} \quad \) drag coefficient contribution from the substrate surface
\( C_{D(RE)} \quad \) drag coefficient contribution from the roughness elements
\( C_{D(u_*)} \quad \) bulk drag coefficient derived from the friction velocity
\( C_{D(\theta)} \quad \) bulk drag coefficient derived from integral momentum theory
\( F_X \quad \) force acting on a roughness element in the \( x \)-direction (N)
\( F_Y \quad \) force acting on a roughness element in the \( y \)-direction (N)
\( F_D \quad \) drag force (N)
\( L_C \quad \) canopy drag length scales (m)
\( L_H \quad \) length scale of homogeneity (m)
\( \tau \quad \) surface shear stress (Nm\(^{-2}\))
Chapter One

Introduction

The Earth’s atmosphere is the layer of gas that surrounds the planet and helps to maintain and protect life on Earth. From the surface the atmosphere extends some 100km in the vertical to the edge of outer space. The temperature and humidity variations at different heights define the different layers of the atmosphere. In terms of mass, almost 80% of the atmosphere is contained in the lowest 11km, in the layer known as the troposphere. It is here where the phenomena associated with daily weather patterns take place. Within the troposphere, the lowest 1-2km is where the Earth’s surface has a direct influence on the atmosphere on timescales of an hour or less (Stull, 1988). This layer is termed the planetary or atmospheric boundary layer (ABL) and responds to the mechanical and thermal forcing of the surface. The variations in the nature of the Earth’s surface result in a boundary layer that is constantly evolving through turbulent vertical mixing. Some of the earliest studies of the structure of the atmospheric boundary layer involved field measurements over flat terrain during the period from 1950 to 1970 (e.g. Lettau and Davidson, 1957; Businger et al., 1971). These field campaigns greatly improved our understanding of the lowest layer of the ABL known as the atmospheric surface layer. For example, the Monin-Obukhov similarity theory emerged as the undisputed scaling law for most surface layer parameters (Kaimal and Wyngaard, 1990). Although the understanding of physical processes in the ABL is now quite advanced, there still remains a broad scope for scientific research. One of the less-well understood parameters is the surface drag which quantifies the resistance of the wind flow due to surface features. The surface drag has a strong influence on the velocity and turbulence structure of the near-surface flow. The research in this thesis focuses on the estimation of surface drag in urban areas due to the geometry and layout of buildings. The flow dynamics of the atmospheric boundary layer has an impact on several areas of study including aeronautics, agriculture, wind engineering, pollution dispersion and numerical weather prediction.
1.1 Urban meteorology

1.1.1 Motivation
Within the general study of boundary layer meteorology there are many sub-fields of study developed largely from our need to understand localised effects. Amongst them is the relatively modern branch of urban meteorology. The world's population is becoming increasingly urbanised and a United Nations estimate suggests that for the first time ever there are now more people living in towns and cities than in rural areas (United Nations, 2003). The urban atmosphere has influence on many aspects of urban life. The surface texture of urban areas results in towns and cities being relatively warmer and drier than the rural surroundings. This results in the now well known phenomenon, referred to as the urban heat island in which the nature of the urban surface results in local temperatures being several degrees warmer. The temperature difference is greater during the night and in the winter months. The warmer climate may be welcome in towns and cities which experience persistent cold weather but the accumulation of heat can have fatal consequences such as those witnessed during the summer heatwave of 2003 in Central Europe. The causes of the urban heat island phenomenon are detailed in Oke (1982) and it is mainly due to (i) the lack of thermal radiation from the surface to the atmosphere due to the blockage effect of tall buildings, (ii) the shortage of vegetation which lowers the rate of evapotranspiration to the atmosphere, (iii) the thermal properties of the materials used to construct buildings which tend to store heat and lower the albedo value and (iv) the blockage of wind by buildings restricting ventilation of heat away from the surface. Thus the physics of the urban boundary layer comprises primarily of (i) the thermal effects involving the energy balance of the surface and (ii) the wind flow dynamics which governs the momentum exchange between the surface and the atmosphere aloft. The focus of the present study is to investigate the latter topic concerning the dynamics of wind flow which is significantly influenced by the roughness of the underlying surface. The surface roughness is quantified in terms of an aerodynamic drag force which arises primarily due to (i) the effects of fluid viscosity as the wind flows across the surface and (ii) the pressure difference generated around surface obstacles. The surface drag results in highly turbulent flow which promotes the vertical and horizontal mixing of the air. The ability to quantify the roughness of the urban surface has important implications in numerical weather prediction and urban pollution dispersion modelling (Best et al., 2006).
The study of urban wind flow extends beyond meteorology into the increasingly important field of wind engineering. Cermak (1975) suggested that ‘wind engineering was best defined as the rational treatment of the interactions between wind in the atmospheric boundary layer and man and his works on the surface of Earth.’ Thus wind engineers are particularly concerned with the structure of the urban boundary layer in order to

- test loads on buildings and structures
- assess the frequency and probability of wind occurrence
- assess internal building ventilation and external heat transfer
- design and identify suitable locations for wind turbines
- maximise pedestrian comfort
- predict the atmospheric dispersion of pollutants

As research in urban meteorology expanded, wind engineers began to gain a better understanding of wind velocity and turbulence within the building canopy. However, in recent years advances in building construction and architecture have led to the development of more complicated and taller buildings. Thus, wind engineers have had to deal with newer problems such as the aerodynamics of irregular building shapes (Calhoun et al., 2004), interference effects induced by local heterogeneous surroundings (Khanduri et al., 1998) and the dynamics of the wind flow much higher in the ABL (Li et al., 1998). These areas of wind engineering research are very much in their infancy and research into urban surface drag in the present study will go some way to complementing existing research in this field.

1.1.2 Horizontal spatial scales

Flow across the urban surface is influenced by features spanning lengthscales ranging from a few metres to tens of kilometres in the horizontal and tens of metres in the vertical dimensions. This range of spatial scales results in temporal scales of the flow ranging from a few seconds to several hours. Britter and Hanna (2003) divide the horizontal spatial extent into four length scales and briefly describe them as follows:

- Street scale / Microscale - (less than 100 to 200m)
  This scale encompasses individual buildings and streets and concerns the understanding of turbulence generated by small scale features such as street furniture and vehicles.
Neighbourhood / Local scale - (up to 1 or 2km)
At the neighbourhood scale the focus is on how the flow interacts with groups of buildings and streets. The flow is significantly influenced by the geometry and layout of obstacles such as buildings.

City scale - (up to 10 or 20km)
This scale encompasses the diameter of the average urban area where the response of the flow due to individual streets and buildings is averaged out into a bulk effect.

Regional Scale / Mesoscale - (up to 100 or 200km)
At the regional scale, the influence of the town or city extends beyond its boundaries and large scale phenomena such as urban heat island circulations occur. Pollution plumes can extend to considerable downstream distances and the urban area represents a perturbation that can decelerate and deflect the wind flow.

Modern numerical weather prediction (NWP) models are now implementing horizontal grid box resolution of the order of the neighbourhood scale. In many urban areas, grid resolutions of that order may encompass a few hundred buildings. The contribution of these buildings to the vertical momentum flux requires accurate representation in NWP. For this purpose the focus of this study is on the aerodynamics of groups of buildings at the neighbourhood scale.

1.2 The urban boundary layer

1.2.1 Vertical structure
The layer of atmosphere above the urban surface is known as the urban boundary layer (UBL). The urban boundary layer (UBL) extends from the ground up to more than a kilometre into the atmosphere depending on the time of day and the meteorological conditions. Compared to the atmospheric boundary layer over flat surfaces, the structure of the UBL is modified near the surface due to the presence of buildings which induce greater drag and increased thermal and momentum fluxes. Towards the top, the presence of a temperature inversion can act to cap vertical motion of the air. The restricted motion of air, can prevent ventilation of pollution out of the UBL. Pollution can be ventilated out if a breakage occurs in the inversion due to extreme convection overcoming the cap or the lifting effect of a front. The depth at which the inversion occurs is often used to define the top of the boundary layer.
The acronym PBL stands for planetary boundary layer and UCL stands for the urban canopy layer. Reproduced from Rotach et al. (2005), modified after Oke (1987).

The structure of the urban boundary layer is divided into three distinct layers known as the outer urban boundary layer (OUBL), inertial sublayer (ISL) and the roughness sublayer (RSL). Each layer represents characteristic differences in the dynamical and thermal structure of the flow. The vertical structure of a typical UBL is illustrated in Figure 1.1.

**Outer Urban Boundary Layer** - The majority of the UBL is made up of the outer urban boundary layer which extends from a few hundred metres above the surface up to the capping inversion. It represents a layer of the atmosphere which is well mixed due to the turbulence generated by convection or strong winds. It occupies the outer 90% of the boundary layer and there are some velocity, temperature and humidity variations within it. The velocity gradient is more pronounced near the bottom of the OUBL compared to the top. However, the change in the velocity with height is much smaller compared to that below the OUBL.
The region between the bottom of the OUBL and the surface occupies approximately 10% of the overall boundary layer depth and is termed the surface layer. In urban areas, the surface layer has a depth of approximately 100-200m and it is within this layer that the strongest vertical gradients in wind, temperature and humidity for the transfer of momentum, moisture and heat exist. It consists of two sub-layers known as the roughness sublayer (RSL) adjacent to the surface and the inertial sublayer (ISL) wedged in between the RSL and the OUBL above.

*Inertial Sublayer* – The ISL is a region where the flow and temperature are horizontally homogeneous but can vary in the vertical. The vertical fluxes of momentum and temperature change very little with height and therefore the ISL is often referred to as the constant-flux layer. The ISL corresponds to the matching layer between the inner layer and outer layer scaling of boundary layer parameters (Tennekes and Lumley, 1972). In numerical weather prediction models the lowest model level is usually assumed to lie within this layer.

*Roughness Sublayer* - Flow within the RSL is directly influenced by the individual buildings and obstacles and is heterogeneous in both the horizontal and vertical dimensions. The geometry and layout of the obstacles have a significant influence on the depth of the RSL which is estimated to be between 2-5 times the average height of the buildings (Raupach et al., 1991; Rafailidis, 1997). The RSL is often further divided into the urban canopy layer (UCL) and the above canopy roughness sublayer. The UCL extends from the surface to a depth equivalent to the average height of the obstacles (buildings and trees). The size, shape and distribution of individual obstacles determine the structure and turbulence of the flow.

### 1.2.2 Wind flow structure in the surface layer

In the surface layer of the UBL, the flow velocity is assumed to be equal to zero at the surface, where the no-slip condition applies, and equal to the velocity of the OUBL at the top. This surface layer in the real atmosphere is equivalent to the engineering concept of a rough wall turbulent boundary layer in which the surface boundary condition is the same and the velocity at the edge of the boundary layer is the equivalent to the free-stream velocity of the flow.

The bulk properties of the mean velocity distribution in a rough-wall boundary layer can be derived using a classical asymptotic matching process (Millikan, 1938; Tennekes and Lumley, 1972) in which the boundary layer is assumed to consist of two overlapping layers. In the outer layer the flow scales with the depth of the boundary layer, $\delta$ and the scaling
velocity referred to as the friction velocity, \( u_* \), where
\[
\frac{\tau}{\rho} = u_*^2
\]
(with \( \tau \) the shear stress at the surface and \( \rho \) the fluid density). In the inner layer, the flow scales with the friction velocity and a set of length scales, \( S \). For smooth surfaces, \( S \) consists of only the viscous length scale \( \frac{V}{u_*} \), where \( V \) is the kinematic viscosity of the fluid. In urban areas, the surface is never smooth and consists of large obstacles such as buildings and is therefore considered a rough surface in which the flow scales on both the viscous length scale and the length scales of the obstacles, \( L_i \), which completely describe the obstacle array. The method of dimensional analysis then gives rise to the law of the wall for the inner layer mean velocity profile, \( \bar{u}(z) \) for which

\[
\frac{\bar{u}(Z)}{u_*} = F\left(\frac{Zu_*}{v}, \frac{L_i u_*}{v}\right)
\]  

[1.1]

where \( Z = z - z_D \) and is the effective height above the surface. This effective height must be considered because the presence of large obstacles on the surface displaces the entire flow upwards by a height equivalent to \( z_D \), known as the displacement height.

For the outer layer, dimensional analysis leads to the velocity defect law where

\[
\frac{\bar{u}(Z) - U_\infty}{u_*} = G\left(\frac{Z}{\delta}\right)
\]  

[1.2]

where \( U_\infty \) is the free-stream velocity.

Tennekes and Lumley (1972) suggest that between the outer layer and the inner layer, there is an overlapping region in which the velocity gradient from equations [1.1] and [1.2] match. This region is known as the inertial sublayer where:

\[
\frac{Z}{u_*} \frac{dZ}{dZ} = \frac{Zu_*}{v} \frac{dF}{d\left(\frac{Zu_*}{v}\right)} = \frac{Z}{\delta} \frac{dG}{d\left(\frac{Z}{\delta}\right)} = \frac{1}{\kappa}
\]  

[1.3]

where \( \kappa \) is the von Karman constant often taken to be equal to 0.4.
Integrating equation [1.3] leads to the general logarithmic velocity equation valid in the inertial sublayer given as

$$\frac{\bar{u}(z)}{u_*} = \frac{1}{\kappa} \ln \left( \frac{z - z_0}{z_0} \right)$$

[1.4]

where $z_0$ is a lengthscale known as the roughness length and is related to the momentum of the flow absorbed by the obstacles and the surface.

The overlapping inertial sublayer is present only if the boundary layer depth, $\delta \gg \left( \frac{V_{u_*}}{u_*}, L_i \right)$. In the urban surface layer there are suggestions that the inertial sublayer may not exist. Jimenez (2004) explained that when the ratio $\delta/z_H$ ($z_H$ is the obstacle height and $\delta$ is the boundary layer depth in laboratory measurements) is less than approximately 80, the influence of the obstacles on the flow penetrates much deeper into the boundary layer. Figure 1.2 shows a sketch of the influence of $z/\delta$ and $\delta/z_H$ on the vertical extension of sub-layers in the surface layer (Rotach, 1999). Here the top of the ISL is chosen to be $0.1\delta$ and the ISL is defined as the region where $z \gg z_0$ and $z \ll \delta$. For $\delta/z_H$ less than 100 (tall obstacles, relative to the depth of the boundary layer), the inertial sublayer is very thin or does not exist because the roughness sublayer extends up to the depth of the surface layer.

**Figure 1.2** – The vertical extension of roughness sublayer and inertial sublayer within the atmospheric surface layer as a function of the $z/\delta$ and $\delta/z_H$. Reproduced from Rotach, (1999).
Only when the obstacle heights are short relative to the boundary layer depth, is there an opportunity for a deep inertial sublayer to grow. This is often the case for forest and crop surfaces. By taking the typical value of the boundary layer depth and building height in urban areas to be $\delta = 1000\text{m}$ and $z_H \approx 5-20\text{m}$ respectively, then the ratio of $\delta/z_H$ would rarely exceed a value of the order of 100. Thus the small ratio of $\delta/z_H$ would restrict the growth of a deep inertial sublayer. Cheng and Castro (2002a) also question the existence of the inertial sublayer over urban areas of extreme roughness. In addition, they found that a significantly large standard deviation in obstacle height leads to a deeper roughness sublayer for the same average $z_H$ which squeezes out the ISL all together.

The presence of the ISL is also dependent on the distance (fetch) available for a sufficiently deep equilibrium layer to develop. This is discussed later in section 1.3.5. Nevertheless, the absence of an inertial sublayer has important significance on characterising the roughness of the underlying surface because the roughness is often quantified using the velocity and shear stress values in this layer.

### 1.3 The aerodynamics of surface-mounted bluff obstacles

#### 1.3.1 The flow field around obstacles

Air flow around an obstacle mounted on a surface involves the impact of highly turbulent shear flow upon the obstacle walls. For sharp-edged obstacles the flow separates from the surface of the obstacles and leads to the formation of a turbulent wake consisting of vortices. The flow field around the obstacle is particularly complex and unsteady and is illustrated in Figure 1.3. The various flow field regions in Figure 1.3 can be summarised as follows:

Figure 1.3 – A schematic representation of the flow field around a surface mounted obstacle. Reproduced and modified from Bottema (1993).
**Front recirculation zone (A)** - The region ahead of the obstacle often consists of a recirculation zone. In this region, the velocity distribution is sheared and near the top of the obstacle the velocity is greater than towards the base. The result is a relatively higher surface pressure towards the top of the obstacle compared to the bottom. The pressure difference is responsible for driving the flow downwards generating a recirculation zone (Melbourne et al., 1971).

**Corner stream zone (B)** - The front corner region at the top of an obstacle is driven by a pressure difference over a building and is a region of increased wind speeds.

**Separation bubble (C)** - At the top corner of the obstacle flow separation occurs due to the large gradient of curvature of the obstacle surface. If the obstacle extends sufficiently downstream, the separated flow can undergo turbulent reattachment on the top surface at some downstream location. The result is a region of trapped air between the separation and reattachment points in which stagnant, constant pressure flow exists upstream ahead of a recirculation zone behind (Houghton and Carpenter, 2003).

**Main recirculation zone (D)** - The wake in the lee of an obstacle is turbulent, highly unsteady and consists of vortices shed off the top and side walls of the obstacles. Several studies (eg. Castro and Robins, 1977; Richards et al. 2001) have shown that the centre-section pressure variation with height across the rear wall is generally constant. On average, there is a return flow towards where the pressure is a minimum near flow separation. Interaction of the flow with shear layers results in a complex flow pattern. The dimensions of the wake are dependent on the dimensions of the obstacle and this is significant because the area of the wake denotes an area of low surface shear stress which reduces the skin-friction drag on the mounted surface (Raupach, 1992).

**Shear layer zone (E)** - The shear layer occurs at the level of the average height of the building and spreads vertically towards the surface and above the obstacle height. In this region there are large velocity gradients caused by the slow flowing sheltered air near the surface and the fast moving flow above the obstacle height. The dimensions of the shear layer are dependent on the ratio of the height of the obstacle to its lateral width, $z_u/L_x$ (Huq et al., 2006).
**Far wake region (F)** – The far wake region consists of lower mean velocity and higher turbulence intensity (Peterka *et al.* 1985). The nature of the flow is dependent on both the nature of the shear layer region and the main recirculation region.

### 1.3.2 Forces acting on bluff obstacles

The size and shape of the obstacle, the properties of the fluid and the velocity and turbulence of the flow govern the forces acting on the obstacle. The streamwise force, \( F_D \) acting on the obstacle restricts the flow of air and is therefore often referred to as the drag force and is given in terms of the following expression

\[
F_D = \frac{1}{2} \rho U^2 A_F C_{D(I)}
\]

where \( \rho \) is the density of the fluid, \( U \) is the flow velocity, \( A_F \) is the frontal area of the obstacle and \( C_{D(I)} \) is the drag coefficient of the obstacle.

The impact of all the complex dependencies concerning the shape of the obstacle and the flow conditions on the streamwise force are taken into account in terms of a single drag coefficient, specific to that obstacle. For surface-mounted obstacles and for obstacles in which the cross-sectional shape is not uniform with height, the flow velocity and forces are not equal at different vertical locations on the obstacle. For this reason, the sectional drag coefficient, \( C_{D(S)}(z) \), is often quoted and is a local drag coefficient at a particular height of the obstacle. The drag coefficient of the isolated obstacle, \( C_{D(I)} \) is simply the integration of the sectional drag coefficient across the height of the obstacle.

For flow around large obstacles the contribution to the drag force comes from two main sources of drag:

*Form drag* – This type of drag is the result of varying surface pressures acting normal to the walls of the obstacle. These pressure variations are caused by the attachment, separation and possible reattachment of the flow along the surface of the obstacle due to its shape as illustrated in Figure 1.3. Differences in the surface pressure result in a net force for which the magnitude can be estimated by integrating the local pressures acting across the entire surface and multiplying by the surface area.
Skin-friction drag – This type of drag is the result of the shear stress acting on the surface of the obstacles. The shear stress acts tangentially to the surface and is directly related to the viscosity of the air in which faster flowing fluid particles rub over slower fluid particles, generating friction. The greater the viscosity of the air, the greater the frictional force acting on the surface. The skin-friction drag is much larger for obstacles with greater surface area parallel to the direction of the flow.

The sum of the form drag and skin friction drag is termed the profile drag. The size and intensity of the wake generated by an obstacle can often indicate the magnitude of the profile drag. This is because the total drag appears as a loss of momentum and an increase of turbulent kinetic energy in the wake. The reduction in average flow speed in the wake represents the loss of momentum and larger eddies and vortices in the wake contain the increased energy.

Obstacles of certain shapes can shed alternate vortices which appear as coherent structures in the wake of the obstacle in a phenomenon commonly known as vortex shedding. In such cases, a lateral fluctuating force, \( F_Y \), is generated on the obstacle (Bearman and Obasaju, 1982) which can cause the obstacle to vibrate if it is not mounted rigidly. There is also experimental evidence that a lift force, \( F_Z \), exists which acts to pull obstacles away from the surface (Einstein and El-Samni, 1949). For surface mounted bluff bodies the streamwise drag force is much greater in magnitude than both the lift and lateral forces combined and therefore is the focus of this study.

1.3.3 The bulk drag of rough wall surfaces
The discussion in section 1.3.2 has focussed primarily on an isolated obstacle mounted on a flat surface. However, when an array of obstacles is mounted on a flat surface, the total drag force acting on each obstacle is not the same as it would be if it appeared in isolation. The primary reason for this is that upstream obstacles influence the velocity and turbulence profile of the flow impacting upon obstacles downstream. This alters the flow field and pressure distribution around the downstream obstacles. From a NWP and dispersion modelling perspective, it is useful to characterise the force components of individual obstacles into a single parameter representing the drag of the surface array. Taylor (1916) suggested that the over a unit area of the Earth's surface, the near surface shear stress, \( \tau \) is given by
\[ \tau = k \rho U^2 \]  

[1.6]

where \( \rho \) is the density of the air, \( U \) is the wind speed and \( k \) is a constant of proportionality.

Taylor (1916) suggested that for a sufficiently large Reynolds number the constant of proportionality, \( k \) would be the same as that obtained from laboratory experiments of flow over a rough plate.

The shear stress based on the friction velocity, \( u_\ast \), is also given as

\[ \tau = \rho u_\ast^2 \]  

[1.7]

Equating equations [1.6] and [1.7] and rearranging the terms

\[ k = \left( \frac{U_\ast}{U} \right)^2 = C_{D(u_\ast)} \]  

[1.8]

Here, the constant of proportionality, \( k \), is referred to as the bulk drag coefficient, \( C_{D(u_\ast)} \). In equation [1.8] the velocity \( U \) is taken as the reference wind speed at a reference height, \( z \) but it is often not clear what the reference height should be.

The friction velocity in equation [1.7] is expressed as a function of the surface shear stress. Since determination of the surface shear stress is difficult for any real surface, the friction velocity is often also determined from a measurement of the Reynolds stress, \( \bar{u}' \bar{w}' \) such that

\[ u_\ast = \left( \frac{\bar{u}' \bar{w}'}{\rho} \right)^{1/2} \]  

[1.9]

The Reynolds stress is determined from within the inertial sublayer where the momentum flux is relatively constant with height. It is assumed that a measurement of \( \bar{u}' \bar{w}' \) within the ISL above a surface equals the surface stress. In section 1.2.2, the presence of an inertial sublayer was questioned over urban areas and therefore the friction velocity may not be defined.

Probably the most accurate method for determining surface drag is to measure it directly. Although this is not practically possible for real urban areas, it is possible in wind tunnel
modelling. Cheng et al. (2007) related the roughness of the surface to the surface shear stress determined from form drag measurements of individual obstacles by method of pressure tapping the front and rear walls. They also used a second method to measure the surface shear stress involving a floating drag balance method in which a section of the array of obstacles was floated on an oil bath and attached to a cantilever spring. The deflection of the spring was proportional to the drag of the array section. Recently, Hagishima et al. (2009) used a similar method to measure surface drag for various arrays of cuboid obstacles.

1.3.4 Aerodynamic parameters and surface morphology

Although equation [1.4] only applies in the inertial sublayer, Ploss et al. (2000) and Cheng and Castro (2002a) have suggested that the logarithmic velocity profile can be extended into the roughness sublayer if the velocity is spatially averaged over obstacle arrays. This is in contrast to other rough surfaces such as plant canopies for which Raupach et al. (1986) came to the conclusion that the velocity in the roughness sublayer is not logarithmic. Thus for urban areas the logarithmic velocity expression applies to a significant portion of the surface layer and depends on two aerodynamic parameters, namely the displacement height, $z_D$ and the roughness length, $z_o$. Thom (1971) and Jackson (1981) defined the zero-plane displacement height as the mean height of momentum absorption by the surface. A totally flat surface with no roughness elements has $z_D = 0$ and for surfaces where buildings are spaced very close together the value of $z_D$ is close to its maximum value equal to the average building height, $z_H$. The surface roughness length is the height above the surface where the mean flow velocity becomes zero when extrapolating the logarithmic velocity profile downwards through the surface layer and is a measure of the roughness of the surface. The surface roughness length and bulk drag coefficient are both measures of the roughness of the surface and can be related to each other by equating equations [1.4] and [1.8] such that the bulk drag coefficient, $C_D$ is:

$$C_D = \left( \frac{u_*}{U} \right)^2 = \frac{\kappa^2}{\ln \left( \frac{z-z_D}{z_0} \right)^2}$$

[1.10]

where $U$ is a reference velocity at a reference height, $z$. 

Alternatively, rearranging equation [1.10] gives the roughness length, $z_0$ as:

$$z_0 = \frac{z - z_D}{\exp \left( \frac{\kappa \sqrt{\nu}}{z_D} \right)}$$

[1.11]

A number of methods have been proposed to estimate the values of these aerodynamic parameters. Grimmond and Oke (1999) describe two classes of approach most commonly used; 1) morphometric methods that use algorithms that relate aerodynamic parameters to measures of surface morphometry and 2) micrometeorological (or anemometric) methods that use field observations of wind or turbulence to solve for aerodynamic parameters. Whereas the latter of the two provides actual values without surface idealisation, practical and financial limitations remain with regards to taking measurements from tall towers in urban areas. Even if the surface layer flow could be determined using measurement towers or remote sensing lidars, only a few vertical measurements would be possible.

**Figure 1.4** – Definition of geometric dimensions used to quantify the area density of building-type obstacles in the urban surface. Reproduced from Grimmond and Oke (1999).

Thus the former method is attractive despite its limitations and requires some method to sufficiently quantify the morphology of the rough surface. Figure 1.4 illustrates definitions of the main geometric dimensions of a simple, idealised obstacle array. Figure 1.4 highlights that even simple obstacle arrays require many morphological dimensions to quantify a complete description of the surface. For real buildings the geometry is of greater complexity and requires an even greater number of morphology parameters to describe the surface. Thankfully, the exact quantification of the urban surface is only necessary for site specific studies such as the dispersion of pollution in a particular part of a town or city. For general NWP and pollution dispersion modelling the more common requirement is for a bulk
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Aerodynamic parameter derived from a relatively simple model of the surface morphology. Often the specific geometry is normalised into simple parameters which are then related to aerodynamic parameters. Two of the most common parameters which describe the morphology of groups of buildings are the non-dimensional area ratios, namely the plan area density ratio, \( \lambda_p \) and the frontal area density ratio, \( \lambda_f \). For an array of obstacles, the plan area density ratio of obstacles, across the total lot area, \( \bar{A}_p \) is defined as:

\[
\lambda_p = \frac{\bar{A}_p}{\bar{A}_r} = \frac{L_x L_y}{D_x D_y}
\]  

[1.12]

Similarly, the frontal area density ratio of obstacles across the total lot area is defined as:

\[
\lambda_f = \frac{\bar{A}_f}{\bar{A}_r} = \frac{z_H L_y}{D_x D_y}
\]  

[1.13]

The plan area density ratio provides a measure of the density of buildings. Buildings further apart have \( \lambda_p \) values close to zero whereas buildings arranged closer together have a value closer to one. The frontal area density ratio provides a measure of the area of building face exposed to the wind, which as equation [1.5] suggests, has significant influence on the drag force acting on the obstacle. Using ground and aerial survey techniques, the geometry of buildings across a large urban area can be catalogued into a database (e.g. Burian et al., 2002a; Ching et al., 2009). In the past, this information was often held by the government for security purposes and therefore difficult to obtain for the purpose of research. However, in recent years building morphology data has been catalogued by commercial organisations and such data has been available for academic research, albeit at significant cost.

