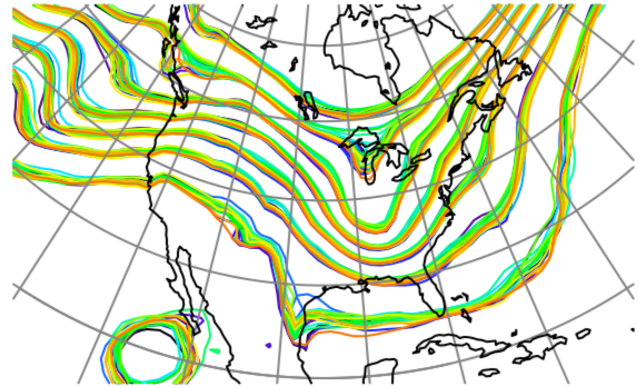


Automated Design of Localization for Ensemble Kalman Filters

Jeff Anderson, NCAR Data Assimilation Research Section

Schematic of an Ensemble Filter for Geophysical Data Assimilation

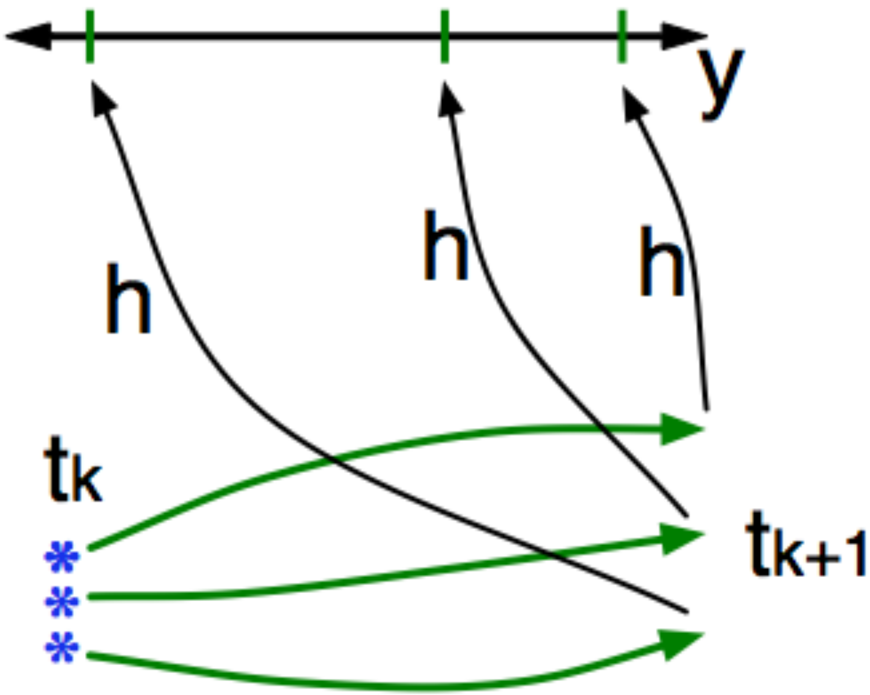
1. Use model to advance ensemble (3 members here) to time at which next observation becomes available.

Ensemble state estimate after using previous observation (analysis)

Ensemble state at time of next observation (prior)

Schematic of an Ensemble Filter for Geophysical Data Assimilation

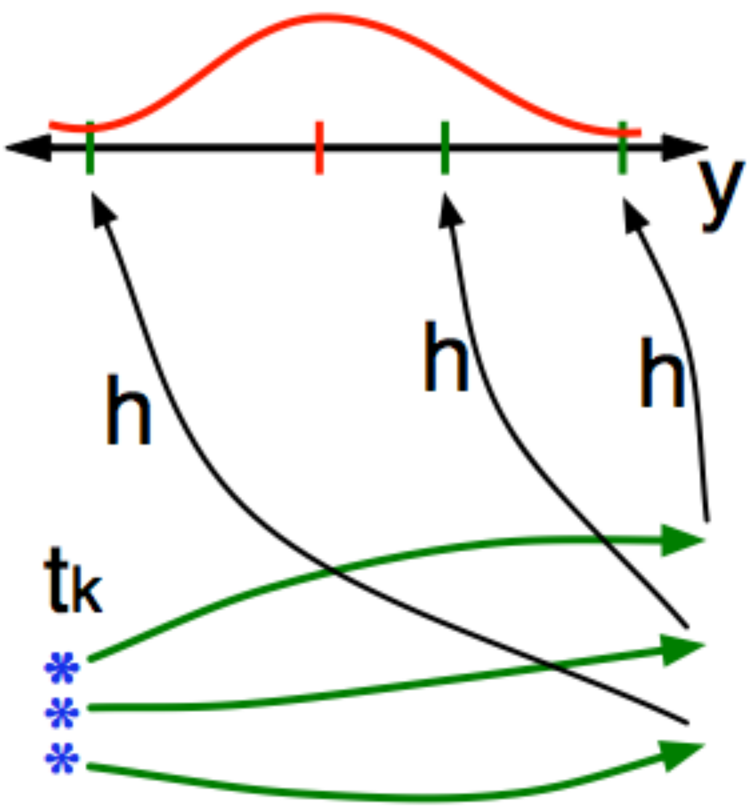
2. Get prior ensemble sample of observation, $y = h(x)$, by applying forward operator **h** to each ensemble member.



Theory: observations from instruments with uncorrelated errors can be done sequentially.

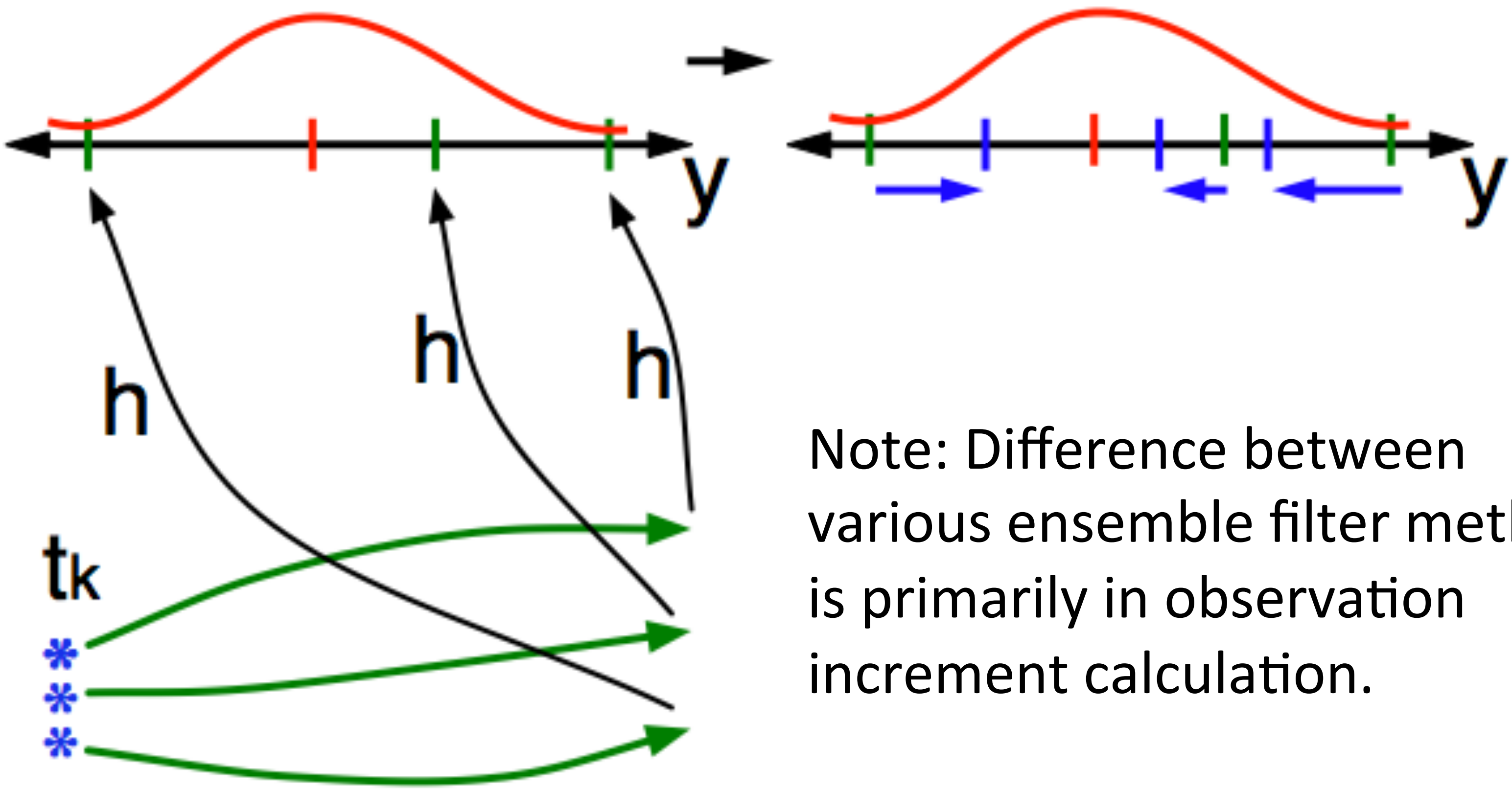
Schematic of an Ensemble Filter for Geophysical Data Assimilation

3. Get observed value and observational error distribution from observing system.



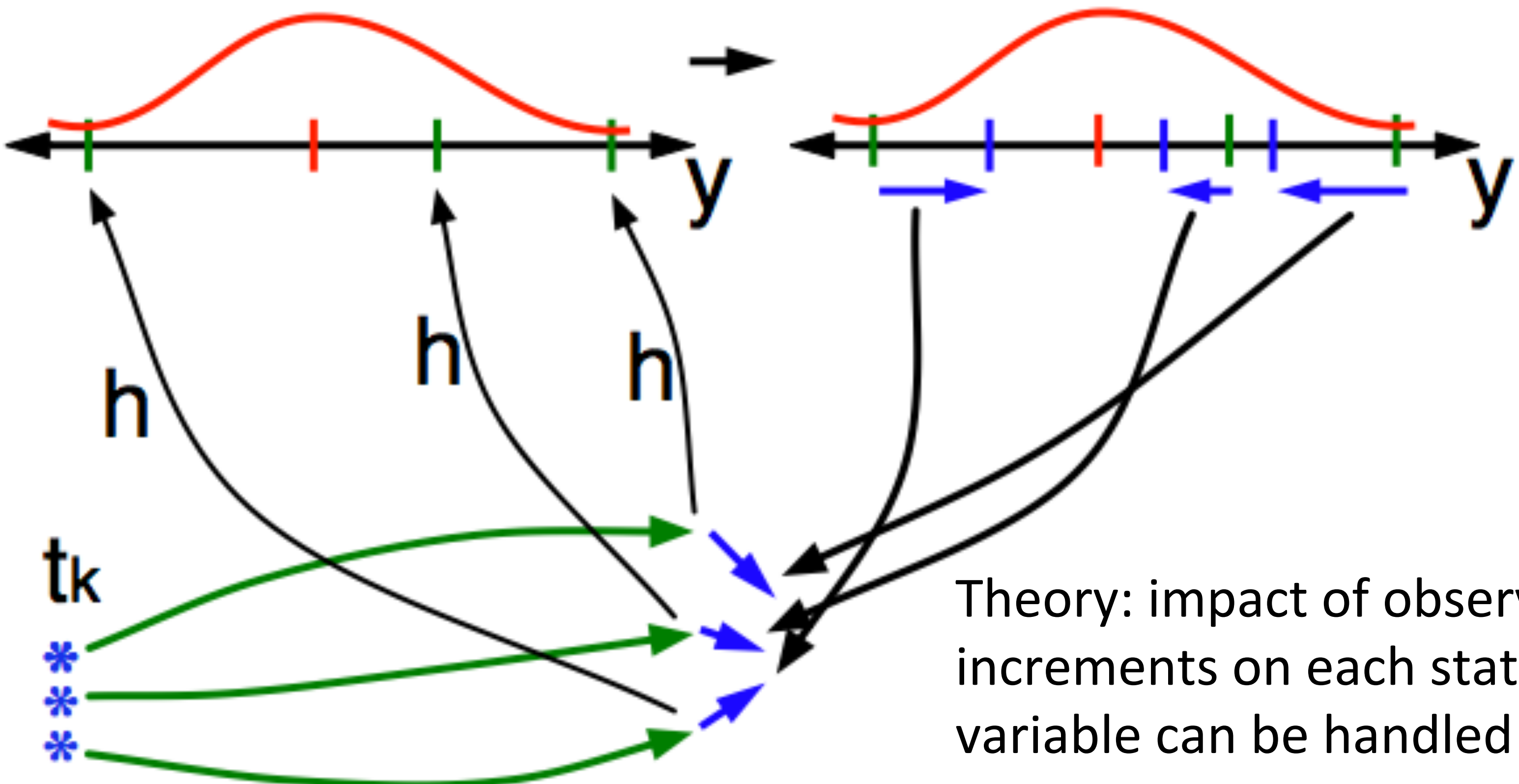
Schematic of an Ensemble Filter for Geophysical Data Assimilation

4. Find the increments for the prior observation ensemble (this is a scalar problem for uncorrelated observation errors).



Note: Difference between various ensemble filter methods is primarily in observation increment calculation.

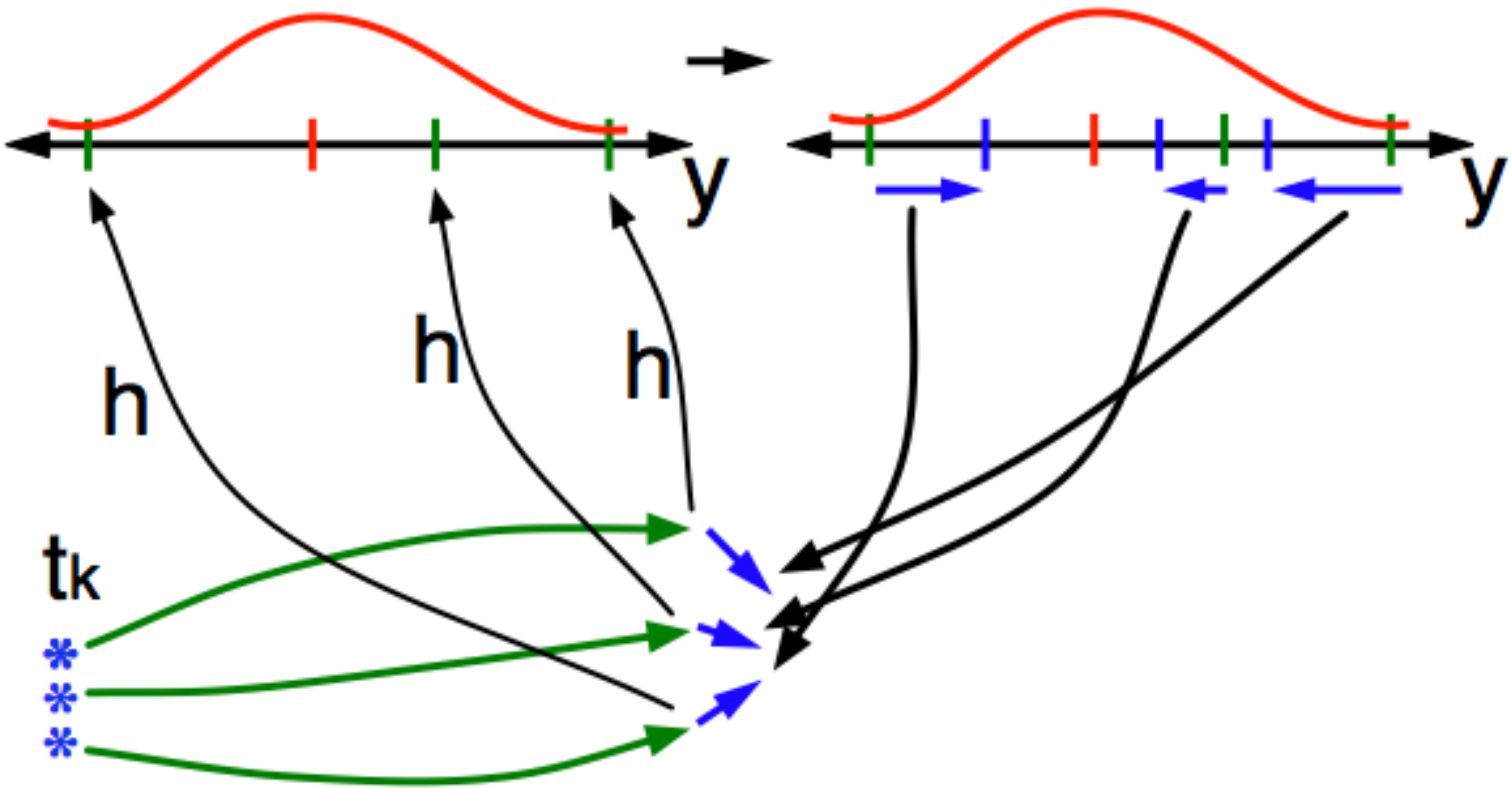
5. Use ensemble samples of y and each state variable to linearly regress observation increments onto state variable increments.



Theory: impact of observation increments on each state variable can be handled independently!

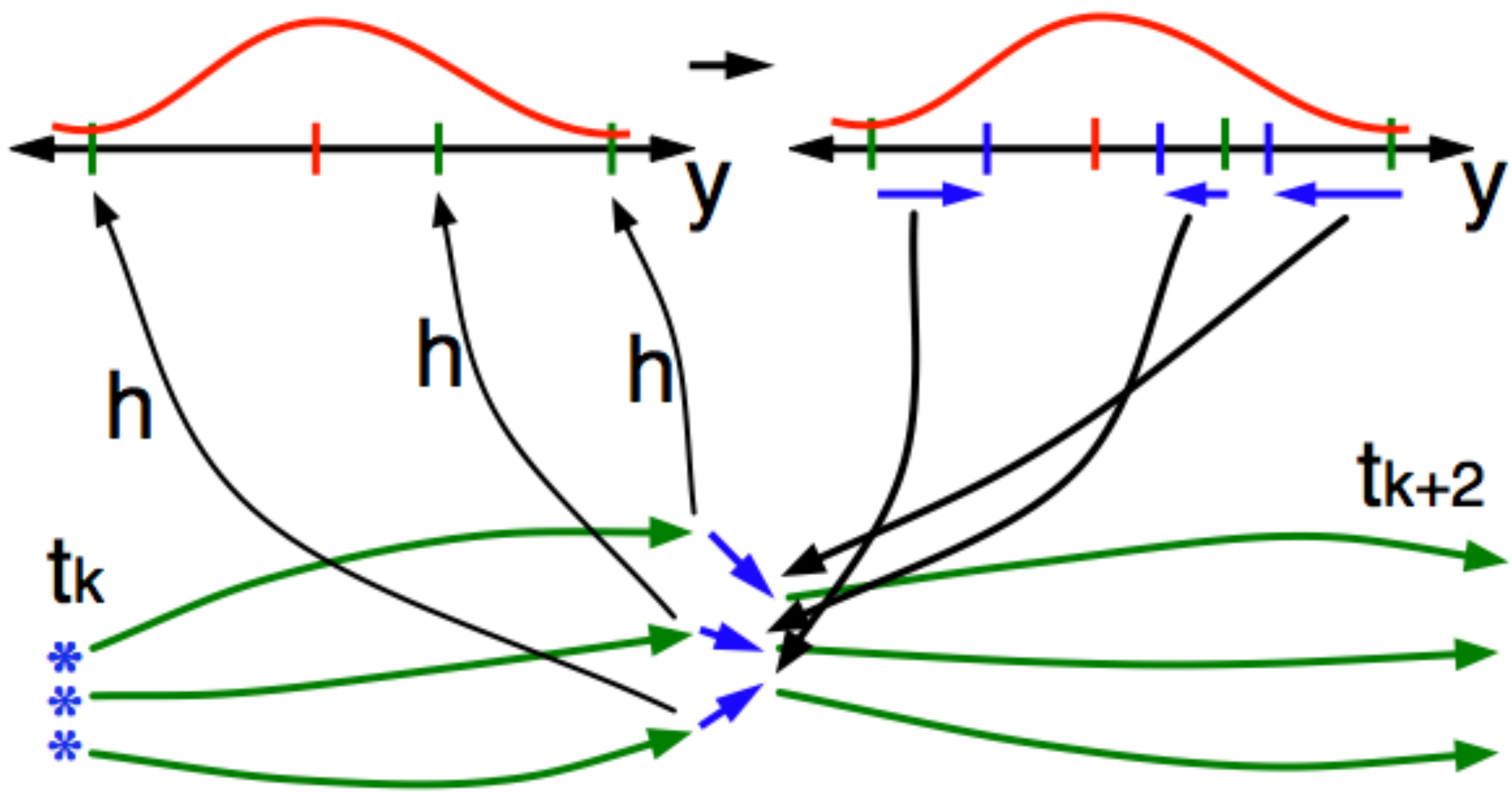
Schematic of an Ensemble Filter for Geophysical Data Assimilation

5. Use ensemble samples of y and each state variable to linearly regress observation increments onto state variable increments.



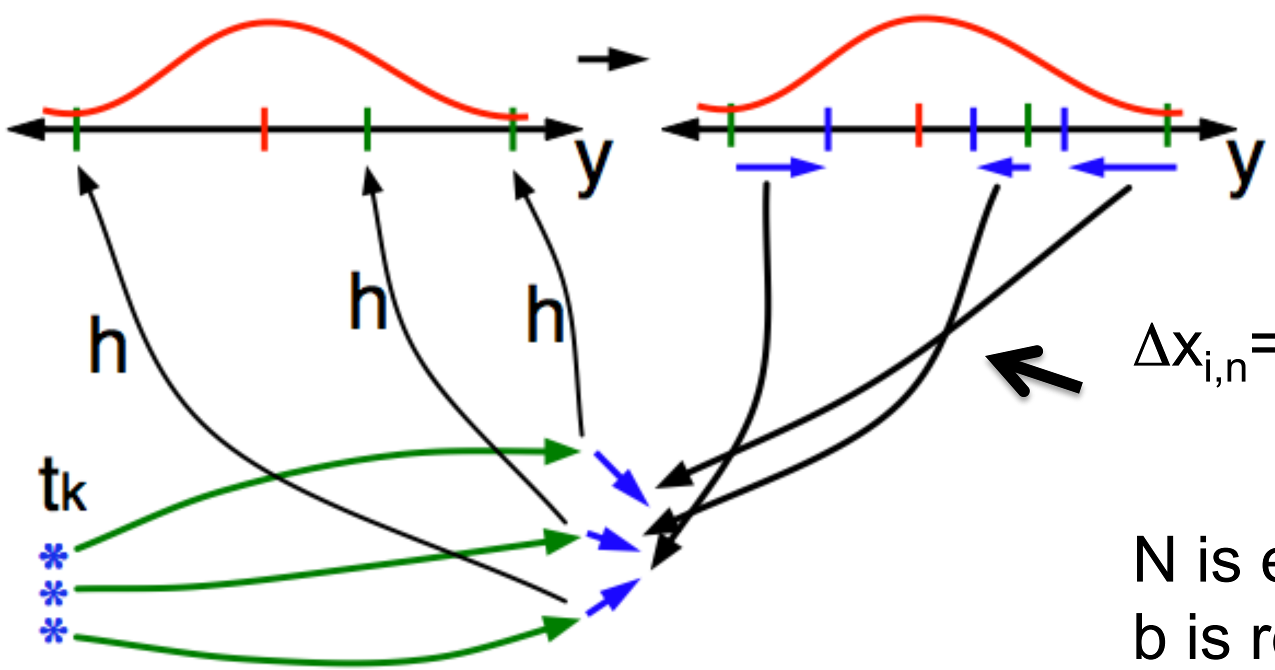
Schematic of an Ensemble Filter for Geophysical Data Assimilation

6. When all ensemble members for each state variable are updated, there is a new analysis. Integrate to time of next observation …



Focus on the Regression Step

Regress y increments onto each state component x_i.



$\Delta x_{i,n} = b \Delta y_n$,
$n=1,\ldots N$.

N is ensemble size.
b is regression coefficient.

Localization is Required for Most Applications

- Localization multiplies regression.
- Increments for N ensemble samples of x are:

  $\Delta x_{i,n}=\alpha b \Delta y_{n,}$ $\qquad$ $n=1, \ldots, N.$

- b is sample regression coefficient.
- α is a localization (normally between 0 and 1).

Defining a Localization Function

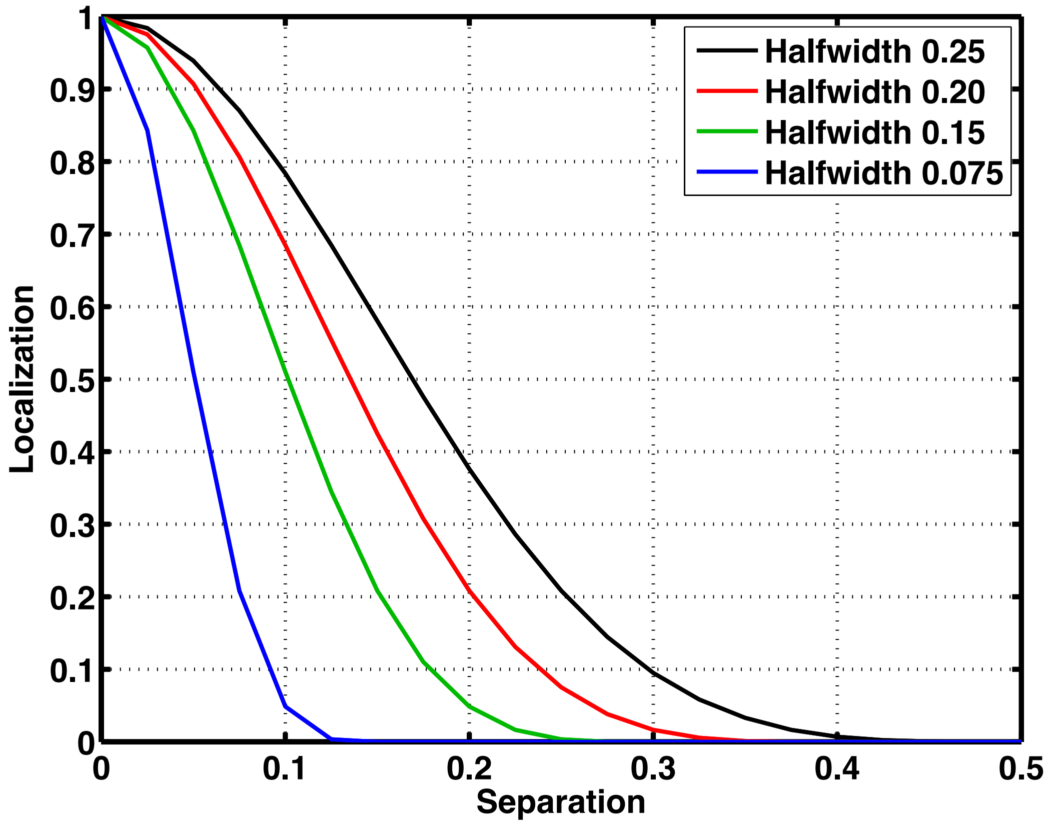
Localization for a given (y, x) might be a function of:

1. Metadata for (y, x) such as:
   - Separation between (y, x),
   - Kind of observation y (temperature, wind, …),
   - Kind of state variable x.

Benchmark Localization: Best Tuned Gaspari-Cohn

- Function of separation between observation y and state x.

- Length scale defined by halfwidth parameter.



Defining a Localization Function

Localization for a given (y, x) might be a function of:

1. Metadata for (y, x) such as:
   Separation between (y, x),
   Kind of observation y (temperature, wind, …),
   Kind of state variable x.

2. Ensemble prior for (y, x) such as:
   Sample Correlation from the Ensemble.

Defining a Localization Function

Define localization function for subsets of (y, x) pairs.

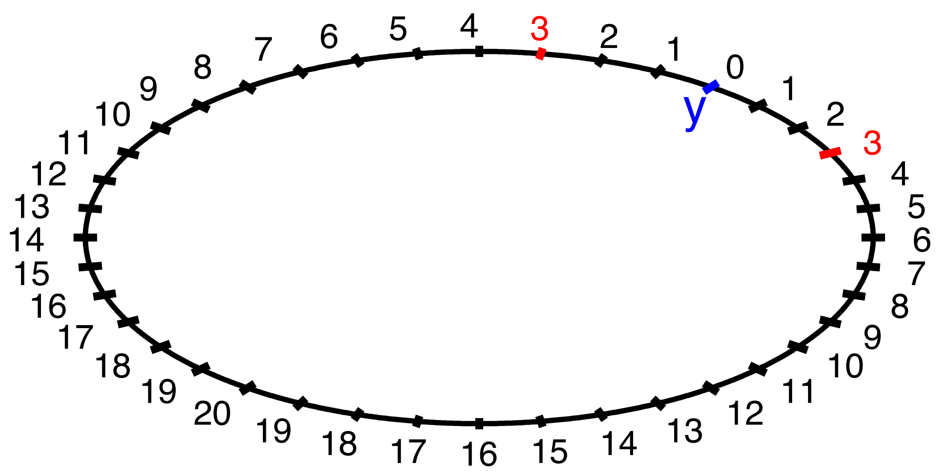
Examples:

- All pairs separated by a given distance range,
- All pairs where y is a temperature and x is a u-wind separated by a given distance range.

Lorenz96 40-Variable Localization Subset Definition

Define subsets of (y, x) pairs by separation:

Example: state x is 3 grid intervals from observation y, (dx = 3).



➢ Estimate localization distribution for each separation.

New Localization Method: Correlation Error Reduction

- Assume all errors are due to ensemble sampling error.

- Focus on regression $b=r(\sigma_x/\sigma_y)$,
  r is correlation,
  σ_x is standard deviation of state,
  σ_y is standard deviation of observation.

Estimates of standard deviation are unbiased
(but estimates of ratio are biased, not discussed here).

- Only <u>correct sampling error in the correlation</u>.

New Localization Method: Correlation Error Reduction

Overview of Algorithm:

- Estimate 'background' correlation distribution for each separation subset.
- Use current sample correlation from assimilation and associated sampling error uncertainty.
- Combine current correlation with background.
- Get 'localized' correlation to update state x.

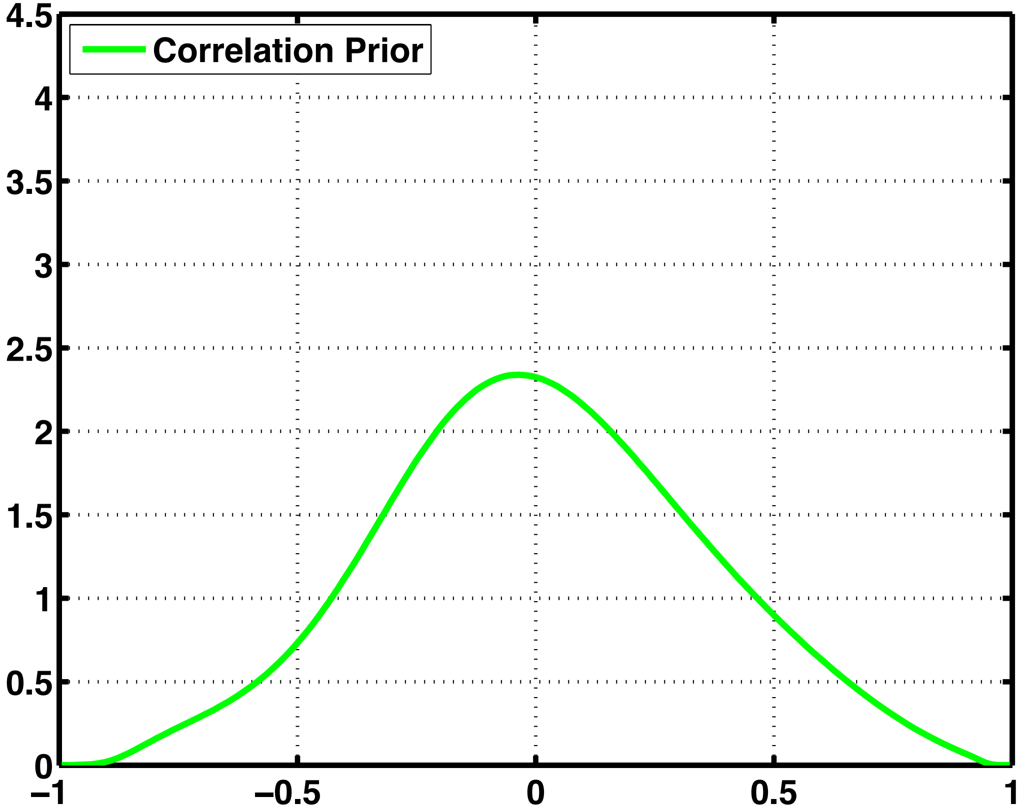
Example: L96 Infrequent High-Quality Observations

Identity observations, error variance 1.

Assimilate every 12th model timestep.

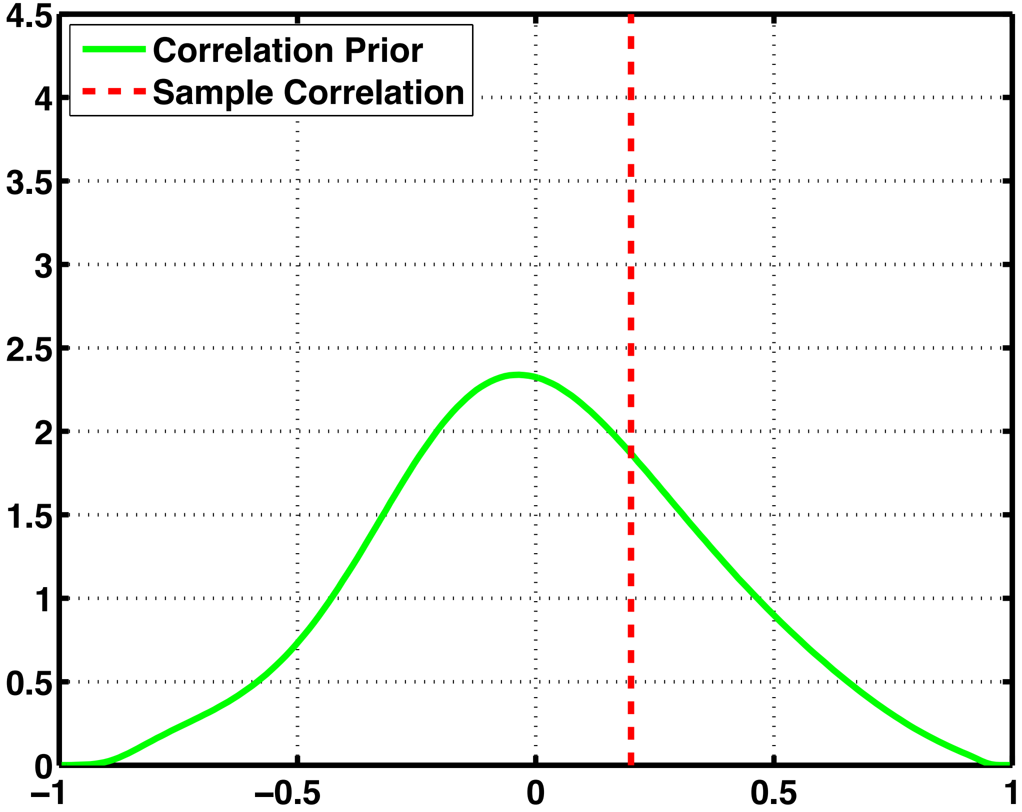
20-member ensemble.

Correlation Error Reduction Algorithm



Start with prior estimate of correlation for a separation (dx = 3 here) between obs and state variable.

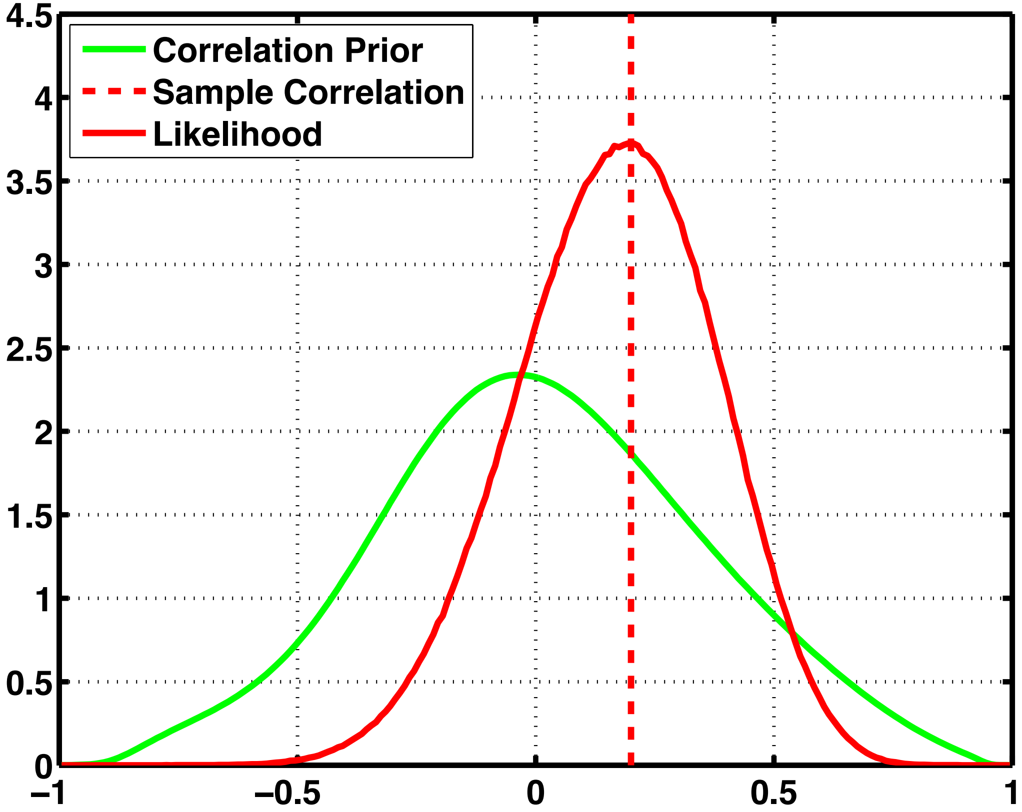
Correlation Error Reduction Algorithm



Ensemble sample correlation between observation and state prior is part of standard ensemble algorithm.

Sample correlation here is 0.2.

Correlation Error Reduction Algorithm

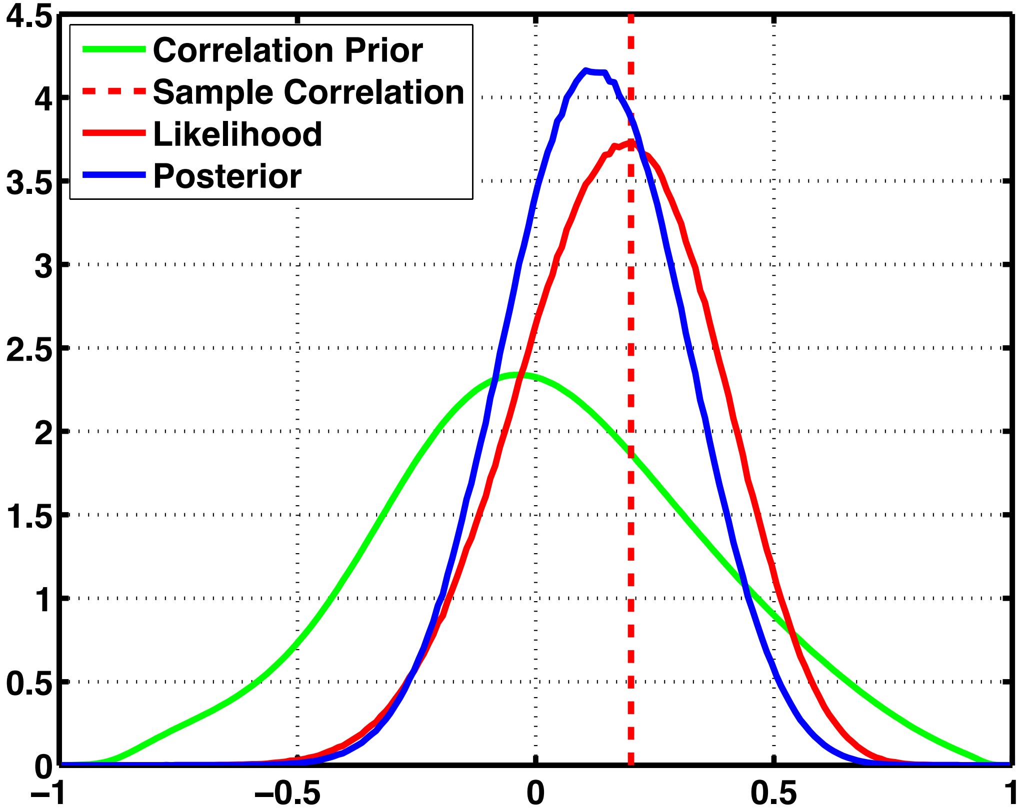


Likelihood for this sample correlation and ensemble size is computed off-line ahead of time.

It is probability of true correlation given the sample correlation.

Note skew to the left.

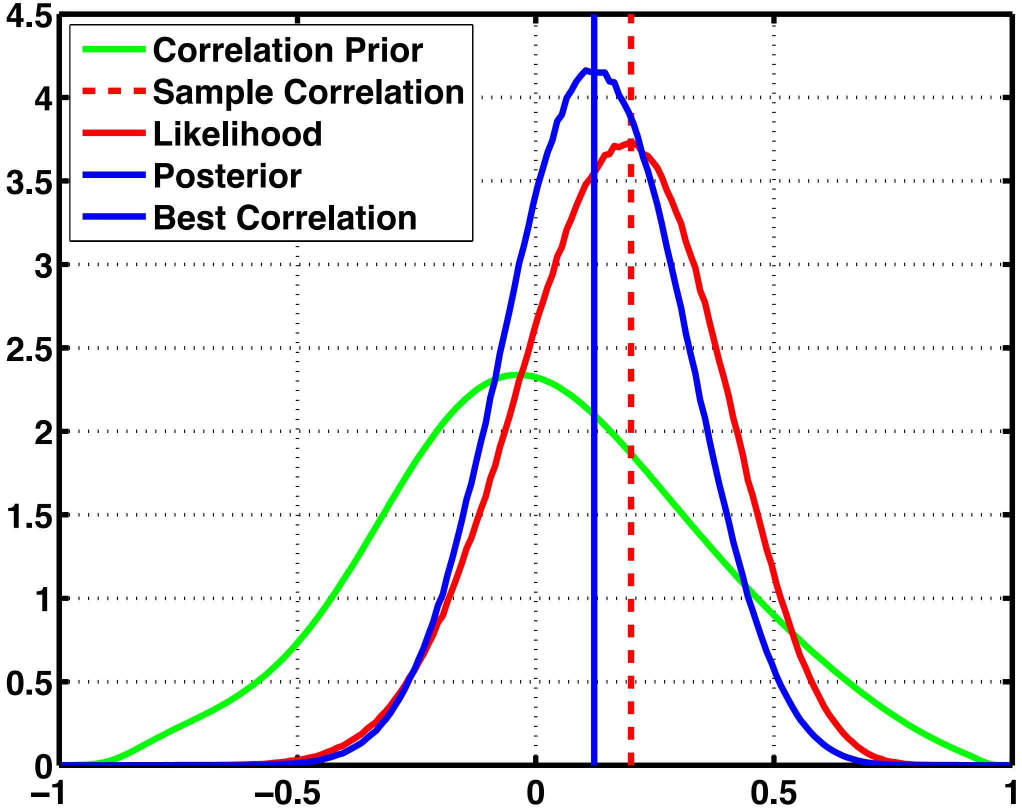
Correlation Error Reduction Algorithm



Posterior is normalized product of prior and likelihood.

This is Bayes rule.

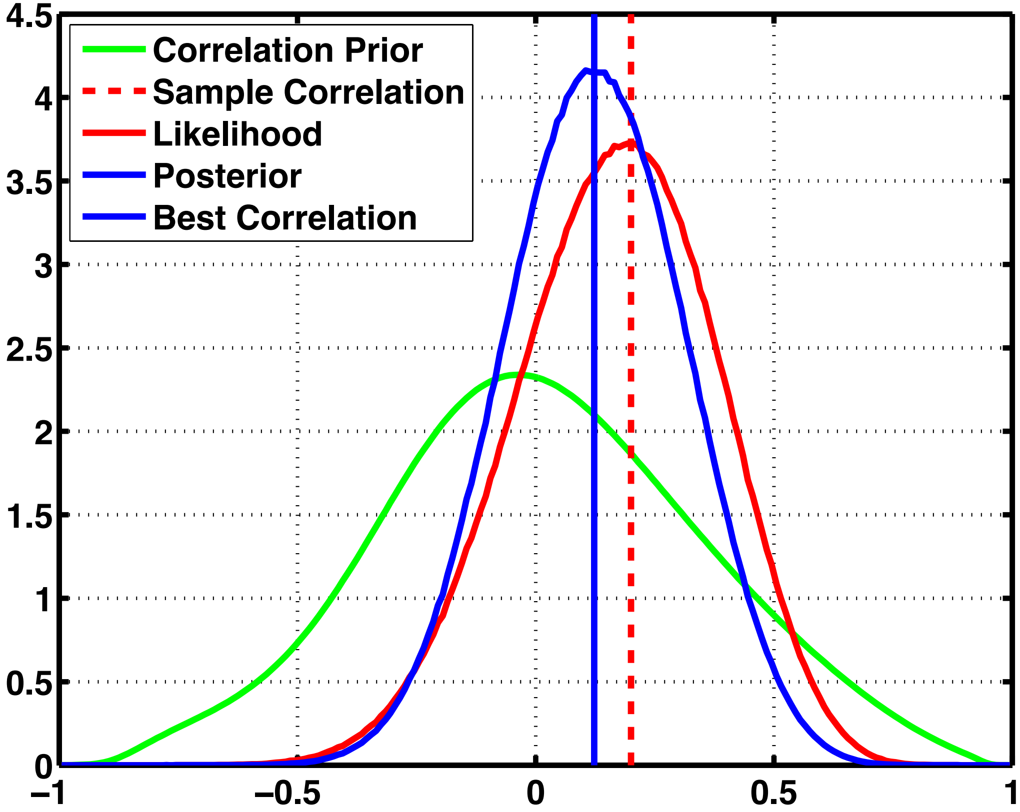
Correlation Error Reduction Algorithm



Use mean value of posterior correlation, (0.1228 here) in the regression to update state.

An equivalent localization is 0.1228 / 0.2 = 0.614.

Correlation Error Reduction Algorithm



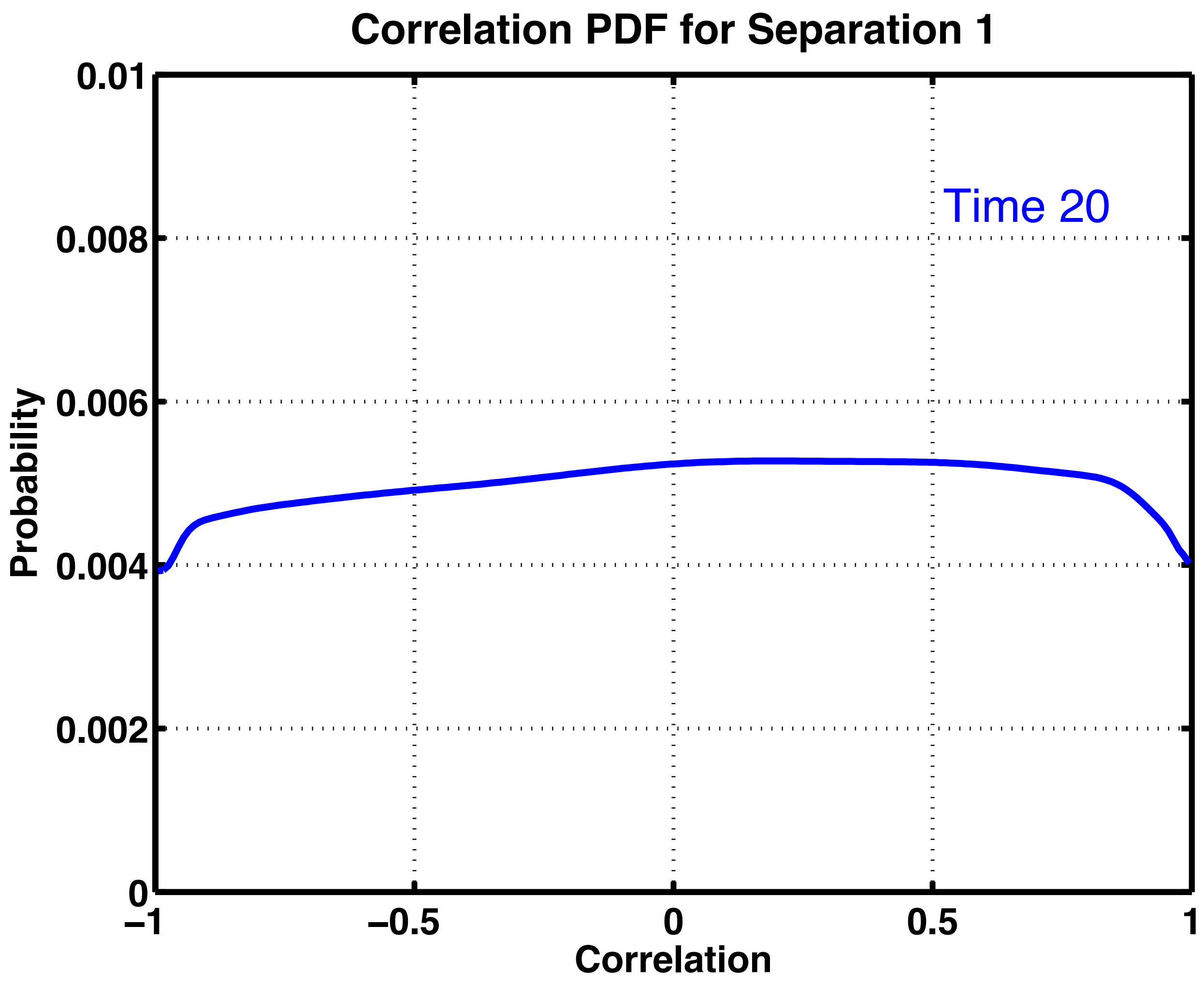
Update prior correlation distribution by adding a small constant times the posterior and normalizing.

Results here use 0.001 for this constant.

This is the only free parameter in the algorithm.

This is NOT Bayes rule.

Evolution of Correlation Distribution



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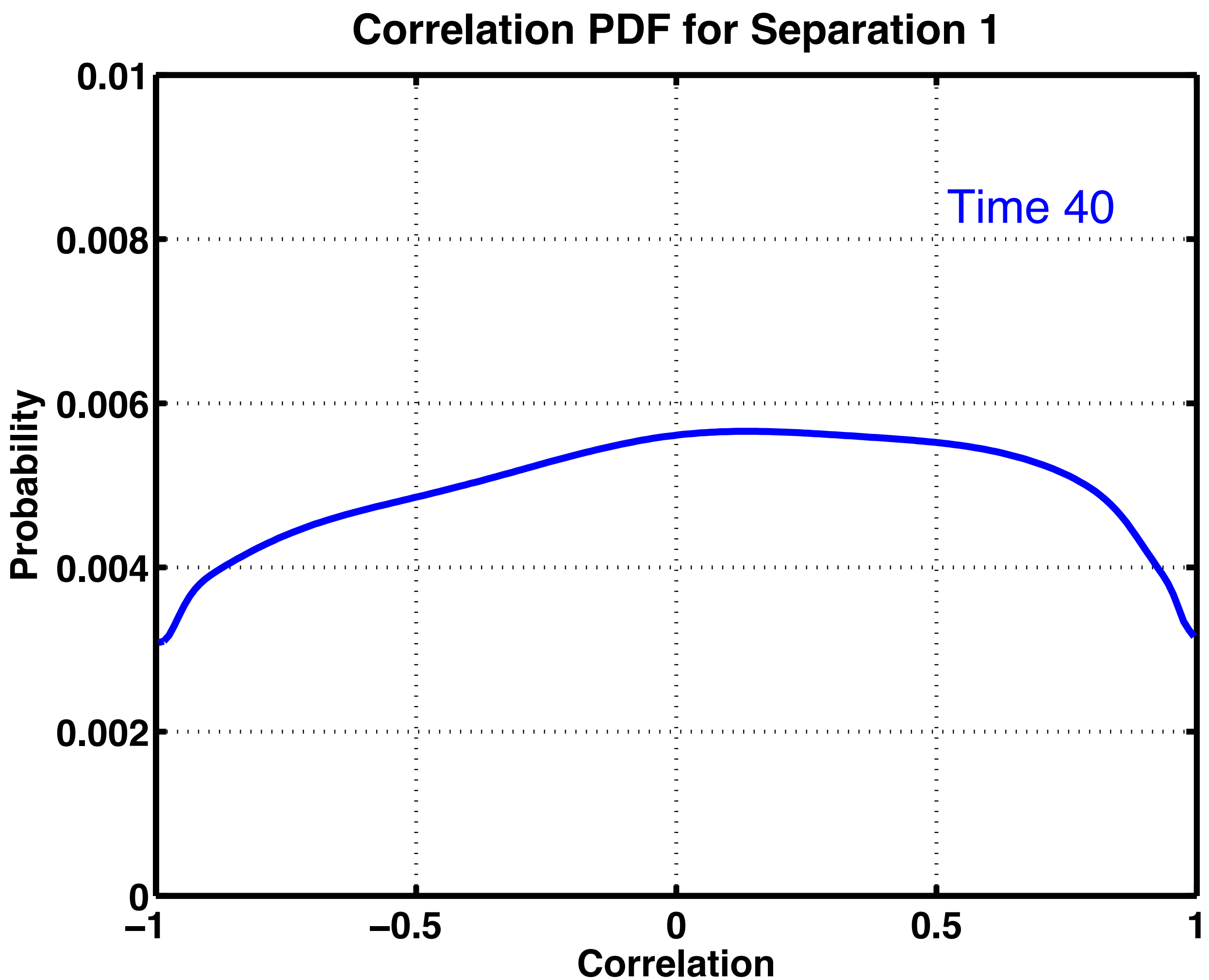

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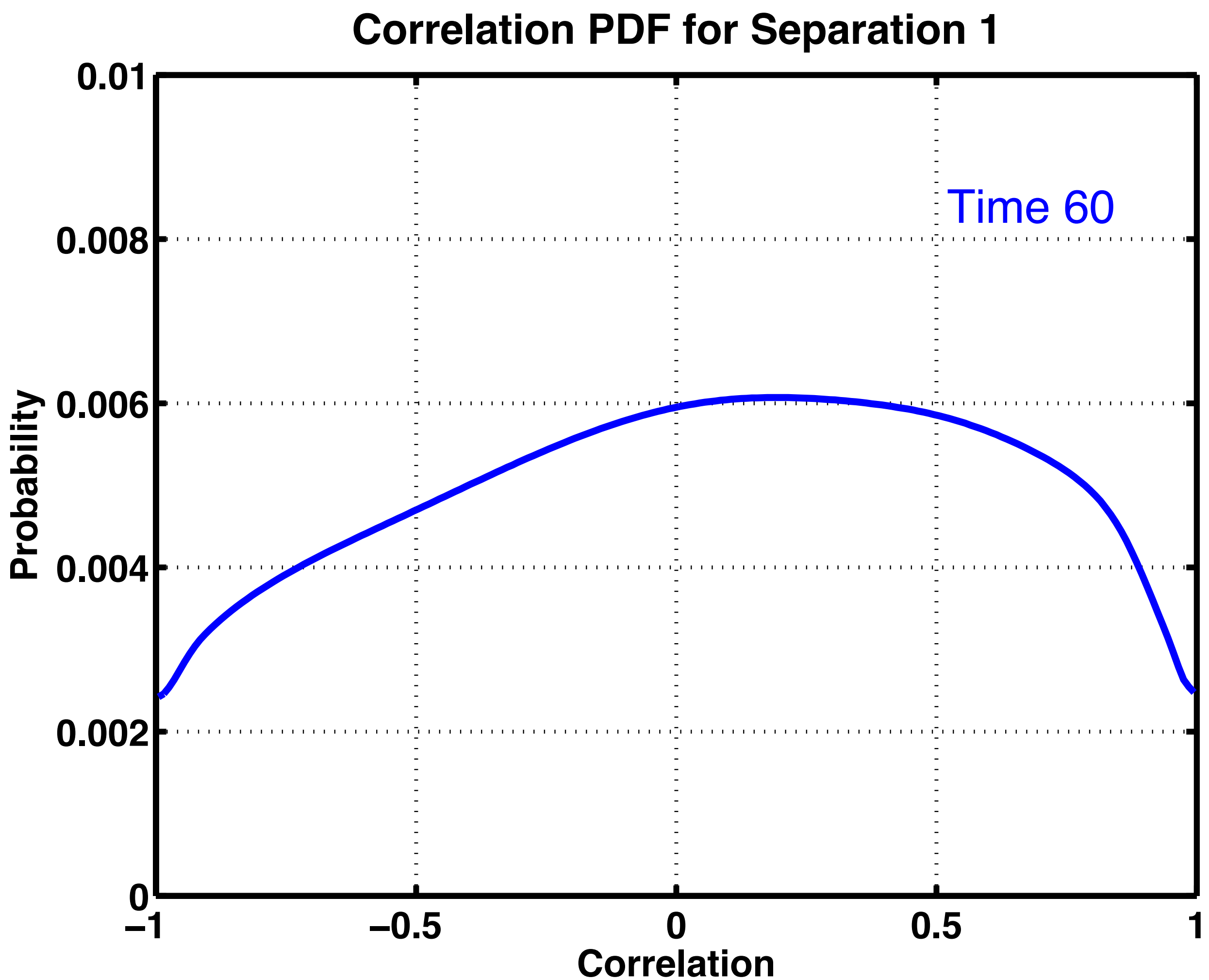

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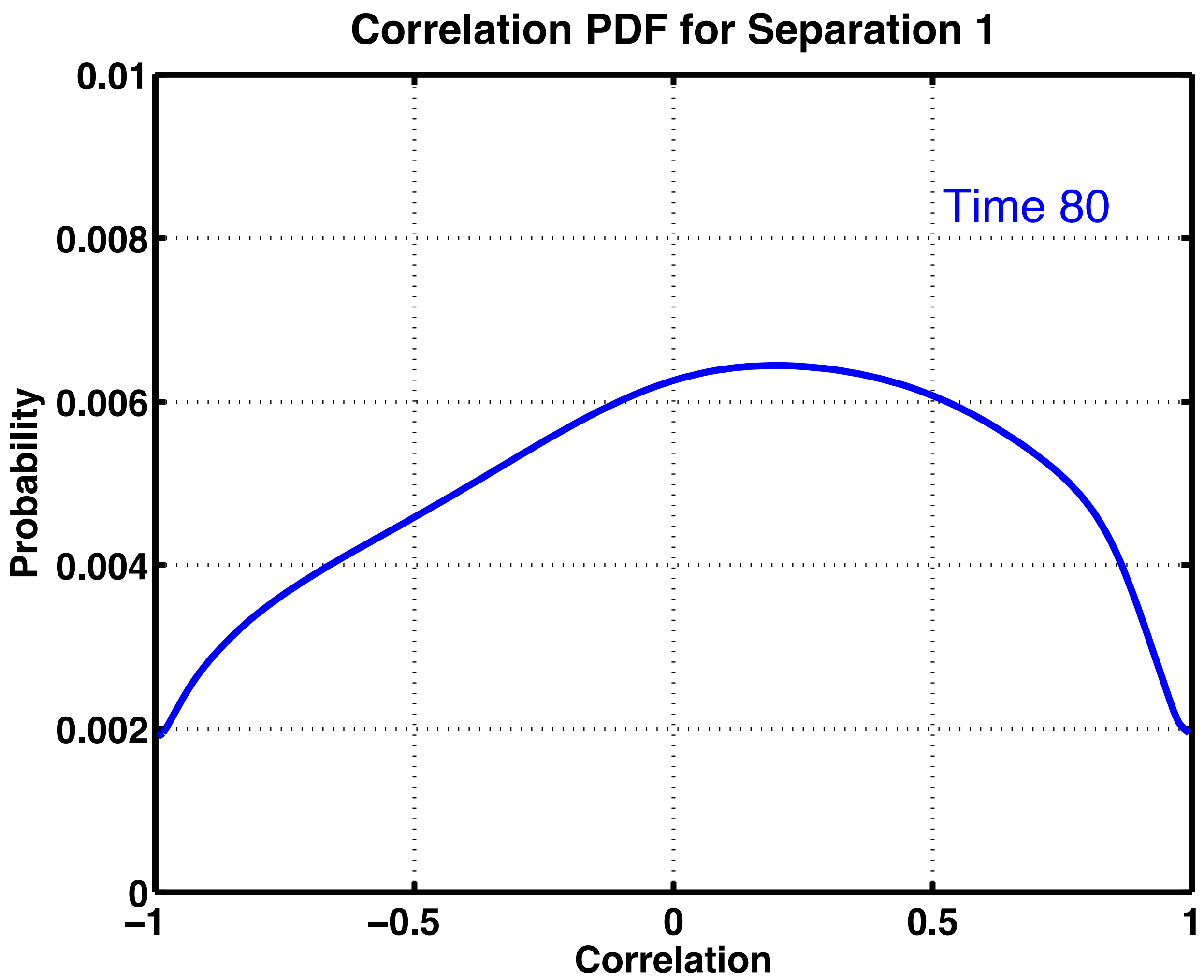

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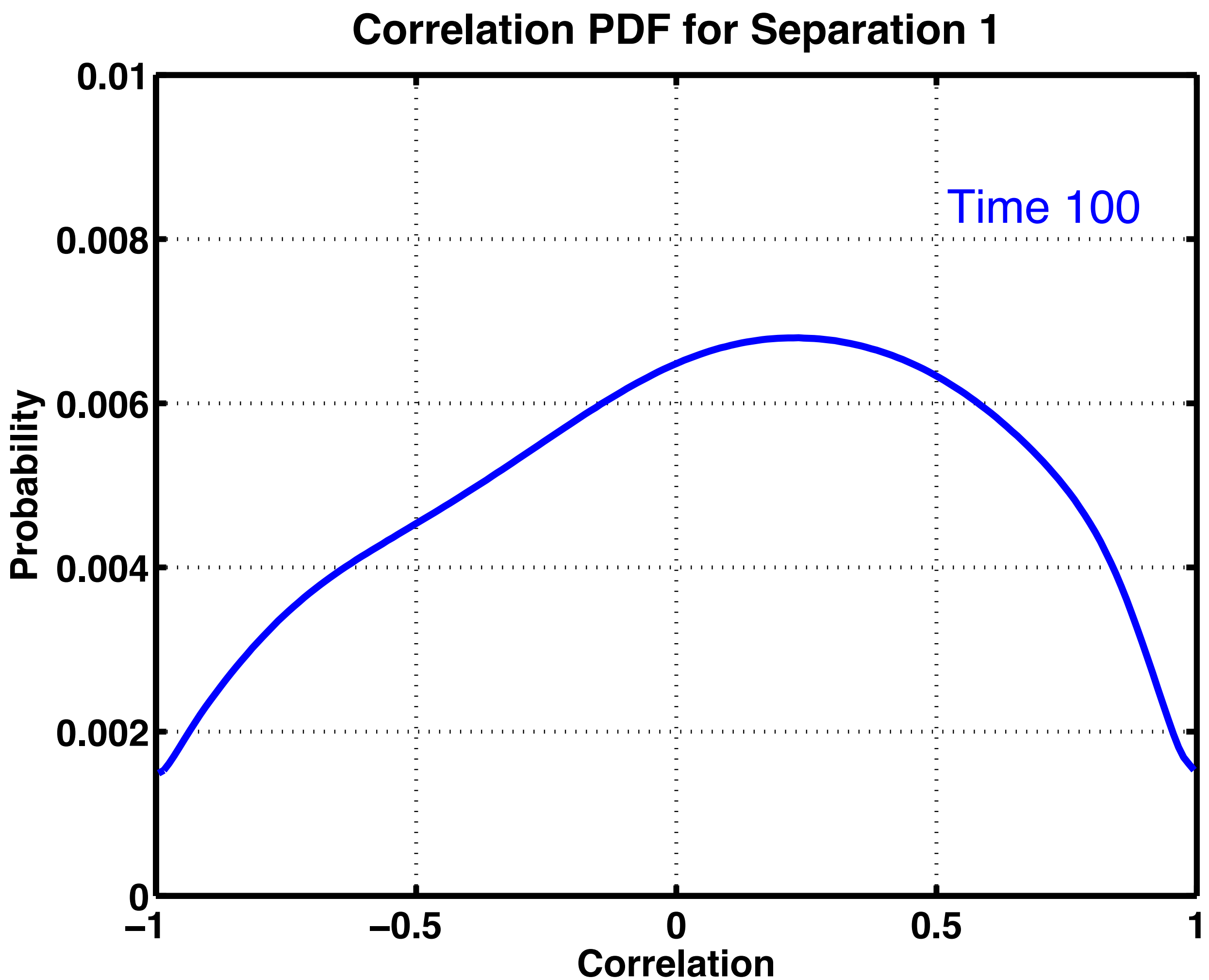

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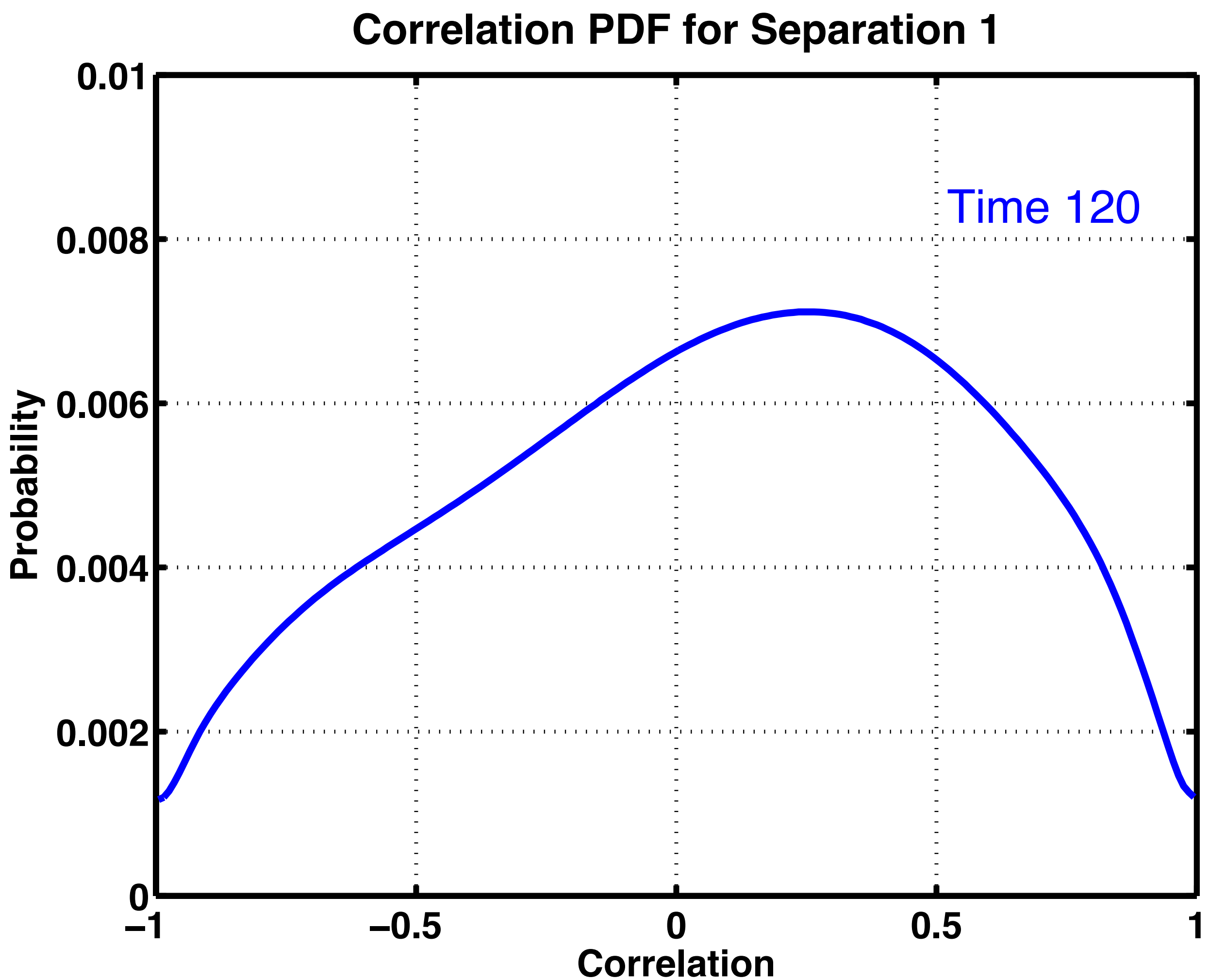