According to Theurer et al. (1992) the plan and frontal area density ratios are the two morphological parameters found to be most important for characterising surface drag. Furthermore, Plate (1995) quoted results which show that the displacement height, \( z_D \) and roughness length, \( z_o \) measured in cities are primarily related to the plan and frontal area density ratios. In response, several expressions and algorithms have been developed using experiments, theory and empirical analysis to relate the area density ratios to the displacement height and roughness length of surface arrays. Figure 1.5 shows a conceptual representation of how \( z_D \) and \( z_o \) vary as a function of \( \lambda_p \) and \( \lambda_f \). Figure 1.5 shows that the displacement height increases with both the plan and frontal area density ratios from its...
theoretical limits of zero to one. The variation of the roughness length differs and involves an increase for sparse area densities followed by a decrease for higher area densities. Oke (1988) suggests that the trend of the roughness length variation is based on the interaction of the upstream wakes with downstream obstacles.

**Figure 1.5** – A conceptual representation of the relationship of the aerodynamic roughness length and the displacement height to the plan and frontal area density ratios. (Reproduced from Grimmond and Oke, 1999).

**Figure 1.6** – Flow regimes associated with a range of building height-to-street width ratio, $\frac{z_H}{W_X}$. The sketches illustrate (a) Isolated roughness flow (b) Wake interference flow and (c) Skimming flow. Reproduced from Oke (1987).
His suggestions are based on three separate flow regimes which are dependent on the ratio of the building height, $z_H$, to the street width, $W_x$, between the upstream and downstream obstacles. The three different flow regimes are illustrated in Figure 1.5 and can be summarised as follows:

a) *Isolated roughness flow* – This regime develops for street canyons with $\frac{z_H}{W_x} < 0.3$ where buildings are well spaced apart such that the flow is similar to that around an isolated building. There is much less interaction between the wake of the upstream obstacles and the downstream obstacle.

b) *Wake interference flow* – For $0.3 < \frac{z_H}{W_x} < 0.65$ flow around an upstream building will influence downstream buildings due to the downstream impingement of the wake that forms around the upstream building. This flow regime is common in urban and suburban areas.

c) *Skimming flow* – For building height to street width ratios of $\frac{z_H}{W_x} > 0.65$ adjacent buildings are so close together that flow over the upstream building roof fails to directly penetrate into the street below. Instead the flow skims over the street just below roof level. The flow is effectively exposed to a smaller frontal area which leads to a reduction in surface roughness. In the region between the ground and roof level, a recirculation region develops which is driven by the flow aloft.

In Figure 1.5, the increase in roughness length for sparse obstacle arrays represents a transition in the flow regime from that of isolated roughness flow to wake interference flow. The decrease in roughness length for dense obstacle arrays represents the transition from wake interference flow to skimming flow.

One of the limitations of the plan and frontal area density ratios is that they are unable to describe the arrangement of obstacles. Thus two surface arrays with equal values of $\lambda_P$ and $\lambda_F$ can have various arrangements of obstacles and therefore exhibit different surface roughness. The two most widely studied arrays are the square and staggered obstacle arrangements (Macdonald *et al.*, 1998; Davidson *et al.*, 1996; Cheng and Castro, 2002a; Cheng *et al.*, 2007). For square arrays, rows of obstacles are placed directly behind each other. For staggered arrays, rows of obstacles are displaced laterally relative to each other. The general consensus is that staggered arrays generate greater surface drag compared to square arrays.
(Cheng et al., 2007; Macdonald et al., 1998) and according to Grimmond and Oke (1999), provide better correlation to real urban building arrays.

For both square and staggered arrays, several morphometric algorithms have been presented by researchers which achieve the expected trends of the aerodynamic parameters in relation to the area density ratios. The various expressions are reviewed in Grimmond and Oke (1999) in terms of their suitability for application to real urban areas and include the works of Kutzbach (1961), Lettau (1969), Counihan (1971), Kondo and Yamazawa (1986), Raupach (1994, 1995), Bottema (1995a), Bottema (1995b, 1997) and Macdonald et al. (1998). Some of the algorithms were criticised for

- requiring a large number of input parameters (Bottema 1995b, 1997; Raupach, 1994).
- not being applicable for the full range of surface morphometry found in real urban areas (Kutzbach, 1961; Lettau, 1969; Counihan, 1971; Kondo and Yamazawa, 1986).
- not achieving the expected decrease in roughness length for dense obstacle arrays (Kutzbach, 1961; Lettau, 1969; Kondo and Yamazawa, 1986).

The expressions of Macdonald et al. (1998) were considered the most practical and best performing and are given as

\[
\frac{z_d}{z_H} = 1 + \alpha^{-\lambda} (\lambda_F - 1) \tag{1.14}
\]

\[
\frac{z_0}{z_H} = \left(1 - \frac{z_d}{z_H}\right) \exp \left\{ -0.5 \beta^\frac{C_{D(I)}}{\kappa} \left(1 - \frac{z_d}{z_H}\right)^{0.5} \right\} \tag{1.15}
\]

where \(\alpha\) and \(\beta\) are empirical constants, \(\kappa\) is the von Karman constant and \(C_{D(I)}\) is the drag coefficient of an isolated obstacle.

The expressions rely on just two surface morphology parameters, namely the plan and frontal area density ratios. The constant \(\alpha\) determines the variation of the displacement height for a varying plan area density ratio. The constant \(\beta\) corrects the drag coefficient, \(C_{D(I)}\) which may vary due to factors such as the shape of the velocity profile within the canopy, the turbulent intensity of the flow and the incident angle of the flow. Macdonald et al. (1998)
configured the empirical constants, \(\alpha\) and \(\beta\), specifically for cube arrays for which the plan and frontal area density ratios are equal. It is very common for cube arrays to be used in the determination of flow velocity, turbulence and aerodynamic parameters. Barlow and Coceal (2009) review no fewer than 31 studies involving flow over cubes, with studies conducted using wind tunnel modelling, computational fluid dynamics and full-scale modelling. In many cases, the results are extrapolated to non-cubic arrays without any physical justification for doing so. With regard to Macdonald et al. (1998), algorithms are used for complex geometries for which the plan and frontal area densities may not be equal. In such cases, the non-cubic shape of the obstacles would result in different values for the empirical parameters, \(\alpha\) and \(\beta\).

**Figure 1.7** – Normalised roughness length for various combinations of the plan and frontal area density, calculated using the expressions presented by Macdonald *et al.* (1998).

However, when researchers have used expressions [1.14] and [1.15] in real urban areas, the values of the empirical constants has often been overlooked (eg. Ratti *et al.*, 2002; Long *et al.*, 2003). Figure 1.7 shows the normalised roughness length for various plan and frontal area densities, calculated by substituting the area density values into the expressions of Macdonald *et al.* (1998). The empirical parameters are kept constant for all area density ratios, with \(\alpha = 4.43\) and \(\beta = 1.0\). Figure 1.7 is an example of extending an expression configured for cube arrays to cuboid arrays more commonly found in actual towns and cities. It is unclear how accurate Figure 1.7 is for estimating the roughness length of non-cubic arrays as there have been no studies which test its validity. However, the expressions display behaviour which appears physically unrealistic for arrays where \(\lambda_p \neq \lambda_F\). For example, the plot
suggests that for arrays with any fixed frontal area density, the roughness length peaks at minimum plan area density and decreases for increasing $\lambda_F$.

The estimates of the aerodynamic parameters from the various algorithms are only valid for uniformly distributed obstacles for arrays sufficiently large such that flow has the opportunity to adjust to the underlying surface. The latter is illustrated in Figure 1.8, showing a conceptual representation of the structure of a developing urban boundary layer.

![Figure 1.8 - A conceptual sketch of the vertical structure of a developing urban boundary layer. Reproduced from Feddersen (2004).](image)

At the location where the surface roughness changes, an internal boundary layer (IBL) begins to develop within the existing atmospheric boundary layer. Above the IBL, the flow field is characteristic of the upstream conditions. Within the IBL, the flow begins an adjustment process and initially the flow field appears as two distinct layers, termed the equilibrium layer, near the surface and the transition layer above. In the equilibrium layer, the wind velocity profile has completely adjusted to the local surface due to the turbulent mixing of the flow. In the transition layer, above the equilibrium layer, the velocity distribution gradually changes from a profile characteristic of that in the equilibrium layer to that outside the IBL. At some distance downstream of the roughness change, the equilibrium layer is sufficiently deep that an ISL begins to grow above the equilibrium layer and below the transition layer. Cheng and Castro (2002b) conducted wind tunnel experiments to show that the thickness of the equilibrium layer is independent of fetch after an initial development region and that the ISL depth increases with fetch, though not as rapidly as the boundary layer depth. In theory, at a significant distance downstream from the initial roughness change, the entire urban boundary layer should be adjusted to the underlying surface. However, in real urban areas,
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the flow encounters multiple changes in surface roughness such that there may not be sufficient fetch for a total boundary layer adjustment to occur and in many cases the development of an inertial sublayer. Cheng and Castro (2002b) observed that the ISL began to grow at a distance of $400z_H$ after a roughness change. In real urban areas, this equates to a fetch of several kilometres within which the surface roughness may change multiple times.

1.3.5 Flow within the urban canopy layer

Knowledge of flow and turbulence in the canopy layer is used to determine the forces acting on obstacles and pedestrians. It is also useful in the prediction of pollution dispersion through groups of buildings. The mean velocity within the canopy layer is dependent on the geometry and layout of the obstacles. Bentham and Britter (2003) suggested that an assumed constant velocity, $U_C$ in the canopy is sufficient for dispersion modelling purposes rather than a complicated model for the variation of velocity with height. They derived a model for the velocity, $U_C$ within the canopy based on the balance of the surface shear stress with the drag force of obstacles in an array. Their model suggested that the in-canopy velocity is a function of the frontal area density (for $\lambda_F > 0.2$) and the roughness length (for $\lambda_F < 0.2$). Wind engineers have often noted that forces on obstacles such as buildings are not uniform with height (Hunt, 1982). A possible reason for this is that the mean velocity in the canopy is not uniform with height. It has long been known that for vegetation canopies, the spatially-averaged mean velocity profile within the canopy varies exponentially with height (Cionco, 1965; Thom, 1971; Raupach and Thom, 1981). Cionco (1965) presented a model of the form:

$$\langle U(z) \rangle = \langle U_H \rangle \exp \left[ a \left( \frac{z}{z_H} - 1 \right) \right]$$  \hspace{1cm} [1.16]

where $\langle U(z) \rangle$ is the spatially-averaged mean velocity, $\langle U_H \rangle$ is the spatially averaged mean velocity at the top of the canopy, $z_H$ and $a$ is an empirical parameter known as the attenuation factor.

The attenuation factor, $a$ is a constant parameter and Macdonald (2000) modified equation [1.16] for an array of cuboidal obstacles to show that the attenuation coefficient is a function of the frontal area density, $\lambda_F$ and the sectional drag coefficient, $C_{D(S)}(z)$. Equation [1.16] suggests that the spatially-averaged mean velocity deep within the canopy is lower than the velocity near the top of the canopy. Experimental data from Cheng and Castro (2002a) also suggests that for a cube in an array, the drag coefficient is lower at the top of the cube.
compared to the bottom. This is expected as the air near the base of an obstacle can only flow around the sides of an obstacle, whereas towards the top, the air can flow both around the obstacle and over the top. The air at the top of the obstacle is less restricted and therefore exerts less force on the obstacle. Thus the in-canopy velocity profile has a significant bearing upon the depth-integrated drag coefficient of the cube which in turn influences the bulk drag coefficient of a cube array.

In much the same way that the flow above the canopy begins to adjust to a new surface roughness (Figure 1.8), the flow within the canopy will also undergo an adjustment process following a roughness change. Coceal and Belcher (2004) introduced a dynamical lengths scale which defines a measure of the distance required by the flow to travel through the canopy before it becomes adjusted. This parameter is known as the canopy drag lengths scale, \( L_c \) and is a function of both the plan and frontal area densities and is given as:

\[
L_c = \frac{2z_H}{C_{D(s)}(z)\lambda_F(1 - \lambda_p)}
\]  

Coceal and Belcher (2004) estimated that the distance, \( x_0 \) required for the flow to adjust to the canopy is equal to

\[
x_0 = 3L_c \ln(K) \tag{1.18a}
\]

where

\[
K = \frac{U_h z_H}{u_c L_c} \tag{1.18b}
\]

Coceal and Belcher (2004) suggest that for typical urban areas, the log factor in equation [1.18] varies in the range, \( 0.5 < \ln K < 2.0 \). From Cheng and Castro (2002b), the mean sectional drag coefficient averaged between the base and the top of a cube is approximately \( C_{D(S)} = 2.5 \). Using idealised urban plan and frontal area density ratios, \( \lambda_P = \lambda_F = 0.25 \) and typical building heights \( z_H = 10m \), the value of \( x_0 \) equates to being of the order of 100m. In comparison, in section 1.3.4 it is discussed that the distance required for an ISL to grow within an equilibrium layer above the canopy is of the order of 2km at least. Thus the flow is well established with the underlying surface much earlier within the canopy layer compared to the flow above. This is as expected because the canopy generates greater turbulent mixing
due to the generation of wakes in the lee of obstacles which increases the shear stress contribution to the turbulent kinetic energy.

The fetch required for the flow to come into turbulent equilibrium with the underlying surface at all heights is significant in numerical weather prediction and urban dispersion modelling. Such models divide the land surface into gridboxes across which the heterogeneity of the land surface must be taken into account. As an example, the UK Met Office’s Unified Model incorporates a surface exchange scheme (Essery et al., 2001, 2003) which assumes independent, one-dimensional vertical fluxes from different surface types. The approach is based on the concept of a blending height (Mason, 1988), below which the fluxes are in equilibrium with the local surface and above which the fluxes are in equilibrium with a uniform, effective surface representing the aggregated effect of the whole surface (Clark et al., 2009). Mason (1988) suggests a rough estimate for the blending height to be $L_H/200$ where $L_H$ is the lengthscale of heterogeneity. This implies that the surface type changes across a distance of at least 100-200m. The assumption is that the size of each surface type is sufficiently large that the error in the fluxes due to the transition in surface type is negligible. Thus for the equilibrium assumption to be valid, the gridbox resolution must be greater than the fetch required for the flow to adjust to the new surface type. As the gridbox resolution increases, the equilibrium assumption may not hold and a representative roughness length value for example, may not be defined. Aerodynamic parameters estimated from algorithms are valid only for surface arrays in which the obstacles are of the same size and shape, distributed uniformly across the surface (homogeneous array) and for which the flow is fully developed and in equilibrium. This is an area of urban meteorology often overlooked and gridbox resolutions often vary greatly because there is no consensus, based on physical reasoning, for the appropriate resolution to use.

1.4 Research objectives and thesis structure

1.4.1 Aims of the thesis

A review of the current understanding of how geometry and layout of buildings influences rough wall turbulence and surface drag, has shown that significant developments have been made in the past few decades. In summary,

- it has become common practice to interpret geometrical features of buildings into a set of morphological parameters such as the plan and frontal area density ratio. However, the relative sensitivity of surface roughness to these parameters is still
unclear. The lack of understanding about how the morphology of buildings influences surface roughness stems from the fact that much of the research in this field has focused specifically on idealised cube arrays. Indeed many of the algorithms used to estimate the surface aerodynamic parameters have been tuned specifically for cube arrays and extending the algorithms beyond the range of permissible morphology values (e.g., wide range of $\lambda_P$ and $\lambda_F$ values) leads to unexpected trends.

- many issues arise when directly quantifying the surface roughness of real urban areas via the method of atmospheric measurements. In most cases, the roughness is determined by assuming the presence of an inertial sublayer and/or a logarithmic velocity profile region. The penetration of buildings and obstacles deep into the surface layer and the constantly changing roughness of the surface may lead to a non-existent inertial sublayer. This limits the ability to determine a friction velocity and restricts application of the log-law to characterise aerodynamic parameters.

The aim of this thesis is to determine the sensitivity of aerodynamic surface roughness to the geometry and layout of buildings in an urban area. The research (i) uses London as a case study to identify the typical morphology of buildings in urban areas, (ii) tests the validity of the integral momentum theory as a method to quantify the roughness of the surface in terms of the surface drag, (iii) presents a modification of an existing algorithm to estimate surface roughness for a wide range of building plan and frontal area density ratios and (iv) considers modification of an existing algorithm to allow for change in surface roughness due to changes in wind direction.

### 1.4.2 Thesis structure

The thesis is organised into six chapters including an introduction to the research, a description of the methodology, a discussion of the results and a summary of the conclusions. The contents of the chapters are summarised as follows:

*Chapter One* has introduced the topic of research and reviewed the current understanding of how the geometry of buildings influences surface drag.
Chapter two describes the methodology and results of the building morphology of London which is used as a case study. In particular Geographical Information Systems (GIS) techniques are employed to analyse typical values of the building area density ratios.

Chapter three describes the methodology used to conduct wind tunnel experiments in which the integral analysis of the mean wind velocity profile of the laboratory boundary layer is used to estimate the bulk drag coefficient of the underlying surface.

Chapter four explains the impact of area density values on the surface drag and presents a model which estimates the roughness of surface arrays with a broad range of area density parameters.

Chapter five investigates the impact of changing wind direction on the roughness of an obstacle array and presents a modified algorithm.

Chapter six summarises the main findings of the research and considers how the results of the present research may shape future research into building morphology and rough wall turbulent boundary layers.
Chapter Two
Building morphology in London – A Case Study

2.1 Introduction

Every major city in the world consists of buildings, vegetation and water features embedded within a network of road, rail and waterways. However, the number, size and distribution of these features vary significantly creating a surface which is heterogeneous. The single most distinguishing feature of urban areas is buildings. In recent years, buildings in towns and cities have been catalogued and sorted into categories such as building usage, age, capacity, geometry, density etc. Motivations for obtaining such information and forming databases have been spurred on by architects, the military, energy providers, environment agencies, insurers and government departments.

A building morphology database is usually assembled using several sources of data. Information can be derived from aerial photographs, remote sensing instruments and ground based surveys. Gathering the data is an expensive and time consuming process and as a result the majority of researchers in the past have focused on a limited number of buildings (Batty et al., 2007) or a small area of a town or city (Ratti et al., 2006) and then extrapolated the data to achieve estimates for the wider spatial domain. Table 2A lists some recent surveys of building geometry conducted for the purpose of urban meteorology and pollution dispersion. All the surveys listed in Table 2A have been conducted for North American and European cities and similar surveys in other continents are either rare or unpublished. As Table 2A highlights, the method and details of individual surveys can vary significantly in terms of the area covered, the grid resolution used and the morphological parameters determined. In terms of areal extent, one of the largest studies ever to be conducted is that of Burian et al. (2003) in which 1653km² of urban surface and 664,861 buildings were analysed for an area of Houston, Texas in the United States. Amongst the smallest studies conducted is that of Ratti et al. (2006) whose spatial extent was just 0.16km² for three different European cities. The
<table>
<thead>
<tr>
<th>Study</th>
<th>Region</th>
<th>Area of Survey (km²)</th>
<th>Number of Buildings</th>
<th>Grid Resolution (km)</th>
<th>Building Parameters of Interest</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ratti et al. (2006)</td>
<td>London Berlin Toulouse</td>
<td>0.16</td>
<td>~32 ~25 ~21</td>
<td>0.4</td>
<td>Plan area density, ( \lambda_P ) Frontal area density, ( \lambda_F ) Mean building height, ( z_H ) Height-to-width ratio, ( \lambda_S )</td>
</tr>
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<td>Long et al. (2003)</td>
<td>St Jerome</td>
<td>16.0</td>
<td>Not Stated</td>
<td>0.2</td>
<td>Plan area density, ( \lambda_P ) Mean building height, ( z_H )</td>
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<tr>
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<td>Toulouse Blagnac Portland</td>
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<td>~150 ~100 18</td>
<td>0.4</td>
<td>Mean height of the canopy layer, ( z_{UCL} ) Variance of the canopy layer height, ( \sigma_{UCL} ) Porosity Factor, ( P_o )</td>
</tr>
<tr>
<td>Bottema and Mestayer (1998)</td>
<td>Strasbourg</td>
<td>5.94</td>
<td>Not Stated</td>
<td>0.45</td>
<td>Maximum building height, ( z_{H(MAX)} ) Plan area density, ( \lambda_P ) Frontal area density, ( \lambda_F )</td>
</tr>
<tr>
<td>Burian et al. (2002c)</td>
<td>Albuquerque</td>
<td>48.5</td>
<td>22,662</td>
<td>0.1</td>
<td>Mean building height, ( z_H ) Variance of the building height, ( \sigma_H ) Plan area density, ( \lambda_P ) Frontal area density, ( \lambda_F ) Surface area to plan area ratio, ( \lambda_B ) Height-to-width ratio, ( \lambda_S )</td>
</tr>
<tr>
<td>Burian et al. (2003)</td>
<td>Houston</td>
<td>1648.6</td>
<td>664,861</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>Burian et al. (2002a)</td>
<td>Los Angeles</td>
<td>12.8</td>
<td>3,353</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>Burian et al. (2005a)</td>
<td>Oklahoma City</td>
<td>27.0</td>
<td>6,333</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>Burian et al. (2002e)</td>
<td>Phoenix</td>
<td>16.8</td>
<td>7,997</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>Burian et al. (2002d)</td>
<td>Portland</td>
<td>9.5</td>
<td>2,000</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>Burian et al. (2002b)</td>
<td>Salt Lake City</td>
<td>140.0</td>
<td>61,669</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>Burian et al. (2005b)</td>
<td>Seattle</td>
<td>40.8</td>
<td>35,971</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>Gal and Sumeghy (2007)</td>
<td>Szeged</td>
<td>25.75</td>
<td>22,000</td>
<td>&lt;0.1</td>
<td>Plan area density, ( \lambda_P ) Frontal area density, ( \lambda_F ) Mean height of the canopy layer, ( z_{UCL} )</td>
</tr>
<tr>
<td>Holland et al. (2008)</td>
<td>Broward County</td>
<td>900.0</td>
<td>Not Stated</td>
<td>1.0</td>
<td>Plan area density, ( \lambda_P ) Frontal area density, ( \lambda_F ) Building width, ( L_Y ) Building length, ( L_X ) No. of buildings per unit area, ( \rho_N )</td>
</tr>
<tr>
<td>Ioannilli and Rocchi (2008)</td>
<td>Rome</td>
<td>8.1</td>
<td>Not Stated</td>
<td>0.1</td>
<td>Mean building height, ( z_H ) Variance of the building height, ( \sigma_H ) Plan area density, ( \lambda_P ) Frontal area density, ( \lambda_F )</td>
</tr>
</tbody>
</table>

Table 2A – List of recent building morphology surveys conducted for the purpose of urban meteorology and pollution dispersion.
majority of surveys have limited the analyses of the urban surface to an area extent of less than 50km\(^2\) and 10000 buildings. These comparatively small surveys are unable to identify difference in neighbourhoods across a city and any trends in the intra-city variations of building morphology.

A common approach in conducting surveys involves selecting an area of a town or city and then dividing the area into gridboxes. The morphological parameters are then averaged over a gridbox. However, the gridbox size is often chosen for matters of convenience rather than with scientific justification. The values of the building morphology parameters can vary depending on the gridbox resolution. If the resolution is too fine, the morphology parameters are not fully representative of distinct neighbourhoods and the geometry of individual buildings begin to dominate over average morphological values. If the resolution is too coarse, the presence of neighbourhood patches becomes smoothed out and the building geometry and distributions exhibit large spatial scatter. Table 2A shows that the choice of gridbox resolution can vary from anywhere between 0.1km to 1km.

The building morphology parameters of interest depend on the purpose of the survey and the availability of practical methods for obtaining the relevant data. In the context of wind flow around buildings, surface drag and the turbulence characteristics of the near-surface flow, the main influencing morphological parameters are those which describe the external geometry and the relative spatial layout of groups of buildings. Table 2A shows that the plan and frontal area densities and the mean height of buildings are common parameters derived in building morphology surveys. Since Theurer et al. (1992) described the significance of the area density ratios on surface roughness, these parameters have appeared in several algorithms relating the roughness length and displacement heights to building morphology. However, since surface drag is dependent on the interaction of individual building wakes with neighbouring obstacles (Raupach, 1992), any morphological parameter which alters the dimensions of the building wake or describes the proximity of neighbouring buildings may have on influence on the bulk drag. The area density ratios and the building height alone are insufficient to illustrate a complete picture and the morphological parameter space is wide.

The purpose of this chapter is to present building area density ratios for neighbourhoods in the city of London. The area density ratios quantify the dimensions of the buildings which in turn quantify the dimensions of their wakes. As explained in chapter one, the spatial extent occupied by building wakes influences the level of form drag generated and the level of interaction of wakes with neighbouring buildings.
This chapter analyses the building morphology data within existing geographical datasets of London to investigate:

(i) the range of the plan and frontal area densities across neighbourhoods in London.

(ii) an estimate of surface roughness for such neighbourhoods based on existing algorithms.

(iii) the spatial size of neighbourhoods based on the plan area density.

The building morphology parameters are calculated for an area covering the 33 London boroughs that make up Greater London. The administrative boundaries of the boroughs together with the River Thames are illustrated in Figure 2.1 for reference.

Figure 2.1 – The 33 London Boroughs of Greater London for which morphological parameters were determined.

2.2 Building morphology datasets

The two datasets involved in the analysis of building morphology in London were the Ordnance Survey (OS) MasterMap® Topography Layer and 1:10000 scale colour raster data. The raw data was read into the ArcGIS (version 9.2) Geographical Information Systems (GIS) software package. The data was then filtered to remove unwanted layers of data (eg. roads, place names, trees etc) and then processed. The following subsections give a description of each dataset together with the methodology for data processing in GIS. Figure 2.2 shows example images of each dataset for a 25km² area of South East London. Both these datasets provide the plan area outline of buildings but do not contain information regarding the heights of buildings. Building height data and therefore the frontal area density ratio data
were sourced from the Virtual London dataset (Evans, 2009), licenced to the Centre for Advanced Spatial Analysis (CASA), University College London. A brief description of the Virtual London dataset is given in the next section.

![Illustrated images of (a) Ordnance Survey MasterMap® Building Topography Layer and (b) Ordnance Survey 1:10000 scale colour raster for an area of south east London. The lower left Easting and Northing of each tile is at (535090, 175050) metres, respectively, with respect to the British National Grid. The extent of the mapped area is 5km x 5km.](image)

**Figure 2.2** – Illustrated images of (a) Ordnance Survey MasterMap® Building Topography Layer and (b) Ordnance Survey 1:10000 scale colour raster for an area of south east London. The lower left Easting and Northing of each tile is at (535090, 175050) metres, respectively, with respect to the British National Grid. The extent of the mapped area is 5km x 5km.

### 2.2.1 Ordnance Survey MasterMap® Topography Layer

The Ordnance Survey MasterMap® dataset is a four-layer feature based dataset accessed from the University of Edinburgh's EDINA® Digimap® map delivery service. The dataset is seamless and tiles are delivered based on the requested Northing and Easting coordinates in 1km² gridbox tiles in uncompressed, open GML format.