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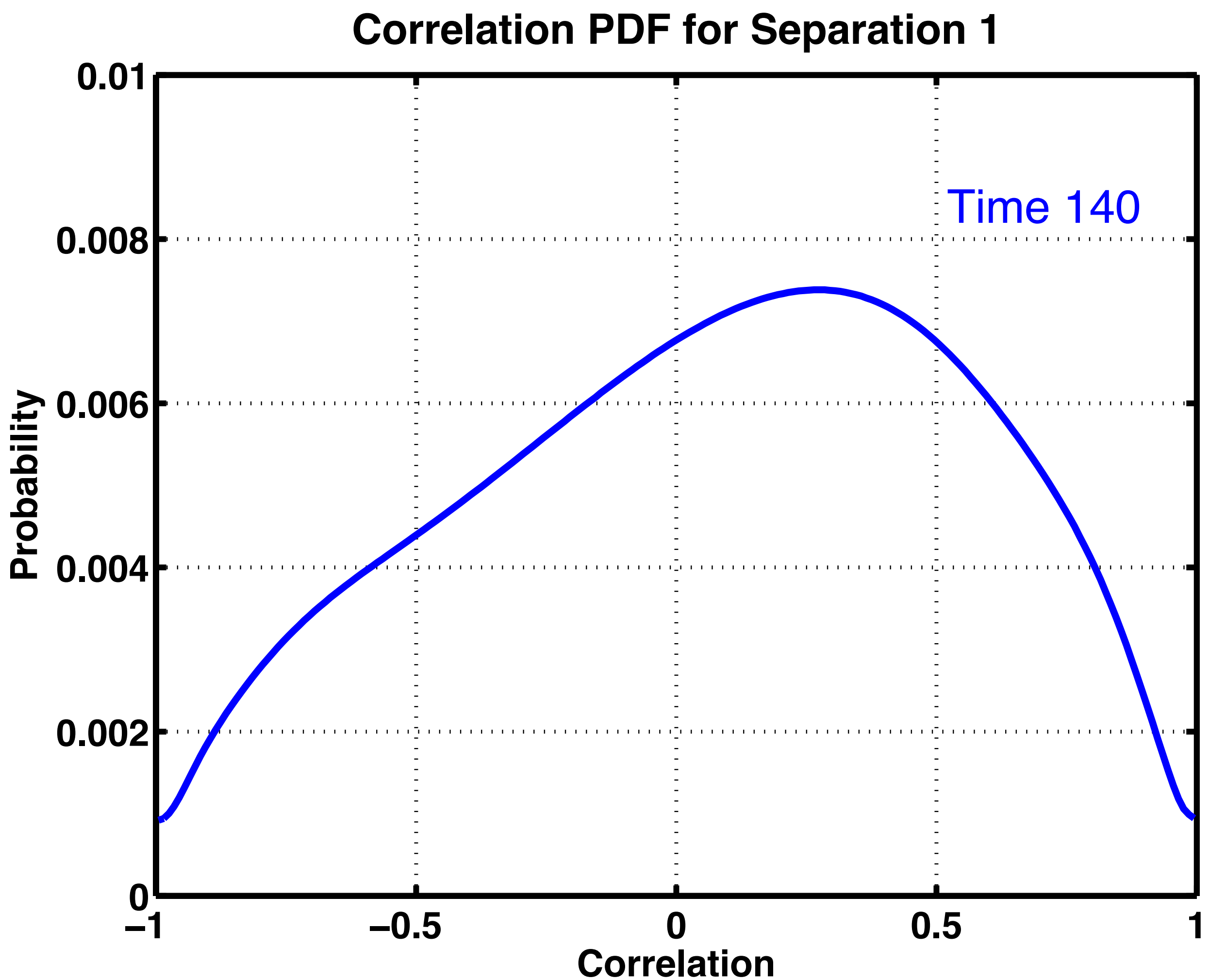

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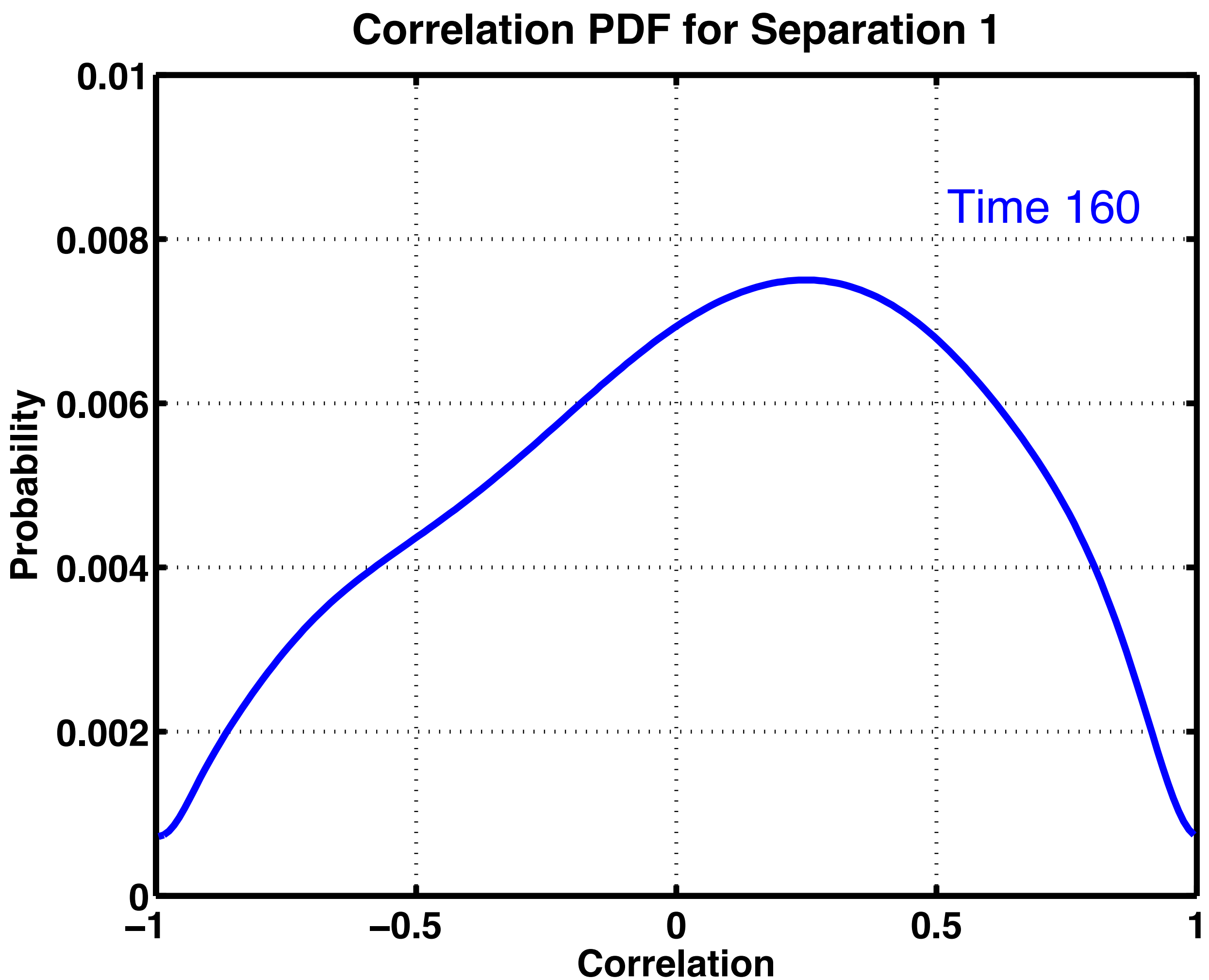

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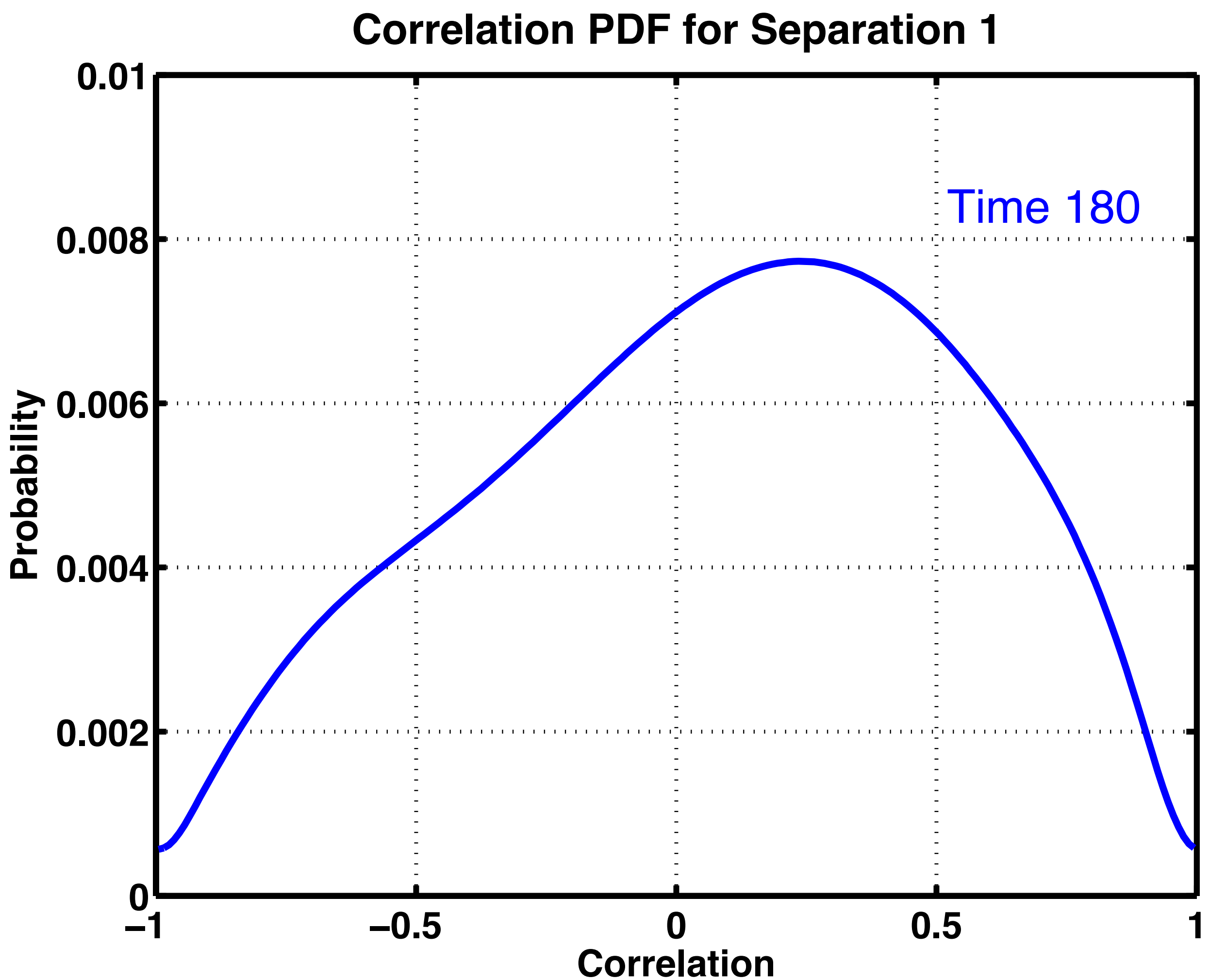

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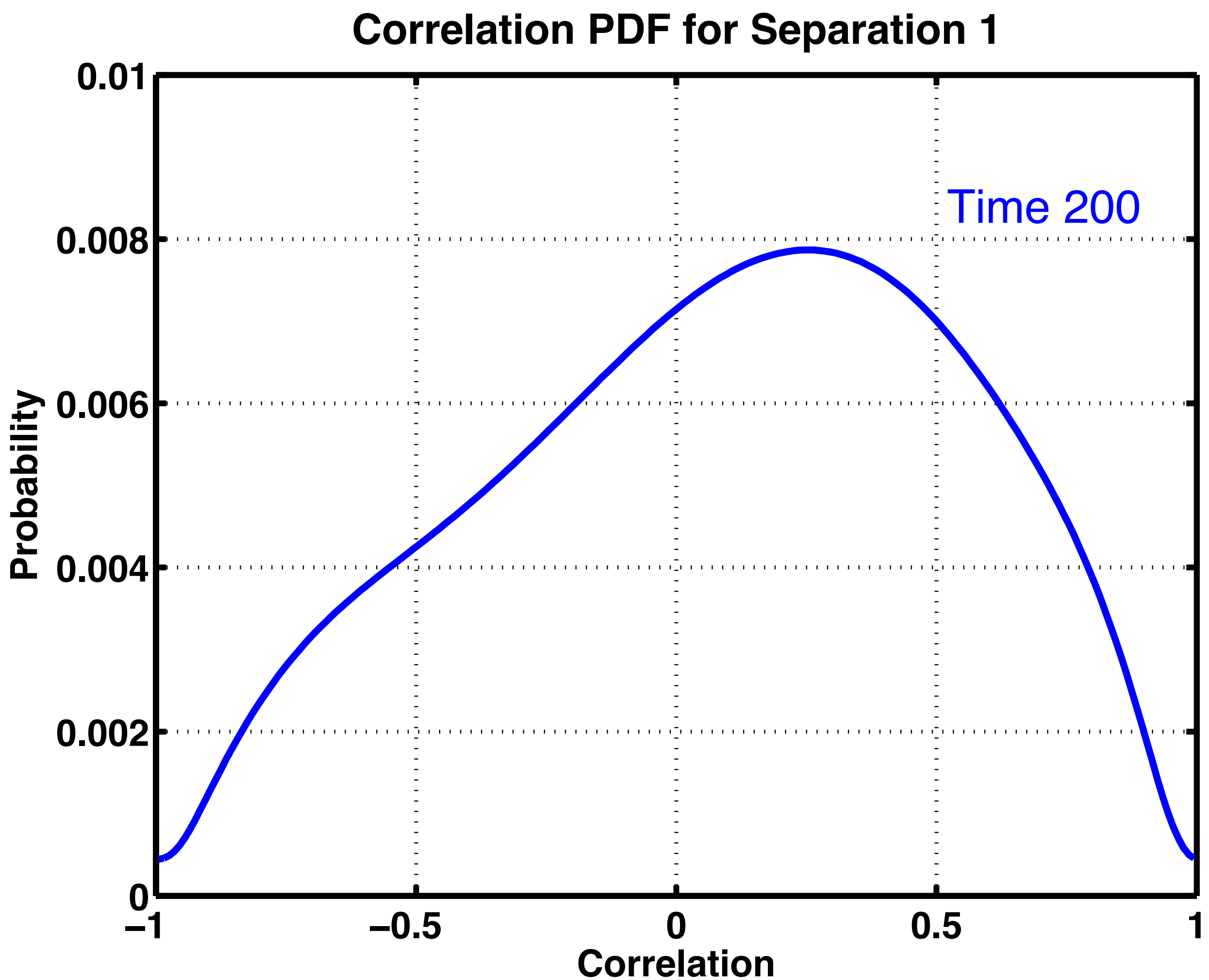
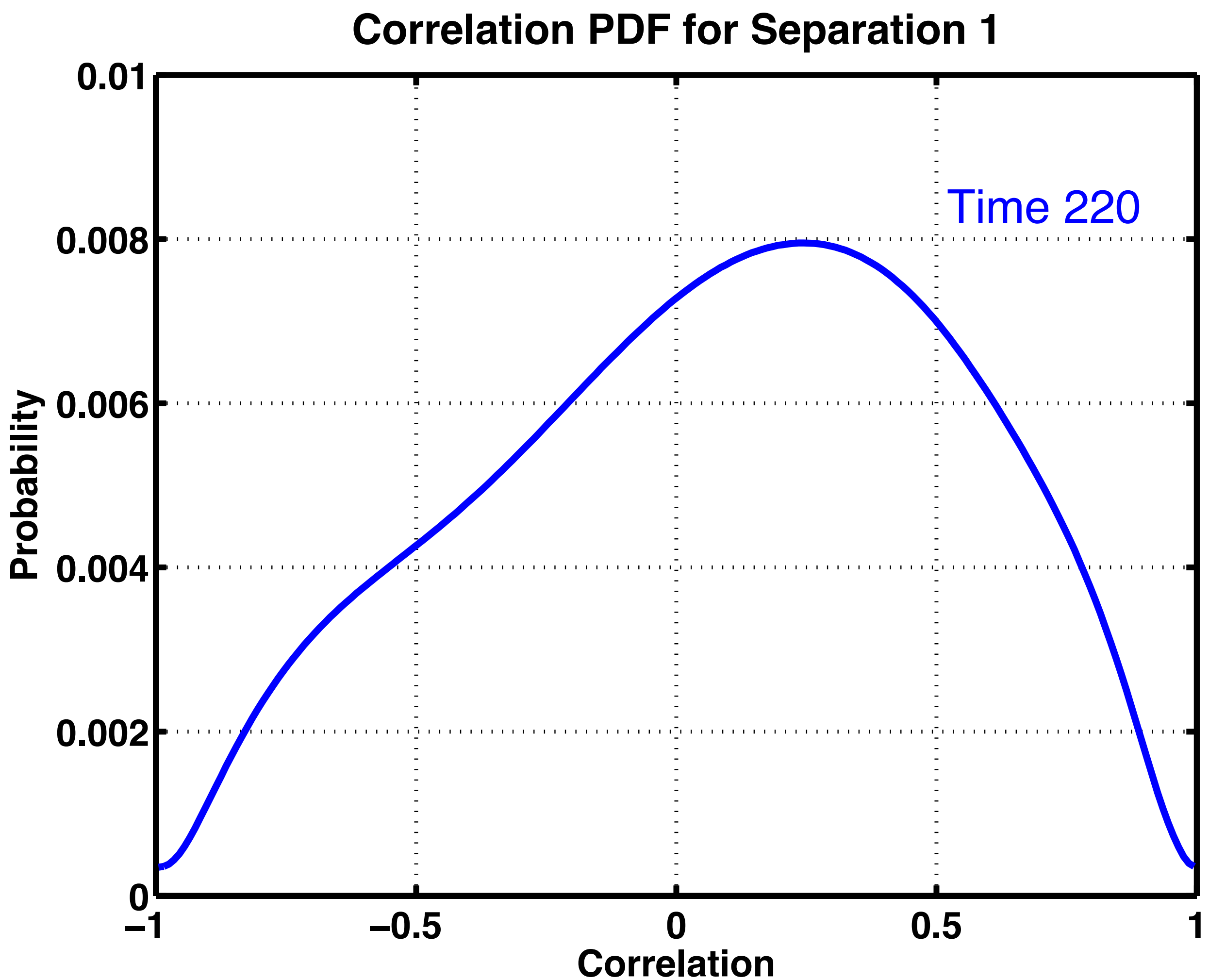
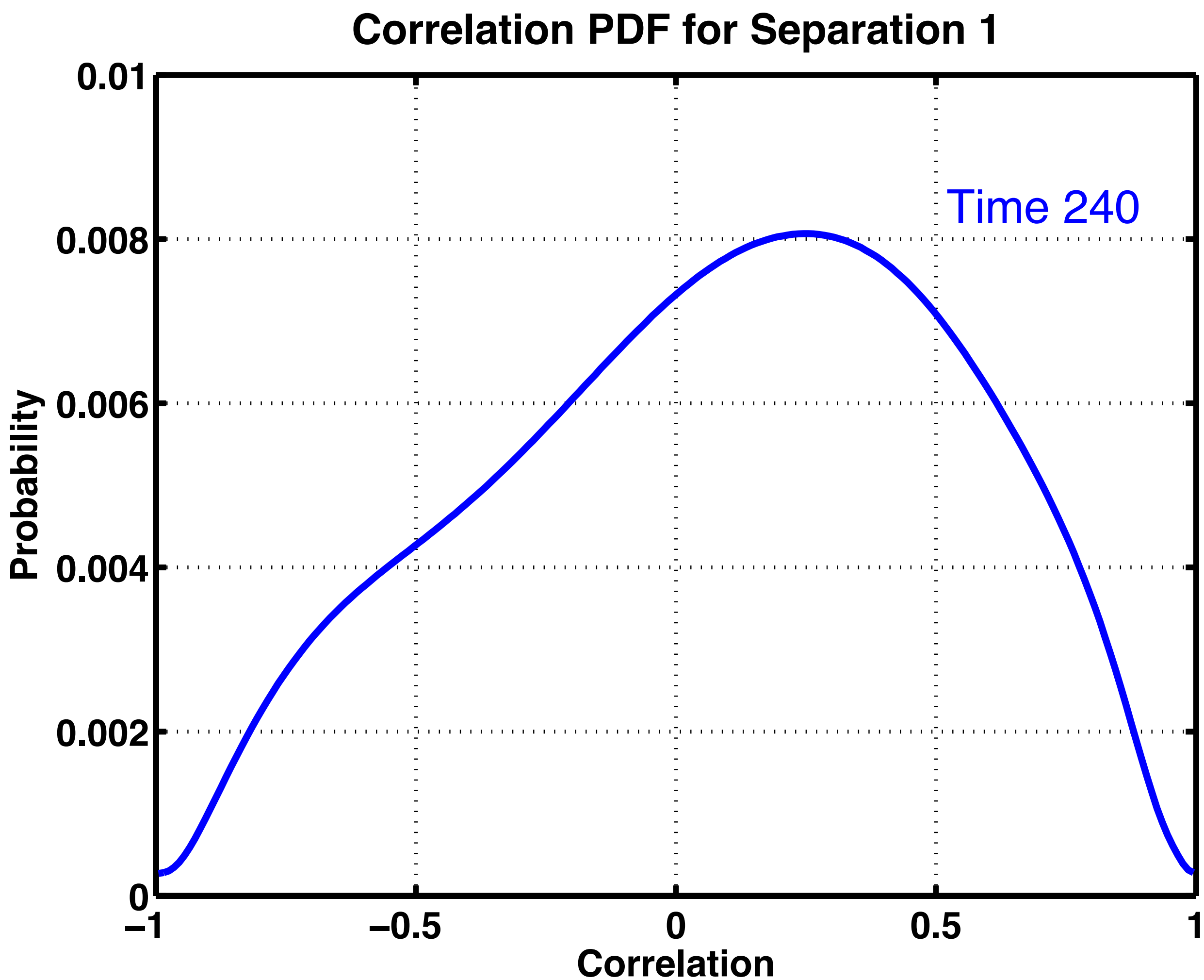




Correlation PDF for Separation 1
Time 220
Probability
Correlation
0.01
0.008
0.006
0.004
0.002
0
–1
–0.5
0
0.5
1

Evolution of Correlation Distribution



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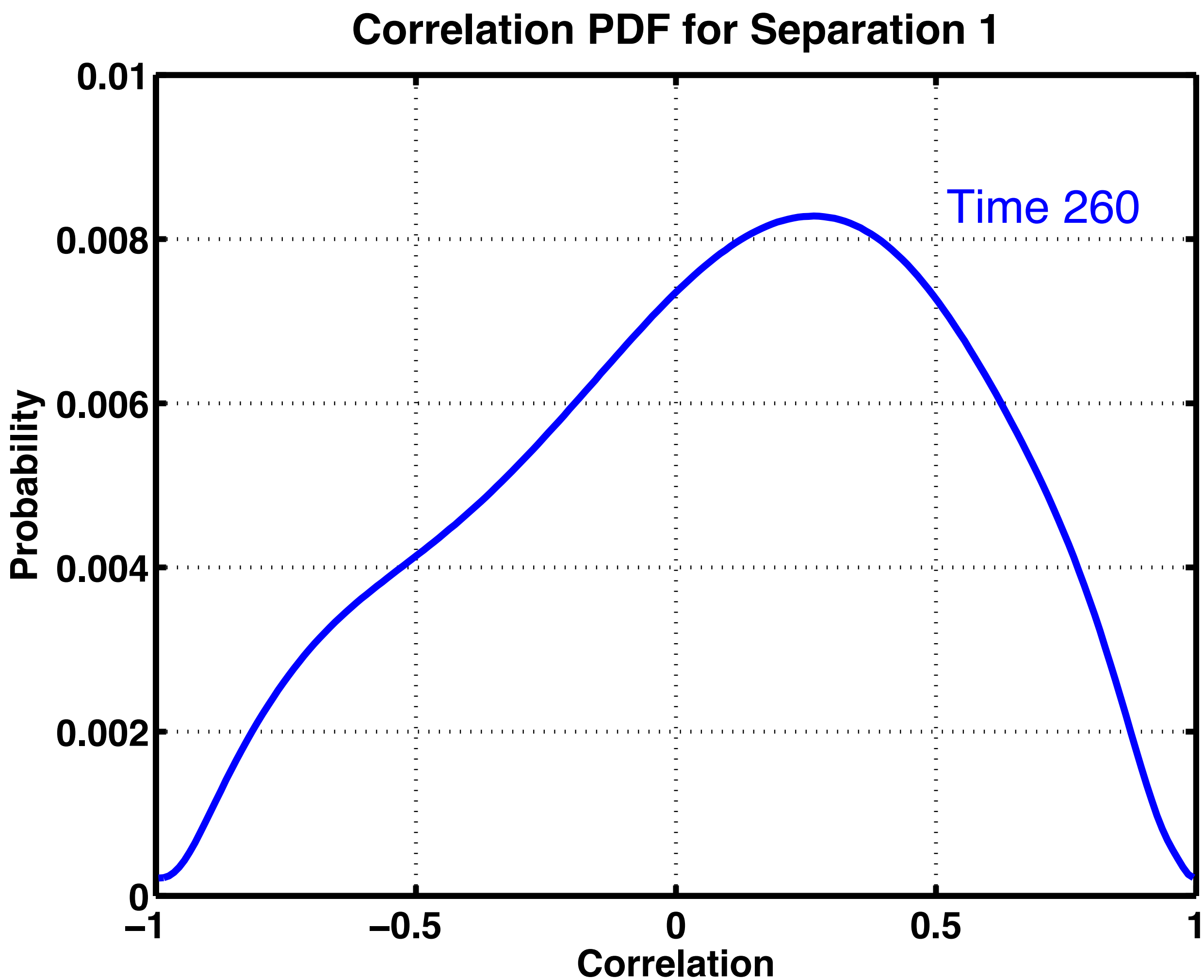

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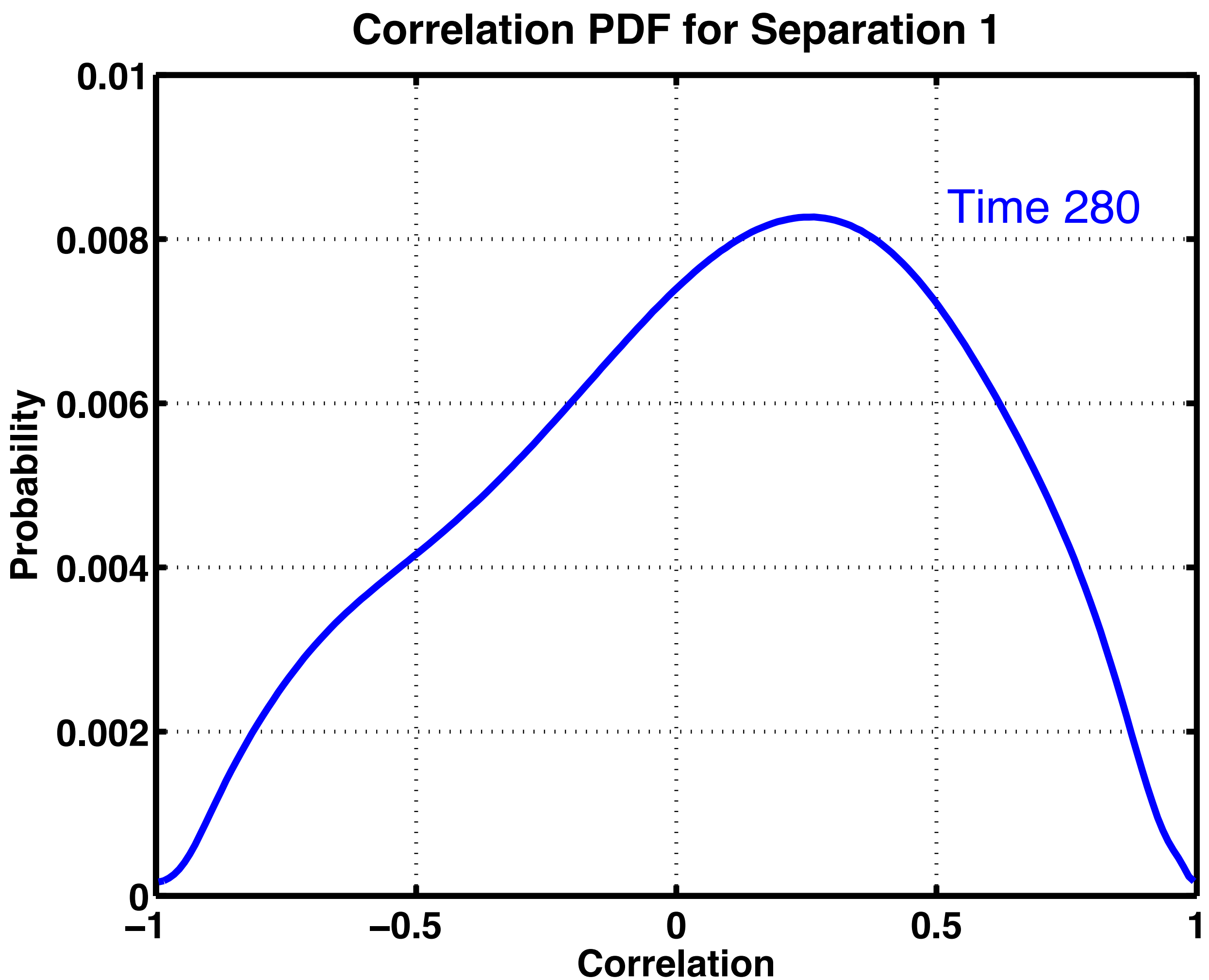

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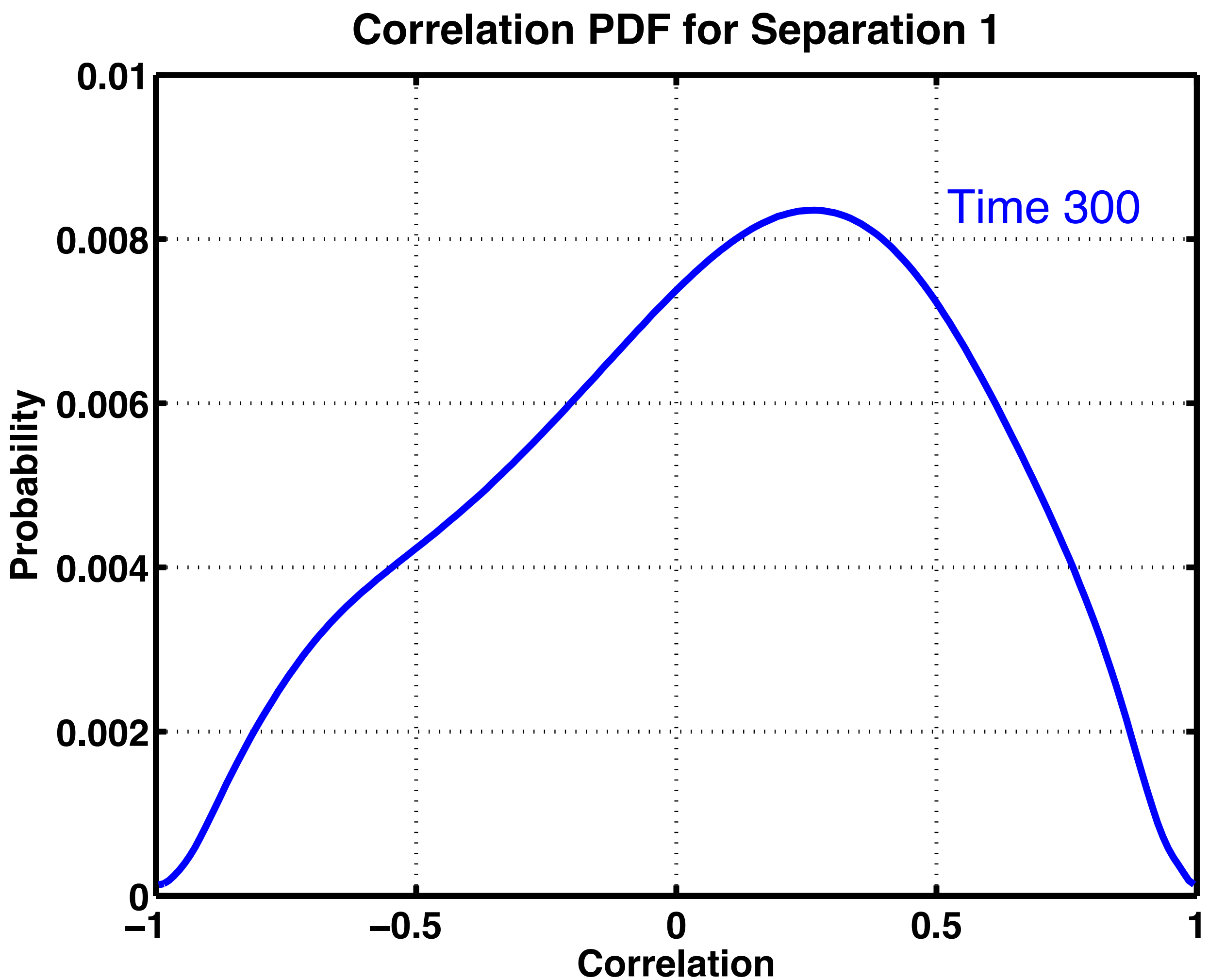

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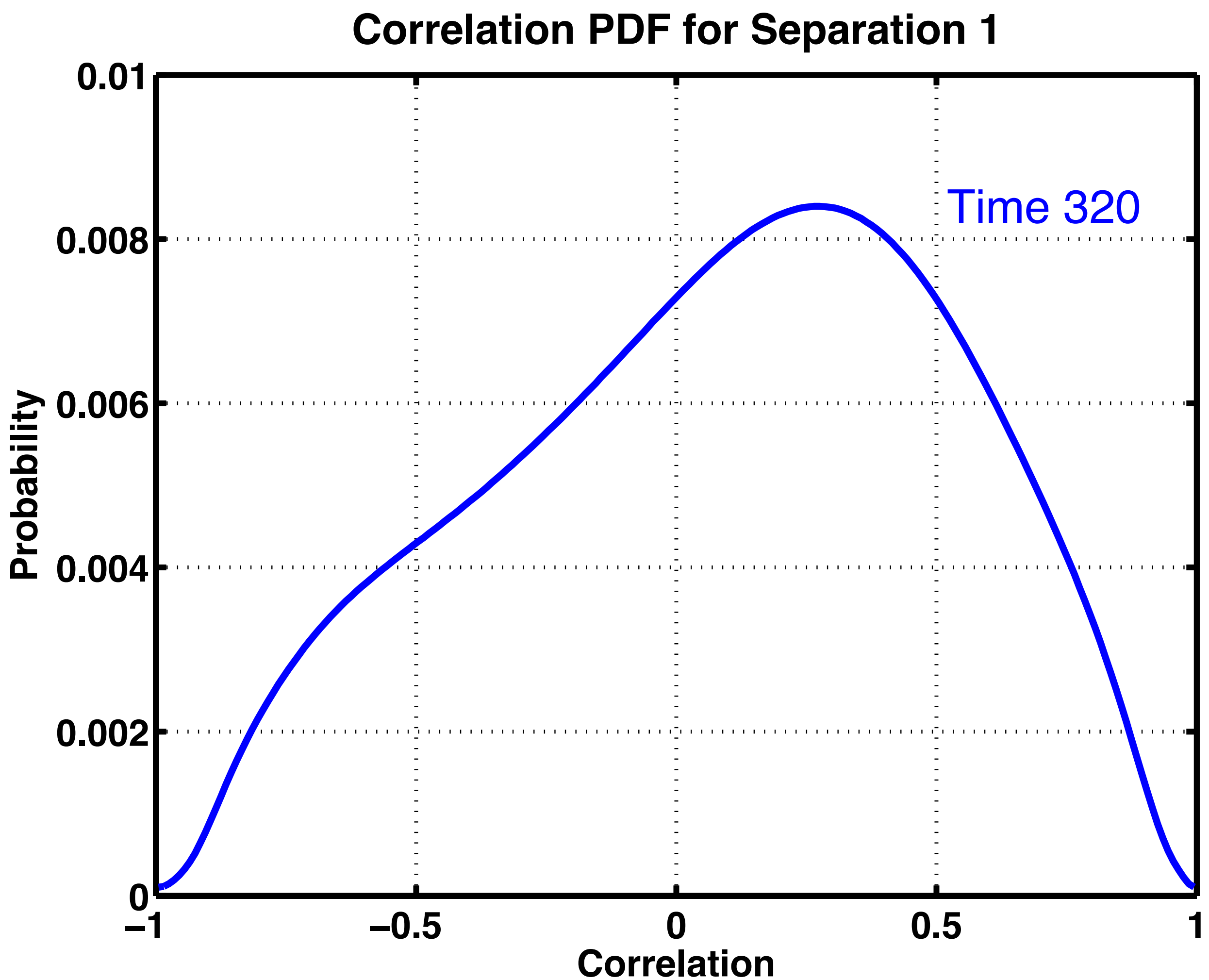

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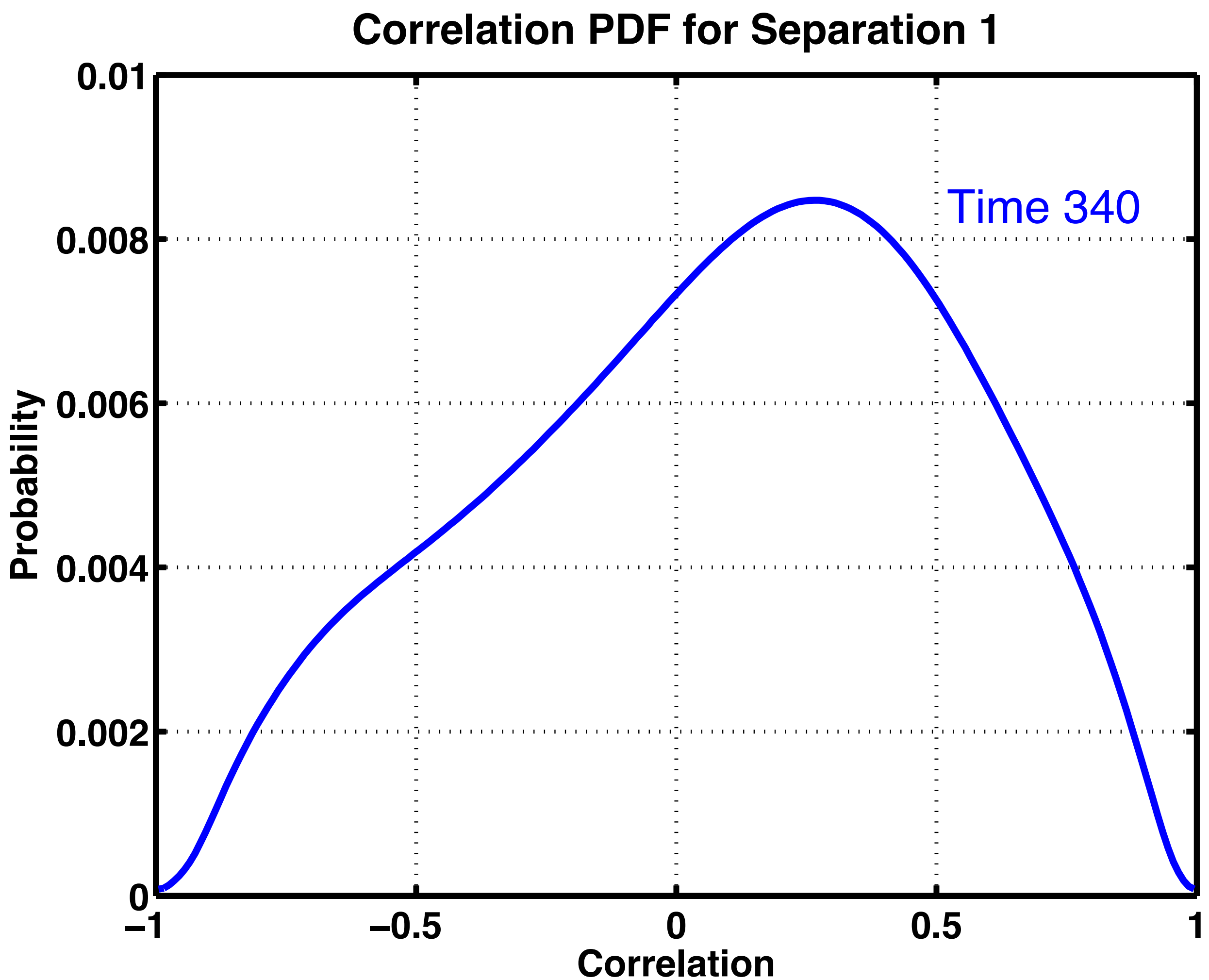

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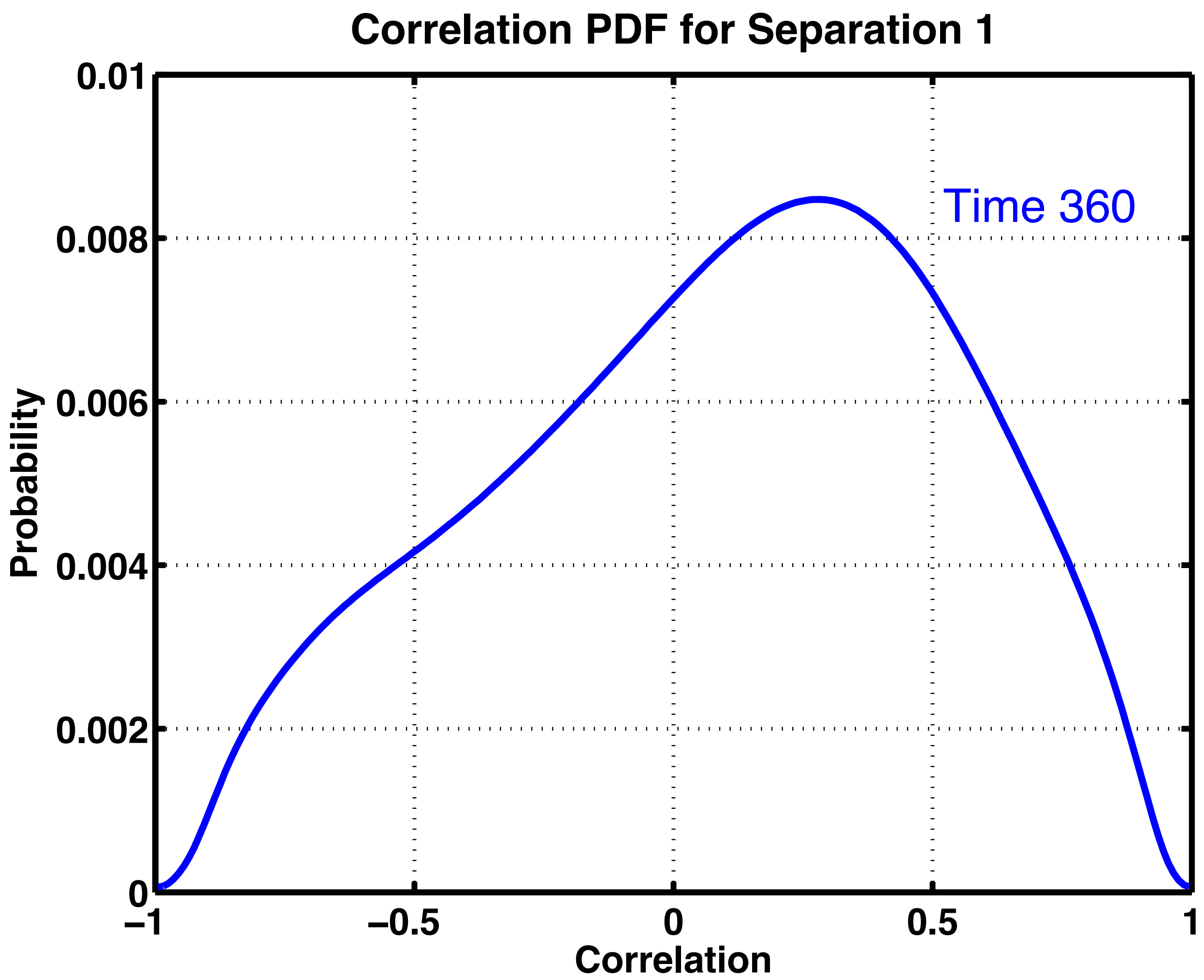

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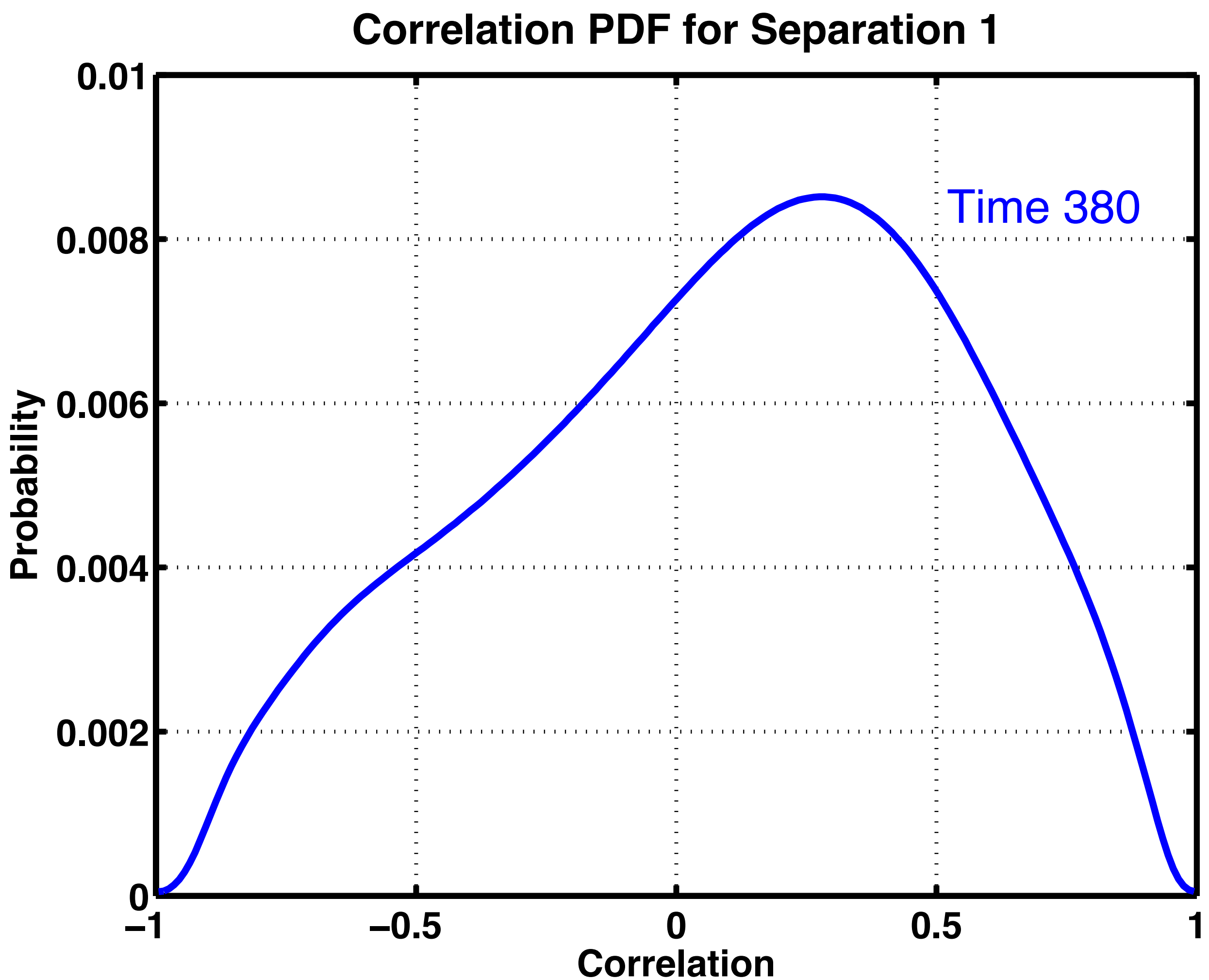

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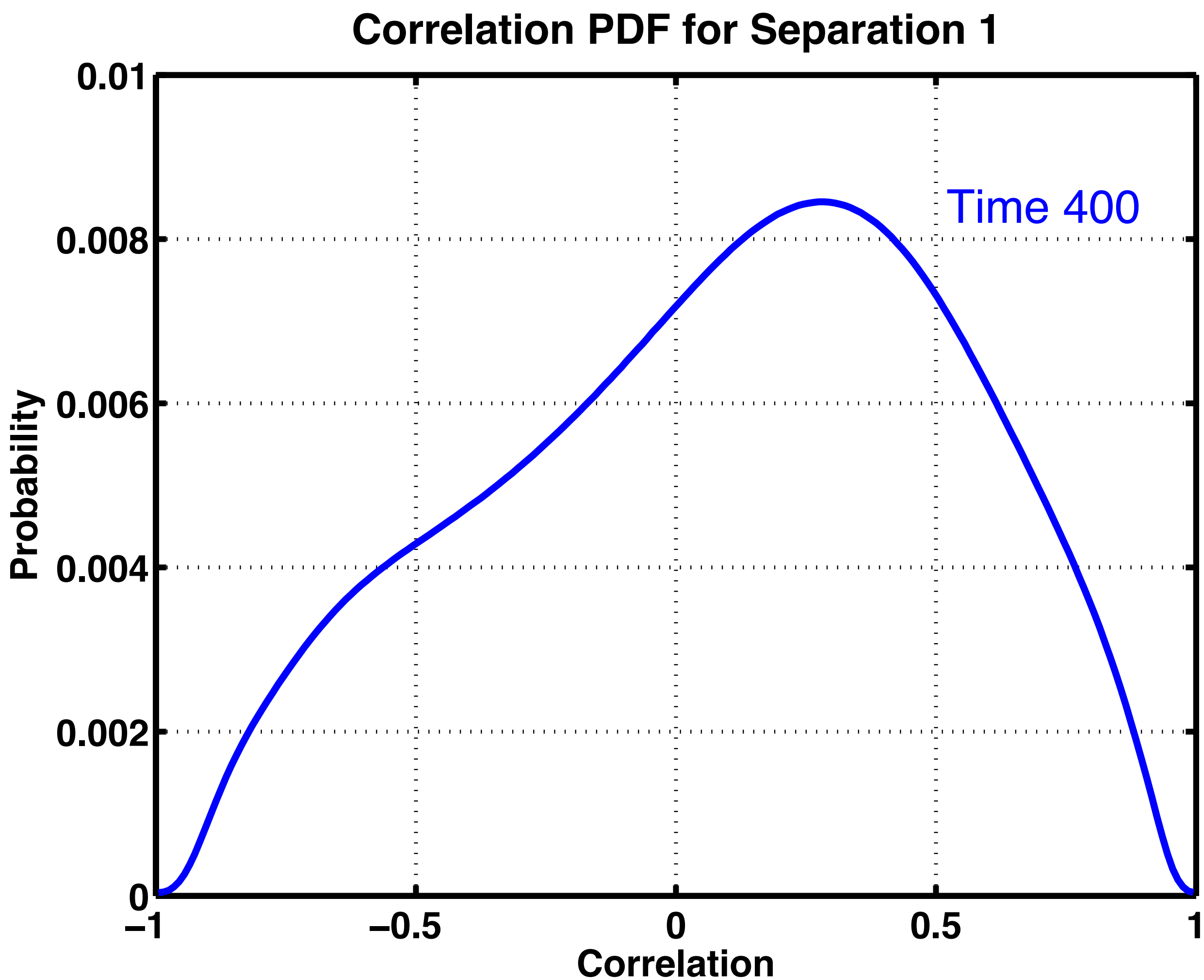

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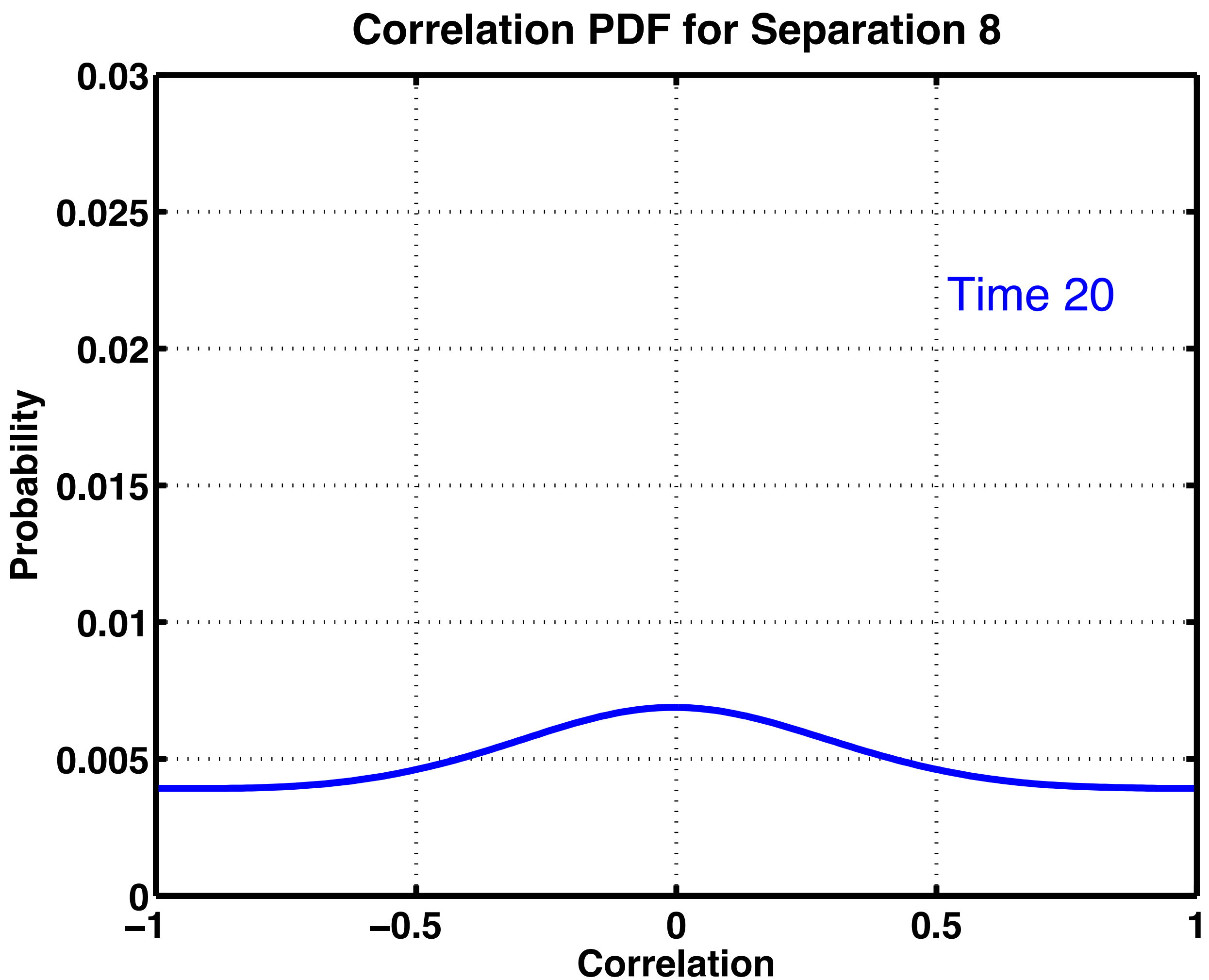

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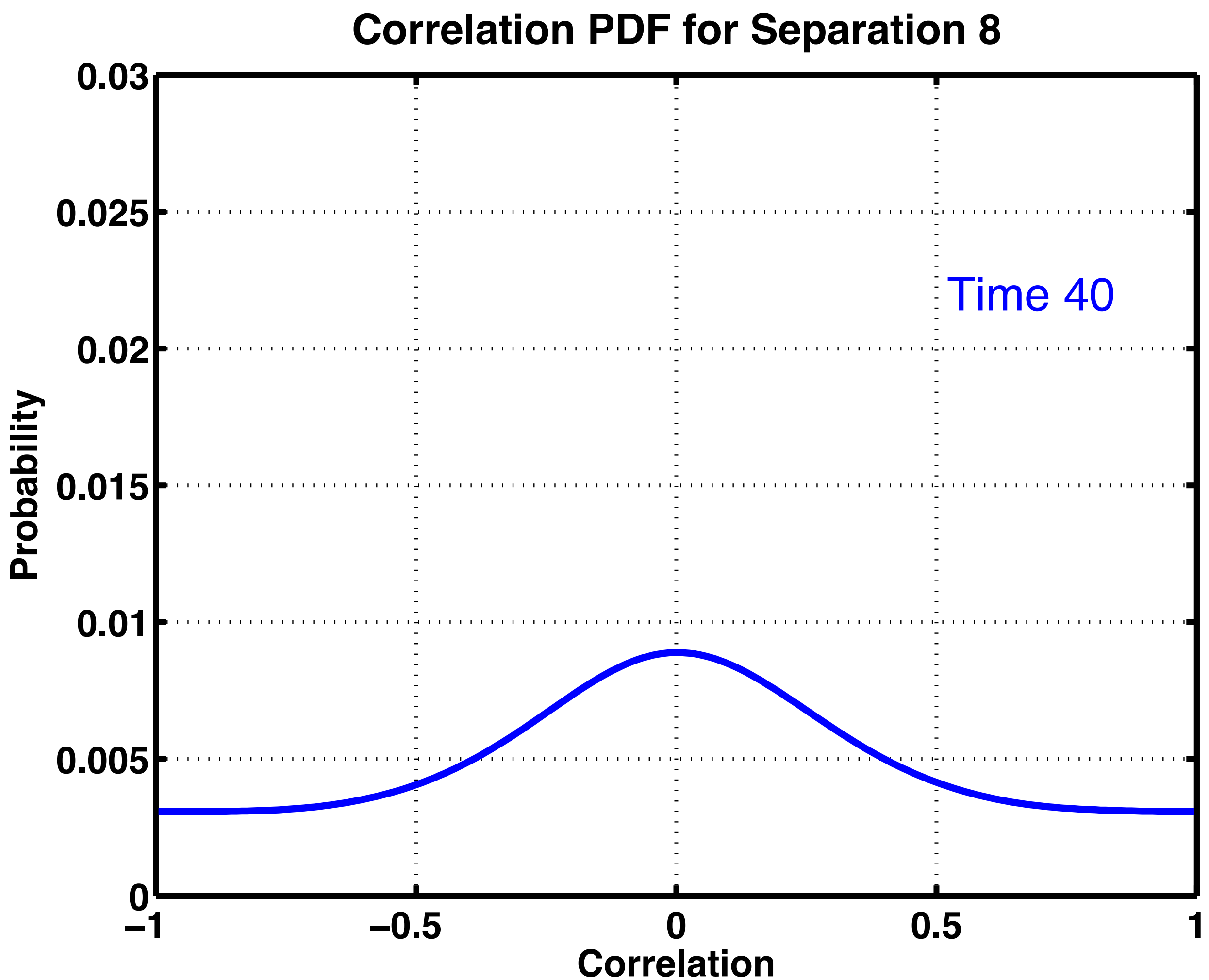

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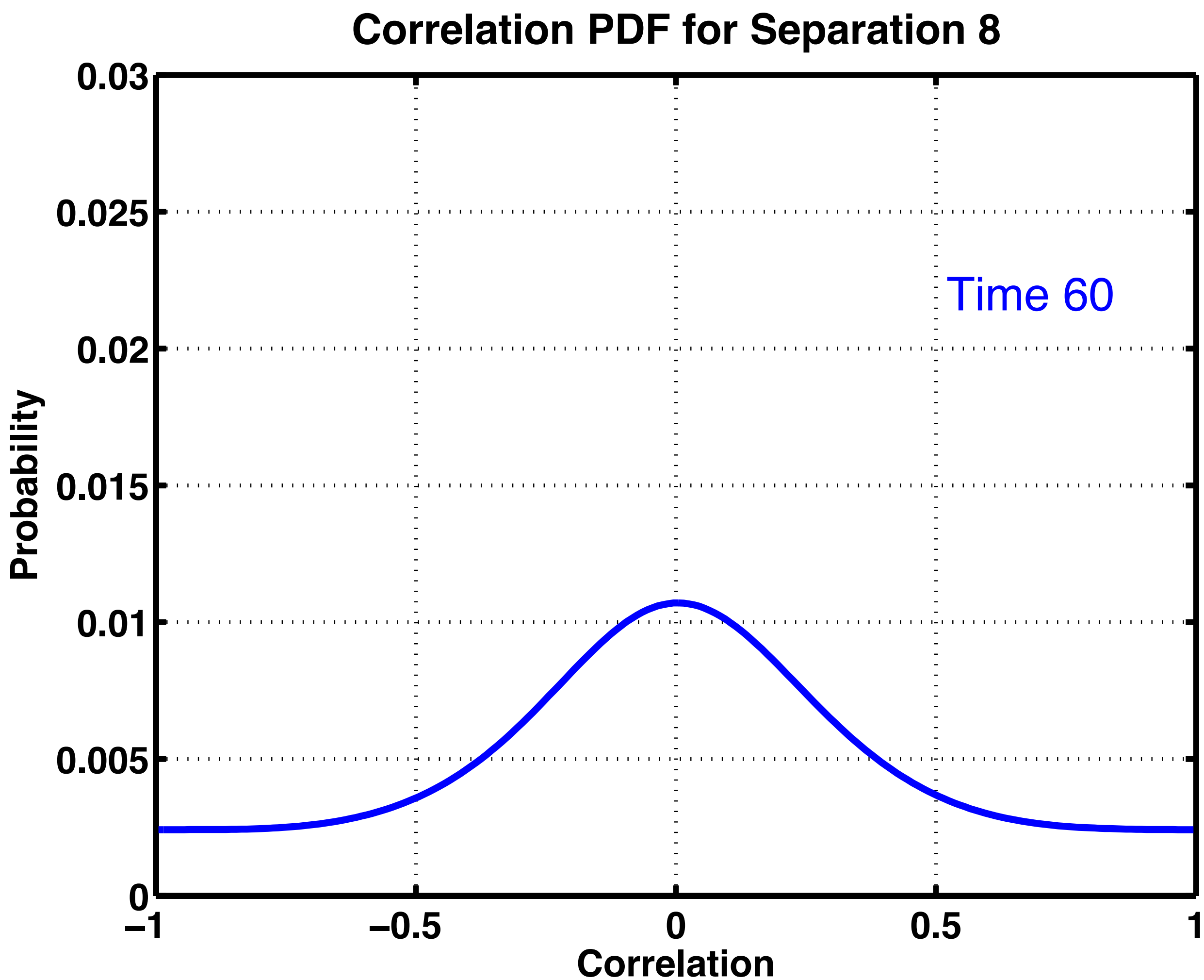

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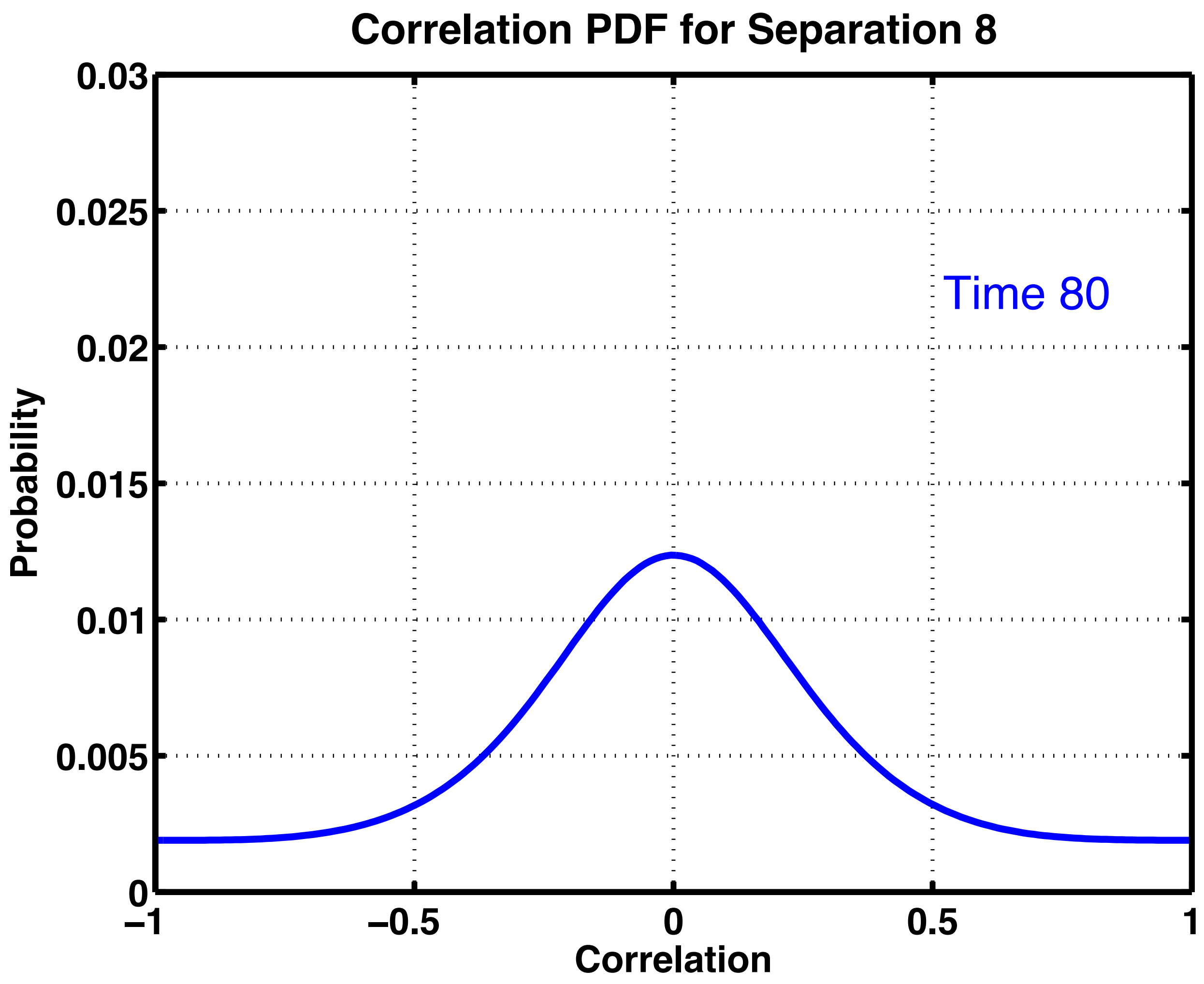