In the present study, only the topographic layer of the dataset was used and this layer displays features which appear in the landscape such as buildings, land, water and roads. The features are sourced directly from a combination of aerial images and ground surveys and are updated on a six-monthly cycle. As of May 2006, the dataset contained 425 million features. For the present purpose, the dataset was filtered to remove all features except those classed as buildings. The OS MasterMap® dataset defines a building as a permanent roofed construction, usually with walls. This includes permanent roofed constructions that exceed 8.0 m² in area (12.0 m² in private gardens). Exceptions are made to this area rule for smaller buildings that, due to their detached position, form relatively important topographic features.
Chapter Two: Building morphology in London – A case study

Features defined as being buildings include:

- roofed buildings
- mobile or park homes that are permanent, residential and have a postal address
- archways and covered passageways where the alignment can be determined from outside the building
- horticultural glasshouses over 50m² and
- covered tanks.

Features such as cooling towers, gas holders, pylons, covered reservoirs, uncovered tanks, bridges and monuments are not defined as buildings but appear in the OS MasterMap® dataset as structures. In the present study, structures were not considered in the analysis. The raw dataset displays buildings as outline features which were then converted into polygons using the 'data conversion' tool within ArcGIS. Each building segment within the dataset was presented as an individual polygon based on the building number and address. The polygon features are held on the database in terms of a vector model in which coordinates denote the location of each vertex of the polygon. The accuracy in the position of each coordinate depends on whether the selected region is defined to be urban, rural or mountainous. In the present analysis, the entire analysed region was defined to be urban and the point position of each coordinate had a 1.0m horizontal nominal accuracy at the 99% confidence level. A whole building can consist of many building segments and therefore many polygon features.

![Figure 2.3](image.png)

**Figure 2.3** – The transformation of building polygon features from segmented blocks to separate whole blocks within the ArcGIS software.

Since wind flow is exposed only to the external walls of buildings, any internal features have to be merged so that a group of adjacent buildings (polygons) are represented by a single
polygon feature. The merge and separation processing stages were carried out in ArcGIS and are illustrated in Figure 2.3. The first step involved merging all the polygon features within a 1km² tile into a single polygon feature. The merging process brings all the polygons into a single group and only one set of feature statistics is available for all the polygons. To be able to consider individual polygons, the ‘explode multipart’ function within the advanced editing tool was used to separate the polygons into separate entities, allowing statistics to be calculated for individual polygons so that each building could be considered independently. The merge and separation process did not cause any loss of information in the original dataset. Figure 2.2a shows a 25km² building topography layer square tile for an area in London showing building features as blue polygons.

2.2.2 Ordnance Survey 1:10000 Scale Colour Raster
The Ordnance Survey 1:10000 scale colour raster (version 2.0) dataset is an image map also accessed through the University of Edinburgh’s Edina® Digimap® service. The data is derived from the Ordnance Survey LandPlan® product and is sourced from a combination of aerial images and ground surveys. The data is converted into raster format at a resolution of 400 dots per inch (dpi) and is received in the form of an image file which shows geographical features such as buildings (shaded yellow), vegetation, water features, administrative boundaries and text. The data is delivered in tile format measuring 5km x 5km with approximately 62 million pixels covering each tile. The length of each pixel on the image is equivalent to a distance of 0.635m on the ground.

Figure 2.4 – The conversion of raster image data to ASCII codes by method of pixilation.
Any permanent structure with a roof is classified as a building with the exception of bus shelters, telephone booths, letter boxes, electricity pylons and cranes which are excluded from the building classification. The raw data was received as a TIFF uncompressed file which was converted into ASCII format using ArcGIS. Each pixel was denoted by an ASCII code which represented the particular land feature. Figure 2.4 shows an illustrated example of the data conversion process. In some cases, the raw data contained text which was superimposed on top of buildings such that the building pixels were masked out. This resulted in an underestimate in the number of pixels denoted as buildings. However, for a 5km tile the maximum error resulting from such instances was less than 2% of the total building area. Figure 2.2b shows a 25km$^2$ colour raster image for an area in London showing buildings in yellow.

### 2.2.3 Virtual London

The Virtual London dataset is a three-dimensional digital model of the Greater London area, licensed to the Centre for Advanced Spatial Analysis (CASA) at University College, London. The dataset is unique in the sense that it contains information about the height of buildings, thereby allowing the computation of the frontal area density, $\lambda_F$. Building morphology information within the dataset is compiled using two different sources of data. Firstly, the Ordnance Survey’s MasterMap® dataset, described in section 2.2.1, is used to provide the outline of buildings and streets to within one metre accuracy. Superimposed upon this data is the obstacle height data obtained from Infoterra’s one metre lidar dataset. The dataset is compiled using a downward-pointing Optech 2033 Laserscanner lidar mounted on an aircraft. The lidar provides a horizontal accuracy of one metre and an absolute vertical accuracy of $\pm 15\text{cm}$ RMSE. The lidar sweep collects the height data of all the obstacles it flies over including building, trees and people. The data is filtered to remove all unwanted obstacles, leaving information about building heights only. Since the underlying terrain of London is not flat, the lidar data is separated into terrain height data and building height data. Subtracting the terrain height data from the building height data forms a single dataset that contains only building height data projected on a pseudo-flat surface.

Building height information within the Virtual London dataset was collected in the summer of 2005 and focuses on the entire Greater London area (Figure 2.1) covering approximately 1650km$^2$. Within this area there are 3,595,689 buildings. Data concerning the frontal area density was analysed and supplied at 1km$^2$ gridbox resolution by Stephen Evans at UCL by method of personal communication. The gridboxes were aligned at the same
location as those within the analysed Ordnance Survey MasterMap® dataset described in section 2.2.1.

2.3 Building morphology results and discussion

2.3.1 The plan and frontal area density

The plan and frontal area density values were calculated for 1km\(^2\) gridbox tiles for a total of 1650km\(^2\). The plan area of buildings was calculated by taking the plan projected area as opposed to the surface area of building tops. The difference is illustrated in Figure 2.5 for the example of slanted roof buildings with a chimney. In all cases the projected plan area is always less than or equal to the surface plan area and previous studies have accounted for the difference in terms of an enhancement factor (see Table 2 of Grimmond and Oke, 1999).

Similarly, the frontal area of buildings, \(A_F\) can be defined in various ways. In the present study all building frontal area calculations made use of the projected frontal area as opposed to the surface frontal area and these definitions are illustrated in Figure 2.6. The illustration suggests that the projected frontal area is dependent on the wind direction. The influence of frontal area changes due to wind direction on the surface drag is discussed further in chapter five. In the present study, the projected plan area density was calculated using the OS MasterMap® dataset. Buildings which lay on the edge of grid box boundaries were treated as partial buildings such that the portion of the building within the grid box was included in the calculation of the building plan and frontal area. For all gridboxes, the area of the buildings falling on the boundary represented less than 4% of the total building area and therefore the impact on the value of the area density ratios was small. Treating buildings on the boundary as partial buildings also eliminates the complication of modifying the footprint area of the grid box which would otherwise be required if whole buildings were considered. Figure 2.7 shows maps of the plan and frontal area density ratios for Greater London. The data suggests that for some gridboxes in central London the area occupied by buildings is more than half the land surface area. Values of \(\lambda_F\) are also much larger in and around central London with a peak value of 0.418. Larger values of \(\lambda_F\) result either from increases in the wall length of buildings and/or the mean building height. For an area such as central London where there is greater competition for a limited ground area, it is likely that the increase in \(\lambda_F\) is the result of an increase in the average building height.
Figure 2.5 – The definition of building plan area showing (a) the surface plan area definition and (b) the projected plan area definition. The solid red line shown in (b) indicates the width of the building roof considered in the calculation of the plan area.

Figure 2.6 – The definition of building frontal area showing (a) the surface frontal area definition and (b) the projected frontal area definition. The solid red line shown in (b) indicates the width of the buildings considered in the calculation of the frontal area.
Figure 2.7 – Map of (a) the plan area density and (b) the frontal area density of Greater London, with a gridbox resolution of 1km\(^2\). The frontal area density plot is presented for a westerly wind direction. The solid black boxes indicate the location of maximum area density values.

Several theories have been proposed about the spatial structure and organisation of cities including the concentric ring model (Burgess, 1924), the sector model (Hoyt, 1939) and the multiple nuclei model (Harris and Ullman, 1945). Often, these models have been largely based on social and economic patterns observed within urban areas. The concentric ring model suggests that distinct differences exist in social and economic trends when moving away from the centre of a town or city to the rural boundary. These trends follow a similar pattern in all directions away from the centre. The maps of Figure 2.7 suggest that the concentric ring model may also extend beyond social and economic parameters to building morphology parameters, for London at least. Both area density ratios are larger in and around the central business district (CBD) and much lower towards the rural boundary. The presence of a concentric ring model to describe the building morphology is further evident in Figure 2.8 which shows four transects of \(\lambda_P\). The plan area density values are for sections running from north to south, west to east, north-west to south-east and south-west to north-east. All sections run through the maximum plan area density gridbox bounded by a solid black line in Figure 2.7a. The plots in Figure 2.7 show that the plan area density values increase between the rural boundary and CBD and decrease between the CBD and rural boundary, in all four directions. As would be expected there is some scatter but the general trend is reasonably clear with symmetry in the plots around the origin.
Figure 2.8 – The cross-section plots of the plan area density ratio for London. The origin of the plot is at the maximum value of the plan area density shown in Figure 2.7a.

Figure 2.9 – The spatially-averaged plan and frontal area density across London. The origin of the plot is where the plan area density ratio is maximum.

A very similar trend is observed for the frontal area density values across London. The symmetry of area density values around the origin in Central London, confirms the concentric organisation of building morphology in London and enables a spatial average of the data to be presented. Figure 2.9 shows the plan and frontal area densities averaged in concentric rings.
of equal radial thickness across London. The origin of the plot is at a location where the plan area density is maximum. The spatial-averaging of the data reduces the scatter of the plots. The clearer trend shows that the increase in the area density ratios from the rural edge to Central London is gradual.

The implications of these area density values for the aerodynamic properties of the surface can be estimated using pre-existing algorithms. The expressions presented by Raupach (1994), Counihan (1971) and Macdonald et al. (1998) are used to estimate the normalised displacement height and roughness length. The expressions of Raupach (1994) suggest that the frontal area density is the only morphological parameter which influences the displacement height and roughness length. Similarly the expressions of Counihan (1971) suggest that the plan area density is the only morphological parameter which influences the two aerodynamic parameters. Macdonald et al. (1998) present expressions which make use of both the plan area and frontal area densities. Figure 2.10a shows estimates of the spatially-averaged normalised displacement height variation across the neighbourhoods of London. All three expressions estimate the displacement height to be greater in central London where buildings are taller and spaced closer together with Counihan (1971) and Macdonald et al. (1998) suggesting that it could be as high as 0.8 of the average building height. Across most of London, the range of the normalised displacement height is quite large with the estimates of Raupach (1994) approximately twice that of Counihan (1971) across Suburban London. Figure 2.10b shows estimates of the normalised aerodynamic roughness length across London. Here the trend across London is very different to that of the normalised displacement height. The initial increase in roughness length between the rural edge and suburban London reaches a peak approximately 5-6 km away from the CBD centre according to the estimate of Counihan (1971) and Macdonald et al. (1998). Counihan (1971) predicts that the roughness length decreases by 27% from the peak in the suburbs to the CBD. The expression of Macdonald et al. (1998) estimates the roughness length to be at a minimum across the whole of London in the CBD. Again the range of the estimates is large amongst the three expressions presented in previous studies. The difference in the peak roughness length estimates of Counihan (1971) and Macdonald et al. (1998) is almost a factor of 2.5. At the city scale, the wind speed from any direction would initially slow down in suburban areas, reach a minimum before the CBD, increase over a short distance, slow down within the CBD and finally increase again as the flow heads towards the rural edge.
Figure 2.10 – Three estimates for (a) the normalised displacement height and (b) the normalised roughness length across London.
Figure 2.11 – Scatter plots showing the correlation between $\lambda_P$ and $\lambda_F$ for tiles from the Virtual London and Ordnance Survey MasterMap® dataset at grid box resolutions of (a) $1\text{km}^2$, (b) $2\text{km}^2$ and (c) $4\text{km}^2$. The dashed red line indicates $\lambda_P = \lambda_F$, representative of a cube array.
For London, Figure 2.10b suggests that even over short distances the aerodynamic parameters can change significantly. The wind tunnel study of Cheng and Castro (2002b) suggests that the inertial sublayer becomes adjusted to the underlying surface at a distance of ~400z_H downstream of a roughness change. For an average building height of 10m in suburban London, a 4km fetch would be required for inertial sublayer adjustment. The roughness length estimate of Raupach (1994) suggests that over a 4km fetch the difference in the normalised roughness length is greater than 15% across the whole of London. The continually changing roughness of London indicates that the surface layer may never fully become adjusted to the underlying surface and that the flow may be in a continuous state of adjustment.

Assuming that the expressions of Raupach (1994) and Counihan (1971) provide good estimates for the aerodynamic parameters, the plots in Figure 2.10 show that the frontal area density estimates larger values of the displacement height than the plan area density. In contrast the plan area density estimates larger values of the roughness length than the frontal area density. Despite more than a dozen studies investigating the influence of area density ratios on aerodynamic parameters, it is not yet clear which of the two area density ratios has greatest influence on the displacement height and roughness length. Part of the reason for this lies in the way the algorithms have been formulated. Many of the experimental and theoretical studies have been applied solely to cube shaped roughness elements (eg. Macdonald et al., 1998; Cheng et al., 2007; Kanda, 2004; Xie and Castro, 2006). Cubes have the unique property of having equal plan and frontal area density for all arrangement of elements. Kutzbach (1961) used bushel baskets as roughness elements yet still found that the plan and frontal area densities were coincidentally similar. Figure 2.11a shows the correlation between plan and frontal area density for all 1650 gridboxes. The plot shows significant correlation between the two parameters but indicates that for Greater London, the plan area density does not equal the frontal area density across any gridbox.

2.3.2 Heterogeneity of building morphology

In Figure 2.11a, the correlation between the plan and frontal area densities calculated for 1km² gridboxes is extended to calculations across 2km² and 4km² resolution gridboxes. Comparing the magnitude of values in the three plots in Figure 2.11 shows that increasing the gridbox area reduces the range of the values of \( \lambda_P \) and \( \lambda_F \). Between the 1km² and 4km² cases the peak values of \( \lambda_P \) and \( \lambda_F \) reduce by 33% and 26% respectively. Applying these changes to the expressions of Macdonald et al. (1998), the difference in the zero-plane displacement
height and the aerodynamic roughness lengths are approximately 20% and 150% respectively. These are significant differences which arise due to the heterogeneity of the urban surface and open the debate about the ‘correct’ gridbox area over which building morphology parameters should be calculated. If the area is too small, the morphology parameters are not fully representative of distinct neighbourhoods and the geometry of individual buildings begin to dominate average morphological values. If the area is too large, the presence of neighbourhood patches becomes smoothed out and the building morphology parameters exhibit large spatial scatter. Researchers in the past have used arbitrary values for the grid-box resolution ranging from 100m (Burian et al., 2003) to 1km (present study). Long et al. (2003) in their analysis of the St Jerome area in Marseille, France suggest that a gridbox of 200m is sufficient to ‘bring out the urban structure of each district without representing a fragment only of the urban fabric.’ However there is no physical justification for this and it is merely explained qualitatively. Bottema and Mestayer (1998) computed large values in the aerodynamic roughness length when they mapped roughness lengths on a 150m grid for the centre of Strasbourg, France. This was due to the fineness of the grid box resolution which was the order of the height of some buildings. They noted that these spikes averaged out when the grid box resolution was increased to >300m.

The use of aerodynamic formulations for numerical weather prediction and dispersion modelling relies on the assumption that building morphology across a grid box is reasonably uniform. In the absence of parameterisations to account for heterogeneity of building morphology within gridboxes, the gridbox resolution itself determines the scale of horizontal heterogeneity across an urban area. Mahrt (1996) explains that the vertical depth of influence of surface features, determined for example by the blending height, increases with increasing horizontal scales of heterogeneity and surface roughness. Since an estimation of the surface drag is itself dependent on the gridbox resolution, identifying an appropriate gridbox resolution would provide a more accurate estimate of the vertical influence of buildings on the wind flow. Based on physical reasoning, an appropriate resolution across which building morphology parameters could be calculated is the lengthscale at which the surface exhibits horizontal homogeneity. In this respect, homogeneity is defined as the condition when the building morphology parameters are relatively independent of horizontal lengthscale. Figure 2.12 shows an illustrative explanation. In an ideal urban surface in which the buildings are of equal size and shape and distributed uniformly, morphology parameter values will be the same for any horizontal lengthscale. For a surface consisting of various size and shape buildings, and distributed non-uniformly, the size of the horizontal lengthscale will
determine the value of a morphological parameter as illustrated for the area density ratios in Figure 2.11. If the lengthscale of the domain across which a morphology parameter is calculated is too small such that it is of the order of the building dimensions, the value of the morphology parameter will vary with the lengthscale of the domain. If the lengthscale of the domain is too large, morphological heterogeneity arising from individual building features will be averaged out. However, the non-uniform layout of the buildings will induce large-scale heterogeneity which too will lead to the morphological parameter being dependent on the lengthscale of the domain. Thus a neighbourhood of buildings may consist of multi-lengthscales of heterogeneity. If the lengthscale of heterogeneity based on small-scale features of the surface is denoted $L_{H(1)}$ and similarly, the lengthscale of heterogeneity based on large-scale features of the surface is denoted $L_{H(2)}$, then in the region $L_{H(1)} < L_{H} < L_{H(2)}$, the surface exhibits homogeneity within which the value of a morphology parameter does not vary with the lengthscale of the domain across which the parameter has been calculated. In such cases, the aerodynamic parameters estimated using morphology based algorithms will also be independent of the lengthscale of the domain.

![Diagram](image_url)

**Figure 2.12** – The homogeneity/heterogeneity of an urban surface due to the horizontal lengthscale of the domain across which morphology parameters are calculated.

To identify whether or not multi-lengthscales of heterogeneity and surface homogeneity were present in real urban areas, a morphology convergence study for the plan area density was carried out using the Ordnance Survey 1:10000 scale raster dataset. A 3600km$^2$ area of London was divided into 144 square tiles of 25km$^2$ as shown in Figure 2.13. Starting from the centre of each tile, the value of $\lambda_P$ was calculated for a segmented square tile measuring 1.27m x 1.27m. The square tile was then increased in size to 6.35m x 6.35m and $\lambda_P$ recalculated. This sequence was repeated with tile size increasing in increments of 5.08m up to 5km. For a single 5km tile, this process resulted in 985 different tile sizes and corresponding values of $\lambda_P$. 
Figure 2.13 – Map of London showing the Ordnance Survey 1:10000 scale colour raster image locations. Each tile is 5km x 5km. The tile highlighted in red corresponds to the location of the convergence plot in Figure 2.14a.

Each of the 144 tiles across London were analysed using this method. Figure 2.14a shows the values of $\lambda_P$ plotted against the tile length for the 5km$^2$ patch of the urban surface shown in Figure 2.14b and highlighted red in Figure 2.13. Values of $\lambda_P$ are highly variable for small tile lengths because the tile area is of the order of individual building areas. As the tile length is increased the number of buildings within the tile increase and the trend of $\lambda_P$ becomes less variable and converges. To quantify the level of convergence, the running coefficient of variation, $c_v$ was calculated across a length, $\Delta x$ such that:

$$c_v\bigg|_{\Delta x} = \frac{\sigma_{\lambda_P}}{\bar{\lambda}_P} \bigg|_{\Delta x}$$

[2.1]

where $\sigma_{\lambda_P}$ is the standard deviation of $\lambda_P$ and $\bar{\lambda}_P$ is the average value of the plan area density, both across an interval length, $\Delta x$.

The red curve in Figure 2.14a shows the running coefficient of variation for an interval length of 500m. For values where the $c_v$ is below 0.0125, we assume that values of $\lambda_P$ have converged over that 500m interval.
Figure 2.14 – (a) The corresponding convergence plot (blue) of the plan area density (left axis) and coefficient of variation plot (red) with $\Delta x = 500m$ (right axis). The black dashed line represents $c_v = 0.0125$. (b) The Ordnance Survey 1:10000 scale colour raster image of the 5km tile highlighted red in Figure 2.13.

Figure 2.14a clearly shows the lengthscale of heterogeneity based on the small-scale and large scale heterogeneous features of the building geometry and layout at tile lengths of approximately 800m and 2100m respectively. Within this tile length range where the value of $\lambda_P$ has converged, the surface exhibits homogeneity of the morphology.

It would be expected that the length scales of heterogeneity are larger in suburban/rural areas where the large spacing between buildings provides the opportunity for less regularity in building distribution. In the central business districts of cities such as London it is economically beneficial to maximise the ratio of total building floor space to total ground area. One method to achieve this is to build tall buildings with many floors. Another method is to group buildings close together so that the space is utilised as efficiently as possible. Buildings with a rectangular plan shape are the most efficient at utilising space. This leads to the hypothesis that the tile length for the convergence of $\lambda_P$ is a function of $\lambda_P$ itself. Figure 2.15 shows the lengthscale of heterogeneity plotted against the converged values of $\lambda_P$ for the length scales of heterogeneity for both small-scale and large scale heterogeneous features. The data has been binned into groups of lengthscales of 200m. Despite the estimated error bars indicating significant values of the standard error, there is a good logarithmic relationship ($R^2 = 0.77$) between the converged values of $\lambda_P$ and the length scales of heterogeneity.
Figure 2.1 – Converged plan area density plotted against the lengthscale of heterogeneity at which convergence has taken place. The points plotted are for all tiles in Figure 2.13 binned into lengthscales of 200m. The black solid line is a logarithmic best fit line. The trend suggests that the smaller the value of $\lambda_P$ the larger the lengthscale of heterogeneity. From a modelling perspective, if the grid box resolution is chosen such that it is of the lengthscale where the surface is homogeneous, then for suburban areas where $\lambda_P$ is relatively small the resolution would be of the order of 4km whereas a finer resolution of the order of 500m would be required in the more densely packed suburban and CBD areas. This would ensure that the grid box averaged $\lambda_P$ values are representative of the underlying heterogeneous surface.

In the present study, the convergence of the frontal area density ratio has not been considered because such an analysis would require high resolution building height data which was unavailable. However, it is likely that the frontal area density would converge at some lot area length. Whether or not that lengthscale would coincide with the lengthscale at which the plan area density converges is difficult to predict.

2.4 Summary
In this chapter, the area of Greater London has been used as a case study to investigate the typical morphology of buildings. High-resolution data has been used to map the plan and frontal area density ratios and based on these parameters, the spatial structure of the
morphology of London appears to be well-described by a concentric ring model, with large area density values close to the central business district and smaller values in the suburban and rural regions. Importantly, the correlation of the two parameters averaged across a lot area, has shown that the buildings are of non-cubic shape with generally a larger plan area compared to the projected frontal area. Using the area density values, the normalised displacement heights and roughness lengths were estimated using the morphology based algorithms of Raupach (1994), Counihan (1971) and Macdonald et al. (1998). The algorithms estimated peak values of the normalised displacement height in the centre of London. However, the algorithms of Counihan (1971) and Macdonald et al. (1998) showed a peak in the normalised roughness length in the suburban areas.

Computation of the plan and frontal area density ratios showed that their values are strongly dependent on the resolution of the gridbox. As expected, the area density ratios decrease with gridbox resolution which leads to smaller values of the aerodynamic parameters as estimated from morphology based algorithms. In the absence of a general consensus on an appropriate gridbox resolution, a method based on the convergence of morphology parameters has been attempted. It is shown that as the lot area is increased, the plan area density converges to a less variable value. The length of the lot area at which the initial convergence takes places defines the lengthscale of heterogeneity of the surface. It is shown that for some tiles, as the lot area is increased further, a second convergence of the plan area density takes place defining the lengthscale of heterogeneity based on larger scale features of the surface. When analysing the convergence of the plan area density across London, it appears that the converged value of $\lambda_P$ is a function of the lengthscales of heterogeneity. The analysis suggests that the larger the value of the plan area density, the larger the lengthscale of heterogeneity.

As the results of this chapter suggest, buildings with a non-cubic shape are a common feature of the urban area of London. A visual inspection of buildings in other towns and cities around the world suggest that London is not alone for constructing cuboid-shaped buildings. Thus, in chapter four a wind tunnel experiment is conducted to determine the surface drag of non-cubic obstacles in turbulent shear flow using the integral momentum method which is described in chapter three along with a detailed description of the experimental set-up.
Chapter Three

Estimating surface drag using wind tunnel experimentation

3.1 Introduction

Methods to estimate and measure the surface drag of rough wall surfaces using wind tunnel modelling have been used by many researchers for a long time. The bulk surface drag coefficient can be measured using several methods. The following is a list of various methods employed in previous studies together with their benefits and limitations.

- $u'w'$ covariance (e.g. Cheng and Castro, 2002a) – measured in the so-called constant-stress layer above the roughness where the friction velocity is determined using equation [1.9]. This is a simple and well-established method used both in the laboratory and in the real atmosphere. However, the method requires a constant-stress layer (ISL) which may not be present above some rough wall surfaces. Also, high frequency instrumentation is required to capture turbulent fluctuations in velocity.

- Slope of the logarithmic velocity profile (e.g. Feddersen, 2004) – measured in the inertial sublayer where $\frac{dU}{dz} = \frac{u_*}{\kappa(z - z_d)}$. This too is a simple and well-established method. However, a logarithmic velocity profile is necessary and often the errors can be significant. For example, Feddersen (2004) estimated errors in the friction velocity of approximately 10% and in the surface roughness length of approximately 24%.

- Velocity defect law (e.g. Raupach et al., 2006) – measured in the outer layer where the flow is dependent on $z, u_*$ and $\delta$, but not on any surface length scales. This method does not require an inertial sublayer and is found to be effective for sparse, heterogeneous arrays. However, the method relies on a canonical boundary layer in
which the surface is uniform, the free-stream is non-turbulent and the boundary layer is grown naturally without any tripping devices.

- **Form drag** (e.g. Cheng *et al.*, 2007) – Involves the direct measurement of the form drag of obstacles by integrating the pressure difference across the obstacle walls. The form drag is then related to the surface shear stress. The direct measurement of the shear stress results in fairly accurate values. However, in practice the technique is limited to the laboratory as external pressure measurements of buildings are difficult to obtain.

- **Floating drag balance** (e.g. Hagishima *et al.*, 2009) – the shear stress is measured directly by floating a section of the surface on a fluid base which is attached to a pre-calibrated strain gauge. This too involves direct measurements of the surface shear stress, however, the technique is limited to the laboratory.

The present research involves a wind tunnel study in which the surface roughness is quantified using the $u'w'$ covariance method. This method is used to validate a method which equates the loss in momentum across a rough wall surface to the surface drag and commonly known as the integral momentum method.

In this chapter, a theory of the integral momentum method is described and the wind tunnel experimentation method is explained. The wind tunnel methodology is consistently used to relate obstacle geometry to surface drag in the chapters of this thesis that follow.

### 3.2 The integral momentum method for drag estimation

#### 3.2.1 Theoretical derivation

Given the velocity and shear stress profiles within a boundary layer growing over a surface, at two locations separated by a distance $\Delta x$, the bulk drag coefficient for the underlying surface can be determined using the integral momentum method and the $u'w'$ covariance method. The former method relies on determining the amount of momentum lost in the fluid as it flows across an array and the underlying theory of the integral momentum method was first presented by Theodore von Karman in 1921.

Figure 3.1 illustrates the growth of the boundary layer between location $X_1$ and $X_2$ on a surface.
The flow of air over a surface forms a boundary layer in which the streamlines become deflected away from the surface to satisfy the mass conservation principle. The boundary layer depth, $\delta$, is here defined as the vertical height from the surface at which the velocity is equal to 99% of the free-stream velocity. As the boundary layer grows with streamwise distance, the streamlines are deflected upwards and there is an increase in the depth of the boundary layer. To reduce the total mass flow rate of a frictionless fluid by the same amount, the surface would have to be displaced outwards by a distance equivalent to the displacement thickness, $\delta^*$ which is calculated from the velocity profile using

$$\delta^* = \int_0^\delta \left(1 - \frac{U}{U_\delta}\right)dz$$  \[3.1\]

where $U_\delta$ is the velocity in the free-stream, $U$ is the flow velocity at height $z$ and $\delta$ is the boundary layer depth.

Between adjacent streamlines there is a loss in horizontal momentum flux. The transverse distance that the flow would have to be displaced to compensate for this loss of momentum flux is equal to the momentum thickness, $\theta$. The momentum flux deficit, $\Phi$ is related to the momentum thickness, $\theta$ as follows,

$$\Phi = \rho U_\delta^2 \theta = \int_0^\delta [(U_\delta - U)\rho U]dz$$  \[3.2\]

where $\rho$ is the density of the air, $U_\delta$ is the velocity in the free-stream, $U$ is the flow velocity at height $z$, $\delta$ is the boundary layer depth and $\theta$ is the momentum thickness.
It is assumed here that the flow is incompressible and therefore the density of the air, $\rho$, is constant throughout. Re-arranging equation [3.2] provides the momentum thickness of the flow as a function of the velocity profile within a growing boundary layer and is given as:

$$\theta = \int_{0}^{\delta} \frac{U}{U_\delta} \left( 1 - \frac{U}{U_\delta} \right) dz$$  \hspace{1cm} [3.3]$$

For a steady, two-dimensional boundary layer with an arbitrary pressure gradient, growing over a porous surface, the general equation can be derived using equation [3.2] as a starting point.

To quantify the rate of loss of momentum, equation [3.2] is differentiated by applying the product rule to give:

$$\frac{d}{dx} \left( \rho U_\theta^2 \theta \right) = \int_{0}^{\delta} \left[ \frac{\partial (\rho U)}{\partial x} (U_\delta - U) + \rho U \frac{dU_\delta}{dx} - \rho U \frac{\partial U}{\partial x} \right] dz$$  \hspace{1cm} [3.4]$$

The two-dimensional mass conservation equation is given as:

$$\frac{\partial (\rho U)}{\partial x} = - \frac{\partial (\rho W)}{\partial z}$$  \hspace{1cm} [3.5]$$

The two-dimensional momentum conservation equation is given as:

$$\rho U \frac{\partial U}{\partial x} = -\rho W \frac{\partial U}{\partial z} + \rho U_\delta \frac{dU_\delta}{dx} + \frac{\partial \tau}{\partial z}$$  \hspace{1cm} [3.6]$$

By substituting equations [3.5] and [3.6] into the first and last terms on the right-hand side of equation [3.4] respectively, we get:

$$\frac{d}{dx} \left( \rho U_\theta^2 \theta \right) = \int_{0}^{\delta} \left[ \frac{\partial (\rho W)}{\partial z} (U_\delta - U) + \rho U \frac{dU_\delta}{dx} + \rho W \frac{\partial U}{\partial z} - \rho U_\delta \frac{dU_\delta}{dx} - \frac{\partial \tau}{\partial z} \right] dz$$  \hspace{1cm} [3.7]$$

Equation [3.7] can be simplified using the product rule and assuming that $U_\delta$ is independent of $z$ such that, $\frac{\partial U_\delta}{\partial z} = 0$: 
Chapter Three: Estimating surface drag using wind tunnel experimentation

\[
\frac{d}{dx} \left( \rho U_{\delta}^2 \theta \right) = -\int_{0}^{\delta} \left( \frac{\partial}{\partial z} \left( \rho W (U_{\delta} - U) + \tau \right) \right) dz - \frac{dU_{\delta}}{dx} \int_{0}^{\delta} (\rho U_{\delta} - \rho U) dz
\]  

[3.8]

Simplifying equation [3.8] gives:

\[
\frac{d}{dx} \left( \rho U_{\delta}^2 \theta \right) = -\left[ \rho W (U_{\delta} - U) + \frac{\partial}{\partial \delta} \left( \rho U_{\delta} \delta^* \right) \right] - \frac{dU_{\delta}}{dx} (\rho U_{\delta} \delta^*)
\]  

[3.9]

The boundary condition at \( z = \delta \) is \( (U_{\delta} - U) = 0 \) and \( \tau = 0 \). Similarly the boundary condition at the surface where, \( z = 0 \) is \( W = W_w \) and \( U = 0 \). Equation [3.9] then becomes:

\[
\frac{d}{dx} \left( \rho U_{\delta}^2 \theta \right) + \rho U_{\delta} \frac{dU_{\delta}}{dx} \delta^* = \tau + \rho W_w U_{\delta}
\]  

[3.10]

which is von Karman’s integral equation relating the momentum balance to the wall parameters.

In Equation [3.10], the first and second terms on the left-hand side are the rate of loss of momentum and the effect of the free-stream pressure gradient respectively. The first and second terms on the right-hand side are, respectively, the surface shear stress and the additional momentum introduced into the boundary layer from any flow that may be entering through the surface if porous. If the wind tunnel surface is non-porous (ie. \( W_w = 0 \)) and the streamwise pressure gradient is small enough to be neglected (ie. \( dU_{\delta}/dx \approx 0 \)), equation [3.10] simplifies to

\[
\tau = \frac{d}{dx} \left( \rho U_{\delta}^2 \theta \right)
\]  

[3.11]

Integrating equation [3.11] from an upstream surface location, \( x_1 \) to a downstream surface location, \( x_2 \) gives the total surface drag between the two locations such that:

\[
D(x) = \int_{x_1}^{x_2} \tau \ dx = \rho U_{\delta}^2 d \theta
\]  

[3.12]
The drag force can also be written as

$$ D(x) = \frac{1}{2} \rho U_x^2 C'_{D(\theta)} $$  \hspace{1cm} [3.13] $$

where $C'_{D(\theta)}$ is the drag coefficient determined using the integral momentum method.

Combining equations [3.12] and [3.13] provides a relationship between the drag coefficient and the momentum thickness (per unit length, $\Delta x$) such that:

$$ C'_{D(\theta)} = \frac{2\Delta \theta}{\Delta x} $$  \hspace{1cm} [3.14] $$

Equation [3.3] shows that the momentum thickness can be determined from the velocity profile of the boundary layer alone. Thus equation [3.14] suggests that for any surface upon which there is a flow of fluid, the bulk drag coefficient can be determined provided that the full velocity profile of the boundary layer is known at two locations a distance, $\Delta x$ apart.

### 3.2.2 The bulk drag coefficient – meteorology versus engineering

In chapter one, section 1.3.3, the bulk drag coefficient, $C_{D(u_\ast)}$, is presented in equation [1.8] to be a function of the friction velocity and a reference velocity. This bulk drag coefficient is often quoted in the field of meteorology. The bulk drag coefficient, $C'_{D(\theta)}$, presented in equation [3.14] and based on the integral momentum method is different to $C_{D(u_\ast)}$ and is often quoted in an engineering context. The difference arises from the inclusion of a factor of 0.5 in the wall shear stress used to derive $C'_{D(\theta)}$, which is absent from the derivation of $C_{D(u_\ast)}$. To enable values of the drag coefficient to be compared, a relationship between the two different drag coefficients needs to be derived.

From section 1.3.3, equating equations [1.6] and [1.8], the surface shear stress can be expressed as:

$$ \tau = \rho U_x^2 C_{D(u_\ast)} $$  \hspace{1cm} [3.15] $$
From equation [3.11] the wall shear stress can also be expressed in terms of the momentum thickness as:

$$\tau = \rho U_\delta^2 \frac{\Delta \theta}{\Delta x}$$

[3.16]

In both the meteorological and engineering context the total surface drag is equal and therefore the surface shear stresses given in equations [3.15] and [3.16] can be equated to give:

$$C_{D(u_\delta)} = \frac{\Delta \theta}{\Delta x}$$

[3.17]

The drag coefficient in equation [3.17] can be related to that determined from the integral momentum method given in equation [3.14] by method of substitution such that:

$$C_{D(u_\delta)} = \frac{C'_{D(\bar{\theta})}}{2}$$

[3.18]

Thus the bulk drag coefficient derived from the friction velocity is half that of the bulk drag coefficient derived from the integral momentum method.

### 3.3 Wind tunnel set-up

In the chapters that follow, experimental estimates of aerodynamic parameters are obtained from mean and fluctuating velocity measurements in a wind tunnel in which several obstacle arrays are considered. In the next sections, the experimental set-up of the wind tunnel and the instrumentation used is described.

#### 3.3.1 Obstacle elements

All surface arrays investigated in this study consisted of obstacles in the form of standard Lego® bricks mounted on Lego® baseboard. The Lego® baseboard is a flat sheet of plastic with cylindrical protrusions upon which Lego® bricks can be securely fixed without movement. The Lego® bricks themselves also have cylindrical protrusions on top, upon which similar bricks can be mounted. The dimensions of both the baseboard and standard Lego® bricks are shown in Figure 3.2. The Lego® bricks and baseboard are manufactured from a plastic known as acrylonitrile butadiene styrene. The manufactured dimension tolerance of each brick is
1/1000th of a millimetre. This provides Lego® with a high degree of experimental reproducibility. The inter-locking and release of the bricks to the baseboard also makes Lego® convenient to use and permits several roughness arrays to be constructed relatively cheaply and quickly.

![Wind Flow](image)

**Figure 3.2** – Dimensions of standard Lego® bricks (light blue fill) and Lego® baseboard (dark blue fill). The faces used in the calculation of the roughness element frontal area (green solid line) and plan area (red solid line) are also shown.

### 3.3.2 Wind tunnel layout

The wind tunnel experiments for the whole of the study took place in the ‘A’ wind tunnel at the Environmental Flow Research Centre (EnFlo) at the University of Surrey, UK. The working section of the wind tunnel is constructed of wood with glass side panels and is 4500mm (length) x 900mm (width) x 600mm (height). It is a blow down, low speed, open-circuit wind tunnel with a free-stream turbulence intensity of < 0.1%. The streamwise, lateral and vertical coordinates are denoted as (x,y,z) respectively, with the origin, x = 0 at the working section inlet, y = 0 at the wind tunnel centreline and z = 0 on the base of the roughness element array. For all experimental runs, the free-stream velocity, \( U_\infty \) was set to 10ms\(^{-1}\). Between the entrance of the working section and a distance x = 1500mm, the roof of the working section was constructed from a combination of wood and a movable acetate sheet. The design of the construction enabled the roof to move under the actions of an automated system yet prevent leakage of the flow from the ceiling.

The primary instrument used to measure the flow velocity and turbulence was a cross-wire anemometer calibrated against a pitot-static tube. The pitot-static tube was positioned at a location (x,y,z) = (491, -260, 420)mm in the free-stream of the wind tunnel working section. At
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this location the velocity measurements made by the pitot-static tube were not influenced by the shallow boundary layers on the walls of the working section, nor did the presence of the pitot-tube influence the flow measured by the cross-wire anemometer. The measurement domain of the cross-wire anemometer was confined to an imaginary box bounded by opposing corner coordinates \((x,y,z) = (491, 110, 15.6)\) and \((986, -110, 96)\). The cross-wire probe was mounted on the end of a rigid boom which itself was mounted on an automated three-dimensional traverse system housed outside the working section of the wind tunnel.

A series of flow filters were positioned at the inlet of the wind tunnel to minimise the presence of dust particles and foreign debris in the working section. Before the air entered the working section it was passed through a contraction to accelerate the flow and minimise the velocity variations within the working section. The contraction area ratio, \(C_R\) (the ratio of the cross-sectional area at the contraction inlet to that at the contraction outlet), was approximately equal to 6.25. Mikhail (1979) suggested that flow velocity fluctuation in the centre of the working section reduces by a factor equivalent to \(1/(C_R)^2\). Thus, it is estimated that the presence of the contraction section of the wind tunnel helps to reduce velocity fluctuations by approximately a factor of 40. Velocity measurements at the centre of the wind tunnel working section, at \((x,y,z) = (491,0,200)mm\) gave values of \(\Delta U/\overline{U_s} = 0.02\). At the outlet, the flow was guided upwards using a series of guiding corner vanes. The corner vanes were at an average distance of 4250mm downstream from the entrance of the wind tunnel working section. At this location the vanes are at a sufficient distance downstream such that their presence has negligible influence on the flow within the measurement domain.

The entire width of the floor of the working section was lined with Lego® baseboard between the streamwise locations \(x = 0 \text{ mm}\) and \(x = 1500\text{mm}\). To generate a turbulent wind velocity profile in the wind tunnel, a flow tripping fence was positioned at the entrance of the working section. It was constructed from Lego® with standard 1 x 1 Lego® bricks placed adjacent to each other to create a single row arrangement. The length, width and height of the fence measured 900mm x 7.8mm x 11.4mm respectively. Lego® bricks were used as roughness elements and were mounted on the Lego® baseboard.

Figure 3.3 shows a side-view schematic of the general wind tunnel set-up. Following the set-up of all the instrumentation and surface arrays, all gaps in the walls of the working section were sealed to prevent flow leakage. Since the flow within a wind tunnel is confined by the solid walls of the working section, the presence of roughness elements generates blockage.
which can influence the flow. With the roughness elements inside the working section, the maximum wind tunnel blockage ratio, $\Phi_A$ was equal to 0.019 (where $\Phi_A$ is the ratio of the cross-sectional area of the working section to the cross-sectional area of the roughness elements). Sahini (2004) showed that the drag on a rectangular block is particularly sensitive to the wind tunnel blockage ratio when $\Phi_A > 0.05$ and therefore it is assumed that the current blockage ratio is within an acceptable range.

### 3.3.3 Working section floor

The age and wooden construction of the wind tunnel resulted in a working section floor which was not flat and level but instead was visibly uneven. The flow streamlines follow the surface of the working section. Since the vertical gradient of the flow velocity is greatest near the surface, small differences in the elevation of the surface can result in significantly different flow velocities if the measuring instrument is fixed relative to the fixed reference frame of the laboratory. To determine the range of the uneven elevation across the measurement domain, an electronic height gauge, designed and manufactured by EnFlo, Surrey was used. The gauge incorporates a movable probe, which when displaced vertically, generates a measurable voltage related to the level of displacement. The sensitivity of the probe permitted displacement measurements to the nearest 1000$^{\text{th}}$ of a millimetre. The unevenness of the floor was insufficient to cause flow separation. However, within the flow measurement domain, the maximum range of the undulations was approximately 4mm. To assess the influence of the surface undulations on the measured flow, a test was carried out with a tripping fence positioned at the entrance of the working section and Lego® baseboard mounted on the surface. The surface did not contain any roughness elements. The test
involved the measurement of the flow velocity using a cross-wire at 32 equally-spaced locations in the horizontal plane spanning the entire measurement domain. The measurement locations included regions at which the minimum and maximum elevations were present. The cross-wire was fixed at a vertical height of $z=15.6\,mm$ relative to the height of the floor at the inlet of the working section. With the free-stream velocity set to $10\,ms^{-1}$, the average difference in the flow velocity across the measured locations was approximately 7%. To reduce the range in the surface elevation of the working section floor, two metal car jacks were positioned under the working section in the vicinity of the flow measurement domain to reduce the surface undulations. Figure 3.4 shows a contour plot of the surface undulations within the flow measurement domain following an attempt to level out the floor.

The downstream measurements show a greater surface elevation than the upstream points. This is due to a thick wooden rib which spanned the working section at $x = 1050\,mm$. Following several attempts to level the surface, the minimum range of surface elevations achieved was 0.8mm. The flow velocity was measured again at the 32 lateral locations. The difference in the velocity was reduced to 0.5% and this was considered good enough to proceed. A sensitive spirit level was employed to level out the working section floor, upstream of the measurement domain, from the inlet to $x = 1500\,mm$.

**Figure 3.4** – Wind tunnel surface elevation contour plot within the flow measurement domain.
3.3.4 Pressure gradient effects

Equation [3.10] shows that the presence of a free-stream pressure gradient increases the wall shear stress. To neglect the effect of a pressure gradient in integral momentum analysis, it must hold true that:

$$\rho_\delta U_\delta \frac{dU_\delta}{dx} \delta^* \ll \frac{d}{dx}\left(\rho_\delta U^2 \theta\right)$$  \[3.19\]

To measure the effect of the pressure gradient, velocity profile measurements were taken at an upstream location of $X_1 = 491\, mm$ and a downstream location of $X_2 = 986\, mm$. Of all experiments conducted for the present study, the maximum displacement thickness, $\delta^*$ was calculated to be $7\, mm$, using equation [3.1]. The term $\frac{dU_\delta}{dx}$ was calculated directly from measurements of the free-stream velocity at $(x,y,z) = (491,0,81)\, mm$ and $(986,0,81)\, mm$. The maximum value of $\frac{dU_\delta}{dx}$ was measured to be $0.03\, s^{-1}$. With $\rho = 1.225\, kgm^{-3}$ the left-hand side term of equation [3.19] is approximately $1\%$ of the term on the right-hand side. Thus it is valid to consider the effect of the free-stream pressure gradient to be negligible. In addition, Cheng and Castro (2002a) used the same tunnel to conduct experiments over rough surface arrays and noted that the pressure gradient in the wind tunnel was too small to have any significant influence on the flow measurements.

3.4 Instrumentation and measurement

In the present study the reference mean flow velocity was measured using a standard pitot-static tube and the velocity and turbulence in the boundary layer was measured using a cross-wire anemometer. The principle of flow measurement of the pitot-static tube is described well in Houghton and Carpenter, (2003). Hot-wire anemometry and specifically, cross-wire anemometry measurement principles are described in detail in Bruun (1995). In the following subsections, the instrumentation set-up is described.

3.4.1 Pitot-static tube measurements

A pitot-static tube determines the flow velocity by measuring the difference between the stagnation and static pressures of the oncoming flow, such that:
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\[ p_0 - p_s = \Delta p = \frac{p_a U^2}{2RT} \]

where the subscripts, ‘0’ and ‘s’ refer to the stagnation and static pressures respectively, \( p_a \) is the atmospheric pressure, \( U \) is the flow velocity, \( R \) is the specific gas constant (=287 Jkg\(^{-1}\)K\(^{-1}\)) and \( T \) is the temperature of the air.

The temperature of the air was measured using a thermocouple located at the inlet of the working section. The atmospheric pressure was measured using a highly accurate electronic pressure transducer located at the National Physical Laboratory in Teddington, 12km from the laboratory. The measurement uncertainty of the pressure transducer was ±30Pa, resulting in the uncertainty of the flow velocity equal to ±0.02ms\(^{-1}\).

The readings of the pitot-static device were connected to a digital differential pressure indicator, model FCO12, manufactured by Furness Controls. The manometer gave an output of 1mV for a 1Pa pressure difference. The pressure tubes were sealed at both ends to ensure there was no pressure leakage and the manometer reading was adjusted to zero after each experimental run.

### 3.4.2 Cross-wire anemometer set-up

The hot-wire anemometer system consisted of two hot-wires mounted on a single probe. This type of probe is known as a cross-wire with the two wires operating independently and aligned at an angle to each other forming an x-configuration. A cross-wire enables two dimensions of the flow velocity to be measured simultaneously. A schematic of the 120° cross-wire used in the present study is shown in Figure 3.5.

![Figure 3.5 – Schematic of a 120° dual sensor cross-wire probe.](image-url)
The two wires were 5µm in diameter, 3mm in length and manufactured from platinum-plated tungsten. The junctions of the wires with the prongs were gold-plated to minimise the conduction of heat to the prongs. This provides a more even temperature distribution across the wires enabling more accurate flow measurements. The active length of each wire was 1.25mm. Each wire was connected to a single channel of a Wheatstone bridge rack system.

<table>
<thead>
<tr>
<th></th>
<th>Wire 1</th>
<th>Wire 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shorting probe resistance, $R_L$</td>
<td>0.7Ω</td>
<td>0.6Ω</td>
</tr>
<tr>
<td>Ambient wire resistance, $R_0 @ 20^\circ C$</td>
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<td>6.3Ω</td>
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<td>Mean operating wire resistance, $R_W$</td>
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<tr>
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<tr>
<td>DC-offset voltage, $E_{os}$</td>
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</tr>
<tr>
<td>Gain, $G$</td>
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<td>6</td>
</tr>
</tbody>
</table>

**Table 3A** - Cross-wire set-up values for the CTA bridge and signal conditioner.

**Figure 3.6** – Schematic of the arrangement of instrumentation for cross-wire and pitot-static tube measurements.
designed and manufactured by the Department of Mechanical Engineering at the University of Newcastle, Australia. Once the voltage signal had passed through the bridge, it was passed through a signal conditioner for filtering and amplification. To remove noise, the signal was passed through a low pass filter with a cut-off frequency of 5.2 kHz. The clean signal was then reduced to reset to 0V for a flow velocity of 0ms\(^{-1}\). This was achieved by applying a DC-offset voltage. The signal was then amplified by setting a gain such that at 10ms\(^{-1}\), the wire voltage read just under 10V. The 10V range for a 0-10ms\(^{-1}\) flow velocity ensured that the recorded velocity was at the greatest resolution possible. It was necessary to ensure that the signal did not exceed 10V as the data acquisition system could only accommodate signals in ±10V range. Signals above that voltage would be subject to clipping. The analogue voltage output from the rack passed through a 4 channel ±10V USB data acquisition and carrier system (model: NI-USB-9162; NI-9215) manufactured by National Instruments Corporation, using BNC connection cables. The digitised signal was then transferred to a computer in which the LabView (v9.2) data logging software was used to analyse the data. To achieve the greatest accuracy in the velocity and turbulence data from the cross-wire, the Wheatstone bridge and signal conditioners were set-up to their optimum conditions. The optimum set-up involved operating the wires at the maximum recommended temperature to improve the sensitivity of the wires to velocity fluctuations. Table 3A lists the electronic set-up of the Wheatstone bridge and signal conditioner for optimum cross-wire conditions. For a complete definition of each set-up parameter, see Bruun (1995). Figure 3.6 shows a schematic of the connections between instruments and data processing equipment.

### 3.4.3 Velocity and angle calibrations

The cross-wire probes were calibrated for velocity against measurements from the pitot-static tube. During the calibration the location of the cross-wire was at \((x,y,z) = (491, 110, 400)\)mm and the pitot-static tube was located at \((x,y,z) = (491, 280, 400)\)mm. The two instruments were located at the same streamwise and vertical locations but physical restrictions prevented the two instruments from being co-located at the same lateral location. However, it was checked that the measured velocities were equal at the two lateral locations. Figure 3.7 shows a typical calibration of the cross-wire with the wire voltages and reference velocity plotted to show a linear trend.

King (1914) conducted a theoretical and experimental study to show that the flow
velocity, $U$ is related to the voltage across the hot-wire, $E$ as follows:

$$E^2 = A + BU^n$$  \[3.21\]

where $A$, $B$ and $n$ are empirical constants and are specific to individual hot wires.

**Figure 3.7** – A typical velocity calibration of a cross-wire against a reference velocity derived from a pitot-static tube. The value of the exponent $n=0.45$.

The exponent, $n$ was taken to be 0.45 for all calibrations since this is the theoretical value expected for heat-loss from a cylindrical object. Thus the linear trend in Figure 3.7 shows that the velocity variation with wire voltage follows equation [3.21] as expected. Both the cross-wire and the pitot-static tube derived velocity values are sensitive to changes in flow temperature. Since the laboratory temperature varied with the outside temperature, velocity calibrations made throughout the day were susceptible to calibration drifts. The percentage drift of the calibration was determined from the differences in the empirical constants, $A$ and $B$, between successive calibrations and was determined from fitting a linear line to the points. For calibrations where the drift exceeded 3%, the cross-wire velocity measurements were rejected. For calibration drifts less than 3%, the measured velocity values were adjusted using linear interpolation to improve the accuracy of the data. The cross-wire velocity and pitot-tube velocities were sampled at a frequency of 600Hz for 12 seconds. Figure 3.5 shows that the wires of a cross-wire probe are not aligned perpendicular to the primary flow.
direction, \( U \). This alignment of the wires enables the mean flow velocity to be decomposed into the \( u \) and \( w \) components. Since the wire axis is not perpendicular to the flow direction, there will be a component of the velocity that is parallel to the axis of the wire. For a wide angle probe such as that used in the present study, the effective cooling velocity that the wire experiences is that which is perpendicular to the wire. The parallel component has a small effect and so the effective flow velocity may be expressed as

\[
\mathbf{u}_{\text{eff}} = U \cos \phi
\]  

[3.22]

where \( \phi \) is the yaw angle between the flow vector and the normal to the axis of the probe.

Figure 3.8 – The angle calibration of the cross-wire for yaw angles between \( \pm 25^\circ \).

Figure 3.8 shows the angle calibration of the cross-wire for yaw angles between \( \pm 25^\circ \) (angle values referenced against the direction of the mean flow velocity in the working section). To carry out the calibration the cross-wire probe was mounted on a rotating plinth which fixed the streamwise, lateral and vertical positions at different yaw angles. This ensured that the same measurement space was being measured for all yaw angles, eliminating the errors due to velocity variations in the working section. The calibration was conducted at a flow velocity of \( 2 \text{ms}^{-1} \).
3.4.4 Measurement sampling time

The velocity measurements taken using a cross-wire were sampled at a frequency of 600Hz for differing sampling times depending on the turbulent variation of the flow. Near the surface the flow is more turbulent since the velocity gradient is large and this leads to greater shear instabilities. To achieve a representative average of the flow velocity, the sampling time has to be longer near the surface than for example, measurements taken in the free-stream outside the boundary layer. Here the generation of turbulence due to shear is much reduced and in such locations a shorter sampling time would provide sufficiently accurate measurements and provide a considerable reduction in experimental time. The accuracy of the velocity measurements was quantified in terms of the standard error of the sample, $\varepsilon_U$, given as:

$$
\varepsilon_U = \frac{\sigma_U}{\sqrt{N}}
$$

where $\sigma_U$ is the standard deviation of sampled measurements and $N$ is the number of measurements within the sample.

![Figure 3.9](image)

**Figure 3.9** – The average sampling time and standard error in the mean velocity against the height of vertical measurement for the S10 surface array. The left-hand axis refers to the blue curve. The right-hand axis refers to the red curve.
Of all the experiments conducted in this study, the maximum standard error did not exceed 0.022 ms$^{-1}$. This was equivalent to <0.4% of the mean velocity. In all the flow velocity measurements obtained in this study the sampling time depended on the vertical height of the measurement above the surface. Figure 3.9 shows the average sampling time and standard error in the mean velocity within the boundary layer for the roughest surface array (S10 surface array) investigated. This surface generated the greatest standard error in the mean at the lowest measurement height of $z = 15.6mm$. The minimum and maximum sampling times were 12 and 60 seconds respectively generating between 7200 and 36000 samples per measurement.

3.5 Summary

Estimates of the drag of a rough wall surface in a wind tunnel can be made using one of several methods. In this chapter, application of the integral momentum method to estimate the bulk drag coefficient in a wind tunnel is explained using theoretical arguments. The integral momentum method is often used in engineering and quantifies the loss in momentum across a rough wall surface from using just the upstream and downstream mean velocity profiles. This loss in momentum is related to the surface drag coefficient. Theoretically, this method has the advantage over the common meteorological $w'u'$ covariance method in that the drag can be quantified across rough wall surfaces for which a constant stress layer is not well defined and/or a logarithmic velocity profile is not present. In the next chapter, estimates of the drag coefficient from the integral momentum method are compared to those estimated from the $w'u'$ covariance method for various rough wall surfaces. However, it is important to realise that the values of the drag coefficient determined from the former method are not the same as that given by the latter method because of the differences in their definition. Instead the mathematics shows that the drag coefficient determined from the $w'u'$ covariance method is half the value of that determined from the integral momentum method.