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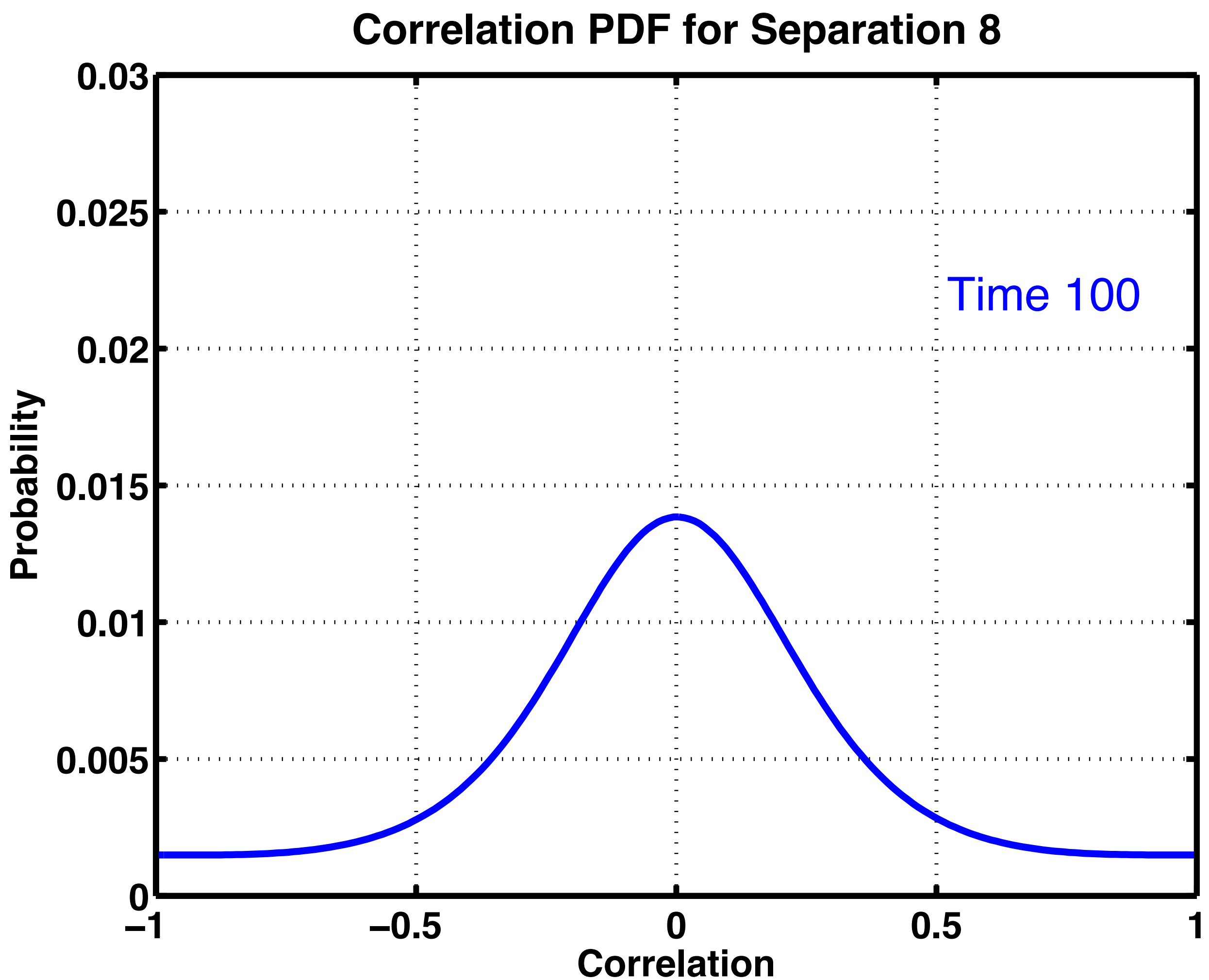

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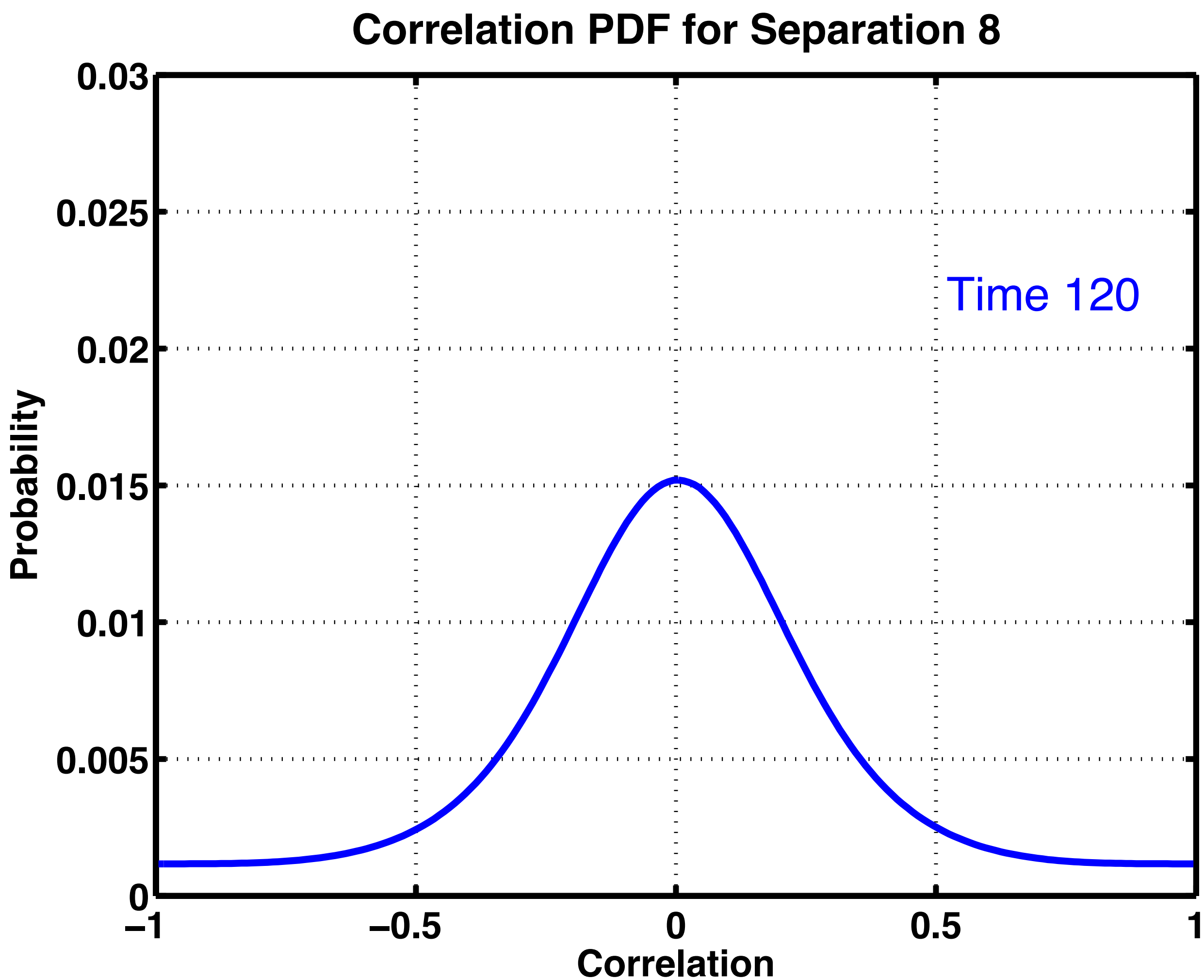

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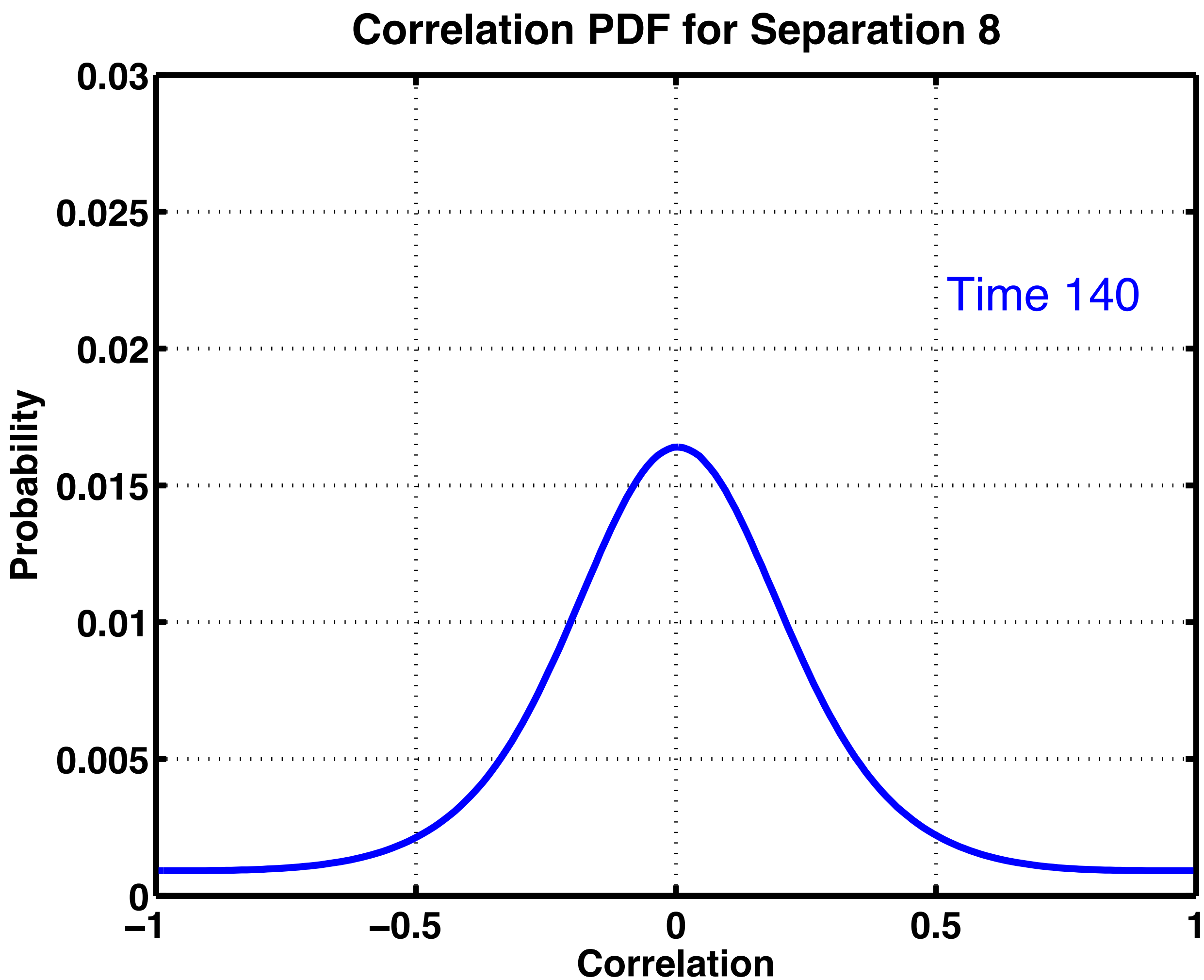

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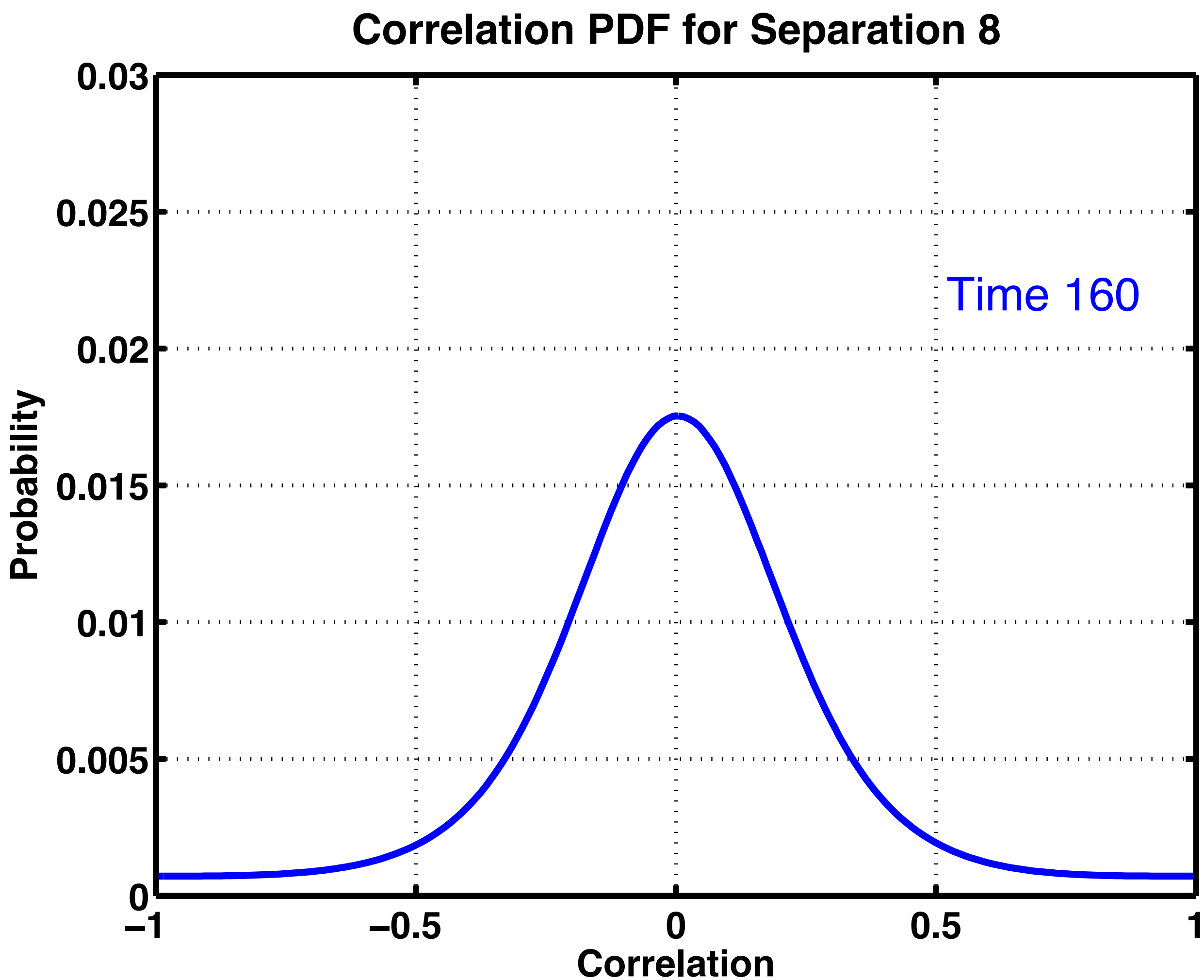

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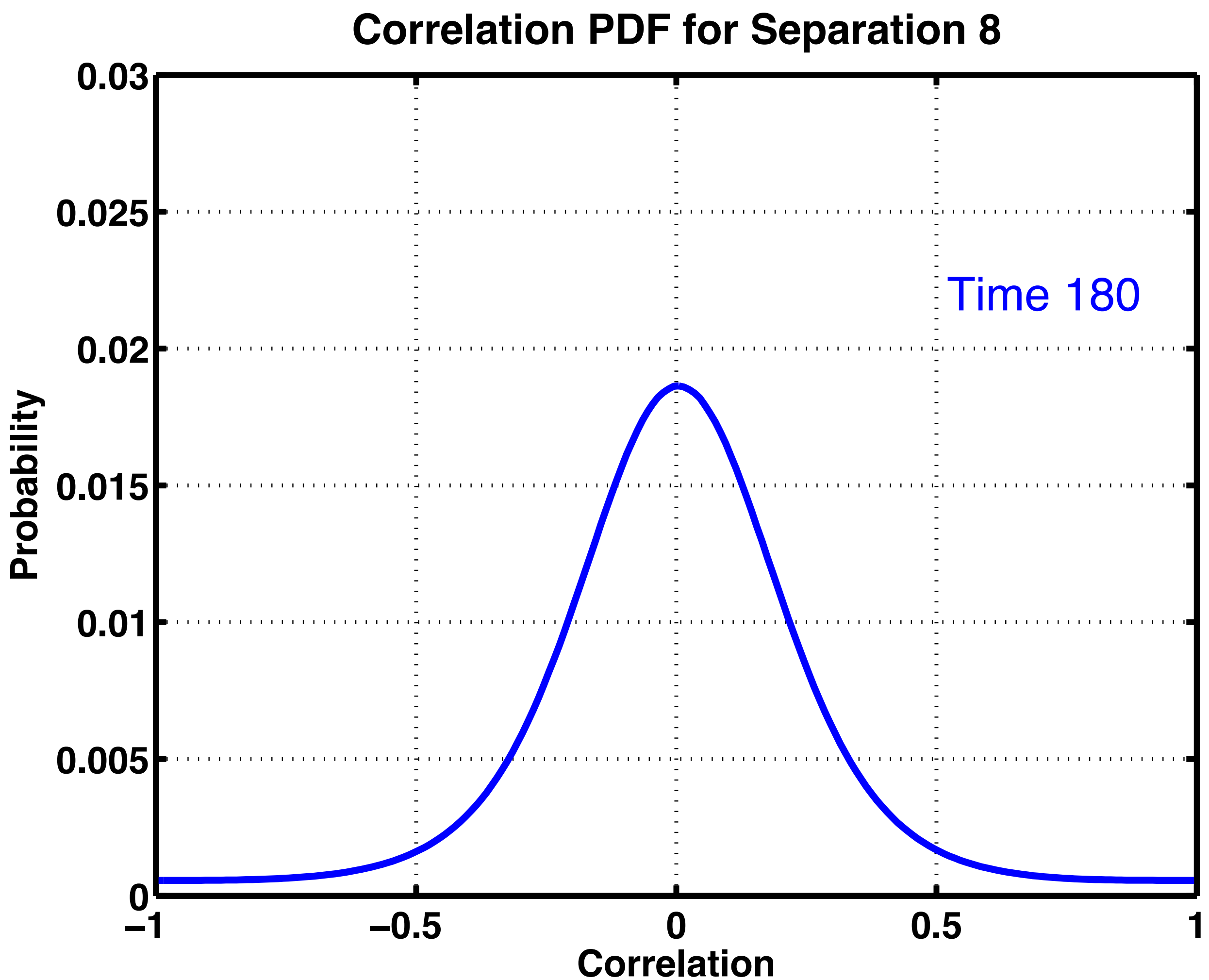

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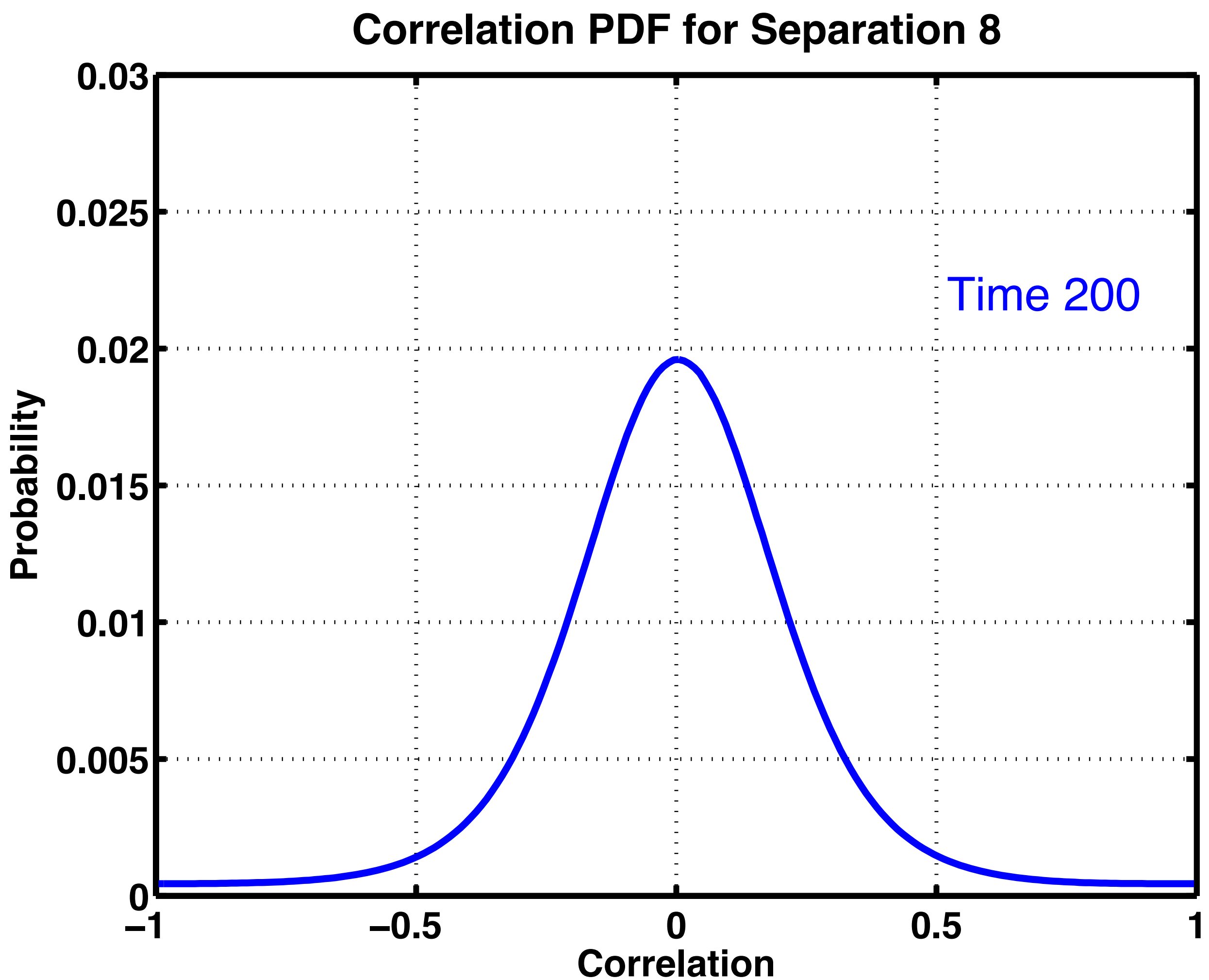

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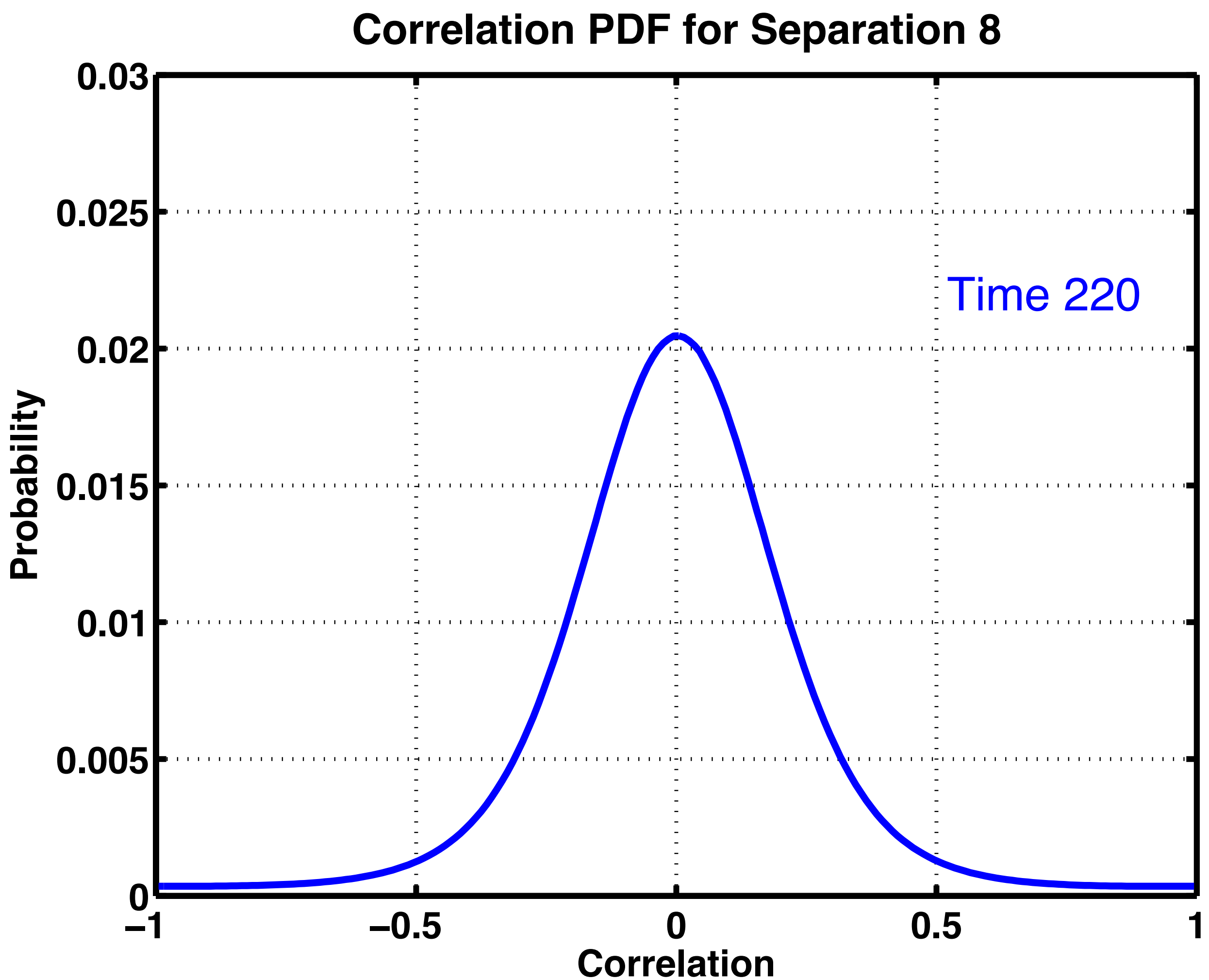

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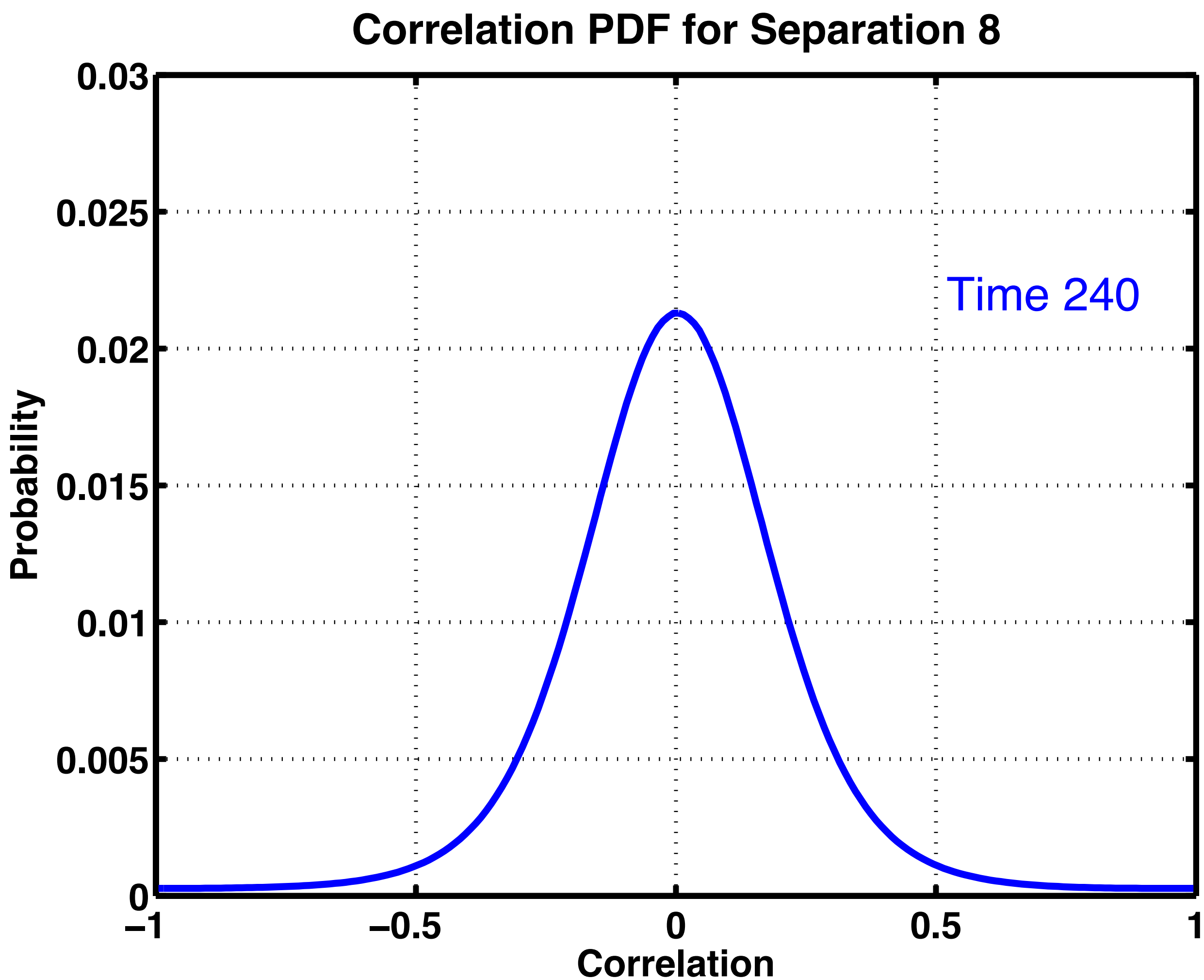

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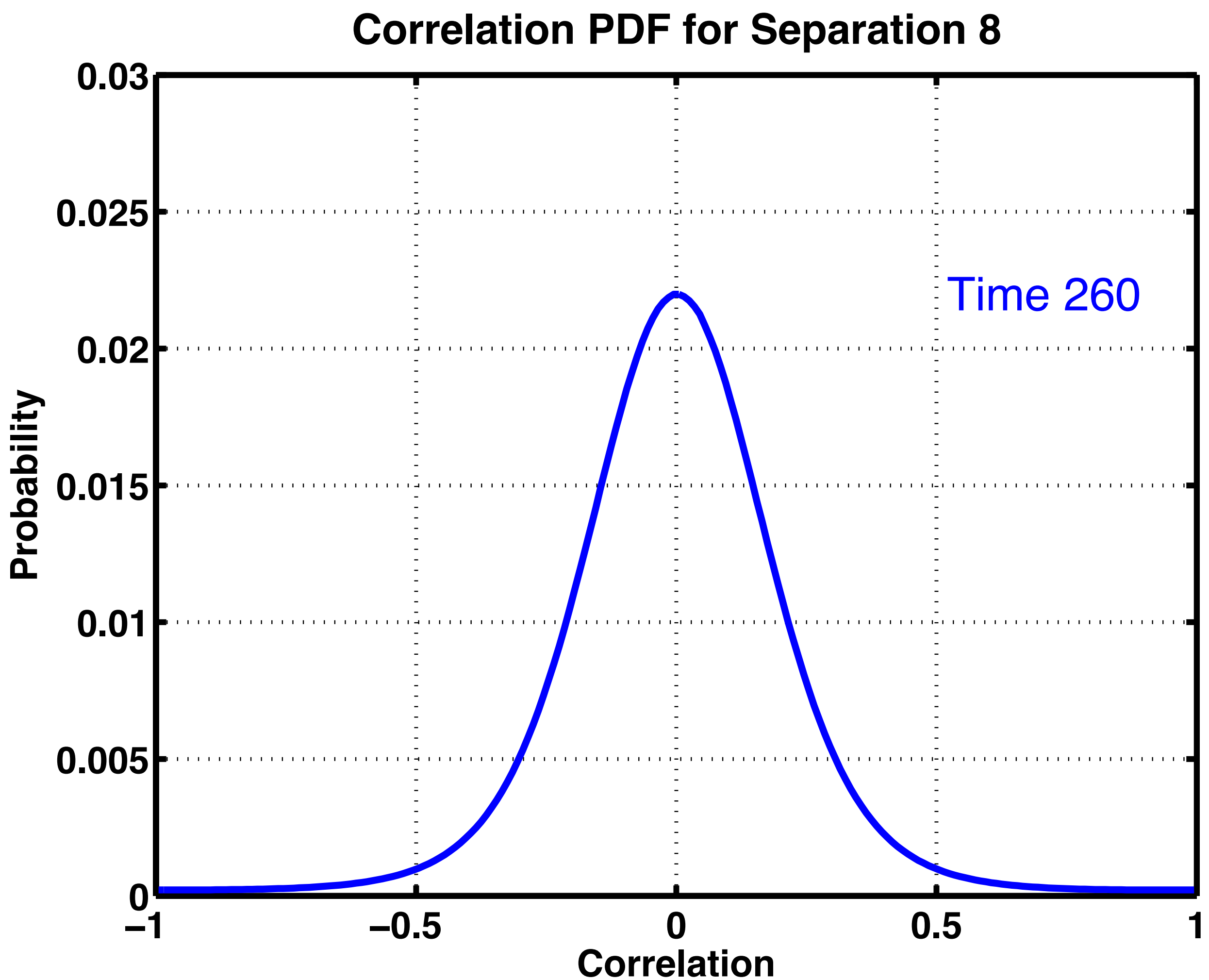

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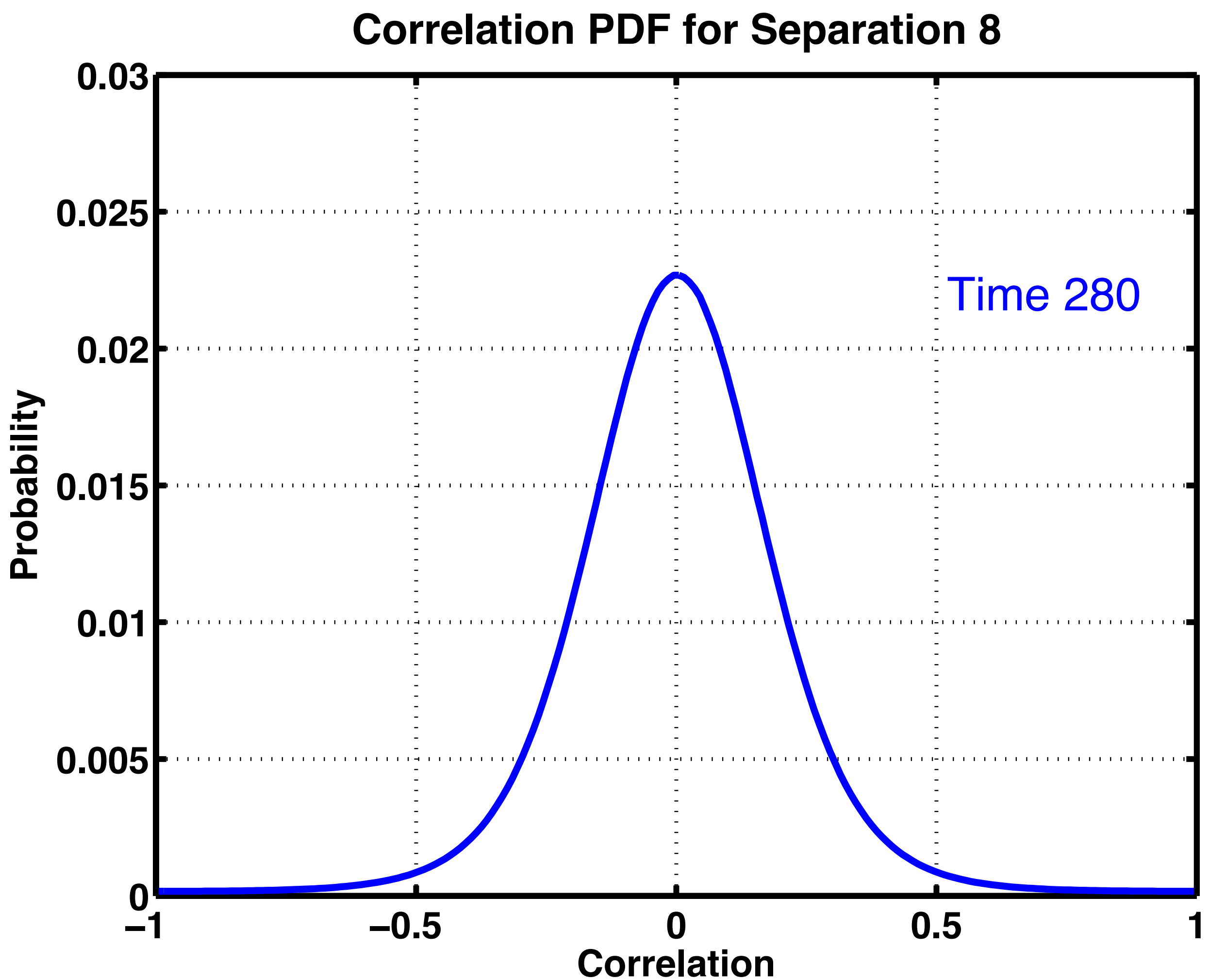

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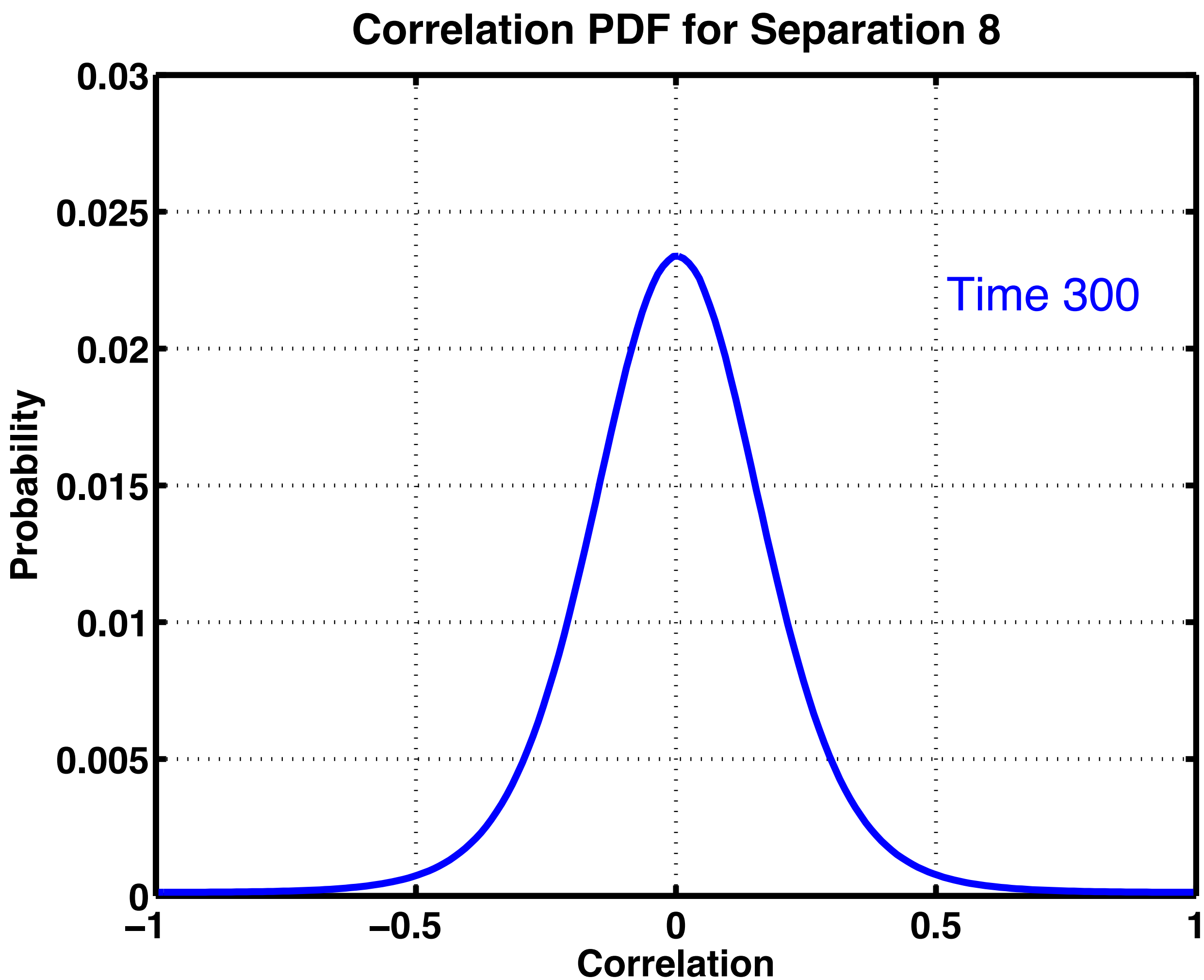

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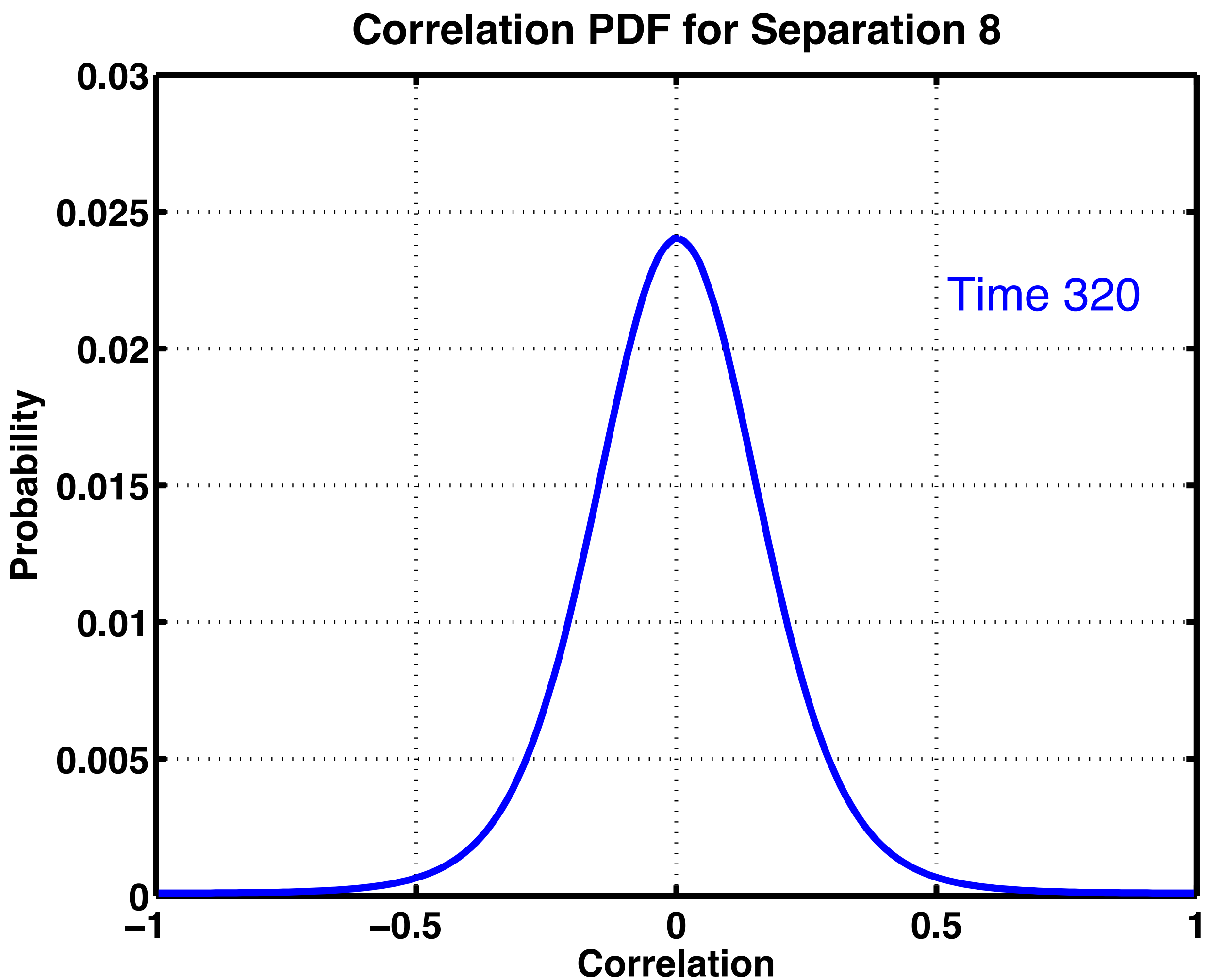

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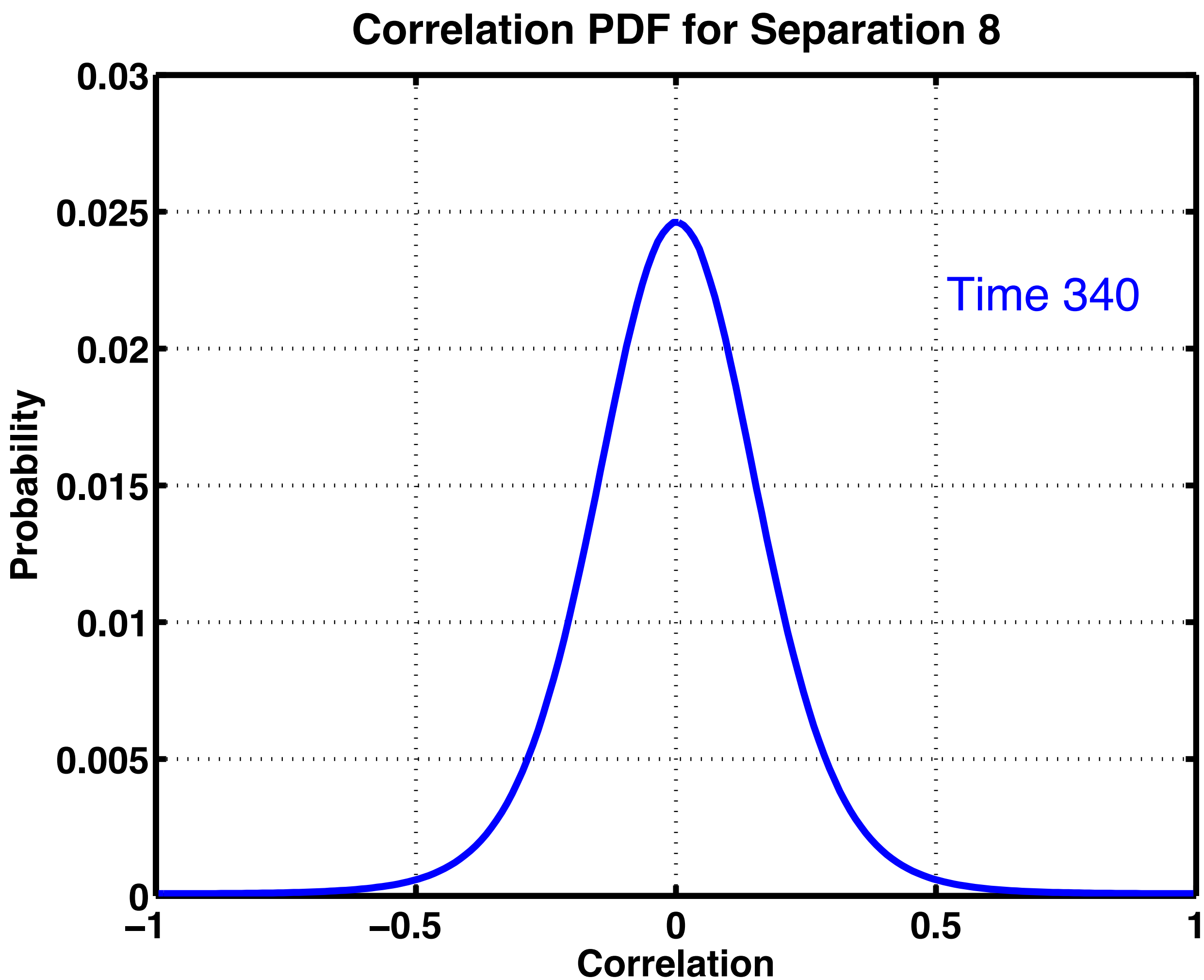

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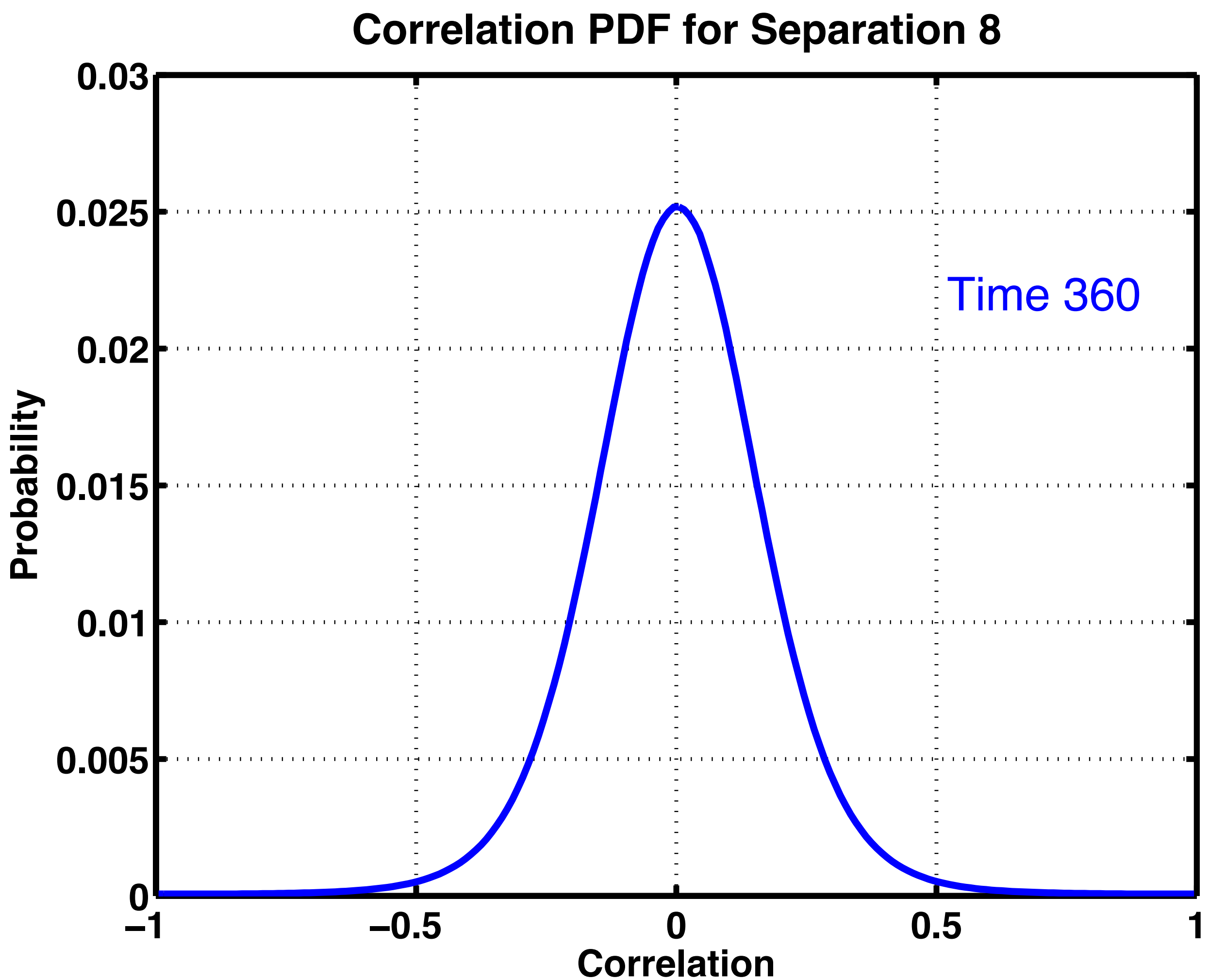

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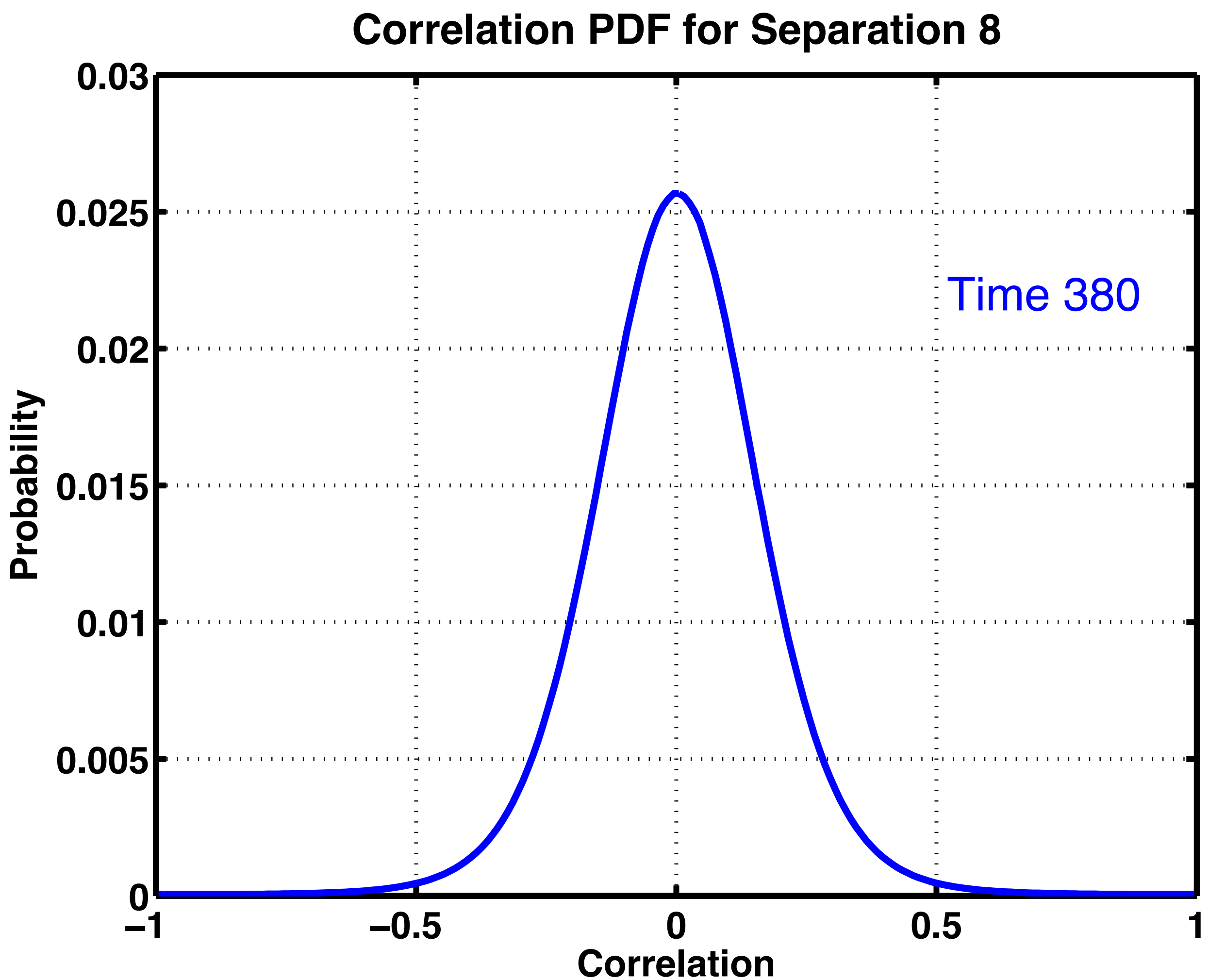

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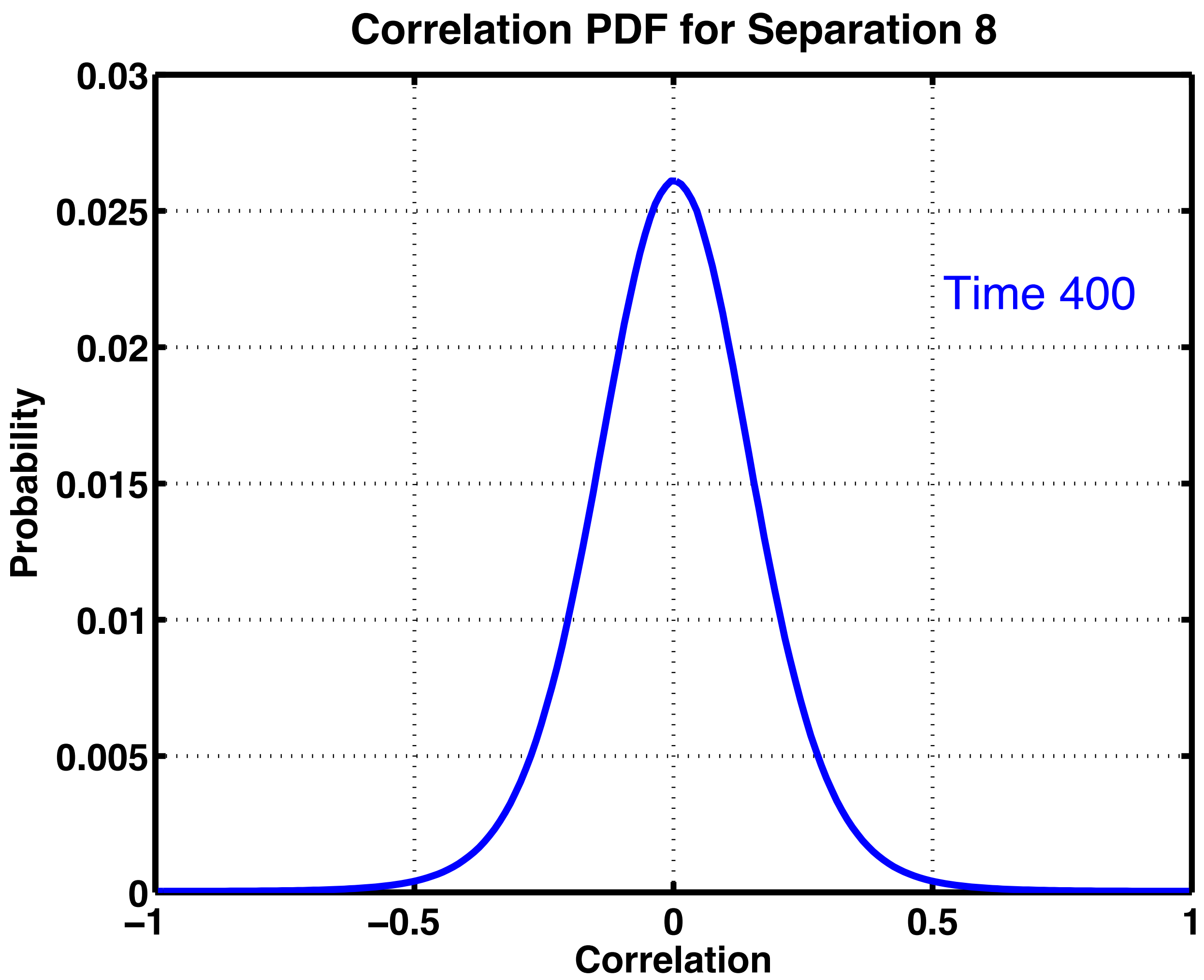
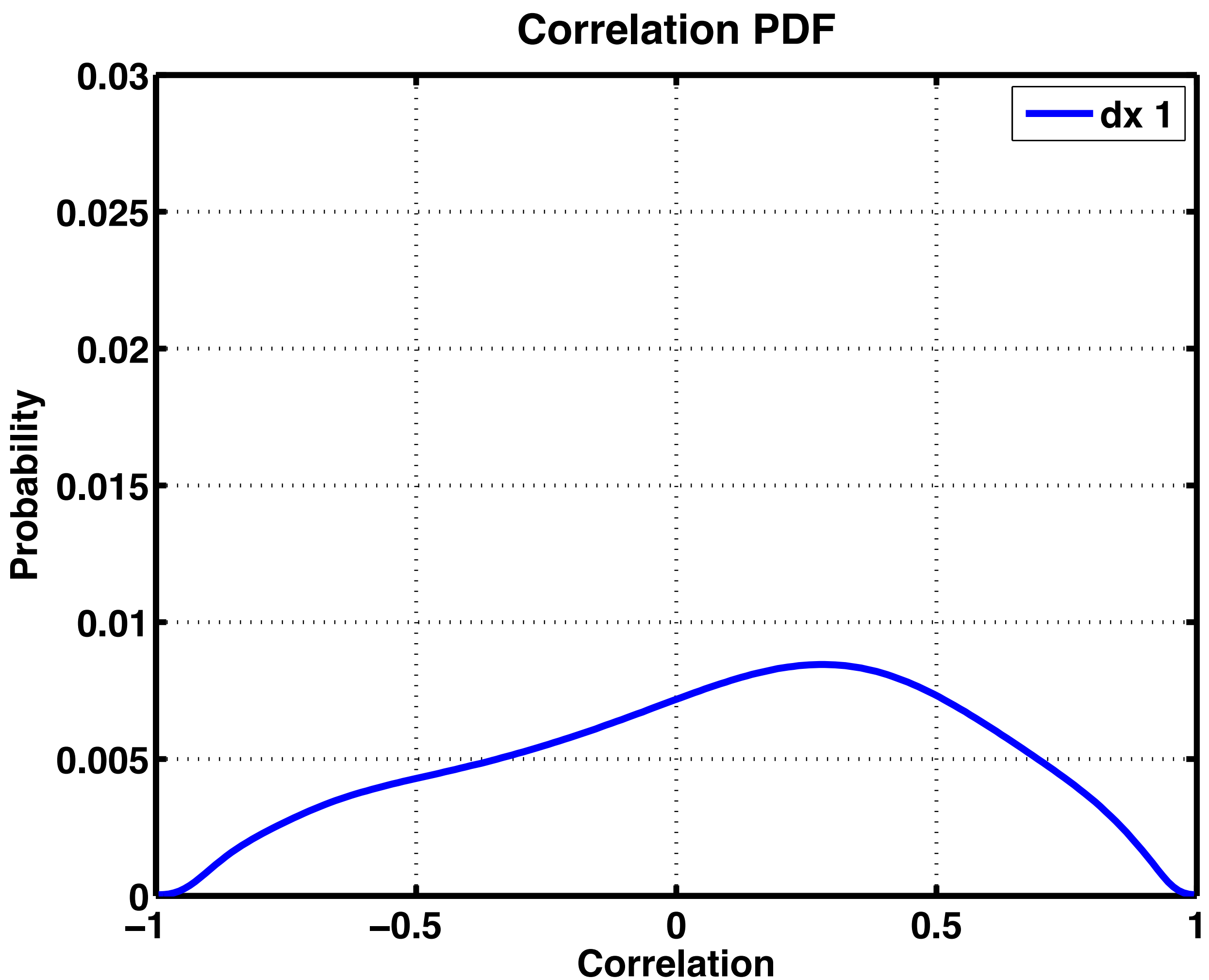
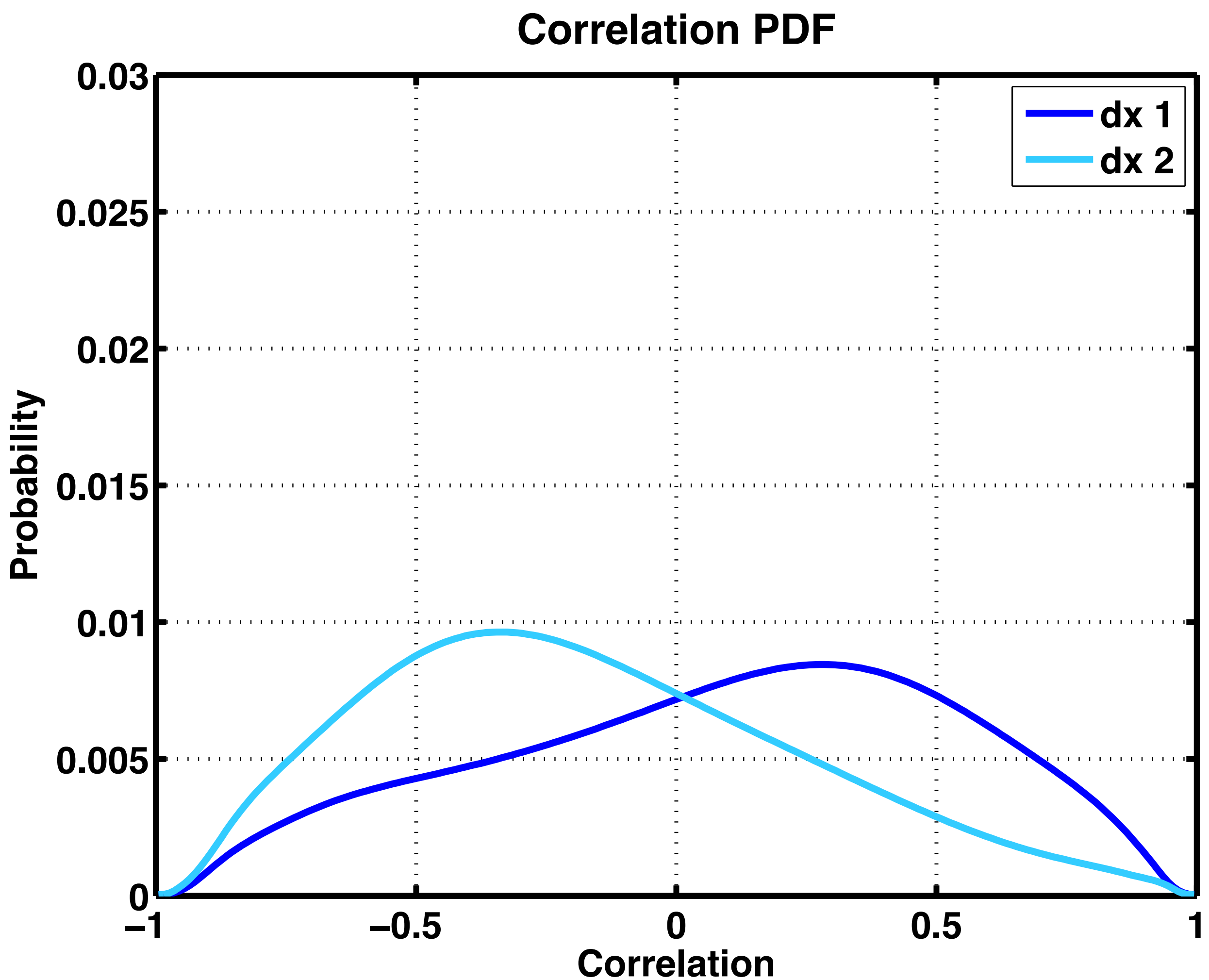