In the following chapter, the validity of the integral momentum method is tested in a wind tunnel using a set of carefully designed experiments. The set-up of the wind tunnel and the associated instrumentation is described in this chapter. A cross-wire (hot-wire anemometer) is used as the primary instrument to measure the mean and fluctuating flow velocity within a rough wall turbulent boundary layer. The boundary layer is generated using a tripping fence positioned upstream of the measurement domain and at the inlet of the wind
tunnel working section. Lego® bricks mounted upon a Lego® baseboard are used as roughness elements.
Chapter Four

Aerodynamic parameters for uniform obstacle arrays

4.1 Introduction

The second chapter highlighted the spatial pattern of the building morphology in urban areas. The morphological pattern is so unique to individual neighbourhoods that it is almost impossible to find two neighbourhoods with an identical morphological structure. This heterogeneity increases the difficulty in formulating a universal methodology which relates the surface morphology to the surface drag and near-surface wind flow characteristics. As a first step, the majority of research effort in the past has focussed on the relatively simpler approach of estimating drag and flow features over uniform surfaces. In this context a uniform rough surface comprising an array of obstacles is defined as that in which

- The dimensions of all obstacles are identical.
- The shape of all obstacles is the same.
- The angle of orientation of those obstacles is identical.
- The spacing between all neighbouring obstacles is equal.
- The obstacles are arranged in a coherent and uniform pattern.

The simplest of uniform arrays consist of cube obstacles for which, if the cubes are aligned perpendicular to the direction of the flow, the plan and frontal area density ratio are equal in all cases. Recent wind tunnel studies (eg. Hagishima et al., 2009; Ricciardelli and Polimeno, 2006) have been conducted for uniform surface arrays involving rectangular obstacles but not necessarily cubes. For such arrays, the plan and frontal area densities are not equal, because either the width or height of the obstacles is varied. The move in research focus away from cube arrays to cuboid arrays is a step towards understanding flow and turbulence over
real urban areas for which neighbourhoods consisting of irregular building shapes and arrangements are more common than simple cubic arrays.

This chapter describes the results of a wind tunnel experiment conducted over uniform surface arrays with differing values of the plan and frontal area density. In particular, two sets of experiments are conducted in which each area density parameter is controlled to determine the sensitivity of aerodynamic parameters to the separate morphology parameters. The experiment was conducted with the aims:

- to compare the bulk surface drag derived from two independent methods namely the shear stress method and the integral momentum method.
- to determine the sensitivity of the surface roughness to the plan and frontal area density ratios.
- to test the hypothesis that existing algorithms which relate the area density of cubic obstacles to aerodynamic parameters can be used for surface arrays with non-cubic obstacles.

4.2 Experimental description

4.2.1 Design of surface arrays
The wind tunnel experiment involved ten idealised uniform surface arrays for which the detailed morphology is listed in Table 4A. The definitions of the morphological dimensions are illustrated in Figure 4.1.

For surface arrays S1 to S5 the plan area density of the array is fixed at 0.243±0.005 with the frontal area density varying between 0.066 and 0.326. For a fixed lot area the frontal area density can be varied by either changing the width of the obstacles or by changing the height of the obstacles. In the present study, the height of all the obstacles in all surface arrays was fixed at \( z_H = 11.4\, \text{mm} \) and only the width of the obstacles, \( L_Y \) was changed to vary the frontal area density. To maintain a square plan cross-sectional shape, the length of the obstacles, \( L_X \) was also changed in proportion. The plan area density of the arrays was fixed by varying the inter-element spacings, \( W_X \) and \( W_Y \) between obstacles. Surface arrays S6 to S10 have a fixed frontal area density at 0.078±0.005 with the plan area density varying between 0.059 and 0.306. The plan area density is varied by changing the width, \( L_Y \) and length, \( L_X \) of the obstacles, yet maintaining a square plan cross-sectional shape.
Table 4A - Morphological characteristics of the ten surface arrays designed for the present study.

<table>
<thead>
<tr>
<th>Surface Code</th>
<th>Obstacle Length $L_x$ (mm)</th>
<th>Obstacle Width $L_y$ (mm)</th>
<th>Obstacle Height $Z_h$ (mm)</th>
<th>Inter-Obstacle Spacing Length $W_X$ (mm)</th>
<th>Inter-Obstacle Spacing Width $W_Y$ (mm)</th>
<th>Lot Area Length $D_X$ (mm)</th>
<th>Lot Area Width $D_Y$ (mm)</th>
<th>Obstacle Plan Area $A_P$ (mm$^2$)</th>
<th>Obstacle Frontal Area $A_F$ (mm$^2$)</th>
<th>Obstacle Lot Area $A_T$ (mm$^2$)</th>
<th>Plan Area Density $\lambda_P$</th>
<th>Frontal Area Density $\lambda_F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>7.8</td>
<td>7.8</td>
<td>11.4</td>
<td>8.2</td>
<td>8.2</td>
<td>16</td>
<td>16</td>
<td>60.84</td>
<td>83.52</td>
<td>256</td>
<td>0.238</td>
<td>0.326</td>
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<tr>
<td>S2</td>
<td>15.8</td>
<td>15.8</td>
<td>11.4</td>
<td>16.2</td>
<td>16.2</td>
<td>32</td>
<td>32</td>
<td>249.64</td>
<td>168.96</td>
<td>1034</td>
<td>0.244</td>
<td>0.165</td>
</tr>
<tr>
<td>S3</td>
<td>31.8</td>
<td>31.8</td>
<td>11.4</td>
<td>24.2</td>
<td>24.2</td>
<td>48</td>
<td>48</td>
<td>566.44</td>
<td>254.4</td>
<td>2304</td>
<td>0.246</td>
<td>0.110</td>
</tr>
<tr>
<td>S4</td>
<td>59.8</td>
<td>39.8</td>
<td>11.4</td>
<td>42.2</td>
<td>42.2</td>
<td>64</td>
<td>64</td>
<td>1011.24</td>
<td>339.84</td>
<td>4096</td>
<td>0.247</td>
<td>0.083</td>
</tr>
<tr>
<td>S5</td>
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<td>7.8</td>
<td>11.4</td>
<td>40.2</td>
<td>40.2</td>
<td>80</td>
<td>80</td>
<td>1584.04</td>
<td>425.08</td>
<td>6400</td>
<td>0.248</td>
<td>0.066</td>
</tr>
<tr>
<td>S6</td>
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<td>15.8</td>
<td>11.4</td>
<td>48.2</td>
<td>48.2</td>
<td>16</td>
<td>16</td>
<td>60.84</td>
<td>83.52</td>
<td>1034</td>
<td>0.059</td>
<td>0.082</td>
</tr>
<tr>
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<td>31.8</td>
<td>11.4</td>
<td>64.2</td>
<td>64.2</td>
<td>32</td>
<td>32</td>
<td>566.44</td>
<td>254.4</td>
<td>3136</td>
<td>0.108</td>
<td>0.073</td>
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<td>S8</td>
<td>31.8</td>
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<td>11.4</td>
<td>56.2</td>
<td>56.2</td>
<td>64</td>
<td>64</td>
<td>1011.24</td>
<td>339.84</td>
<td>4096</td>
<td>0.247</td>
<td>0.083</td>
</tr>
<tr>
<td>S9</td>
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<td>31.8</td>
<td>11.4</td>
<td>72.2</td>
<td>72.2</td>
<td>72</td>
<td>72</td>
<td>1584.04</td>
<td>425.08</td>
<td>5184</td>
<td>0.306</td>
<td>0.082</td>
</tr>
</tbody>
</table>

Figure 4.1 – An idealised morphology illustration (Reproduced from Grimmond and Oke, 1999).
Figure 4.2 – The position and size of obstacles within the measurement domain as seen from a plan view. Red shaded blocks represent the obstacles. The white regions are the inter-obstacle spaces.
A scatter plot of the plan and frontal area density ratios for London calculated for 1km\(^2\) gridbox areas. The solid red line indicates the range of $\lambda_P$ and $\lambda_F$ values for surface arrays $S1$ to $S5$ tested in the wind tunnel. Similarly, the solid black line indicates the range of $\lambda_P$ and $\lambda_F$ values for surface arrays $S6$ to $S10$.

With the exception of the $S6$ surface array, the distance between obstacles is fixed and the width of the obstacles changed in proportion to maintain a fixed frontal area density. The position and relative sizes of the obstacles within the measurement domain for all ten arrays is shown in Figure 4.2. Figure 4.3 shows the range of the area density values tested in the wind tunnel in relation to actual values for London. Comparing the area density values of the idealised surface arrays tested in the wind tunnel to those calculated for Greater London, the arrays in the present study reflect Suburban London which occupies 76% of the Greater London metropolis. The remaining 24% of London is occupied by areas where the area density values are greater than approximately 0.35 or less than approximately 0.05. These regions represent the highly dense and the rural/suburban districts, respectively.

### 4.2.2 Velocity profiles and boundary layer structure

Measurements of the velocity and shear stress profiles were measured within the measurement domain described in section 3.4.3. Before the idealised surface arrays were tested, profiles of the average velocity in the lateral direction were measured using a cross-
wire probe, with the floor of the wind tunnel working section containing only Lego® baseboard and an upstream tripping fence. Figure 4.4 shows the average velocity profile measured in the lateral direction between $y=\pm 110mm$ at $(x,z)=(491,15.6)mm$ and $(986,96)mm$. It can be seen that the profile was not totally uniform and that there was a slight gradient in the trend in which the flow was faster for positive $y$ values and relatively slower for negative $y$ values. The maximum range of the variation was near the surface at the most upstream location within the measurement domain and equal to approximately 3.50% (solid red line in Figure 4.4). Further away from the surface and at the most downstream measurement location ($x=986mm$), the average velocity range was approximately 2.02% (solid blue line in Figure 4.4). It is likely that the cause of the velocity variation across the measurement domain is either due to some upstream blockage of the flow in the contraction of the wind tunnel or due to the misalignment of flow filters at the inlet of the wind tunnel. To counter the influence of the lateral velocity variations in the analysis of the results, a local free-stream velocity, $U_{\delta L}$ was measured at each lateral location. The velocity measurement at each vertical height, $U$ was normalised to the reference pitot-static tube velocity, $U_0$ and the local free-stream velocity, $U_{\delta L}$ using the relationship:

$$U_{\delta L} = \frac{U}{U_0} \times \frac{U_0}{U_{\delta L}}$$  \[4.1\]

Figure 4.5 shows the laterally-averaged wind velocity profile variation with height at $x=491mm$ and $x=986mm$ over the Lego® baseboard surface, without Lego® bricks and with the tripping fence present. The lateral-average is from 45 profiles equally spaced by $\Delta y=5mm$ from $y=-110mm$ to $y=110mm$. Velocity measurements in the vertical coordinate were taken at 31 heights spaced unequally. The concentration of measurements was greatest near the surface and at the edge of the boundary layer. Fewer measurements were taken in the outer layer. A greater concentration of measurements was taken near the surface to achieve accurate determination of the surface shear stress so that the drag coefficient could be calculated from the friction velocity. The value of drag from the integral momentum method varies by approximately 2% for a 1.5mm change in boundary layer depth. To minimise the errors, measurements at the edge of the boundary layer were made at $\Delta z = 3mm$. The depth of the boundary layer was taken to be the value of $z$ at which $U/U_\delta = 0.99$. Linear interpolation was applied between the nearest upper and lower measurement point. Figure 4.5 also shows the various layers of the rough wall turbulent boundary layer simulated in the wind tunnel. The roughness sublayer encompasses the canopy layer and lies below the inertial sublayer.
Figure 4.4 – Normalised lateral velocity profile within the measurement domain at \((x,z) = (491,15.6)\text{mm}\) and \((986,96)\text{mm}\).

Figure 4.5 – Normalised line-averaged velocity and shear stress profiles within the measurement domain at \(x = 491\text{mm}\) (blue diamonds) and \(x = 986\text{mm}\) (red squares). The floor of the working section consisted of an upstream tripping fence and Lego® baseboard only. The red solid lines indicate the various layers of a rough wall turbulent boundary layer and relate to the profiles at \(x = 986\text{mm}\).

The inertial sublayer is a vertical region characterised by a relatively constant shear stress. Experimental data suggests that the shear stress actually decreases slightly with increasing height in the inertial sublayer. However the decrease in the shear stress is markedly less than
the drop off in shear stress in the outer layer. In the present study the vertical extent of the inertial sublayer is defined to be the region where the variation in shear stress is less than 3%. The region below this layer was then defined to be the roughness sublayer. Many researchers (eg. Cheng et al. 2007; Cheng and Castro, 2002a) have often defined the top of the RSL/base of the ISL to be where the spatially-averaged vertical profiles converge. However an agreed tolerance of convergence is not well-defined and can lead to large errors. For example, in the present study defining the tolerance of convergence to be within 5% and 10% of the average shear stress, results in a RSL height difference of tens of millimetres for some surface arrays.

4.2.3 Estimation of the momentum thickness

In calculating the surface bulk drag coefficients for the ten surface arrays the integral momentum method described in section 3.2 was employed. The method relies on the calculation of the momentum thickness at two streamwise locations and equation [3.2] suggests that the momentum thickness can be determined from the mean streamwise velocity profile at different axial locations. Therefore velocity profiles measurements were taken at \( x=491 \text{mm} \) and \( x=986 \text{mm} \), separated by a distance of \( \Delta x=495 \text{mm} \). To calculate the momentum thickness at each location, the variation of velocity with height was formulated using

\[
\eta = \frac{z}{\delta} \quad \text{and} \quad \frac{U}{U_{\delta}} = f(\eta) \quad [4.2a,b]
\]

A second-order polynomial line of best-fit was overlaid to a plot of \( \frac{U}{U_{\delta}} \) against \( \frac{z}{\delta} \) to provide a representative function of the velocity profile of the form

\[
\frac{U}{U_{\delta}} = A\eta^2 + B\eta + C \quad [4.3]
\]

where \( A, B \) and \( C \) are constants.

Substitution of equation [4.3] into equation [3.2] then provides the momentum thickness at a particular location and is given as:
\[ \theta = \int_0^\delta \frac{U}{U_\delta} \left(1 - \frac{U}{U_\delta}\right) dz = \delta \int_0^\delta \left[A \eta^2 + B \eta + C(1 - A \eta^2 - B \eta - C)\right] \eta \]  
\[ \theta = \left[\left(-\frac{A^2}{5}\right) \eta^5 + \left(-\frac{AB}{2}\right) \eta^4 + \left(\frac{A - 2AC - B^2}{3}\right) \eta^3 + \left(\frac{B - BC}{2}\right) \eta^2 + (C - C^2) \eta \right] \delta \]  

Figure 4.6 shows the fit of a second-order polynomial to the velocity profiles at \(x = 986\text{mm}\) for the S8 surface array. The plot shows that a second-order polynomial line provides a good and sufficient fit to the measured data with a correlation coefficient close to unity. For all surface arrays at both the upstream and downstream locations, the correlation coefficients of the polynomial line fits were greater than 0.98. When allowing the measured data to vary so that the correlation coefficient of the polynomial fit decreased to 0.9, the difference in the value of the momentum thickness varied by less than 3%. This is evidence of the robustness of the second-order polynomial line and shows that even with a poor velocity profile, the momentum thickness can be calculated within reasonable accuracy.

Figure 4.6 – Normalised velocity plotted against normalised height for the S8 surface array at \(x=986\text{mm}\). The solid black line is a second-order polynomial fitted to the measured data.

To achieve a total momentum thickness at a particular location, the momentum thickness above and within the canopy must be summed. Thus it is important to measure the mean velocity profile from the base of the substrate surface to the edge of the boundary layer.
However, due to practical limitations, velocity measurements could not be made within the canopy layer. To overcome this hurdle, the laterally-averaged logarithmic velocity profile was extrapolated through the roughness sublayer to obtain a laterally-averaged velocity, $U_h$ at the height of the roughness element. Cheng and Castro (2002b) show that this is justified for rough-wall urban canopies but not for plant canopies. At the base of the substrate surface the no-slip condition is assumed such that the velocity is zero. The average in-canopy velocity is then estimated by halving the velocity at the top of the canopy, $U_H$, to compute an average in-canopy velocity, $U_C$. If the velocity profile within the canopy is then modelled to be uniform with height and equal to $U_C$ (similar to the approach adopted by Bentham and Britter, 2003), then the momentum thickness of the flow within the canopy can be determined using equation [4.5]. This approach also assumes that the roughness elements extract enough momentum from the in-canopy flow that for all surface arrays, $U_H$ is never greater than the flow velocity above the canopy. Furthermore, it is also assumed that the in-canopy flow between $X_1 = 491\text{mm}$ and $X_2 = 986\text{mm}$, is fully developed so that any additional momentum to the flow (e.g. due to a pressure gradient) is negligible. This is a valid assumption because Figure 3.3 showed that the flow above the canopy is near enough fully-developed within a fetch of $x = 491\text{mm}$. Since the turbulent mixing is greater within the canopy, one would expect the in-canopy velocity to have adjusted within a shorter fetch than that required for the above-canopy velocity. Furthermore, Coceal and Belcher (2004) provided an estimate for the fetch required for the in-canopy velocity to adjust following a roughness change based on the roughness element geometry and is given in equation [1.17]. Their estimate of the adjustment fetch, $x_0$ is repeated here and estimated to be

$$x_0 = 3L_C \ln K$$  \hspace{1cm} [4.6]$$

where $K = \frac{U_H z_H}{u_* L_C}$ and $L_C = \frac{z_H (1 - \hat{\lambda}_p)}{\hat{\lambda}_f}$.  \hspace{1cm} [4.7a,b]$$

$L_C$ is the canopy-drag length scale and is a function only of the roughness element geometry in an array. The ‘$\ln K$’ parameter varies between 0.5 and 2.0 for typical urban arrays. For very sparse canopies, the value of $\ln K$ becomes negative and equation [4.6] is no longer valid for estimating the adjustment fetch. Of the arrays investigated in the present study, the adjustment fetch estimate was only valid for the $S1$ and $S2$ surfaces. In both cases the
adjustment fetch was less than 100mm and therefore the assumption that the in-canopy velocity would have adjusted before a fetch of $x = 491mm$ is valid.

In the present study, the total momentum thickness was determined at two streamwise locations, $X_1$ and $X_2$. The bulk drag coefficient of the underlying surface was then determined using equation [3.13] such that:

$$C'_{D(\theta)} = \frac{2\Delta \theta}{\Delta x} = \frac{2(\theta|_{x=X_2} - \theta|_{x=X_1})}{X_2 - X_1} \quad [4.8]$$

The bulk drag coefficient was computed using the integral momentum theory for all surface arrays.

### 4.3 Results and Discussion

#### 4.3.1 Surface drag

The primary aim of the wind tunnel experiments was to determine the sensitivity of the surface drag to the controlled area density ratios. Figure 4.7a shows the variation of the bulk drag coefficient with the frontal area density for surfaces S1 to S5 for which the plan area density was fixed at approximately 0.245. Figure 4.7b shows the variation of the bulk drag coefficient with the plan area density for surfaces S6 to S10 for which the frontal area density is fixed at approximately 0.08. Figure 4.7 shows the bulk drag coefficient values estimated from both the friction velocity method, $C_{D(u_*)}$, and the integral momentum method $C'_{D(\theta)}$.

Since equation [3.17] suggests that the value of $C'_{D(\theta)}$ is equal to twice the value of $C_{D(u_*)}$, the values of $C'_{D(\theta)}$ plotted in Figure 4.7 have been corrected to the meteorological convention so that values are comparable to $C_{D(u_*)}$. Thus,

$$\sqrt{2} C'_{D(\theta)} = C_{D(\theta)} = C_{D(u_*)} \quad [4.9]$$

The reference velocity used in calculating the drag coefficient, $C_{D(u_*)}$, was equal to the free-stream velocity of the air flow in the wind tunnel and equal to 10m s$^{-1}$. 
The drag coefficient variation with frontal area density, $\lambda_F$, derived from the integral momentum method shows an increasing trend for $0.066 < \lambda_F < 0.110$ for which the drag coefficient increases by approximately 20%. Between $0.110 < \lambda_F < 0.165$ the drag peaks but it is not clear at what value of the frontal area density ratio the peak occurs. Although not possible in the present study, an investigation of more arrays spanning this range would provide a more accurate indication of where the peak occurs. For $\lambda_F > 0.165$, the drag coefficient decreases by approximately 12% from the measured peak drag coefficient.

The drag coefficient variation with plan area density shows a similar trend but the value of plan area density at which the peak drag coefficient is derived is much clearer. Figure 4.7b shows that the integral momentum method derived drag coefficient increases by approximately 22% for $0.059 < \lambda_P < 0.181$. The peak drag coefficient occurs at $\lambda_P = 0.181$ before a decreasing trend observed at $\lambda_P > 0.181$.

Comparison between the drag coefficients determined from the integral momentum method with that determined from the friction velocity method is very good in both Figures 4.7a and 4.7b. With the exception of the drag coefficient estimate for the S5 surface array ($\lambda_P = 0.248$, $\lambda_F = 0.066$), both values fall within the respective error limits. In Figure 4.7a, the largest percentage difference in drag coefficients is 21%. The average percentage difference is just 7%. Further comparison of the drag coefficients suggests that the percentage differences tend to increase for lower measured values of $C_D$. This is particularly noticeable in Figure 4.7a where the two largest drag coefficients compare well for both methods. This suggests that the integral momentum method and the friction velocity methods may not be in good agreement for sparse and very dense area densities where surface roughness is smaller. In general the drag coefficient variations of $C_{D(\theta)}$ are smoother than $C_{D(u)}$. Indeed the estimated error in the calculation of $C_{D(\theta)}$ is almost half that of $C_{D(u)}$. For the integral momentum method the error in the calculation of the momentum thickness arises from errors in the measured velocity values and errors in the depth of the boundary layer which forms the limit of integration for determining the rate of loss of momentum. As stated in section 3.4.4 the maximum error in the measured velocity was found to be less than 0.4%. The coarse vertical resolution of the vertical velocity measurements introduced errors in determining the boundary layer depth of approximately ±3.8%. By combining the two errors using the quadrature rule, the total error in the momentum thickness is found to be approximately ±4.1%. The dominant source of error in the calculation of $C_{D(u)}$ is in estimates of the friction velocity.
Figure 4.7 – Drag coefficient variation with (a) frontal area density for surface arrays $S1$ to $S5$ and (b) plan area density for surface arrays $S6$ to $S10$. The blue line shows the drag coefficient estimate determined from the integral momentum method. The red line shows the drag coefficient estimate determined from the friction velocity method.

The value of the friction velocity is dependent on the tolerance levels chosen to indicate a constant shear stress in the inertial sublayer. Despite selecting a 3% tolerance limit, the friction velocity can vary by up to 5.6%. Combining this error with the measurement error of the shear stress, the resulting total error in $C_{D(\epsilon)}$ is approximately 9.3%. Thus the average
error in determining $C_{D(\theta)}$ is estimated to be $\pm 4.1\%$, whereas the average error in $C_{D(u_\infty)}$ is estimated to be $\pm 9.3\%$.

For cube arrays, the physical reasoning for the characteristic trend in the normalised roughness length can be explained by the three different flow regimes that exist depending on the packing density of the obstacles (Oke, 1987). These flow regimes have been previously described in section 1.3.4. However, for non-cubic arrays, the geometry and layout of the obstacles may not always result in these three different flow regimes for varying values of $\lambda_p$ and $\lambda_o$. Therefore there may be other physical reasons for the trends observed in Figure 4.7.

For surfaces $S1$ to $S5$, the layout of obstacles is such that the ratio of the height of obstacles to the inter-obstacle spacing length, $z_H/W_X$ is different in each case. For these particular surface arrays, the ratio $z_H/W_X$ decreases as the frontal area density decreases. Oke (1987) suggests that the ratio $z_H/W_X$ governs the type of flow regime within the canopy as illustrated in Figure 1.6. In particular, the height of the obstacles determines the downstream extent of the wake, $G_X$ generated behind the obstacles and this is illustrated in Figure 4.8. Taller obstacles generate a wake which extends further downstream than shorter obstacles.

\[ \text{Figure 4.8} \quad \text{The dependence of the extent of the downstream wake length, } G_X \text{ on the height of obstacles.} \]

If the wake of the upstream obstacle impinges on the downstream adjacent row of obstacles, the influence of sheltering takes place and the flow regime advances from the isolated flow regime to the wake interference flow regime and then to the skimming flow regime. Thus for surfaces $S1$ to $S5$ the most likely reason for the trend in the surface drag is due to the transition of the flow from the isolated flow regime to the skimming flow regime.
For surface $S6$ to $S10$ the ratio $z_H/W_X$ remains fixed at approximately $0.354$ at which, according to Oke (1987), the flow regime is at the low end of the wake interference flow. The fixed ratio suggests that the trend in the drag coefficient shown in Figure 4.7b is likely not due to transition of the classic flow regimes. Instead the increase in the drag coefficient in the range $0.059 < \lambda_P < 0.181$ is most likely due to the presence of free-flowing channels in the canopy layer of the obstacle array. These free-flowing channels are best explained by the illustration in Figure 4.9 and are defined as regions of the flow in which the wind can pass through the obstacle canopy without any obstruction from the obstacles. To quantify the level of channelling in an array of obstacles it is best to introduce an additional morphological parameter which can be referred to as the channelling ratio, $\lambda_C$ defined as

$$
\lambda_C = \frac{\sum_{i=1}^{n} W_C(i)}{D_Y}
$$

[4.10]

where $\sum_{i=1}^{n} W_C(i)$ is the sum of the widths of all free-flowing channels across a repeating unit width, $D_Y$.

**Figure 4.9** – An illustration of free-flowing channels (grey shading) across (a) the $S6$ obstacle array and (b) the $S8$ obstacle array. For the $S6$ surface array the channelling ratio, $\lambda_C = 0.5$ and for the $S8$ surface array the channelling ratio, $\lambda_C = 0.25$. 
Schlichting (1936) suggested that the contribution to the total surface drag coefficient of an obstacle array arises from two separate sources. These two contributory sources are the drag coefficient due to the substrate surface upon which the obstacles are mounted, $C_{D(SUB)}$ and the drag coefficient contribution due to the obstacles themselves, $C_{D(RE)}$ such that the total surface drag coefficient is given as

$$C_D = C_{D(RE)} + C_{D(SUB)}$$

[4.11]

Since the contribution to the drag coefficient in the free-flowing channel regions is only from the substrate surface it can be suggested that if $C_{D(SUB)} > C_{D(RE)}$, then an increasing value of the channelling ratio, $\lambda_C$ will increase the total surface drag coefficient. If however, $C_{D(SUB)} < C_{D(RE)}$, then an increasing value of the channelling ratio will reduce the total surface drag coefficient. Equation [4.11] represents the theory of drag partitioning. In the present study, the substrate surface is the Lego® baseboard which consists of cylindrical protrusions. Since the wake formed by these protrusions is of smaller dimensions than the Lego® brick roughness elements, it can be assumed that for all surfaces investigated, the drag partition condition $C_{D(SUB)} < C_{D(RE)}$ applies. For surfaces $S6$ to $S8$ for which $0.059 < \lambda_P < 0.181$, the value of the channelling ratio decreases with increasing frontal area density. Therefore the surface drag coefficient increases. For surface $S9$ and $S10$ free-flowing channels are not present in these arrays and the channelling ratio is zero. The reason for the decrease in the drag coefficient with increasing frontal area density for obstacle arrays in the range $0.181 < \lambda_P < 0.306$ is not clear. Nevertheless, the physical reasoning hypothesised to explain the trends in the surface drag coefficient suggests that the channelling ratio is an additional obstacle morphology parameter which has significant influence on the bulk surface drag coefficient. An experiment to assess the sensitivity of the surface drag to the channelling ratio is discussed later in section 5.4.2.