Equilibrated Correlation Distribution as Function of Separation



Equilibrated Correlation Distribution as Function of Separation



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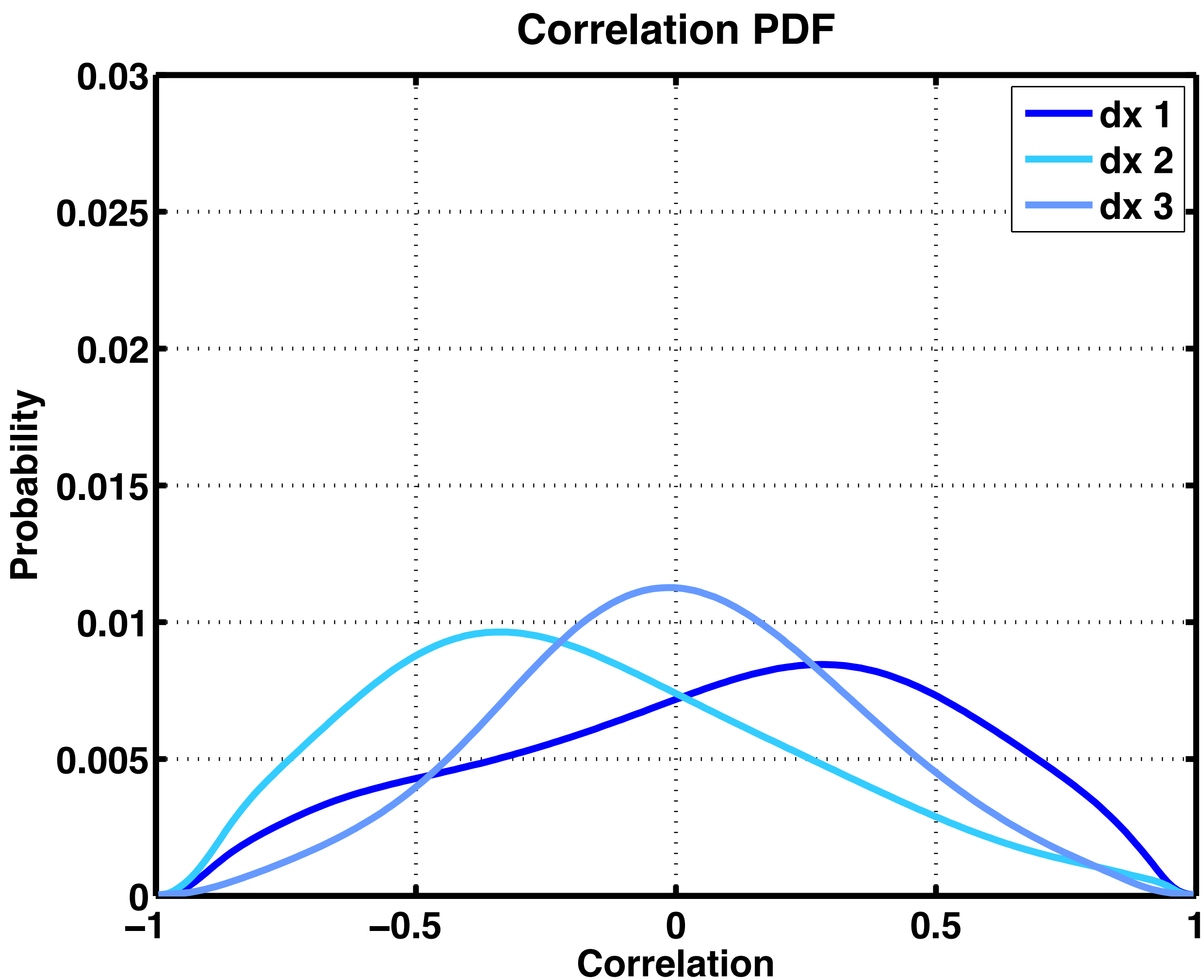

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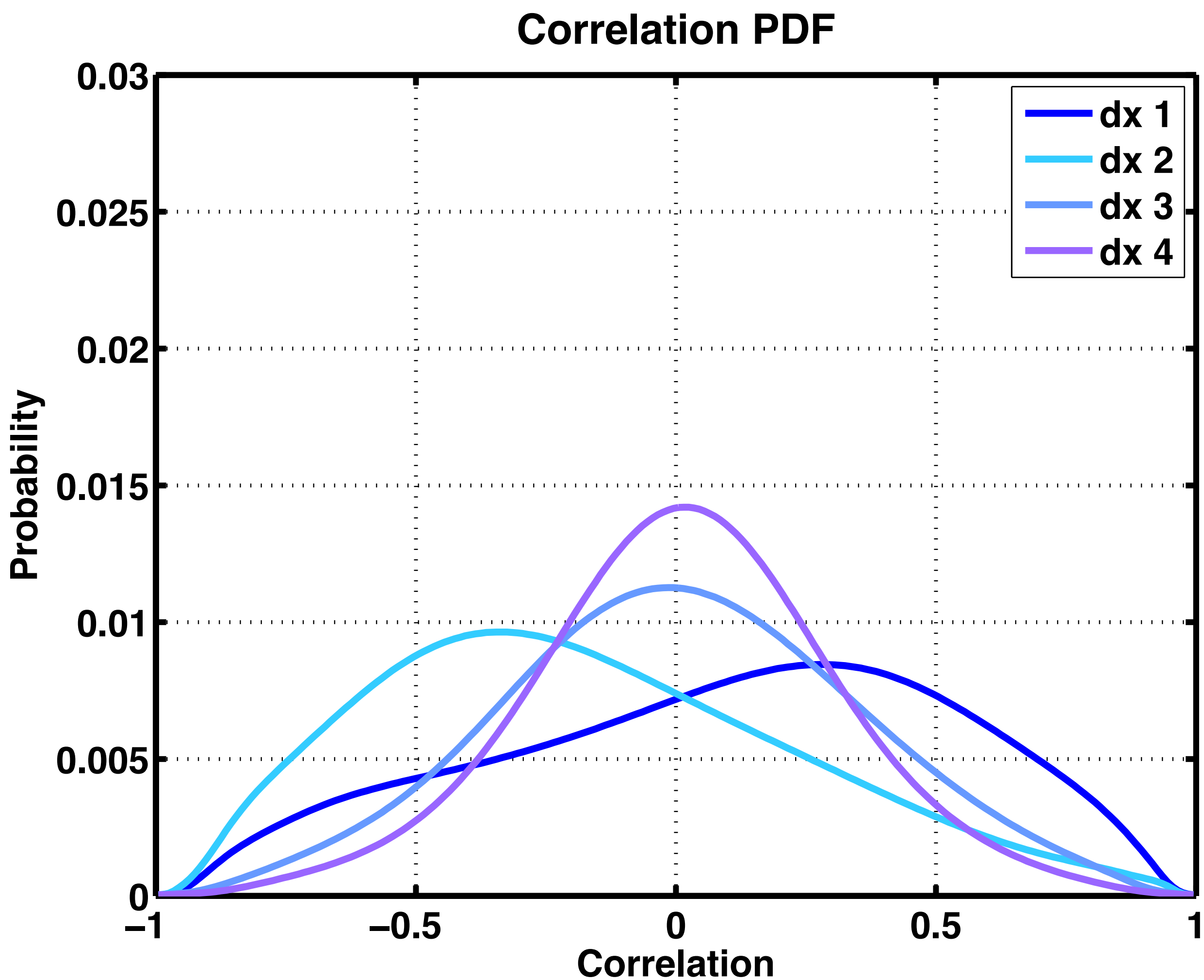
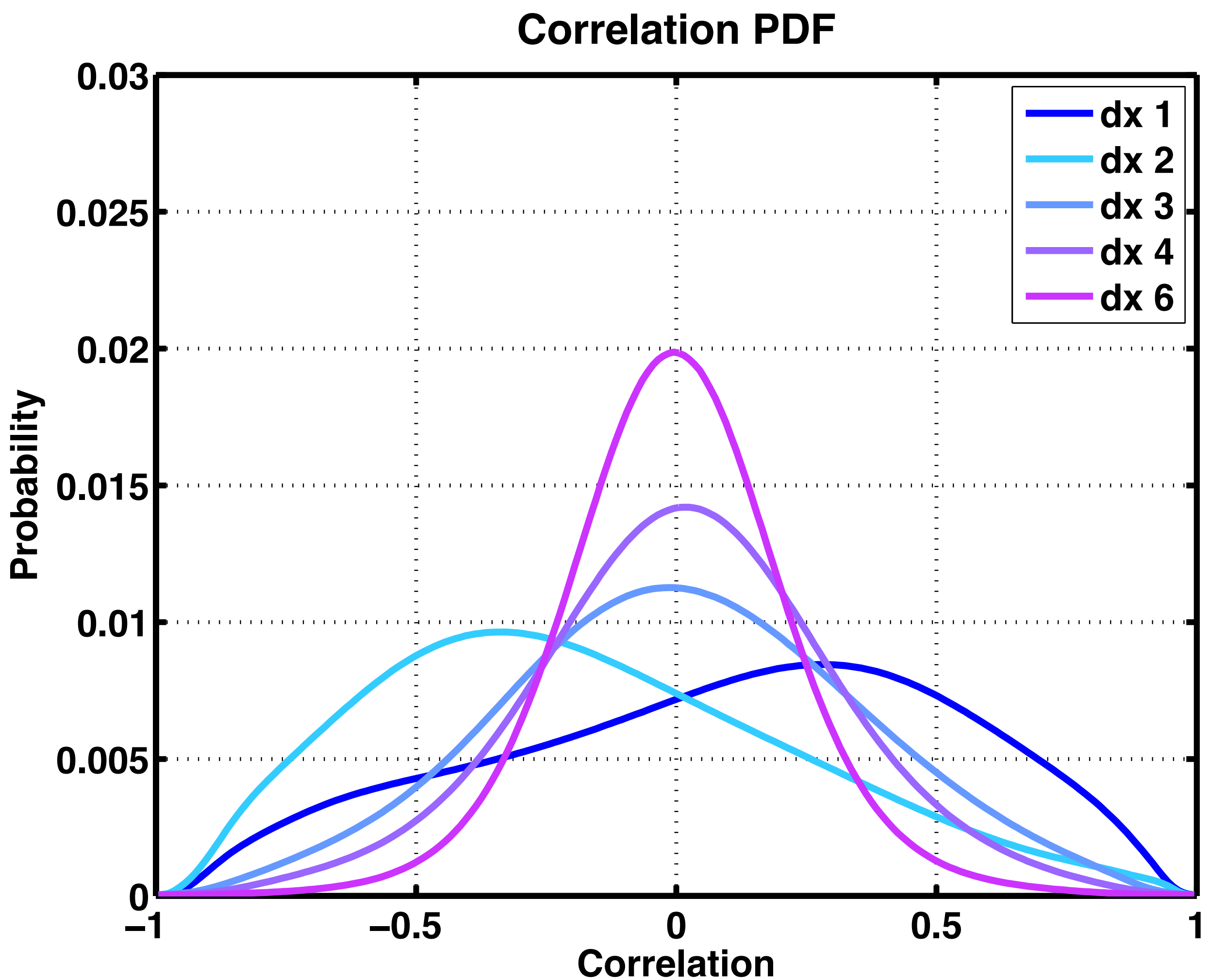
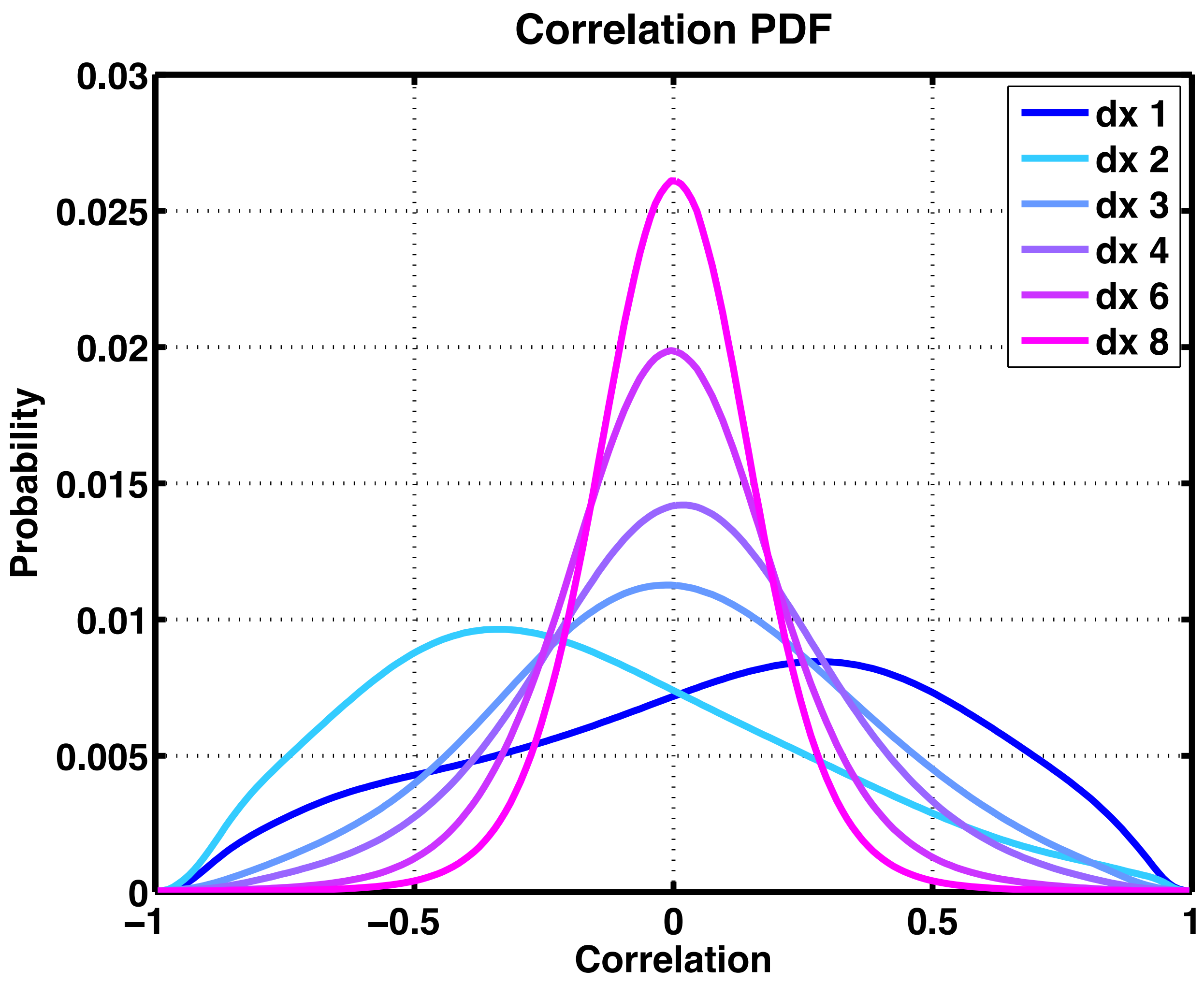


Equilibrated Correlation Distribution as Function of Separation



Equilibrated Correlation Distribution as Function of Separation



L96 Case 1: Infrequent high-quality obs

Identity observations, error variance 1.

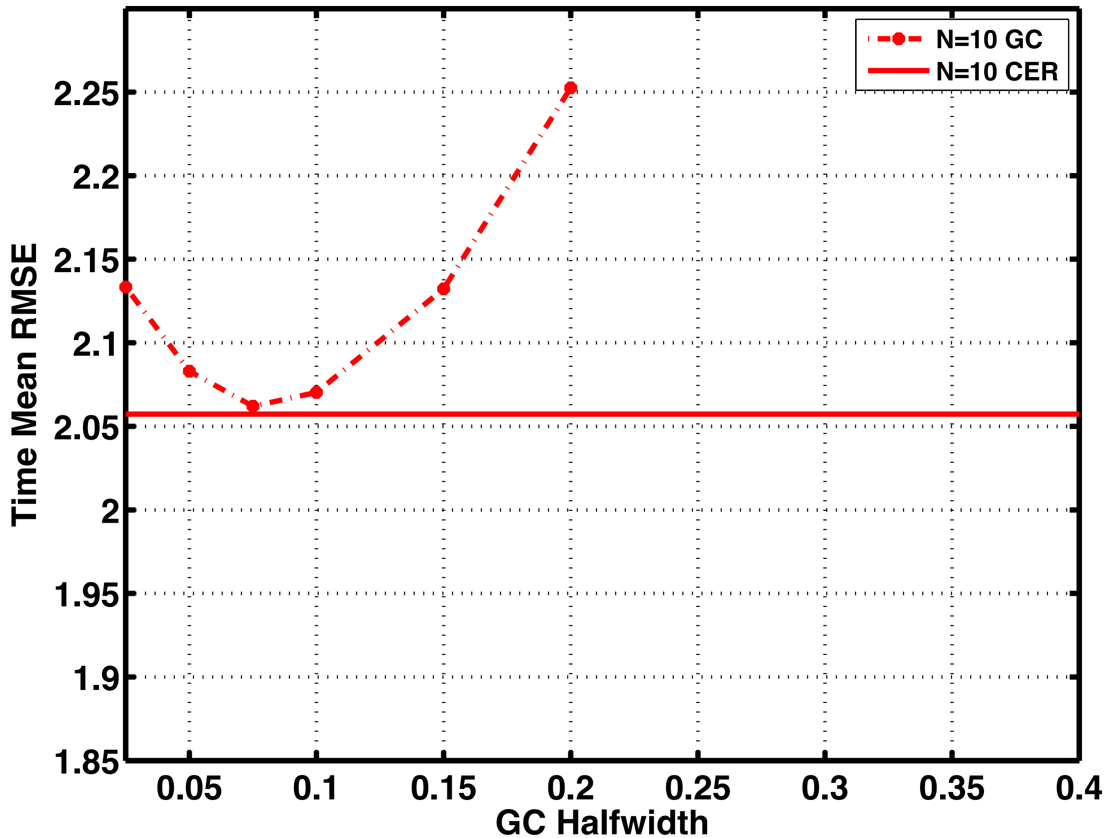
Assimilate every 12th model timestep.

20-member ensembles.

All cases use same adaptive inflation settings.

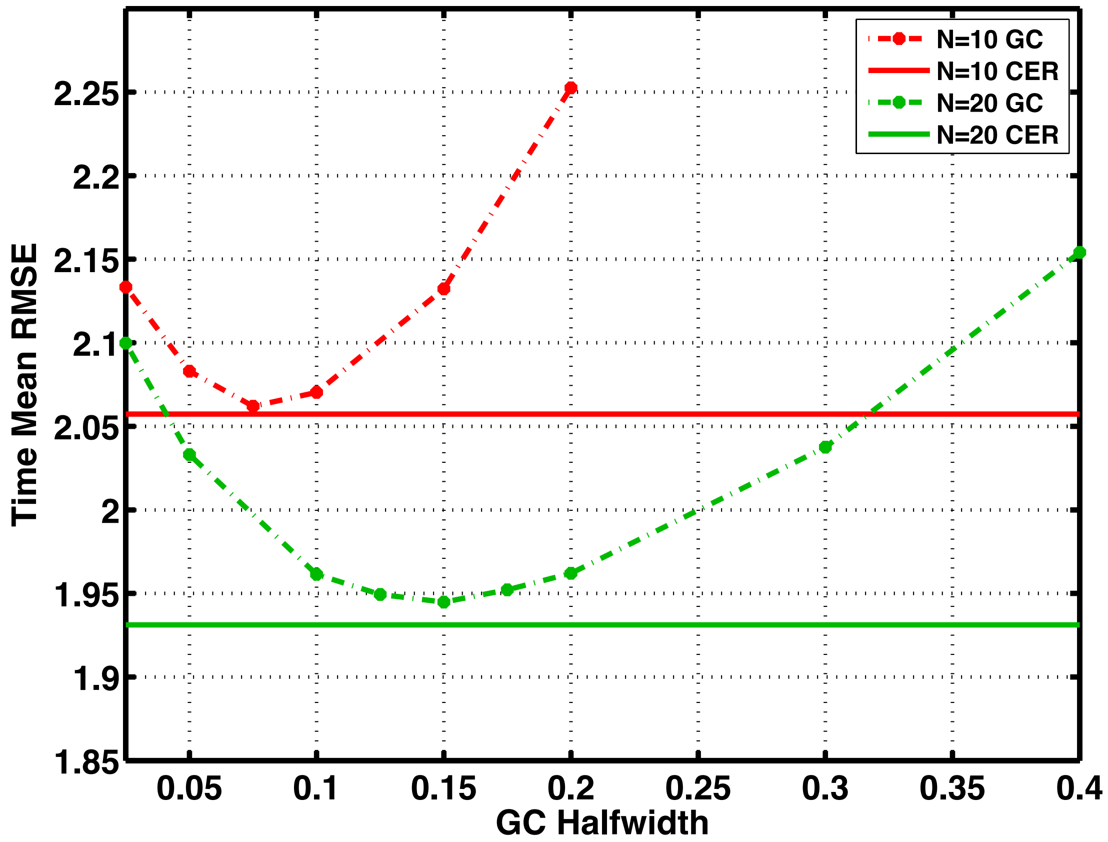
Prior RMSE: Obs. every 12 Hours, Error Variance 1

Comparison to Gaspari Cohn Localization Cases
Ensemble Size 10



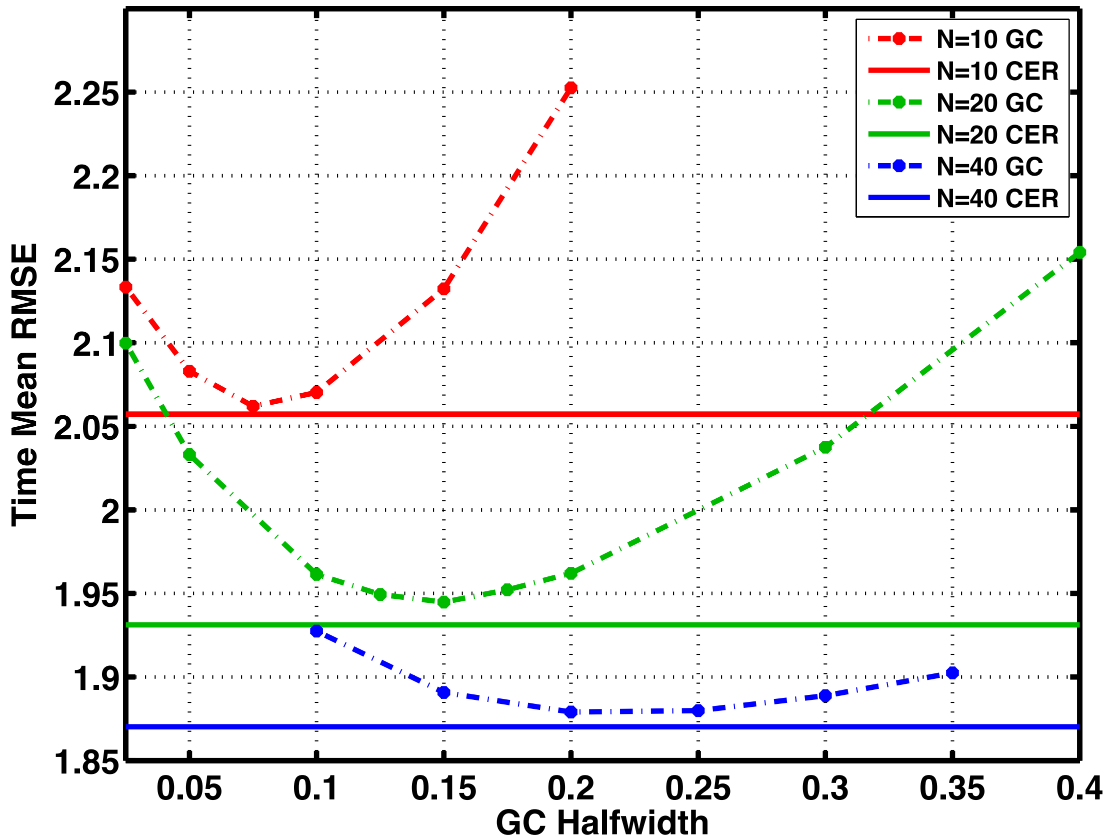
Prior RMSE: Obs. every 12 Hours, Error Variance 1

Comparison to Gaspari Cohn Localization Cases
Ensemble Size 10, 20



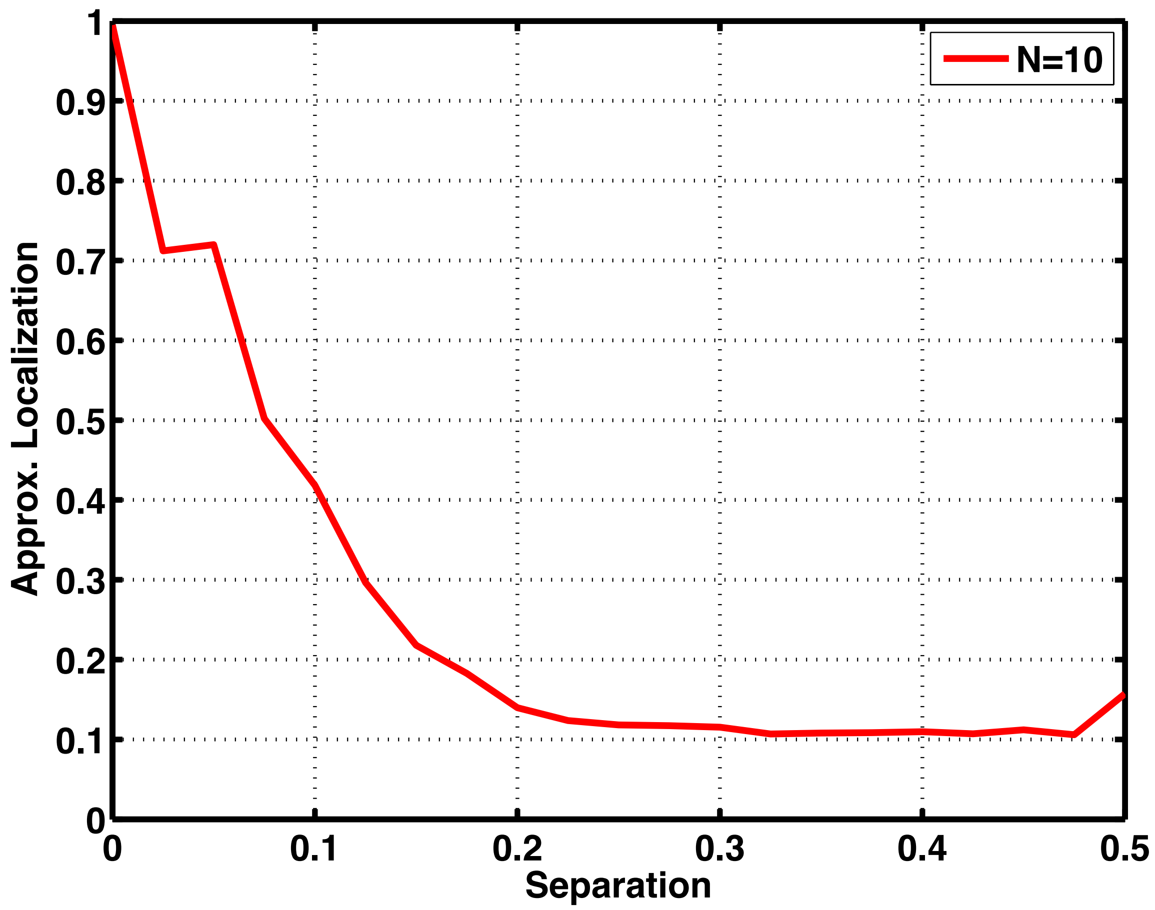
Prior RMSE: Obs. every 12 Hours, Error Variance 1

Comparison to Gaspari Cohn Localization Cases
Ensemble Size 10, 20, 40



Equivalent Localization: Obs. every 12 Hours, Err. Var. 1

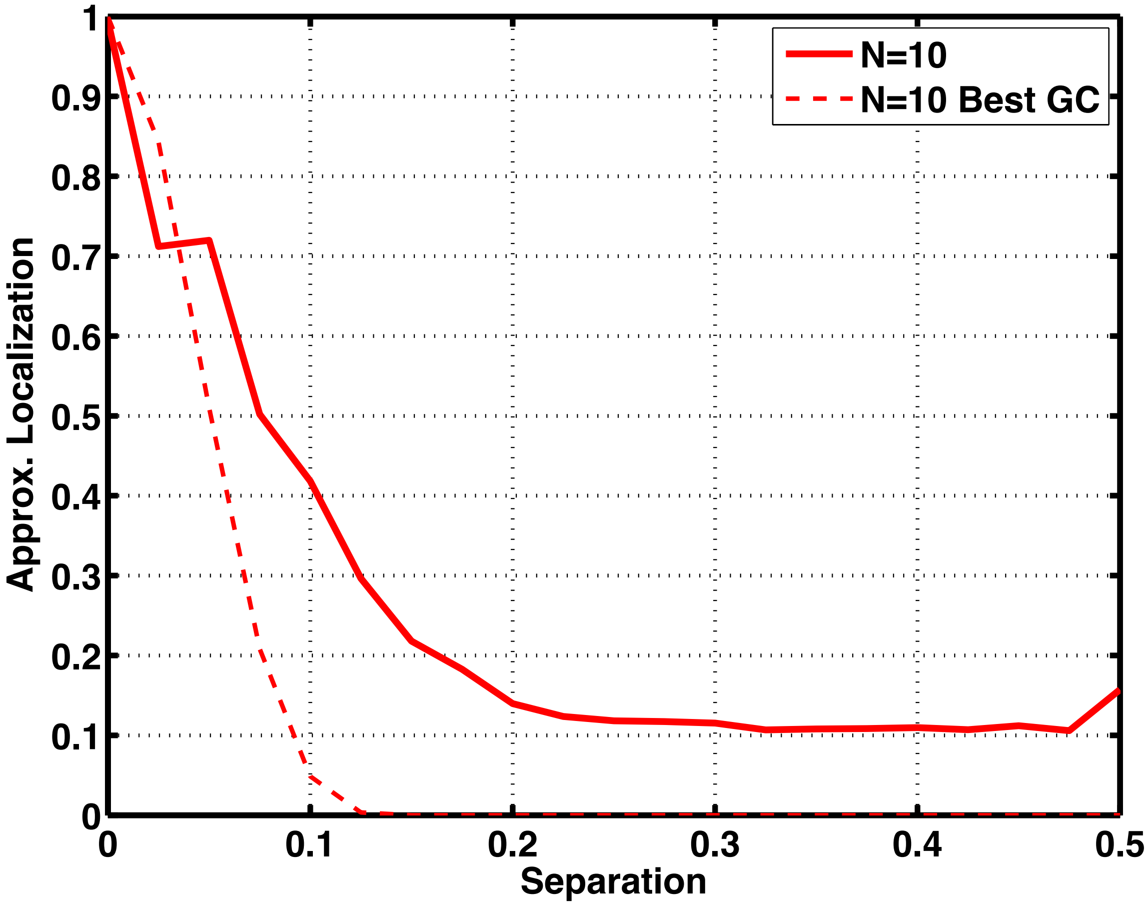
Ensemble Size 10



Equivalent Localization: Obs. every 12 Hours, Err. Var. 1

Ensemble Size 10

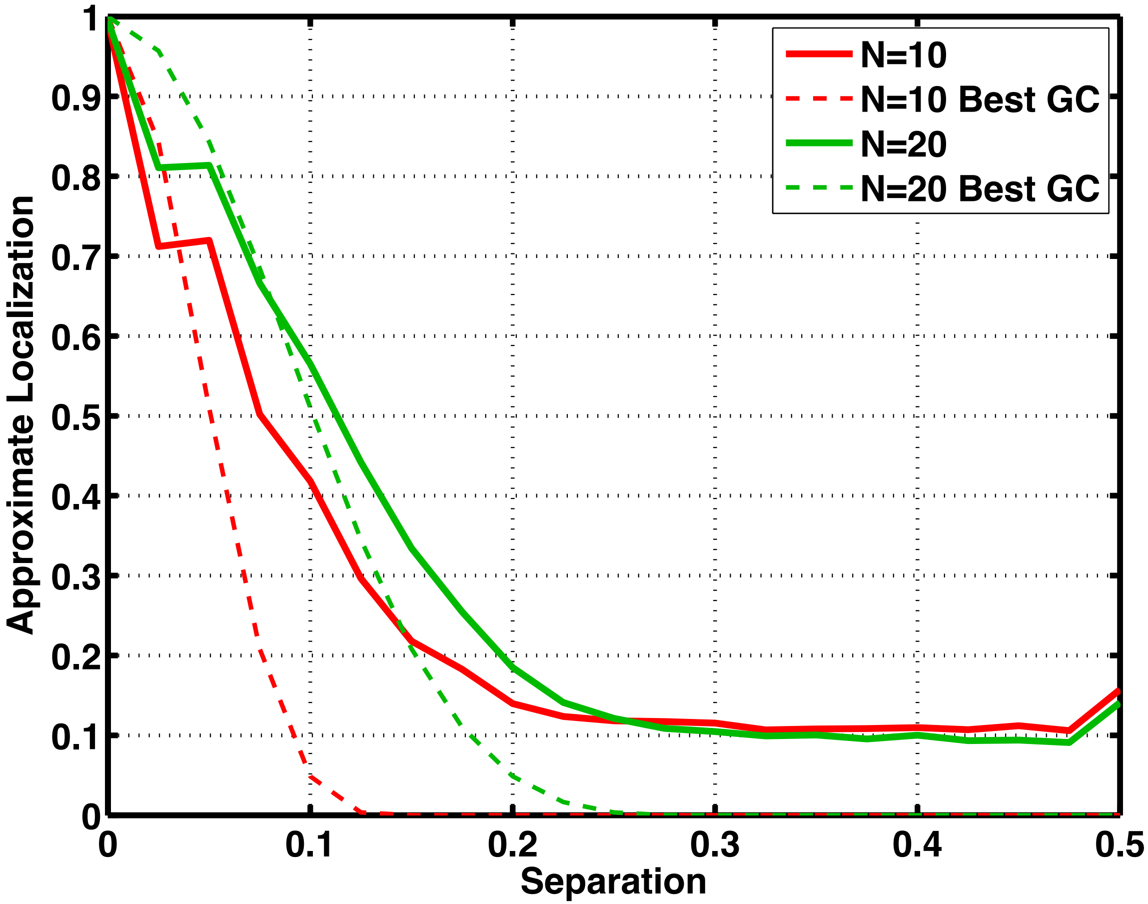
Plus Best Gaspari Cohn



Equivalent Localization: Obs. every 12 Hours, Err. Var. 1

Ensemble Size 10, 20

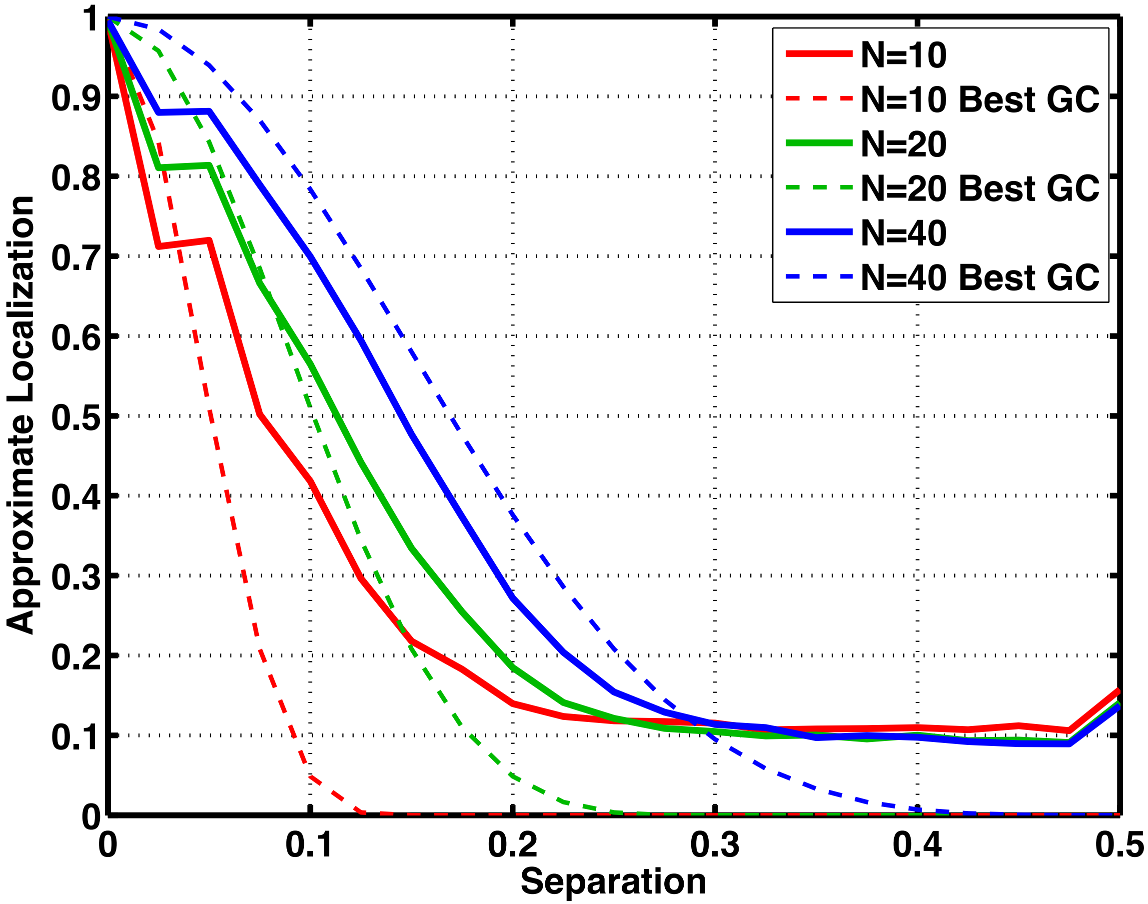
Plus Best Gaspari Cohn



Equivalent Localization: Obs. every 12 Hours, Err. Var. 1

Ensemble Size 10, 20, 40

Plus Best Gaspari Cohn



Case 2: Frequent low-quality obs

Identity observations, error variance 16.