### 4.3.2 Roughness sublayer depth

The roughness sublayer is defined as the layer of the urban boundary layer which is directly influenced by the individual roughness elements. According to Mahrt (1996), the vertical depth of influence of surface features increases with the value of the drag coefficient. Therefore it is reasonable to assume that the roughness sublayer depth must be related to the drag coefficient. Figure 4.10 shows the velocity and shear stress profiles of all ten obstacle arrays with the obstacle height, roughness sublayer height and inertial sublayer height.
marked on the plots. The profiles are for all 45 lateral profile measurements obtained at the most downstream measurement location of $x = 986\text{mm}$. At this location the various layer depths are more distinct because the boundary layer flow develops over a greater distance. The vertical limits of the inertial sublayer have been determined by the shear stress variations with height as explained in section 4.2.2. Both the velocity and shear stress profiles show greater variability near the tops of the obstacles. This is likely due to two reasons. Firstly, the turbulent kinetic energy near the surface is larger because the fluctuations in velocity are greater resulting in greater experimental uncertainty. This scatter can be reduced by increasing the sampling time as was done in the present study. Secondly the layout of the obstacles results in lateral variations in the flow which are greater near the surface because it is there that the obstacles have greater influence on the flow. This is due to the spatial heterogeneity of the flow in the roughness sublayer and velocity and shear stress variations are expected. Figure 4.11 shows the variations in the normalised roughness sublayer depth with the plan and frontal area density. The depth of the RSL was determined from the average RSL depth of the 45 lateral shear stress profiles measured for each surface array and thus the RSL depth is a line-average. For all surface arrays, the roughness sublayer depth is less than twice the height of the obstacles. This is less than the commonly quoted range of 2-5 times the average obstacle height (Raupach et al., 1991; Rafailidis, 1997). The trend in the normalised RSL depth variation with area density seems to follow the same trend as the drag coefficient variation with area density. Thus the greater the drag coefficient of the surface array, the greater the depth of the roughness sublayer. Thus the present study indicates that the RSL depth may be ascertained from the morphology of the obstacles. This is in contrast to Barlow and Coceal (2009) who suggested that the relationship between RSL depth and urban morphology cannot easily be generalised. The generalisation may not be possible because in some studies, especially field studies, the RSL depth is determined from a single measurement profile. As shown in the present study the relationship between RSL depth and obstacle geometry may be much clearer if the RSL depth is based on a line (or spatial) average of many measurement profiles. Variation in the depth of the inertial sublayer does not seem to be related to the morphology of the obstacle arrays and therefore suggests that the vertical influence of the obstacles is limited to below the top of the inertial sublayer.
(a)

(b)

(c)

(d)

(e)
Figure 4.10 – The normalised velocity and shear stress profiles at \( x = 986mm \) for (a) \( S1 \), (b) \( S2 \), (c) \( S3 \), (d) \( S4 \) and \( S9 \), (e) \( S5 \), (f) \( S6 \), (g) \( S7 \), (h) \( S8 \) and (i) \( S10 \) surface arrays. The solid green line indicates the top of the obstacles. The solid red line indicates the top of the RSL. The black solid line indicates the top of the ISL.
4.3.3 Aerodynamic Parameters

The general logarithmic expression for surface layer flow is given in equation [1.4] and includes the two aerodynamic parameters, namely the displacement height and aerodynamic roughness length.

The surface drag coefficient can be determined from equation [1.4] by method of rearrangement such that

Figure 4.11 - Normalised roughness sublayer depth variation with (a) the frontal area density and (b) the plan area density.
Thus these two aerodynamic parameters influence the bulk surface drag and are themselves a function of the obstacle morphology. From a practical context the displacement height and roughness lengths can be determined by considering the line-averaged velocity profile and plotting the left-hand side of equation [4.12] against the right-hand side of the equation. Adjusting the displacement height by method of trial and error controls the slope of the line. A displacement height is chosen which produces a best fit to a one-to-one slope in the ISL. Once this parameter is set, the roughness length can be adjusted such that the logarithmic portion of the velocity profile overlays a line with a slope of one intersecting the origin. This method is known as the log-law fitting process and has been widely used in previous studies (Feddersen, 2004; Cheng et al., 2007). The log-law fitting process requires the friction velocity to be pre-determined and in the present study was calculated using the same method used to determine $C_{D(u_*)}$ from the relatively constant shear stress in the inertial sublayer.

Figure 4.12 shows an example of the log-law fitting process applied to the line-averaged velocity profile of the S8 surface code at the most downstream measurement location, $x=986mm$. As the plot shows, the log-law does not extend throughout the boundary layer but is confined to the roughness sublayer and inertial sublayer only. In the example shown in Figure 4.12, the normalised zero-plane displacement height, $\frac{z_D}{z_H} = 0.622$ and the normalised roughness length, $\frac{z_0}{z_H} = 0.133$. The method was repeated for all surface arrays. When the velocity profile is plotted on a semi-log plot, the y-axis intercept of the line is the roughness length. This makes the roughness length susceptible to large errors because even differences of less than 5% in the slope of the line will result in a roughness length variation of almost 20%. Therefore the normalised roughness length derived from the log-law fitting process must be treated with some caution. Furthermore Cheng et al. (2007) suggest that when all three unknown roughness parameters ($z_D$, $z_0$ and $u_*$) are derived from the log-law fitting process using the mean velocity profiles, the final results obtained are sensitive to the initial values assigned to the unknowns. To minimise the errors, roughness length values were calculated using equation [1.11] which makes use of the bulk drag coefficient values,
Chapter Four: Aerodynamic parameters for uniform obstacle arrays

$C_{D(0)}$ calculated using the integral momentum method and presented in Figure 4.7. The displacement height values were obtained from the log law fitting process.

![Figure 4.12](image)

**Figure 4.12** – The log-law fitting process applied to the velocity profile at $x = 986$mm for the S8 surface array.

The reference height used is taken to be the boundary layer depth because the drag coefficient values are scaled on this height. Using this method results in roughness length values which are more stable than those obtained simply from the log-law fitting process and therefore suggest that it is less prone to error.

Figures 4.13 and 4.14 present the results of the normalised displacement height and roughness lengths determined for surface arrays S1 to S10 from the present study with those estimated using pre-existing algorithms. As explained in chapter one the majority of these algorithms have been formulated for surface arrays consisting of cube-shaped roughness elements. Amongst the well-behaved algorithms (see the review by Grimmond and Oke, 1999) are the algorithms presented by Counihan (1971), Raupach (1994), Bottema (1995) and Macdonald *et al.* (1998). Table 4B lists the algorithms relating the displacement height and roughness length to the surface array morphology. These particular algorithms are considered in this study because the review of Grimmond and Oke (1999) suggest that they have wide applicability to real urban areas. The work of Counihan (1971) suggests that both $z_D$ and $z_0$ are functions of the plan area density alone. In contrast the study of Raupach (2004) suggests that the aerodynamic parameters are a function of the frontal area density alone.
Both Bottema (1995) and Macdonald et al. (1998) suggest that both area density ratios have an influence of $z_D$ and $z_0$.

Figure 4.13 shows that for the present study the displacement height increases for an increasing frontal area density. This trend is also estimated by the algorithms of Raupach (2004) and Bottema (1995). The physical reasoning for this trend lies in the smaller ratio of $W_X/z_H$ for larger frontal area densities. This is true for the present study but may not be true for all surface arrays of this type because an alternative method for varying the frontal area density is to increase the height of obstacles. The mutual sheltering of the downstream row of elements by the upstream row decreases the depth to which the flow penetrates into the inter-element spacing. The result is a rise in the effective height of the surface upon which the drag acts. The results of the present study show higher normalised displacement heights compared to those estimated by Raupach (1994) and Bottema (1995) the reasons for which are not obvious. However the current experimental study rejects the algorithms of Counihan (1971) and Macdonald et al. (1998) which fail to realise the influence of the frontal area density on the displacement height.

Figure 4.13b shows that for the present study the displacement height also increases for an increasing plan area density. The physical reasoning for this trend lies in the fact that increasing the plan area density of obstacles increases the substrate surface area covered by obstacles. The volume of the buildings deflects the bulk of the flow upwards resulting in an increasing displacement height. This trend is estimated by all the pre-existing algorithms with the exception of Raupach (1994). In the expressions presented by Raupach (1994), both the displacement height and roughness lengths are functions of the frontal area density only and therefore is limited in its applicability to estimate the aerodynamic parameters. Figure 4.14a shows that the variation of the normalised roughness length with frontal area density follows the same trend as the surface drag coefficient as would be expected. The pre-existing algorithms, with the exception of that presented by Counihan (1971), do well to predict an increasing trend in surface roughness length for $\lambda_F < 0.108$. However the decrease in roughness length for $\lambda_F > 0.165$ is not estimated by any of the algorithms. The physical reasoning for the trends in the roughness length for variations with frontal area density is the same as that explained in section 4.3.1. In Figure 4.14b, the expression of Counihan (1971) follows the same trend as the roughness length from the present study for $\lambda_F < 0.181$. For values of $\lambda_F > 0.181$, the expression of Macdonald et al. (1998) follows the roughness length trend from the present study. However none of the algorithms are able to estimate the correct trend for the whole range of area density values tested in the present study. Figure
4.14 suggests that there currently does not exist any algorithm or model which is able to correctly predict the trend in normalised roughness length for non-cubic obstacle arrays for the range of area density values found in real urban areas.

**Figure 4.13** – Variation in the normalised displacement height with (a) the frontal area density and (b) the plan area density.
Figure 4.14 – Variations in the normalised roughness length with (a) frontal area density and (b) plan area density.
<table>
<thead>
<tr>
<th>Study</th>
<th>Normalised Zero-Plane Displacement Height</th>
<th>Normalised Aerodynamic Roughness Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>Counihan (1971)</td>
<td>( \frac{z_D}{z_H} = 1.4325 \lambda_p - 0.0463 )</td>
<td>( \frac{z_0}{z_H} = 1.08 \lambda_p - 0.08 )</td>
</tr>
<tr>
<td>Raupach (1994)</td>
<td>( \frac{z_D}{z_H} = 1 - \left( 1 - \exp \left( \frac{15 \lambda_p}{\lambda} \right)^{0.5} \right) )</td>
<td>( \frac{z_0}{z_H} = \left( 1 - \frac{z_D}{z_H} \right) \exp \left( - \frac{U}{u_<em>} + 0.193 \right) ) where ( \frac{u_</em>}{U} = \min \left[ \left( 0.003 + 0.3 \lambda_F \right)^{0.5}, 0.3 \right] )</td>
</tr>
<tr>
<td>Bottema (1995)</td>
<td>( \frac{z_D}{z_H} = \frac{L_x + 0.33 \left( \frac{4 L_\lambda z_H}{0.5 L_\lambda + z_H} \right)}{2 D_x} )</td>
<td>( \frac{z_0}{z_H} = \frac{z_H - z_D}{z_H} \exp \left[ - \frac{\kappa}{\left( 0.5 \lambda_F C_{DH} \right)^{0.5}} \right] ) where ( C_{DH} = 1.2 \max \left[ 1 - 0.15 \left( \frac{L_x}{z_H} \right)^{0.82} \right] \min \left[ 0.65 + 0.06 \left( \frac{L_x}{z_H} \right)^{1.0} \right] )</td>
</tr>
<tr>
<td>Macdonald et al. (1998)</td>
<td>( \frac{z_D}{z_H} = 1 + 4.43^{-\lambda_F} (\lambda_p - 1) )</td>
<td>( \frac{z_0}{z_H} = \left( 1 - \frac{z_D}{z_H} \right) \exp \left[ - \frac{0.6}{\kappa^2} \left( 1 - \frac{z_D}{z_H} \right) \lambda_F^{-0.5} \right] )</td>
</tr>
</tbody>
</table>

**Table 4B** – A list of existing algorithms relating the zero-plane displacement height and the aerodynamic roughness length to obstacle morphology.
4.3.4 COMbined Cube Arrays (COCA) model

Figure 4.14 shows that increasing both the plan and frontal area density results in the same trend for the normalised roughness length. The characteristic increase in roughness leading to a peak and then a decrease in roughness for increasing area density is a feature that any acceptable model or algorithm must estimate. Because existing algorithms for cube arrays already provide the characteristic trend, it is likely that this is a good starting point to construct a new set of algorithms for non-cubic arrays.

The results of the present study have shown that both the obstacle plan and frontal areas contribute to the roughness of the surface. Thus any new algorithm for estimating surface roughness should incorporate both these morphological parameters either implicitly or explicitly.

One method by which the plan area and frontal area may be incorporated explicitly into an algorithm is to investigate the hypothesis that the morphology of a non-cubic obstacle array can be separated into two pseudo-cube arrays and then the respective roughness length values of the two cube arrays can be combined to give a representative roughness length value. For example, consider a non-cubic obstacle array with plan area density, \( \lambda_P = 0.2 \), frontal area density, \( \lambda_F = 0.1 \) and a roughness length, \( z_0 \). Now consider the roughness length, \( z_0^{(1)} \), for a cube array with \( \lambda_P = \lambda_F = 0.2 \) and the roughness length, \( z_0^{(2)} \), for a cube array with \( \lambda_P = \lambda_F = 0.1 \). The hypothesis stated above suggests that these two roughness lengths may be combined to provide a single roughness length, \( z_0 \) equivalent to that of the original non-cubic array for which \( \lambda_P = 0.2 \) and \( \lambda_F = 0.1 \).

The simplest method to combine \( z_0^{(1)} \) and \( z_0^{(2)} \) is to take an arithmetic average of the two values such that

\[
\frac{A z_0^{(1)} + B z_0^{(2)}}{A + B}
\]

where \( z_0^{(A)} \) is the roughness length of a cube array with an area density equal to the plan area density of the non-cubic array, \( z_0^{(B)} \) is the roughness length of a cube array with an area density equal to the frontal area density of the non-cubic array and \( A \) and \( B \) are the respective weighting coefficients.

The weighting coefficients are included because it is not known \textit{a priori} which of the two cube array roughness lengths has greater influence on the non-cubic array roughness length. In the
absence of a theoretical framework, the weighting coefficients can only be determined empirically by fitting the model to experimental data. It is shown later that the best fit to the experimental data of the present study results in \( A = 1 \) and \( B = 1.15 \). Thus the roughness length of non-cubic arrays is more sensitive to the frontal area density than the plan area density.

The model presented in equation [4.13] is referred to here as the COCombined Cube Arrays model (COCA). Using the expression for staggered cube arrays proposed by Macdonald et al. (1998), estimates, using the COCA model, of the normalised roughness length variation with the plan and frontal area density ratios is shown in Figure 4.15.

Figure 4.15 – Normalised roughness length variation with the plan and frontal area density as estimated using the COCA model.

Figure 4.15 shows that the COCA model estimates the characteristic increase, peak and decrease trend in roughness length for any controlled variation of the area density ratios. Also a positive feature of Figure 4.15 is that the COCA model estimates the same roughness length values for cube arrays as the original Macdonald et al. (1998) estimate. Figure 4.16 shows the comparison of the COCA model estimate for normalised roughness length with data obtained from two separate experimental studies. Figures 4.16a and 4.16b compares the COCA model to the present study and Figures 4.16c and 4.16d compares the
Figure 4.16 – (a) Comparison of the normalised roughness length variation with frontal area density between the present study and the COCA Model; (b) Comparison of the normalised roughness length variation with plan area density between the present study and the COCA model; (c) Comparison of the normalised roughness length variation with frontal area density between data from Hagishima et al. (2009) and the COCA model; (d) Comparison of the normalised roughness length variation with plan area density between data from Hagishima et al. (2009) and the COCA model.
COCA model to the wind tunnel study of Hagishima et al. (2009) who measured the drag coefficient and roughness lengths for several non-cubic obstacle arrays.

The weighting coefficients in equation [4.12] were fit to the data from the present study and shown in Figures 4.16a and 4.16b. Using several combinations of the weighting coefficients, the values of $A = 1.0$ and $B = 1.15$ were shown to give the best fit. Figure 4.16a shows that the trend in the normalised roughness length is fairly good. The maximum percentage difference of 15% occurs at $\lambda_F = 0.11$, but the average percentage difference is less than 10%. Data for the roughness length variation with plan area density in Figure 4.16b shows very good comparison of the trends and the average percentage error is less than 9%. The COCA model correctly predicts the value of the plan area density at which the maximum roughness length is achieved.

The COCA model also behaves well for the experimental data of Hagishima et al. (2009). In Figure 4.16c, the COCA model correctly predicts the roughness length trend but does not correctly predict the location of the peak roughness length. However, because the range of roughness lengths around the peak value is small, the COCA model may be within the errors of the experimental data. For $\lambda_F > 0.26$, the slope of the trend achieved by the COCA model is excellent compared to the experimental data. In general the difference in roughness length values is larger for sparse obstacle arrays compared to more dense arrays. In Figure 4.16d, the variation of roughness length with plan area density obtained from the experimental study compares very well with that estimated by the COCA model. The COCA model correctly predicts the plan area density at which maximum roughness length occurs and the trends compare well. The average percentage difference between experimental data and the model estimates is approximately 20% due largely to the fact that the model overestimates the roughness length values for $\lambda_P > 0.1$. Despite these differences, it is fair to suggest that the COCA model compares favourably to the experimental data. The roughness length estimates from the COCA model are significantly better than the other algorithms tested in Figure 4.16. The COCA model is however limited to obstacles arrays with uniform building heights and to validate the model further, the model estimates need to be compared with more experimental or numerical data.
4.4 Summary
In this chapter, the sensitivity of the surface roughness to the plan and frontal area density ratios has been determined. The test was conducted using a wind tunnel experiment in which the area densities of the roughness element arrays were controlled such that in one set of experiments the plan area density was fixed for varying frontal area density and vice versa for the second set of experiments. The range of area density values chosen were typical of the range found across Greater London. The surface roughness was quantified in terms of the bulk drag coefficient determined using two different methods, namely the friction velocity and the integral momentum method. Once the value of the drag coefficient from the latter method was corrected from the engineering to the meteorological definition, the two methods provided drag coefficient estimates which were comparable in both value and trend across the area density range tested. For both the plan and frontal area density ratio, the surface drag coefficient increased for increasing area density for sparse arrays. For dense arrays, the drag coefficient decreased for increasing area density with a peak occurring at medium array density. This is a well known characteristic trend for cube arrays ($\lambda_P = \lambda_F$).

However, for non-cubic arrays ($\lambda_P \neq \lambda_F$), the lack of previous studies has failed to verify the trend in surface roughness for varying area density ratios. Existing morphological algorithms which aim to estimate surface roughness length as a function of roughness element morphology (Macdonald et al., 1998; Counihan, 1971; Raupach, 1994 and Bottema, 1995) have failed to reproduce the trend observed in the wind tunnel experiment of the present study for non-cubic arrays. This has prompted the development of the Combined Cube Arrays (COCA) model which reproduces the observed roughness length trends for non-cubic arrays with various combinations of plan and frontal area density ratios. The model compares well to the recent wind tunnel study conducted by Hagishima et al. (2009) who made direct measurements of the surface drag for several non-cubic arrays.

The results from this chapter also highlight that the depth of the roughness sublayer is dependent on the plan and frontal area density ratios with the trend of the variation similar to that of the bulk drag coefficient. Table 4C lists the results of the wind tunnel experiments for all ten surface arrays investigated.

The present chapter also highlighted the sensitivity of surface drag to a third morphological parameter, namely the channelling ratio, $\lambda_C$. This parameter quantifies the level of channelling present in roughness element arrays due to the arrangement of the roughness elements. An in-depth analysis of the influence of the channelling ratio on the surface drag is presented in the chapter that follows.
Table 4C – A table of results for the ten surface arrays investigated in the wind tunnel.
Chapter Five

The influence of wind direction on surface roughness

5.1 Introduction

For a group of buildings in an urban neighbourhood the morphology of the roughness elements does not entirely govern the bulk drag of the surface. The direction from which the flow interacts with the elements can also have a significant bearing on the surface roughness. For example, Grimmond et al. (1998) used anemometers to observe a range of roughness length values from 0.2m to 1.6m for different wind directions at a suburban site in Chicago. Whilst the number of studies concerning the influence of wind direction on tracer gas concentrations are numerous (eg. Dabberdt and Hoydysh, 1991; Kastner-Klein and Plate, 1999; Somerville et al. 1998) the number of studies concerning wind direction and surface roughness is relatively small. As numerical weather prediction and urban dispersion models begin to use higher resolution grids, the need to understand the influence of building scale features on aerodynamic parameters is becoming more important. Wind engineers have long known that slight changes to the wind direction can have a significant influence on the pressure forces across the walls of a building (eg. Grosso, 1992). The resulting pressure differences cause the drag force, $F_D$, on the object to vary and is often estimated using the aerodynamic drag force equation:

$$ F_D = \frac{1}{2} \rho U^2 A_F C_{D(I)} $$

[5.1]

where $\rho$ is the density of the fluid, $U$ is the flow velocity, $A_F$ is the frontal area of the obstacle and $C_{D(I)}$ is the drag coefficient of the obstacle.

This chapter investigates the morphological and aerodynamic factors influencing changes to the drag force acting on an obstacle for different wind directions.
Figure 5.1 – Features of an idealised surface array at (a) flow perpendicular to the roughness element walls and (b) flow at an oblique angle.

The resulting change in obstacle drag force would vary the roughness length and bulk drag coefficient of an array of obstacles. Figure 5.1 shows an idealised array of cubic roughness elements for two different wind directions. Four factors are identified which are expected to have a significant impact on the bulk drag of an array of obstacles for different wind directions. They are:

- **The frontal area density, \( \lambda_F \)** – The projected roughness element width, \( L_Y \) is a function of the wind direction and gives the frontal area density, \( \lambda_F \) when multiplied by the average height of the obstacle and divided by the lot area. The shape of the obstacle defines the level of variation in \( \lambda_F \) with wind direction. For example, if the obstacle is of cylindrical shape, the frontal area density will be equal for all wind directions.
• The drag coefficient of the individual obstacles, $C_{D(i)}$ – The obstacle drag coefficient is a dimensionless quantity which represents all the complex features of the flow and obstacle which contribute to the total drag force. The drag coefficient is significantly dependent on the separation and reattachment of the flow on the obstacle which will be dependent on the flow direction.

• The width of the free-flow channels through the array, $W_C$ – For certain wind directions, channels appear in the array in which the flow is totally unobstructed by the obstacles. The presence of the channels and their width is governed entirely by the arrangement of the obstacles in the array. The ability of the air to flow unimpeded should reduce the bulk drag coefficient.

• The displacement height of the array, $z_D$ – When the wind direction across an array changes, morphologically the plan area density is unchanged but the frontal area density will vary. Figure 4.13a from the previous chapter suggests that in such situations the displacement height can change and therefore must be accounted for when estimating the surface roughness.

The aim of the present chapter is to modify the normalised roughness length formulation of Macdonald et al. (1998) to provide roughness length estimates for different wind directions. The modifications take into account the four factors listed above. The focus of the roughness length estimates are upon idealised cubic arrays. Even for such a simple array, there currently exists no method for estimating the surface roughness for an array with different incident wind directions.

5.2 Wind direction and frontal area density

5.2.1 The frontal area density variation with wind direction

With the exception of cylindrical obstacles, the frontal area density, $\lambda_F$ will vary with respect to changes in wind direction for obstacles of all shapes. Aerial surveys of urban areas suggest that the plan cross-sectional shape of most buildings is predominantly cuboidal. It is therefore instructive to firstly consider cuboidal obstacles in trying to quantify the variation in frontal area density for different wind directions. For additional simplicity, the cuboids are assumed to have the same orientation with the adjacent streets, although the cuboids may be randomly arranged within the array. This assumption leads to the situation where streets are
orientated along one of two perpendicular directions, which define reference directions \( \hat{A}_x \) and \( \hat{A}_y \). For this simple but fairly realistic building configuration, it is easy to derive an expression for the variation of \( \lambda_F \) with incident wind direction. Figure 5.2 shows wind incident at an acute angle \( \theta_I \) (with respect to one of the street reference directions, \( \hat{A}_y \)) on a cuboid with sides of length \( L_{Xi} \) and \( L_{Yi} \) and height \( z_{Hi} \).

**Figure 5.2** – Definition of angles and lengths used to formulate frontal area density variations with wind direction.

The frontal area is the projected area perpendicular to the wind direction, and thus frontal area density \( \lambda_F \) is

\[
\lambda_F (\theta_I) = \frac{1}{A_T} \sum_{i=1}^{n} (L_{Xi} \cos \theta_I + L_{Yi} \sin \theta_I) z_{Hi} = \left( \frac{1}{A_T} \sum_{i=1}^{n} L_{Xi} z_{Hi} \right) \cos \theta_I + \left( \frac{1}{A_T} \sum_{i=1}^{n} L_{Yi} z_{Hi} \right) \sin \theta_I
\]

[5.2]

given that there are \( n \) buildings on a total site area \( A_T \), and \( 0 \leq \theta_I < \pi/2 \). Recognising that

\[
\left( \frac{1}{A_T} \sum_{i=1}^{n} L_{Xi} z_{Hi} \right) = \lambda_F \bigg|_{\theta_I=0} \quad \text{and} \quad \left( \frac{1}{A_T} \sum_{i=1}^{n} L_{Yi} z_{Hi} \right) = \lambda_F \bigg|_{\theta_I=\pi/2}
\]

[5.3a,b]

it is possible to write
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\[ \lambda_F(\theta_i) = \lambda_F \mid_{\theta_i=0} \cos \theta_i + \lambda_F \mid_{\theta_i=(\pi/2)} \sin \theta_i \quad [5.4] \]

If the incident wind direction \( \theta_i \) is obtuse, i.e. \( \pi/2 \leq \theta_i < \pi \), similar considerations show that

\[ \lambda_F(\theta_i) = -\lambda_F \mid_{\theta_i=0} \cos \theta_i + \lambda_F \mid_{\theta_i=(\pi/2)} \sin \theta_i \quad [5.5] \]

Equations [5.4] and [5.5] can be expressed in the form

\[ \lambda_F(\theta_i) = \begin{cases} \sqrt{\left(\lambda_F \mid_{\theta_i=0}\right)^2 + \left(\lambda_F \mid_{\theta_i=(\pi/2)}\right)^2} \sin(\theta_i + \gamma), & \text{for } 0 \leq \theta_i < \pi/2 \\ \sqrt{\left(\lambda_F \mid_{\theta_i=0}\right)^2 + \left(\lambda_F \mid_{\theta_i=(\pi/2)}\right)^2} \sin(\theta_i - \gamma), & \text{for } \pi/2 \leq \theta_i < \pi \end{cases} \quad [5.6a,b] \]

where \( \gamma = \tan^{-1}\left(\frac{\lambda_F \mid_{\theta_i=0}}{\lambda_F \mid_{\theta_i=(\pi/2)}}\right) \quad [5.7] \)

A full derivation of equations [5.6] and [5.7] is given in the Appendix. Equations [5.6] and [5.7] show that the characteristic variation of \( \lambda_F(\theta) \) is sinusoidal with an amplitude equal to \( \sqrt{\left(\lambda_F \mid_{\theta_i=0}\right)^2 + \left(\lambda_F \mid_{\theta_i=(\pi/2)}\right)^2} \) and phase-shifted by an angle of \( \gamma \). Both the amplitude and angle of phase shift are dependent on the average aspect ratio of the cuboidal elements. The expression is simple and has the advantage that the frontal area density of a cuboidal array, can be calculated for all wind directions from simply knowing the average frontal area density at two perpendicular wind directions. Modern GIS software packages are able to compute the frontal area density for two wind directions quite easily and within a relatively short space of time compared to the calculation of morphological parameters for all wind directions. The sinusoidal pattern of variation with constant amplitude results in the maximum value of \( \lambda_F(\theta) \) being given by

\[ \lambda_{\max} = \sqrt{\left(\lambda_F \mid_{\theta_i=0}\right)^2 + \left(\lambda_F \mid_{\theta_i=(\pi/2)}\right)^2} \] that occurs at an incident wind angle of \( \theta_{\max}^f = \left(\pi/2 - \gamma\right) \). This is significant because if the surface drag is most sensitive to the frontal area density, it corresponds to the wind direction for which maximum drag is exerted by the building canopy. The minimum value of \( \lambda_F(\theta) \) would be the smaller frontal area density value of \( \lambda_F \mid_{\theta_i=0} \) and \( \lambda_F \mid_{\theta_i=(\pi/2)} \).
It is important to note that for all arrays the frontal area density is the same for opposing wind directions. Thus the frontal area density at a wind direction angle of $45^\circ$ is the same as that $225^\circ$ since it is the silhouette of the array which is important. This is true for all arrays regardless of building size, shape, orientation and arrangement.

### 5.2.2 Application to a real urban canopy

For real urban areas, the plan cross-sectional shape of all buildings is not always exactly rectangular. Buildings can be $L$-shaped, have slanted walls and courtyards. Figure 5.3a shows an aerial view of a typical urban area in Central London for which this is the case. The site is the location of the DAPPLE project (Arnold et al. 2004; Wood et al. 2009) which commenced in 2002 for which more details can be found at www.dapple.org.uk. The red dashed line in Figure 5.3 represents the lot area and is equal to approximately $0.35\text{km}^2$. The plan area density, $\lambda_F$ within the lot area is equal to 0.5. Within the lot area most buildings are mainly rectangular and aligned parallel to the street network. Figure 5.3b shows the area chosen for detailed wind tunnel investigations (eg. Carpentieri et al. 2009) for which building height data were available. For this simplified version of the real urban area, courtyards have been removed, exterior building walls have been smoothed to a flat wall and the roofs of the buildings have been flattened such that they are uniform and represent the mean height of the building. The obstacles in Figure 5.3b show that whilst 42 (80%) of the buildings are cuboidal with walls parallel to the adjacent streets, 5 (10%) are not cuboidal and have at least one wall which is slanted and 5 (10%) are $L$-shaped. In Figure 5.4, the values of the frontal area density are plotted for all wind directions for the central London site as estimated from the expressions given in equations [5.6] and [5.7]. This estimate is plotted together with the actual value of $\lambda_F$ for each wind direction angle as calculated by summing the frontal areas of all buildings and dividing by the total lot area. This takes into account slanted walls and $L$-shaped buildings. The agreement between the actual and estimated variation of $\lambda_F(\theta)$ is very good, especially for angles less than $\pi/2$. The maximum discrepancy is $5.4\%$ at an angle of $138^\circ$. The lack of symmetry about $\theta_i = \pi/2$ in the actual curve, and the resulting discrepancy for angles in the range $\pi/2 < \theta_i < \pi$, is due to $L$-shaped buildings and those with slanted walls. The simple expressions of equations [5.6] and [5.7] provide a potentially useful way of obtaining an estimate of $\lambda_F(\theta_i)$ from actual morphological datasets, avoiding the need for the time-consuming development and application of algorithms that compute $\lambda_F(\theta_i)$ explicitly for each angle. Ratti et al. (2006) recently computed $\lambda_F(\theta_i)$ using image processing algorithms applied to digital elevation models (DEMs) of case study sites in London, Berlin and Toulouse.
A plan view of the sites is shown in Figure 5.5. The variation of $\lambda_F(\theta_I)$ obtained appears qualitatively similar to the curves in Figure 5.4 (see Figure 8 in Ratti et al., 2006). By extracting values of $\lambda_F|_{\theta_I=0}$ and $\lambda_F|_{\theta_I=(\pi/2)}$ from their published curves, the maximum value of $\lambda_F$ and the wind direction for which it occurs were computed using the simple expressions of equation [5.6] and [5.7].

**Figure 5.3** – (a) an aerial view of a typical urban site in central London; (b) a simplified rectangular representation of the buildings. (Colour coding represents the mean heights of buildings and the dashed red line indicates the boundary of the lot area).

**Figure 5.4** – The variation of the frontal area density, $\lambda_F$ with wind direction for the central London urban site shown in Figure 5.3.
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Figure 5.5 – Plan view of the London, Toulouse and Berlin sites used in the study by Ratti et al. (2006). All sites measure 400m x 400m.

Table 5A – Values of $\lambda_{F}^{\text{max}}$ and $\theta_{I}^{\text{max}}$ calculated using equations [5.6] and [5.7], compared with their actual magnitudes, for the London site of the DAPPLE project and for the three sites studied by Ratti et al. (2006).

<table>
<thead>
<tr>
<th></th>
<th>$\lambda_{F}^{0}$</th>
<th>$\lambda_{F}^{(\pi/2)}$</th>
<th>$\gamma$</th>
<th>$\lambda_{F}^{\text{max}}$ Estimated</th>
<th>$\lambda_{F}^{\text{max}}$ Actual</th>
<th>$\theta_{I}^{\text{max}}$ Estimated</th>
<th>$\theta_{I}^{\text{max}}$ Actual</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Present Study - London</strong></td>
<td>0.166</td>
<td>0.225</td>
<td>36.4°</td>
<td>0.279</td>
<td>0.276</td>
<td>54°</td>
<td>54°</td>
</tr>
<tr>
<td><strong>Ratti et al. (2006) - London</strong></td>
<td>0.254</td>
<td>0.284</td>
<td>41.8°</td>
<td>0.381</td>
<td>0.35(^{(1)})</td>
<td>48°</td>
<td>50°(^{(2)})</td>
</tr>
<tr>
<td><strong>Ratti et al. (2006) - Berlin</strong></td>
<td>0.208</td>
<td>0.186</td>
<td>48.2°</td>
<td>0.279</td>
<td>0.25(^{(1)})</td>
<td>42°</td>
<td>40°(^{(2)})</td>
</tr>
<tr>
<td><strong>Ratti et al. (2006) - Toulouse</strong></td>
<td>0.292</td>
<td>0.307</td>
<td>43.6°</td>
<td>0.424</td>
<td>0.34(^{(1)})</td>
<td>46°</td>
<td>50°(^{(2)})</td>
</tr>
</tbody>
</table>

\(^{(1)}\) read from chart in Ratti et al. (2006) paper to accuracy of ±1%
\(^{(2)}\) read from chart in Ratti et al. (2006) paper to accuracy of ±5°.

They are shown together with the actual values in Table 5A, as well as values taken from Figure 5.4 for the London site of the DAPPLE project. The agreement is very good for both the London sites and reasonable for Berlin. However, there are fairly large discrepancies for Toulouse. This is readily understood by looking at the plan view of the sites shown in Figure 5.5. Whilst London and Berlin consist of mainly cuboidal buildings with similar orientation, Toulouse consists of very few cuboidal buildings and building orientations vary substantially. Thus equations [5.6] and [5.7] have their limitations and work best for an array of cuboidal...
buildings in which the orientation of buildings is parallel and perpendicular to streets aligned in a gridded format.

5.3 Drag coefficient of an isolated cube
The drag coefficient of an isolated object is a dimensionless quantity which quantifies the drag force acting on the object. It is dependent on the shape of the object, the angle at which the flow approaches the object and the conditions of the flow itself such as the turbulence intensity.

![Diagram of flow around a cuboidal structure](image)

**Figure 5.6** – A sketch of flow around a cuboidal structure at an oblique wind direction angle. (Modified reproduction from Akins *et al.*, 1977).

Akins *et al.* (1977) observed changes to the location of flow separation and reattachment on the surface of obstacles for different wind directions. Figure 5.6 shows a schematic, based on Akins *et al.* (1977), of the typical flow field around a cuboidal obstacle. The flow usually separates at a point on the surface where there is a sharp edge. At some point further along the obstacle wall, the turbulent flow reattaches. The space between the point of separation and turbulent reattachment develops a separation bubble in which the flow is trapped close to the surface and is of a slower velocity than the driving flow. The slower velocity flow creates an area of relatively high pressure in the vicinity of the separation bubble. The result is a significantly different pressure distribution around the obstacle which has a significant bearing on the form drag of the obstacle. Akins *et al.* (1977) measured the mean and fluctuating surface pressures and observed that the dimensions of the separation bubble and the corresponding surface pressures around an obstacle change with the incident wind direction. The Engineering Sciences Data Unit (ESDU) conducted wind tunnel experiments in
which they measured the mean forces around rectangular structures in turbulent shear flow for different incident wind directions (ESDU-80003, 1979).

![Sketch of shear flow around a cuboidal obstacle to illustrate axes and geometry notation.](image)

**Figure 5.7** – Sketch of shear flow around a cuboidal obstacle to illustrate axes and geometry notation.

Using the notation in Figure 5.7, the mean force, \( F_X \) acting on the obstacle in the \( x \)-direction is given as:

\[
F_X = \frac{1}{2} \rho U^2 L_y z_H \left( C_X k_{s(x)} k_{l(x)} k_{r(x)} k_{g(x)} k_{r(x)} \right)
\]

where \( C_X \) is the basic force coefficient for the flow incident to the wind direction for low turbulence, non-sheared flow over a sharp-edged obstacle in a wind direction perpendicular to a wall in the \( z-y \) plane. The coefficients \( k_{s(x)} \), \( k_{l(x)} \), \( k_{r(x)} \), \( k_{g(x)} \), \( k_{r(x)} \) are correction factors to account for the effects of the shear flow velocity profile, turbulence scale, turbulence intensity, wind direction incidence and curvature of the wall edges, respectively.

The term \( C_X k_{s(x)} k_{l(x)} k_{r(x)} k_{g(x)} k_{r(x)} \) is then equal to the drag coefficient of the isolated obstacle, \( C_{D(I)} \), for incident flow in the \( x \)-direction. Assuming that the only correction to the drag coefficient of the isolated obstacle is due to wind direction incidence changes, equation [5.8] simplifies to:

\[
F_X = \frac{1}{2} \rho U^2 L_y z_H \left( C_X k_{g(x)} \right)
\]

\[5.9\]
Similarly, the mean force, \( F_y \) acting on the obstacle in the \( y \)-direction is given as:

\[
F_y = \frac{1}{2} \rho U^2 L_x z_H (C_y k_{\phi(y)}) \tag{5.10}
\]

Figure 5.8 shows the ESDU experimental data of the correction to the drag coefficient due to changes in wind direction for a cube in shear flow. The patterns of variation are symmetrical about 180° and therefore the variations are the same between \( 180^\circ \leq \theta \leq 360^\circ \).

\[ \text{Figure 5.8} \] – Variation of the correction factor for the drag coefficient with wind incident angle for a cube in turbulent shear flow. (Re-plotted from ESDU-80003, 1979).

For incident flow at an oblique angle, the resultant force, \( F_R \) is equal to

\[
F_R = \sqrt{F_x^2 + F_y^2} = \frac{1}{2} \rho U^2 A_Y C_R k_{\phi(R)} \tag{5.11}
\]

where \( C_R \) is the drag coefficient of the resultant force and \( k_{\theta(R)} \) is the resultant correction factor due to oblique wind incidence and is equal to

\[
k_{\phi(R)} = \sqrt{k_{\phi(x)}^2 + k_{\phi(y)}^2} \tag{5.12}
\]

Figure 5.9 shows the resultant correction factor variation with incident wind angle for a cube in turbulent shear flow. The following sinusoidal function is also plotted for comparison:
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\[ k_{\phi(R)} = \begin{cases} 
1 + \frac{1}{4} \sin(2\theta) & \text{for } 0 \leq \theta \leq \frac{\pi}{2} \\
1 - \frac{1}{4} \sin(2\theta) & \frac{\pi}{2} \leq \theta \leq \pi 
\end{cases} \]  \[ [5.13] \]

Figure 5.9 shows that the sinusoidal function is a very good fit to the wind tunnel experiment data and illustrates that the drag coefficient of an isolated cube increases when the incident wind angle changes from \( \theta = 0 \) to \( \theta = \frac{\pi}{4} \). Symmetry in the geometry of a cube results in a decrease in the drag coefficient from \( \theta = \frac{\pi}{4} \) to \( \theta = \frac{\pi}{2} \).

**Figure 5.9** – Variation of the resultant correction factor for the basic drag coefficient with wind incident angle for a cube in turbulent shear flow. \( \theta = 0^\circ \) along the \( x \)-direction and \( \theta = 90^\circ \) along the \( y \)-direction.

### 5.4 Channelling ratio and surface roughness

As shown in Figure 5.1, the arrangement of obstacles in an array can lead to the presence of channels within the canopy at certain wind directions. To quantify the influence of these channels on the surface drag, we must first quantify the geometry of the channels. Equation [4.10] in the previous chapter introduced the channelling ratio, \( \lambda_c \) which is the ratio of the widths of the unobstructed channels to the width of the repeating units. It was proposed in section 4.3.1 that the smaller the value of the channelling ratio, the greater the surface drag and this hypothesis was used to explain the physical reasoning for the surface drag coefficient trend with increasing plan area density. However, the variation of surface drag with
channelling ratio has yet not been quantified. To do so, a separate wind tunnel experiment was conducted in which the experimental set-up was the same as that explained in Chapter three, using the integral momentum method to estimate the surface drag. In this experiment, the surface drag was estimated for four different surface arrays with the same values of the plan and frontal area density but with varying values of the channelling ratio. It is important to fix the values of the plan and frontal area densities for all four arrays to ensure that the variations of the surface drag are due to the variations in the channelling ratio alone. Figure 5.10 shows the plan view of the four arrays.

![Figure 5.10](image)

The S6 surface array used in the experiments described in Chapter four was used for these experiments. All the morphological parameters for the obstacle arrays in Figure 5.10 are the same as those listed in Table 4A. The channelling ratio varies in the range $0 < \lambda_c < 0.75$. The S0 obstacle array (no obstacles and baseboard only) for which the drag has already been measured is used to denote an obstacle array with a channelling ratio of one.

It is worth noting here that the arrangement of obstacles in Figure 5.10 shows that it is possible to vary the channelling ratio, $\lambda_c$ without varying the plan and frontal area density. This suggests that the channelling ratio is independent of the area density ratios. However, the channelling ratio and the plan area density ratio both quantify the blockage of the flow through the canopy. The larger the plan area density, the greater the blockage of the flow within the canopy due to the presence of the obstacles. Similarly, the smaller the channelling ratio, the greater blockage of the flow through the canopy. Thus it would be expected that the plan area density and the channelling ratio are related. Indeed, they are related but only for surface arrays in which the obstacles are arranged in a square formation in which rows of obstacles are placed directly behind each other. For such surface arrays, $\lambda_c = 1 - \sqrt{\lambda_p}$.
However, this relationship is not valid if the obstacles are arranged in any other formation as is the case in the present study. Figure 5.11 shows the normalised roughness length for varying channelling ratio in the range $0.0 < \lambda_c < 1.0$ from the $S6A$, $S6B$, $S6C$, $S6D$ and $S0$ surface arrays. The roughness length values are obtained from the integral momentum method to reduce the errors that would otherwise be large if the log-law fitting process were applied. The trend shows that as expected, the surface roughness length reduces for increasing channelling ratio. There are two physical reasons for this trend. Firstly, the unobstructed channels provide a smoother path through the canopy provided that the drag coefficient contribution from the substrate surface is less than that from the obstacles. Secondly, for a fixed number of obstacles, increasing the channelling ratio increases the lateral ‘clumping’ of buildings. For example, in Figure 5.10, the distance between an upstream obstacle and the obstacle directly behind it is far greater for the $S6A$ surface array than the $S6D$ surface array. Thus for the $S6A$ surface array, the flow regime is more likely to be that of isolated roughness or wake interference and as the channelling ratio is increased, the flow regime tends towards the wake interference/skimming flow regime. Thus for larger channelling ratio values, obstacle sheltering reduces the surface roughness. Figure 5.11 shows that the rate of decrease in roughness length is small for the range $0.0 < \lambda_c < 0.6$. For $\lambda_c > 0.6$, the reduction in roughness length is greater. The surface roughness length for $\lambda_c$ equal to one is simply the roughness length of the substrate surface. In the present study, Figure 5.11 shows that the substrate surface alone has a normalised roughness length of 0.33 which is approximately 37% of the $S6A$ surface roughness length. Thus flow channelling has a significant influence on the surface drag and it is important to include it as the third parameter along with the area density ratios. For the purpose of being able to correct the surface roughness length for the presence of flow channels, it is beneficial to construct an expression relating a correction factor, $k_C$ to the channelling ratio. Figure 5.12 shows a correction factor, $k_C$ for specific channelling ratios quantified from the present study. As with all corrections, a reference must be quoted to which the correction factor applies. In the case of Figure 5.12, the reference is the roughness length of an obstacle array with a channelling ratio of 0 (ie. full lateral staggering), which in this case is the roughness length of the $S6A$ surface array. As an example, for a surface array with $\lambda_c = 0$ and a roughness length, $z_0$, if the arrangement of the obstacles were changed such that the new array had a channelling ratio of $\lambda_c = 0.7$, the roughness length would have to be multiplied by correction factor, $k_C$ equal to 0.9 to correct it for the presence of channels. What
would be more attractive is an expression which relates the correction factor, $k_C$ to the channelling ratio, $\lambda_C$.

![Figure 5.11](image1.png)

**Figure 5.11** – Normalised roughness length variation with channelling ratio.

![Figure 5.12](image2.png)

**Figure 5.12** – Correction factor, $k_C$ for varying channelling ratios. The correction factor is applied to a reference obstacle array with a channelling ratio, $\lambda_C = 0$. The empirical constants $M$ and $N$ are equal to 0.614 and 4.5 respectively.
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For Figure 5.12, the following expression is found to best fit the experimental data.

\[ k_C = 1 - M \lambda_C^N \]  \[5.14\]

where \( M = 1 - \frac{z_0|_{\lambda_C = 1}}{z_0|_{\lambda_C = 0}} \) and is equal to 0.614; \( N \) is an empirical constant, \( N = 4.5 \).

The parameter \( M \) is the ratio quantifying the roughness contribution of the obstacles to the total surface roughness. Expression [5.14] requires an empirical parameter, \( N \), which when set to 4.5, provides the best fit to the experimental data. The correlation coefficient between the best-fit line and experimental data in Figure 5.12 is 0.96 and it is therefore considered a good fit. For relatively smooth substrate surfaces, the ratio \( M \) tends towards one and the correction factor would decrease more across a particular channelling ratio range.

Cheng et al. (2007) carried out a wind tunnel experiment to investigate the influence of staggered and aligned cube arrays. Without the authors specifically realising, the experiment in effect investigated the roughness length variation with channelling ratio. Two staggered surface arrays, with the same plan and frontal area density of \( \lambda_P = \lambda_F = 0.25 \) were investigated. The first array had a channelling ratio, \( \lambda_C = 0 \), the second array had a channelling ratio, \( \lambda_C = 0.5 \). Cheng et al. (2007) presented data (see their Figure 13b) showing that the normalised displacement height of both arrays was the same. Thus only the channelling ratio is likely to have influenced the surface drag. The normalised roughness length for the array with \( \lambda_C = 0.5 \) was found to be 6% less than the normalised roughness length for the array with \( \lambda_C = 1.0 \). The expression presented in equation [5.14] with \( M = 1.0 \) suggests that a correction factor of 0.956 implying a roughness length decrease of approximately 4.5%. Thus the correction factor formulation in equation [5.14] is not too far off the experimental data of Cheng et al. (2007). Of course more experimental data of this sort is required to validate the expression.

5.5 Displacement height variation with frontal area density

There is very little experimental data to explain the variation of the wind direction on the displacement height. However, as explained in the introduction, a change in wind direction changes the frontal area density of an obstacle array but not the plan area density. The displacement height variation with frontal area density for a fixed plan area density has already been presented in Figure 4.14b of section 4.3.4. Figure 4.14 shows that the
normalised displacement height variation with plan area density is larger than with the frontal area density. Nevertheless, the frontal area density does influence the displacement height with more significant variations for sparse arrays. The figure is reproduced in Figure 5.13 showing that the normalised displacement height variation is small for $\lambda_F > 0.15$ with the increase in displacement height equal to approximately 3%. To link the variation of the displacement height with the frontal area density, it is possible to construct a line of best fit through the experimental data for which the expression is

$$\frac{z_D}{z_H} = \frac{P}{\lambda_F^Q}$$  \[5.15\]

where $P (=0.75)$ and $Q (=0.12)$ are empirical constants.

The empirical constants are chosen to construct a line which best fits the experimental data. The correlation coefficient between the line of best fit and the experimental data is 0.87 and therefore considered to be a good fit. It is expected that when the frontal area density tends towards zero, the normalised displacement height must also tend towards zero. The expression in equation [5.15] does not achieve this condition and therefore the expression is only valid in the range $0.066 < \lambda_F < 0.326$. However, Figure 4.3 suggests that this range is frequently present in real urban environments.

Figure 5.13 – Normalised displacement height variation with frontal area density for fixed plan area density. The empirical constants $P$ and $Q$ are equal to 0.75 and 0.12 respectively.
5.6 Roughness length variation with wind direction
In sections 5.2 to 5.5, the various physical parameters which vary with wind direction have been described and quantified. Existing algorithms relating surface roughness to obstacle morphology for a fixed wind direction can be modified to achieve a formulation which makes the surface roughness length a function of wind direction. Such a modification would have to include various physical parameters which vary with wind direction, namely the frontal area density, obstacle drag coefficient, channelling flow and displacement height. Applying the modification to the normalised roughness length expression of Macdonald et al. (1998) for a staggered cube array with a fixed wind direction the modified expression becomes

$$\frac{z_o}{\bar{z}_H} = k_c \left( 1 - \frac{z_D}{\bar{z}_H} \right) \exp \left\{ - \left[ \frac{0.5 k_{\phi(R)} C_{D(I)}}{\kappa^2} \left( 1 - \frac{z_D}{\bar{z}_H} \right) \lambda_F \right]^{-0.5} \right\}$$

[5.16]

where \(\kappa\) is the von Karman constant (=0.4); \(C_{D(I)}\) is the drag coefficient of an isolated obstacle (≈1.2 for a cube); \(k_c\) is a correction factor for channelling flow; \(k_{\phi(R)}\) is the correction factor for the isolated obstacle drag coefficient, \(\frac{z_D}{\bar{z}_H}\) is the normalised displacement height and \(\lambda_F\) is the frontal area density.

![Graph](image-url)

**Figure 5.14** – Normalised roughness length variation with wind direction angle for an array of staggered formation cubes with a plan area density equal to 0.25.
In expression [5.16], the frontal area density variation with wind direction is given in equation [5.6], the correction factor $k_{\phi(R)}$ variation with wind direction is given in equation [5.13], the correction factor $k_C$ is given in equation [5.14] and the normalised displacement height is given in equation [5.15]. To assess the roughness length variation with wind direction, equation [5.16] was applied to an idealised cube array with the cubes arranged in a staggered formation for a wind direction of zero degrees. Such an arrangement is shown in Figure 5.1. Figure 5.14 shows the roughness length variation with wind direction for the range $0^\circ < \theta_I < 90^\circ$. The symmetry of a cube obstacle results in a roughness length variation which repeats at an angle of $90^\circ$ and therefore it is not necessary to consider the full $360^\circ$ wind angle range. For this idealised cube array, the channelling ratio, $\lambda_C = 0$ for all wind angles except $\theta_I = 26.57^\circ$ and $\theta_I = 90^\circ$ for which $\lambda_C = 0.211$ and 0.5 respectively.

Figure 5.14 shows that the roughness length increases with wind direction between $0^\circ < \theta_I < 45^\circ$ by approximately 35%. The surface roughness length reaches a peak at $\theta_I = 45^\circ$ before decreasing.

At $\theta_I = 90^\circ$, the roughness length dips slightly because of the presence of channelling flow. Hagishima et al. (2009) conducted wind tunnel experiments in which they first measured the surface drag coefficient across a staggered and aligned cube array and then rotated the aligned cube array by $45^\circ$ to measure the difference in drag coefficient of the array. They found that rotating the aligned cube array by $45^\circ$ increased the drag coefficient by an approximate average of 46%. This is greater than the 35% increase estimated by the expression in equation [5.16]. This is expected because taking an aligned cube array and rotating the array by $45^\circ$ changes the formation of the array from aligned to staggered, as illustrated in Figure 5.15. The channelling ratio then also decreases from $\lambda_C = 0.5$ to $\lambda_C = 0.0$. 

\[\theta = 0^\circ \quad \theta = 45^\circ\]

**Figure 5.15** – The arrangement of obstacles for an aligned array for wind direction $\theta_I = 0^\circ$ and $\theta_I = 45^\circ$. 
This change in formation will result in a much larger increase in roughness length. When comparing the Hagishima et al. (2009) drag coefficient for the staggered cube array at 0° with the 45° aligned array, the difference in the drag coefficient is on average approximately 32% which is more comparable to the 35% increase in normalised roughness length estimated by the expression in equation [5.16]. Ratti et al. (2006) computed roughness lengths for sites in the cities of London, Toulouse and Berlin for different wind direction angles, also using the expressions of Macdonald et al. (1998). However, they only modified the expressions for changes in the frontal area density. Although, the trends computed from their digital elevation model are very similar to that in Figure 5.14 with peaks occurring at 45° for all three cities, the magnitude of the normalised roughness lengths are unlikely to be accurate in comparison to those estimated in the present study.

It is likely that some parameters discussed in section 5.2 to 5.5 have a greater influence on the surface roughness than others. Figure 5.16 shows the sensitivity of surface roughness to the frontal area density, drag coefficient for an isolated cube and the normalised displacement height. The sensitivity plots are constructed using controlled values of $\lambda_F = 0.25$, $C_D(I) = 1.2$ and $z_D/z_H = 0.5$.

![Figure 5.16 - Normalised roughness length sensitivity to the frontal area density, normalised displacement height and the drag coefficient of an isolated cube.](image_url)
Figure 5.16 shows that the roughness length variation with wind direction is most sensitive to the frontal area density. In the wind direction range $0^\circ < \theta < 45^\circ$, the frontal area density increases causing the roughness length to increase too. The drag coefficient of the isolated cube also has a significant influence on the trend of the roughness length which also increases for the wind direction range $0^\circ < \theta_I < 45^\circ$. However, an increase in the displacement height for the wind direction range $0^\circ < \theta_I < 45^\circ$ causes the roughness length to decrease which results in a much shallower trend in the surface roughness length. The expression in equation [5.16] is strictly only valid in the range $0.066 < \lambda_F < 0.326$ because the normalised displacement height variation with frontal area density has yet to be quantified outside this range. Since the displacement height must tend towards zero as the frontal area density tends towards zero, the sensitivity of the roughness length to the displacement height will be much greater for obstacle arrays for which $\lambda_F < 0.066$. In this range, the displacement height alone could switch the trend in the roughness length with wind direction such that the roughness length reaches a minimum at $\theta_I = 45^\circ$ rather than a maximum.

5.7 Observed variations in wind direction in London

Figure 5.14 has shown that changes in wind direction can have significant changes on urban roughness and drag. Thus it is useful to determine how large changes in wind direction are for an urban area. Large scale meteorological variability causes wind direction to change over a few hours. Data measured on the rooftop of a building near the centre of the DAPPLE site is analysed here to give typical wind direction changes. Wind velocity was measured using a sonic anemometer over a 36 day period between 24th April and 3rd June 2004 at a height of 17m above the ground (the average, plan-area weighted building height at the DAPPLE site is 20.4m). Descriptions of the instrumentation and layout are detailed in Barlow et al. (2009). Figure 5.17a shows 10 minute averages of wind direction for 27th May 2004. The large change in wind direction between 05:00 and 07:00 is due to a trough which affected the whole of the UK. The change in wind direction over the same period is up to $120^\circ$. During the rest of the day the wind direction changes by up to $80^\circ$. Figure 5.17b shows the fluctuations in the wind direction determined from the standard deviation of the ten-minute average wind directions for all 36 days. It can be seen that during the measurement period wind direction over a ten minute period fluctuated by up to a standard deviation of $130^\circ$ on the 24th April 2004. Average standard deviation over the 36 day period is more than $45^\circ$. In Figure [5.14], the maximum range of $z_0 / \bar{z}_H$ for any $45^\circ$ change in wind angle occurs between $0^\circ$ and $45^\circ$. For this wind angle range, $z_0 / \bar{z}_H$ changes by 35%.
Chapter Five: The influence of wind direction on surface roughness

Figure 5.17 - (a) Ten-minute average wind direction for 27th May 2004. (b) Standard deviation of the ten-minute average wind direction for each of the 36 days. All data obtained using a sonic anemometer positioned at a height of 17m from the ground. The incident wind angle, $\theta_i = 0^\circ$ is referenced to westerly flow along Marylebone Road.

Thus, depending on the change in wind direction, there could be a significant change in roughness and drag between ten minute periods for an urban area such as the DAPPLE site. Calculating the standard deviation of the wind direction for a ten-minute averaging period as opposed to timescales less than ten minutes is more useful because parameters such as the roughness length depend on mean wind characteristics. The variation of the wind direction on a timescale of ten-minutes shown in Figure 5.17b demonstrates large variability in the wind direction.
5.8 Summary

This chapter has dealt with the variation of the surface roughness length with changes in the incident wind angle, $\theta_i$ of the flow. In terms of building morphology, changes to the incident wind angle result in changes to the frontal area density ratio (for non-cylindrical obstacles) whilst the plan area density ratio remains fixed. Along with the frontal area density, the drag coefficient of obstacles, the channelling ratio, $\lambda_C$ and the displacement height of the surface were identified as the four physical parameters that influence the roughness length due to changes in incident wind direction. The frontal area density varies because the shape of the obstacle changes the projected frontal area. The drag coefficient of an isolated obstacle varies because the pressure forces around the obstacle change with incident angle of the flow. The channelling ratio of the obstacle array change due to the presence of unobstructed flow channels through the array for certain wind directions. Also it is hypothesised that the displacement height changes because it has been shown in Chapter four that the value of $z_D$ changes for surface arrays where the frontal area density is changed but the plan area density value is fixed. The variations of these parameters with wind direction were quantified using a combination of experimental, analytical and empirical analysis and then used to modify the normalised roughness length expression presented by Macdonald et al. (1998) for cube arrays. A sensitivity study shows that the estimated normalised roughness length is most sensitive to changes in the frontal area density. For an idealised array of cube obstacles, the normalised roughness length increases by a maximum of approximately 35% for flow perpendicular to a cube face ($\theta_i = 0^\circ$) compared to an oblique incident flow angle of $\theta_i = 45^\circ$. 
Chapter Six

Conclusions

6.1 Thesis Summary

Buildings are a common feature of all urban areas and their size and shape play an important role in the dynamics of the mean wind flow, the turbulence structure and the exchange of momentum between the surface and the atmospheric boundary layer. The key parameter which determines these physical processes is the surface drag often expressed in terms of the bulk surface drag coefficient or the aerodynamic roughness length. The aim of the research in this thesis has been to quantify the sensitivity of surface drag to the geometry and layout (morphology) of buildings in an urban neighbourhood. The complex geometry of buildings means that it is often not possible to resolve the individual influence of each building. In the context of numerical weather prediction and pollution dispersion modelling, it is not necessary to know how each intricate feature of the building morphology modifies the near surface flow and turbulence. Instead it is sufficient to group the features of multiple buildings into a few bulk morphology parameters which then lead to a few bulk aerodynamic parameters. Previous research has identified the plan and frontal area density of a group of buildings to be the most important morphological parameters for determining the surface drag. These parameters have effectively been applied to idealised building arrays and the behaviour of the trends of the roughness length and displacement height is now well established. However, despite more than three decades of research effort into idealised obstacle arrays, we have to concede that the real urban environment is heterogeneous and in some respects very different to the idealised notion. Thus the aim of this research project was to

(i) quantify the morphology of buildings in a real urban area.

(ii) determine the sensitivity of the surface drag of an obstacle array to the plan and frontal area densities independently.
Consider the physical processes which alter the surface drag of an obstacle array when the wind direction changes.

The first aim was achieved in the second chapter in which the urban area of Greater London was used as a case study. Building morphology data was derived from geographical map data which was then processed using GIS techniques. The strength of the research was that a relatively large urban area was investigated enabling building morphology trends to be identified across a city. The non-uniformity of building geometry and layout suggested that the surface drag across the city may also be non-uniform. This prompted the idea that towns and cities should be split into neighbourhoods for which the size is defined by the morphology of the buildings within it. The spatial extent of the neighbourhood was described in terms of the gridbox resolution which was found to be a function of the plan area density.

One of the key points arising from the GIS study was that \( \lambda_P \neq \lambda_F \) (ie. on average, the buildings are not cubes) and that the two parameters can vary independently. To determine the sensitivity of the surface drag to the plan and frontal area density separately, a wind tunnel study was conducted. The literature review in chapter one highlighted that the traditional method of estimating surface drag from the friction velocity calculated in the inertial sublayer may not be possible. This is because the ISL may not always be distinct in the urban surface layer due to the excessive roughness of the surface and the non-equilibrium structure of the surface layer due to a continually changing underlying surface. This prompted the application of the long-standing integral momentum method as an alternative method to estimate surface drag from the velocity profile alone. The method was used to test ten different surface arrays in which the plan and frontal area density values were controlled such that five of the ten obstacle arrays had a fixed value of \( \lambda_P \) for various values of \( \lambda_F \) and vice versa for the remaining five arrays. The results of this study prompted the modification of Macdonald et al. (1998) which generated the characteristic trends observed in experimental data for obstacle arrays for which \( \lambda_P = \lambda_F \) and more importantly obstacle arrays for which \( \lambda_P \neq \lambda_F \). The wind tunnel study highlighted that for some obstacle arrays it is important to consider an additional parameter known as the channelling ratio, \( \lambda_C \) which is defined to be the ratio of the total width of all channels within an obstacle array for which the flow is uninhibited by obstacles to the total width of the array.

Results of the roughness length sensitivity to the channelling ratio were used in chapter five to assess the correction required for obstacle arrays for which unobstructed flow channels are present at certain wind directions. By including the change in frontal area
density, obstacle drag coefficient and displacement height, the expression presented by Macdonald et al. (1998) was again modified to relate the surface roughness length to the wind direction. This is a novel formulation and was found to agree well with experimental data.

6.2 Conclusions

In terms of the flow dynamics, the surface drag of an urban area governs the mean wind velocity profile and the turbulence structure. The turbulent mixing defines the depth of the roughness and inertial sublayers. From an NWP and a dispersion modelling viewpoint, the surface drag quantifies the exchange of momentum at the first computational level. The introductory first chapter presented a summary of existing research concerned with the relationship between the geometry and layout of buildings with the surface roughness. The main findings from the literature survey were that

- existing geometric methods have extensively considered homogeneous idealised obstacle arrays to formulate algorithms for estimating aerodynamic parameters.
- extrapolating these algorithms to real urban heterogeneous neighbourhoods can result in unphysical trends and therefore is likely to be inaccurate.
- the presence of an ISL in rough urban areas is questioned due to the small ratio of the surface layer depth to the average height of the obstacles and also due to the limited fetch.

The second chapter in which real building morphology was analysed highlighted that

- the plan and frontal area densities are strongly correlated for London, but, on average, not equal.
- the value of each area density ratio is strongly dependent on the size of the area across which the morphology is averaged. Consideration of the convergence of the plan area density with increasing lot area suggests that for suburban neighbourhoods, the area density ratios should be averaged across an area of the order of 4km, whereas for dense urban areas, spatial averaging could take place across a much finer area of the order of 0.5km. This difference in spatial averaging area is a direct result of the non-uniformity in the size and layout of buildings in real urban areas. The size of the spatial averaging area represents the lengthscale of heterogeneity of the
building morphology, based on the plan area density. It appears that the greater the value of $\lambda_p$, the greater the lengthscale of heterogeneity.

The wind tunnel experiments described in chapter three, for which results were presented in chapter four, show that

- theoretical considerations prove that the surface drag can be estimated using the integral momentum method. Experimental data proves that this method achieves surface drag coefficient values and trends similar to those obtained from the calculation of the friction velocity determined from the ISL.

- the surface drag coefficient variation with the plan and frontal area density is similar, resulting in a characteristic trend in which the surface roughness increases for sparse to medium density arrays and decreases for medium to dense arrays. These trends are replicated by the COmbined Cubes Arrays (COCA) model which provides estimates for the surface roughness length for non-cubic obstacle arrays. The COCA model is developed from a modification of the staggered cube array algorithm presented by Macdonald et al. (1998). Fine tuning of the empirical parameters of the model suggest that the surface roughness is more sensitive to the frontal area density than the plan area density by a factor of 1.15. The model provides roughness length estimates which agree very well with the present wind tunnel study and that of Hagishima et al. (2009), for non-cubic obstacle arrays. For the first time we have a broad map of the surface roughness length for non-cubic idealised obstacle arrays. However, the model is not without its limitations and it is important to state that the estimates derived from the model are only for uniform height obstacles. Xie et al. (2008) have shown results which suggest that for an array of random height obstacles, the tallest obstacles make the greatest contribution to surface drag. The validity of the COCA model for random height arrays is yet to be tested. Nevertheless, the COCA model does improve on existing geometric algorithms, in that it provides reasonable roughness length estimates for a wider range of plan and frontal area density values.

- in trying to justify the physical processes which produce the characteristic trends in surface roughness, a third morphology parameter is identified which is found to have a significant influence on the surface roughness. This parameter denoted as the channelling ratio accounts for unobstructed passages or channels within the canopy
within which the flow can pass relatively uninterrupted due to the absence of obstacles. Experimental data shows that the surface drag decreases for obstacle arrays with a large channelling ratio.

- a positive trend is found between the value of the surface drag coefficient and the depth of the roughness sublayer. This trend opens up the possibility for the roughness sublayer depth to be estimated implicitly from the surface roughness and therefore is of benefit to dispersion modellers concerned with the vertical profile of turbulence which governs the lateral and vertical spread of pollution.

The fifth chapter shows that for an obstacle array with a fixed morphology, the surface drag can vary with wind direction.

- the frontal area density, the obstacle drag coefficient, the displacement height and the channelling ratio are identified as the physical parameters which increase the surface drag coefficient for oblique wind angles compared to the wind flowing perpendicular to the obstacle walls. Of these parameters the roughness length is found to be most sensitive to the frontal area density.

- by modifying the Macdonald et al. (1998) expressions for an idealised staggered cube array, the analysis estimates that the percentage increase in surface drag may be approximately 35%. Although the 35% increase in drag is an analytical estimate, this figure is comparable to the wind tunnel experimental data of Hagishima et al. (2009). However further evidence is required to validate this estimate.

6.3 Impacts on the wider field of urban meteorology
The field of urban meteorology can be broadly separated into the thermal influences and the mechanical processes. The thermal processes are mostly associated with the surface energy balance whilst the mechanical processes are associated with the wind flow and turbulence particularly in the surface layer. One of the key physical parameters determining the wind flow and turbulence is the bulk surface drag coefficient or roughness length. This parameter is determined either via direct field measurements giving values which are representative of the real urban surface and atmospheric interaction or by method of algorithms which relate the geometry and layout of obstacles to the surface roughness. Almost all the research effort put into formulating such algorithms has considered idealised homogeneous obstacle arrays. Such algorithms have often been blindly used to estimate surface roughness in the real urban
environment. The development of the COCA model in the present study narrows the gap between the idealised obstacle array and the real urban environment. The location of the present study in terms of the broader field of urban meteorology is illustrated in Figure 6.1 showing how the different sub-branches are interrelated. The present study makes an important contribution to the field of urban meteorology in that it provides a framework for including non-idealised effects. The work also identifies underlying processes and parameters (eg. channelling ratio) which influence surface roughness.

**Figure 6.1** – A schematic representation of the inter-related fields of urban meteorology in the context of the present study.

In addition, the COCA model is a simple formulation for estimating the roughness length for surface arrays with a broad range of plan and frontal area densities, commonly found in real urban areas. The good comparison between the model estimates and the experimental data makes the model attractive for use in NWP and urban dispersion models which make use of building morphology data.

The wind tunnel research showing the valid use of the integral momentum method as an alternative method for surface drag estimation provides a scope for its use in the real atmosphere. Prior to the present study, the integral momentum method was only valid theoretically for rough wall turbulent boundary layers. However, the present study has shown that the method is also practically valid and therefore increases the chances of its
success in the real atmosphere. Of course in the real atmosphere, there are more variables and therefore more terms to include in the integral equation for momentum balance given in equation [3.9]. The effects of the Coriolis force are not negligible in a real urban atmosphere because the Coriolis force generates a Coriolis acceleration given as

$$\frac{dU}{dt}_{\text{Coriolis}} = -2\Omega \times U$$

[6.1]

where $\Omega$ is the angular velocity and $U$ is the velocity of the flow.

The acceleration of the flow introduces an increased rate of change of momentum in the boundary layer which must be accounted for to balance the rate of loss of momentum and the pressure gradient force. For the wind tunnel experiment in the present study, the pressure gradient term was neglected because the flow of air was largely due to the momentum of fluid parcels. In the real atmosphere, local pressure gradient forces may be present which accelerate fluid parcels and therefore add momentum to the flow. To be able to apply the integral momentum method to the real urban atmosphere would require lateral velocity measurements at two streamwise locations. Whilst, this may be difficult with current technology, future remote sensing techniques may allow this to be possible. Surface drag coefficient values derived from point measurements of the velocity profile at two streamwise locations may be sufficiently accurate, but this method is yet to be validated.

The variation of the neighbourhood lengthscale of heterogeneity based on plan area density leads to an appropriate resolution for the spatial averaging area of building morphology. This work fits into the broader field of meteorology and numerical modelling.

### 6.5 Scope for future work

Although the present study has answered some key questions about the relationship between building morphology and urban aerodynamic parameters, the narrow focus of the study has overlooked some avenues. Some of the novel results in the present study have also opened up the path to new avenues which need to investigated. Thus there remains scope for future work and certain aspects of the research need more detailed investigation.

Foremost the convergence of the plan area density parameter with increasing lot area leads to an estimated spatial area considered suitable for averaging morphological features of groups of buildings. It is not obvious how this guideline spatial area would compare if the
frontal area density parameter were considered. This of course is more difficult to accomplish because high resolution building height data is required. Nevertheless the present study introduces the scope to investigate this further. Considering the convergence of the morphology parameters is one method for determining correct gridbox resolution across an urban area. It is important to investigate how this method compares with results from other techniques.

The wind tunnel study for controlled obstacle arrays has highlighted the relative sensitivity of the surface drag to the area density ratios. However, for the surface arrays $S1$ to $S5$, the plan area density was fixed at approximately $\lambda_P = 0.243$. The variation of the bulk drag coefficient for sparser or denser plan area density values is unknown. Similarly for surfaces $S6$ to $S10$ the frontal area density ratio was fixed at approximately $\lambda_F = 0.078$. This is considered a fairly sparse frontal area density value typical of suburban neighbourhoods. The trend in the bulk drag coefficient is unknown for denser obstacle arrays. Further work would involve the measurement of surface drag for numerous obstacle arrays covering a broad range of plan and frontal area density values. This work would also complement the COCA model in terms of its validation.

Like the many wind tunnel experiments that have been conducted for obstacles within a rough wall turbulent boundary layer, the present study has only considered the neutral case. The obvious next step is to consider the momentum and thermal roughness lengths for both the unstable and stable boundary layer cases. Such a study would further narrow the divide between real urban neighbourhoods and the model idealised data.

Investigating non-neutral cases as future work also extends to the measurement of surface roughness for various wind directions. In addition, the four flow processes which change with wind direction may not be the only ones that change. The lack of wind tunnel data and numerical data for idealised arrays makes it difficult to validate the modified algorithm presented in chapter five.
Appendix

The variation of frontal area density with wind direction

The variation of the frontal area density with wind direction is determined primarily by the shape of the obstacles. The following is a derivation of a set of formulae describing the frontal area density as a function of the wind direction for a set of randomly-arranged cuboid-shaped obstacles. The orientation of the obstacles is the same as that of the adjacent streets and all streets are aligned perpendicular and/or parallel to each other.

For incident wind direction, $0 < \theta_i < \pi / 2$:

The frontal area is the projected area perpendicular to the wind direction. For an array of $n$ buildings, the frontal area $A_F$ is

$$A_F(\theta_i) = (L_{x_i} \cos \theta_i + L_{y_i} \sin \theta_i) z_{i,i} \quad [A.1]$$

$$A_F(\theta_i) = \left( \frac{L_{x_i} L_{y_i} \cos \theta_i}{L_{y_i}} + \frac{L_{y_i} L_{x_i} \sin \theta_i}{L_{x_i}} \right) z_{i,i} \quad [A.2]$$
Estimating the sensitivity of urban surface drag to building morphology

\[
A_F(\theta_i) = \left(\frac{L_X}{L_Y}\right) \left(\frac{\cos \theta_i}{L_X} + \frac{\sin \theta_i}{L_Y}\right) \quad \text{[A.3]}
\]

\[
A_F(\theta_i) = \left(\frac{L_X L_Y z_{hi}}{\sqrt{L_X^2 + L_Y^2}}\right) \left(\sqrt{\frac{(L_X)^2 + (L_Y)^2}{L_Y}} \cos \theta_i + \sqrt{\frac{(L_X)^2 + (L_Y)^2}{L_X}} \sin \theta_i\right) \quad \text{[A.4]}
\]

Recognising that:

\[
A_F(\theta_i) = \left(\frac{L_X}{\sqrt{L_X^2 + L_Y^2}}\right) = \sin \gamma \quad \text{and} \quad A_F(\theta_i) = \left(\frac{L_Y}{\sqrt{L_X^2 + L_Y^2}}\right) = \cos \gamma \quad \text{[A.5a,b]}
\]

where \( \gamma = \tan^{-1}\left(\frac{L_X}{L_Y}\right) \) \quad \text{[A.6]}

we get

\[
A_F(\theta_i) = \left(\frac{L_X L_Y z_{hi}}{\sqrt{L_X^2 + L_Y^2}}\right) \left(\frac{\cos \theta_i + \sin \theta_i}{\cos \gamma} \right) \quad \text{[A.7]}
\]

\[
A_F(\theta_i) = \left(\frac{L_X L_Y z_{hi}}{\sqrt{L_X^2 + L_Y^2}}\right) \left(\frac{\sin \gamma \cos \theta_i + \cos \gamma \sin \theta_i}{\sin \gamma \cos \gamma}\right) \quad \text{[A.8]}
\]

Using the trigonometric identity:

\[
\sin(\theta_i + \gamma) = \sin \theta_i \cos \gamma + \sin \gamma \cos \theta_i \quad \text{[A.9]}
\]

\[
A_F(\theta_i) = \left(\frac{L_X L_Y z_{hi}}{\sqrt{L_X^2 + L_Y^2}}\right) \left(\sin(\theta_i + \gamma)\right) \quad \text{[A.10]}
\]

Using [A.5a,b]

\[
\sin \gamma \cos \gamma = \frac{L_X}{\sqrt{L_X^2 + (L_Y)^2}} \times \frac{L_Y}{\sqrt{L_X^2 + (L_Y)^2}} = \frac{L_X L_Y}{(L_X^2 + L_Y^2)} \quad \text{[A.11]}
\]
Estimating the sensitivity of urban surface drag to building morphology

\[ A_F(\theta_i) = \left( \frac{L_X L_Y z_H}{\sqrt{(L_X)^2 + (L_Y)^2}} \right) \sin(\theta_i + \gamma) \]  

[A.12]

\[ A_F(\theta_i) = z_H \sqrt{(L_X)^2 + (L_Y)^2} \sin(\theta_i + \gamma) \]  

[A.13]

\[ A_F(\theta_i) = \left( z_H L_X \right)^2 + \left( z_H L_Y \right)^2 \sin^2(\theta_i + \gamma) \]  

[A.14]

Given that there are \( n \) buildings on a total site area \( A_T \)

\[ \lambda_F(\theta_i) = \left\{ \frac{1}{A_T} \sum_{i=1}^{n} \left[ (z_H L_X)^2 + \frac{1}{A_T} \sum_{i=1}^{n} \left[ (z_H L_Y)^2 \right] \right] + \sin^2(\theta_i + \gamma) \right\} \]  

[A.15]

Recognising that:

\[ \left( \frac{1}{A_T} \sum_{i=1}^{n} z_H L_X \right) = \lambda_F \bigg|_{\theta=0} \quad \text{and} \quad \left( \frac{1}{A_T} \sum_{i=1}^{n} z_H L_Y \right) = \lambda_F \bigg|_{\theta=(\pi/2)} \]  

[A.16a,b]

\[ \lambda_F(\theta_i) = \left( \lambda_F \bigg|_{\theta=0} \right)^2 + \left( \lambda_F \bigg|_{\theta=(\pi/2)} \right)^2 \sin^2(\theta_i + \gamma) \]  

[A.17]

\[ \lambda_F(\theta_i) = \sqrt{\left( \lambda_F \bigg|_{\theta=0} \right)^2 + \left( \lambda_F \bigg|_{\theta=(\pi/2)} \right)^2} \sin(\theta_i + \gamma) \]  

[A.18]

where \( \gamma = \tan^{-1}\left( \frac{L_X}{L_Y} \right) = \tan^{-1}\left( \frac{\lambda_F \bigg|_{\theta=0}}{\lambda_F \bigg|_{\theta=(\pi/2)}} \right) \)  

[A.19]
For incident wind direction, \( \pi/2 < \theta_i < \pi \):

\[
\begin{align*}
A_F(\theta_i) &= (L_{x_i} \sin \phi + L_{y_i} \cos \phi)z_{ij} \\
\text{where } \phi &= \theta_i - \frac{\pi}{2}, \text{ and so} \\
A_F(\theta_i) &= \left[ L_{x_i} \sin \left( \theta_i - \frac{\pi}{2} \right) + L_{y_i} \cos \left( \theta_i - \frac{\pi}{2} \right) \right]z_{ij} \\
\text{Since } \sin \left( \theta_i - \frac{\pi}{2} \right) &= -\cos \theta_i \text{ and } \cos \left( \theta_i - \frac{\pi}{2} \right) = \sin \theta_i
\end{align*}
\]

\[
A_F(\theta_i) = \left[ L_{y_i} \sin \theta_i - L_{x_i} \cos \theta_i \right]z_{ij} \\
A_F(\theta_i) = \left( L_{y_i} \frac{L_{x_i} \sin \theta_i}{L_{x_i}} - L_{x_i} \frac{L_{y_i} \cos \theta_i}{L_{y_i}} \right)z_{ij} \\
A_F(\theta_i) = \left( L_{x_i} L_{y_i} z_{ij} \right) \left( \frac{\sin \theta_i}{L_{x_i}} - \frac{\cos \theta_i}{L_{y_i}} \right) \\
A_F(\theta_i) = \left( L_{x_i} L_{y_i} z_{ij} \right) \left( \frac{\sqrt{L_{x_i}^2 + L_{y_i}^2}}{L_{x_i}} \frac{\sin \theta_i}{L_{x_i}} - \frac{\sqrt{L_{x_i}^2 + L_{y_i}^2}}{L_{y_i}} \cos \theta_i \right)
\]

Recognising that:

**Figure A.2** - Definition of angles and lengths to used to derive the formulation of frontal area density variations with wind direction for incident wind direction, \( \pi/2 < \theta_i < \pi \).
\[ A_F(\theta_i) = \left( \frac{L_{yi}}{\sqrt{(L_{xi})^2 + (L_{yi})^2}} \right) = \sin \gamma \quad \text{and} \quad A_F(\theta_i) = \left( \frac{L_{yi}}{\sqrt{(L_{xi})^2 + (L_{yi})^2}} \right) = \cos \gamma \]

(A.28a,b)

where \( \gamma = \tan^{-1} \left( \frac{L_{xi}}{L_{yi}} \right) \)

(A.29)

we get

\[ A_F(\theta_i) = \left( \frac{L_{xi} L_{yi} z_{hi}}{\sqrt{(L_{xi})^2 + (L_{yi})^2}} \right) \begin{pmatrix} \sin \theta_i - \cos \theta_i \\ \sin \gamma - \cos \gamma \end{pmatrix} \]

(A.30)

\[ A_F(\theta_i) = \left( \frac{L_{xi} L_{yi} z_{hi}}{\sqrt{(L_{xi})^2 + (L_{yi})^2}} \right) \begin{pmatrix} \sin \theta_i \cos \gamma - \cos \theta_i \sin \gamma \\ \sin \gamma \cos \gamma \end{pmatrix} \]

(A.31)

Using the trigonometric identity:

\[ \sin(\theta_i - \gamma) = \sin \theta_i \cos \gamma - \sin \gamma \cos \theta_i \]

(A.32)

\[ A_F(\theta_i) = \left( \frac{L_{xi} L_{yi} z_{hi}}{\sqrt{(L_{xi})^2 + (L_{yi})^2}} \right) \begin{pmatrix} \sin(\theta_i - \gamma) \\ \sin \gamma \cos \gamma \end{pmatrix} \]

(A.33)

Using [A.5a,b]

\[ \sin \gamma \cos \gamma = \frac{L_{xi}}{\sqrt{(L_{xi})^2 + (L_{yi})^2}} \times \frac{L_{yi}}{\sqrt{(L_{xi})^2 + (L_{yi})^2}} = \frac{L_{xi} L_{yi}}{(L_{xi})^2 + (L_{yi})^2} \]

(A.34)

\[ A_F(\theta_i) = \left( \frac{L_{xi} L_{yi} z_{hi}}{\sqrt{(L_{xi})^2 + (L_{yi})^2}} \right) \begin{pmatrix} (L_{xi})^2 + (L_{yi})^2 \\ (L_{xi})^2 + (L_{yi})^2 \end{pmatrix} \sin(\theta_i - \gamma) \]

(A.35)

\[ A_F(\theta_i) = z_{hi} \sqrt{(L_{xi})^2 + (L_{yi})^2} \sin(\theta_i - \gamma) \]

(A.36)

\[ A_F(\theta_i) = \left( z_{hi} L_{xi} \right)^2 + \left( z_{hi} L_{yi} \right)^2 \sin^2(\theta_i - \gamma) \]

(A.37)

Given that there are \( n \) buildings on a total site area \( A_T \)
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\[
\lambda_F (\theta_t) = \left\{ \frac{1}{A_T} \sum_{i=1}^{n} \left[ z_{hi} L_{xi} \right]^2 + \frac{1}{A_T} \sum_{i=1}^{n} \left[ z_{hi} L_{yi} \right]^2 \right\} \sin^2 (\theta_t - \gamma) \quad [A.38]
\]

Recognising that:

\[
\left( \frac{1}{A_T} \sum_{i=1}^{n} z_{hi} L_{xi} \right) = \lambda_F \bigg|_{\theta = 0} \quad \text{and} \quad \left( \frac{1}{A_T} \sum_{i=1}^{n} z_{hi} L_{yi} \right) = \lambda_F \bigg|_{\theta = (\pi/2)} \quad [A.39a,b]
\]

\[
\lambda_F (\theta_t) = \left[ \lambda_F \bigg|_{\theta = 0} \right]^2 + \left[ \lambda_F \bigg|_{\theta = (\pi/2)} \right]^2 \sin^2 (\theta_t - \gamma) \quad [A.40]
\]

\[
\lambda_F (\theta_t) = \sqrt{\left[ \lambda_F \bigg|_{\theta = 0} \right]^2 + \left[ \lambda_F \bigg|_{\theta = (\pi/2)} \right]^2} \sin(\theta_t - \gamma) \quad [A.41]
\]

where \( \gamma = \tan^{-1} \left( \frac{L_{xi}}{L_{yi}} \right) = \tan^{-1} \left( \frac{\lambda_F \bigg|_{\theta = 0}}{\lambda_F \bigg|_{\theta = (\pi/2)}} \right) \quad [A.42] \)

In summary:

\[
\lambda_F (\theta_t) = \begin{cases} 
\sqrt{\left[ \lambda_F \bigg|_{\theta = 0} \right]^2 + \left[ \lambda_F \bigg|_{\theta = (\pi/2)} \right]^2} \sin(\theta_t + \gamma), & \text{for } 0 \leq \theta_t < \pi/2 \\
\sqrt{\left[ \lambda_F \bigg|_{\theta = 0} \right]^2 + \left[ \lambda_F \bigg|_{\theta = (\pi/2)} \right]^2} \sin(\theta_t - \gamma), & \text{for } \pi/2 \leq \theta_t < \pi 
\end{cases} \quad [A.43a,b]
\]

where \( \gamma = \tan^{-1} \left( \frac{\lambda_F \bigg|_{\theta = 0}}{\lambda_F \bigg|_{\theta = (\pi/2)}} \right) \quad [A.44] \)
References


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ESDU (1986). Mean fluid forces and moments on rectangular prisms: surface-mounted structures in turbulent shear flow. *Engineering Sciences Data Unit*, Item Number **80003**.