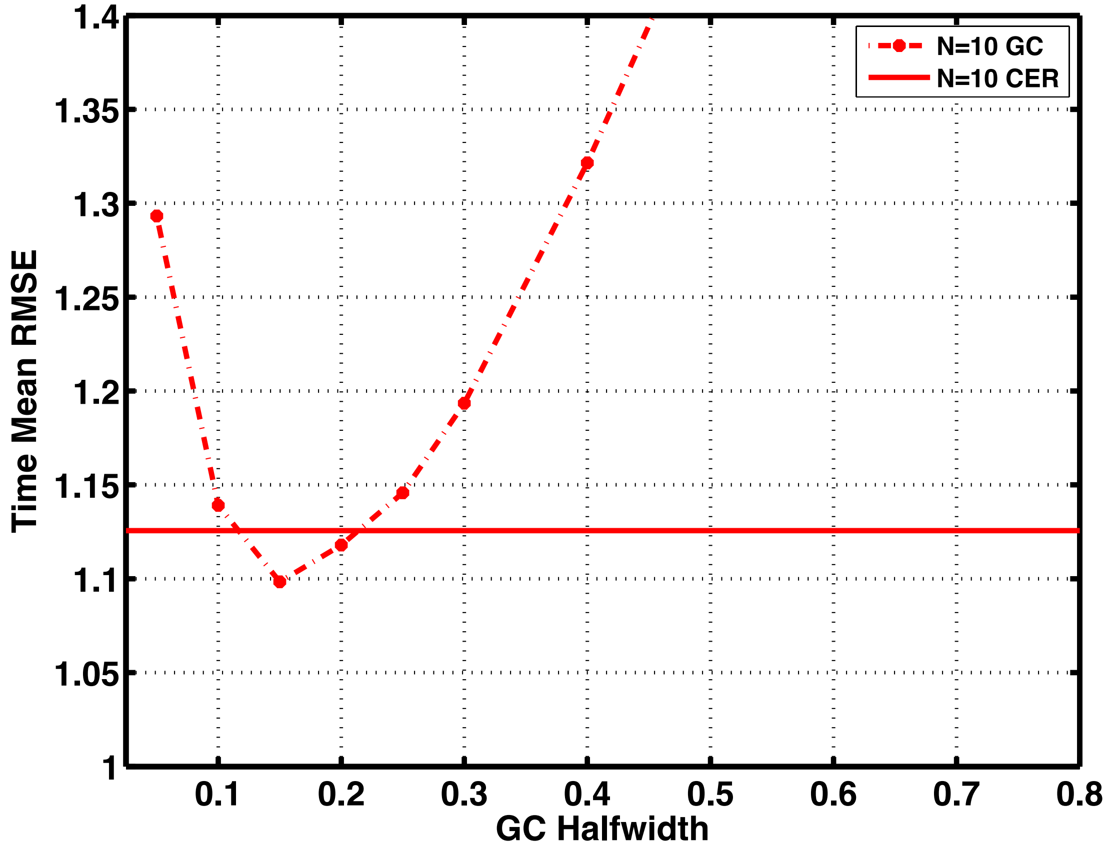
Assimilate every model timestep.

20-member ensembles.

All cases use same adaptive inflation settings.

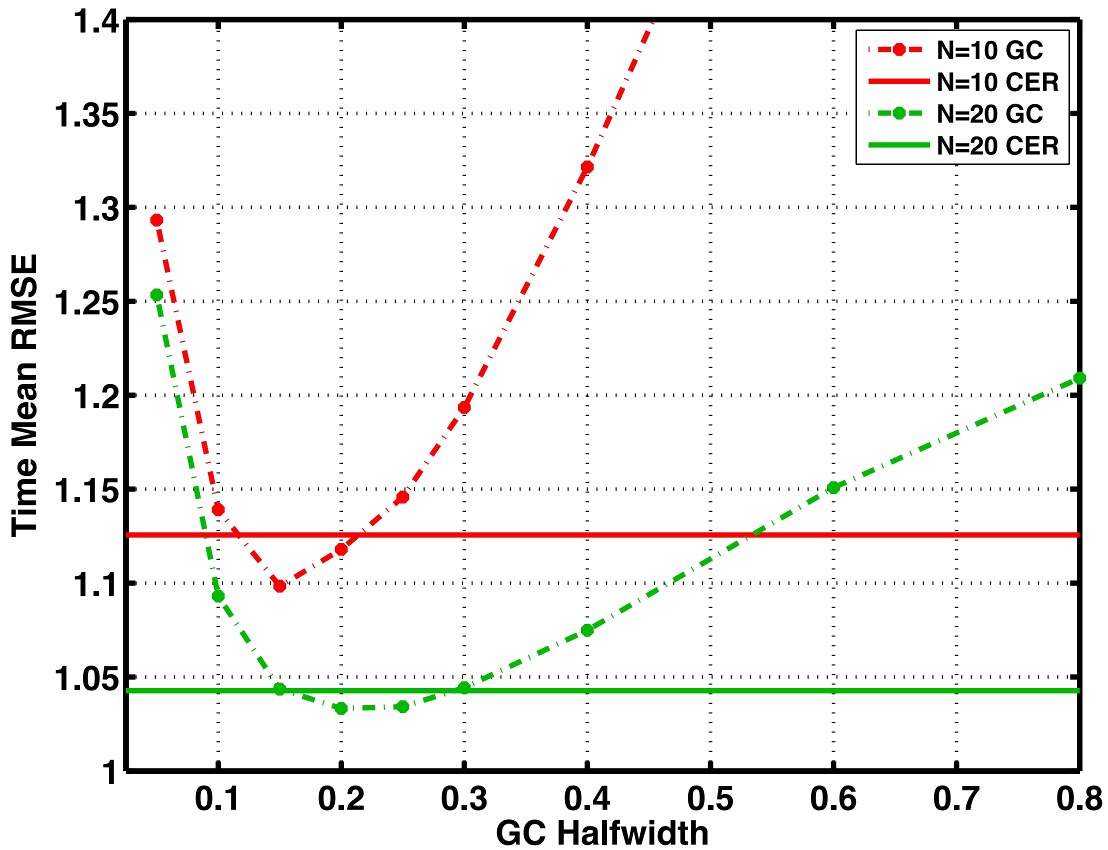
Prior RMSE: Obs. every Hour, Error Variance 16

Comparison to Gaspari Cohn Localization Cases
Ensemble Size 10



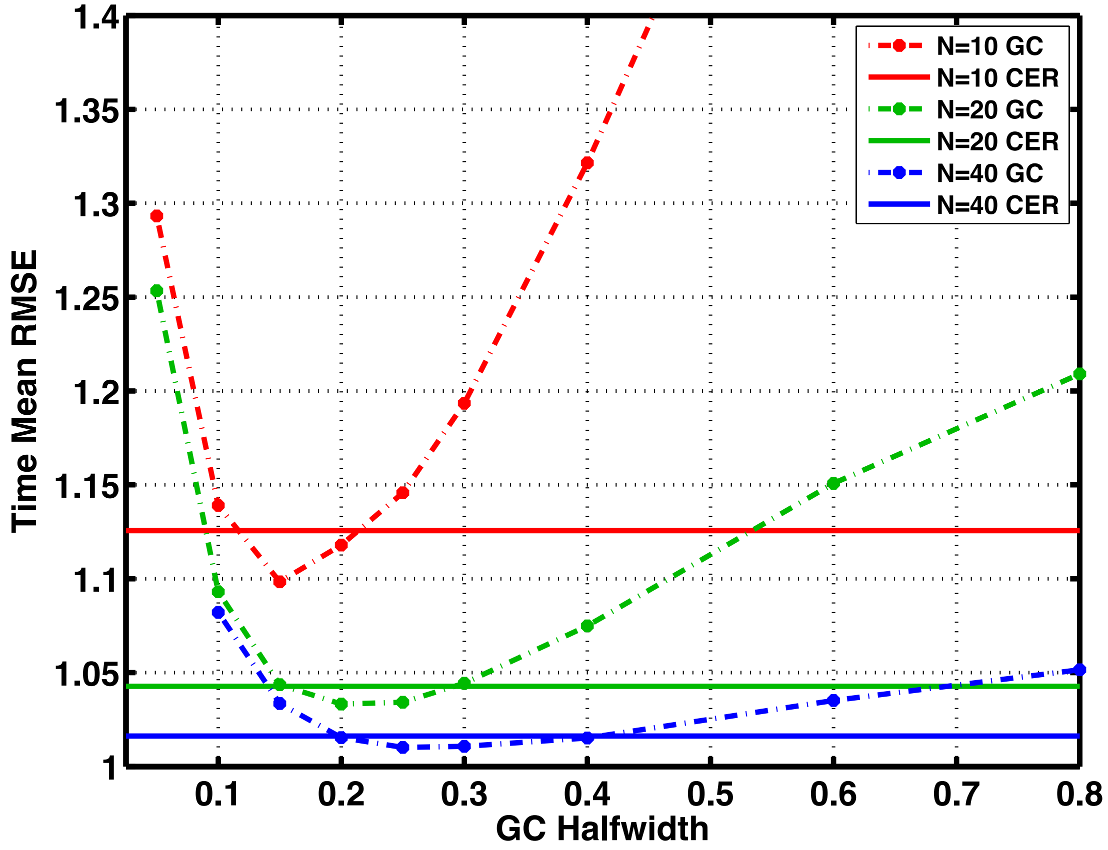
Prior RMSE: Obs. every Hour, Error Variance 16

Comparison to Gaspari Cohn Localization Cases
Ensemble Size 10, 20



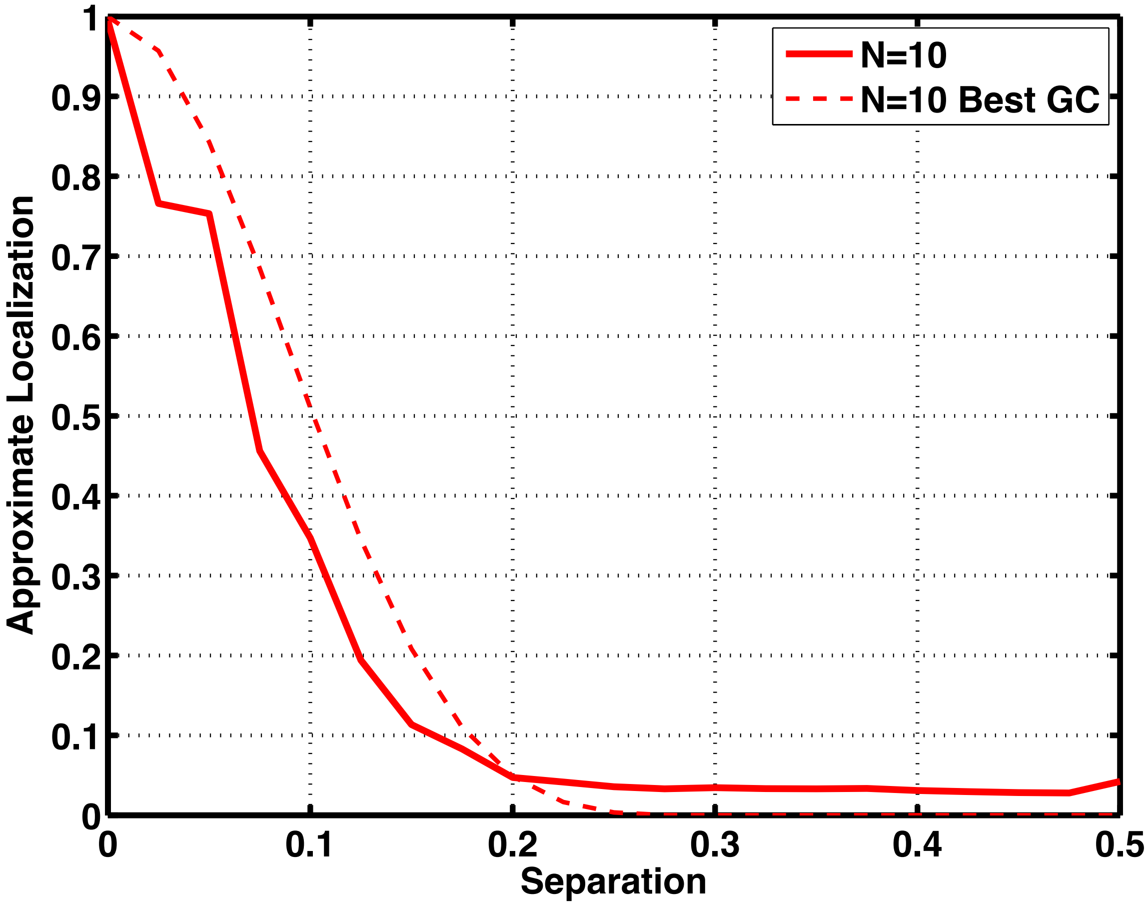
Prior RMSE: Obs. every Hour, Error Variance 16

Comparison to Gaspari Cohn Localization Cases
Ensemble Size 10, 20, 40



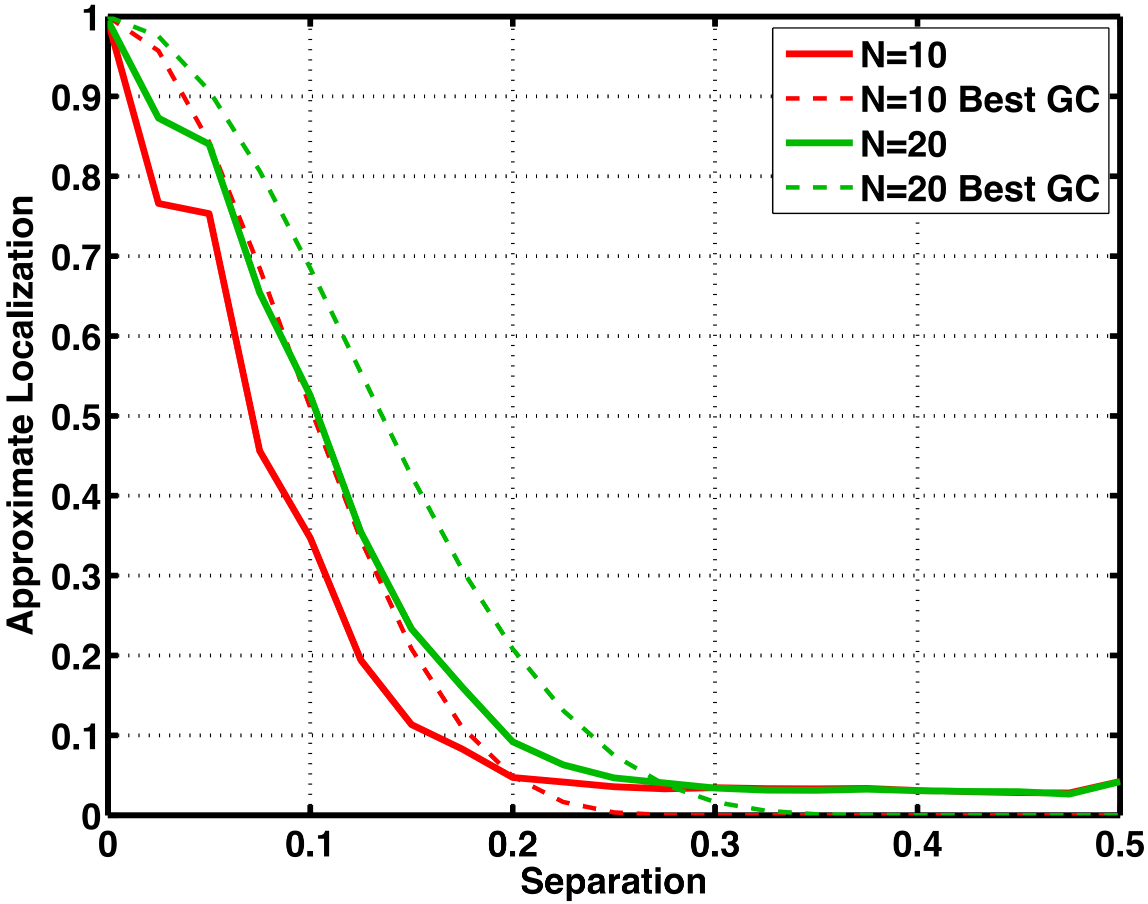
Equivalent Localization: Obs. Every Hour, Err. Var. 16

Ensemble Size 10



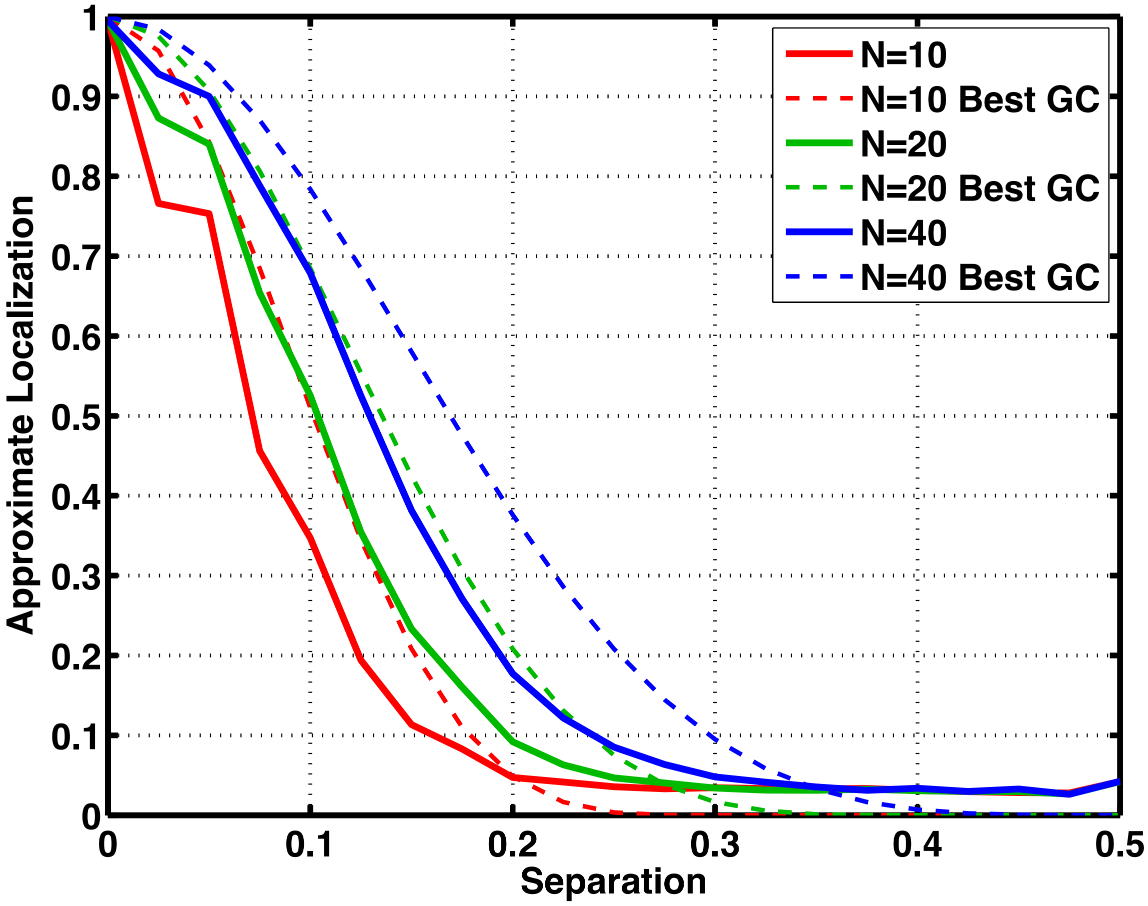
Equivalent Localization: Obs. Every Hour, Err. Var. 16

Ensemble Size 10, 20



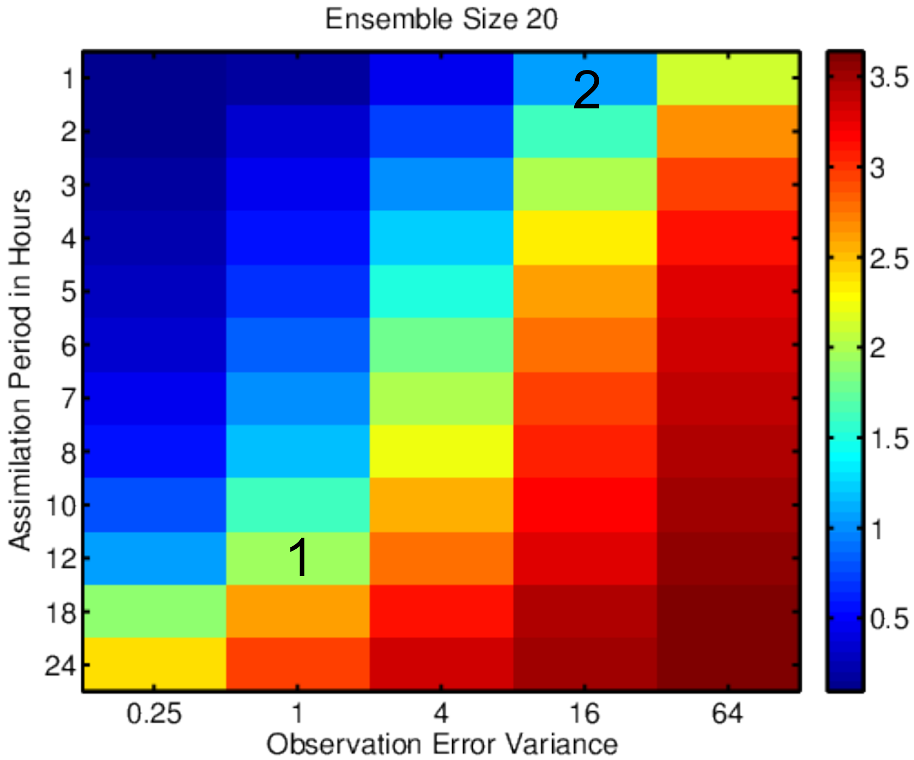
Equivalent Localization: Obs. Every Hour, Err. Var. 16

Ensemble Size 10, 20, 40



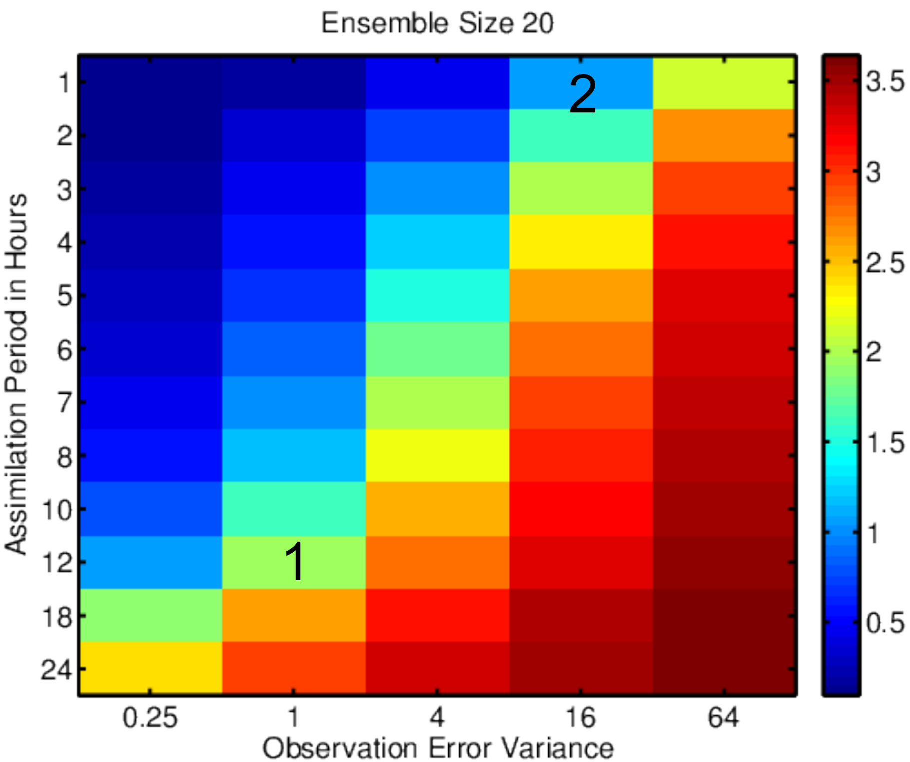
Lorenz96 Identity Observations Summary (N=20)

RMSE for Best GC

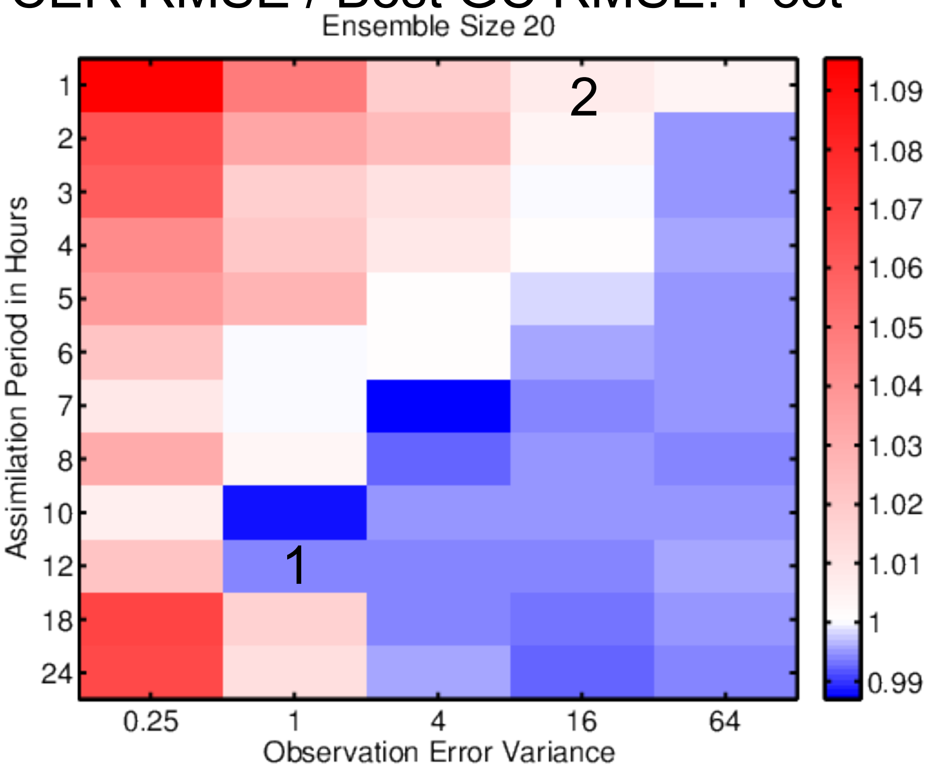


Lorenz96 Identity Observations Summary (N=20)

RMSE for Best GC

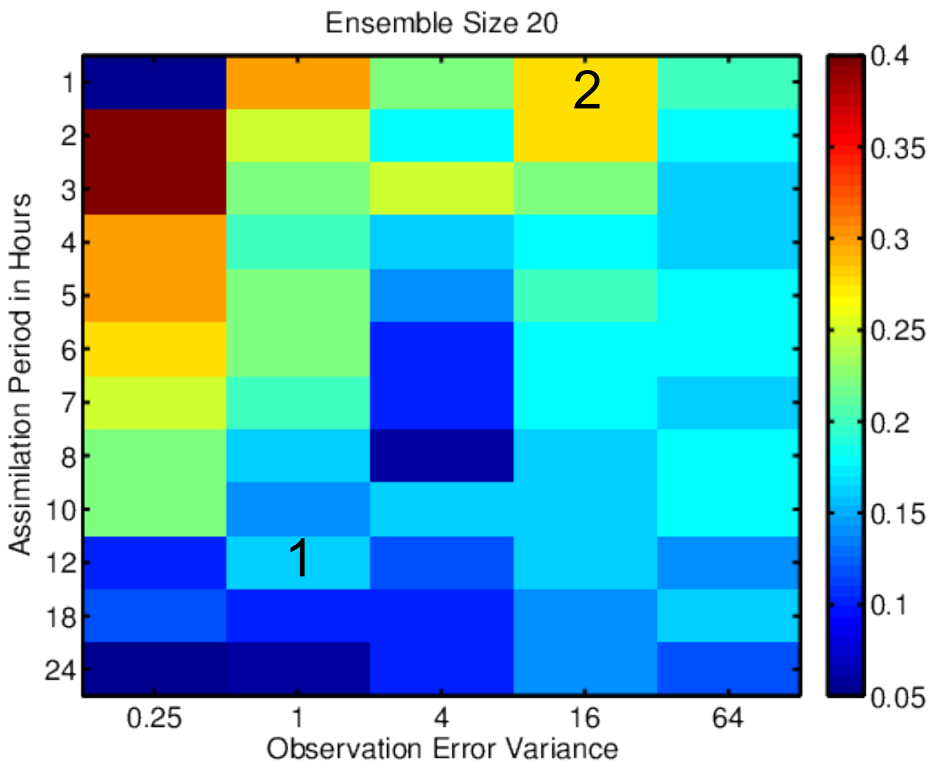


CER RMSE / Best GC RMSE: Post

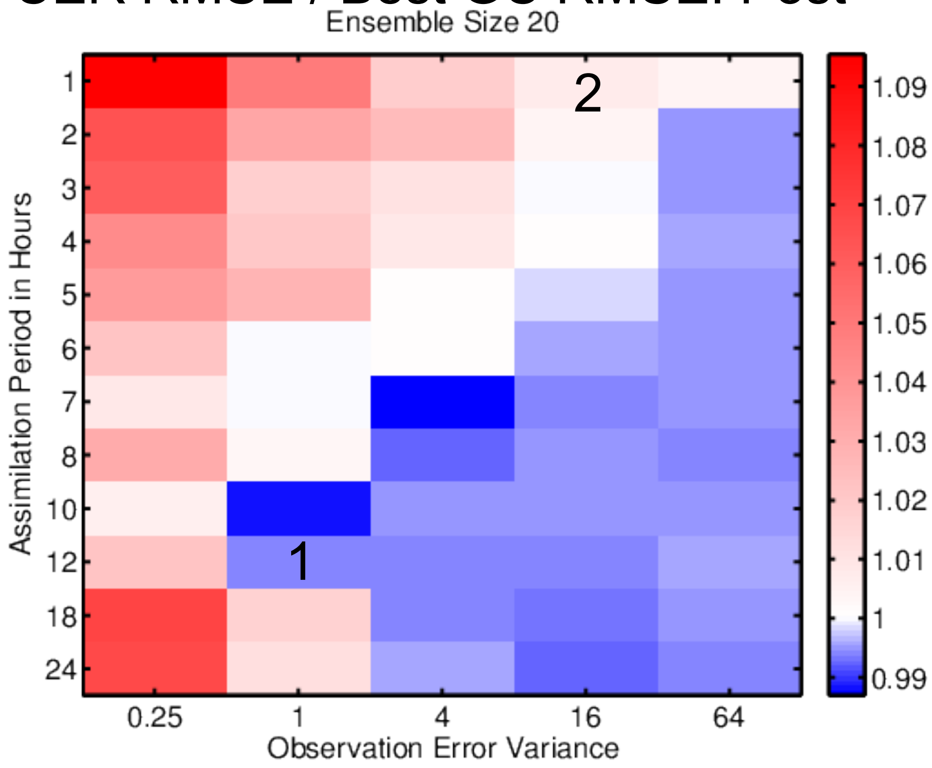


Lorenz96 Identity Observations Summary (N=20)

GC Halfwidth for Best RMSE

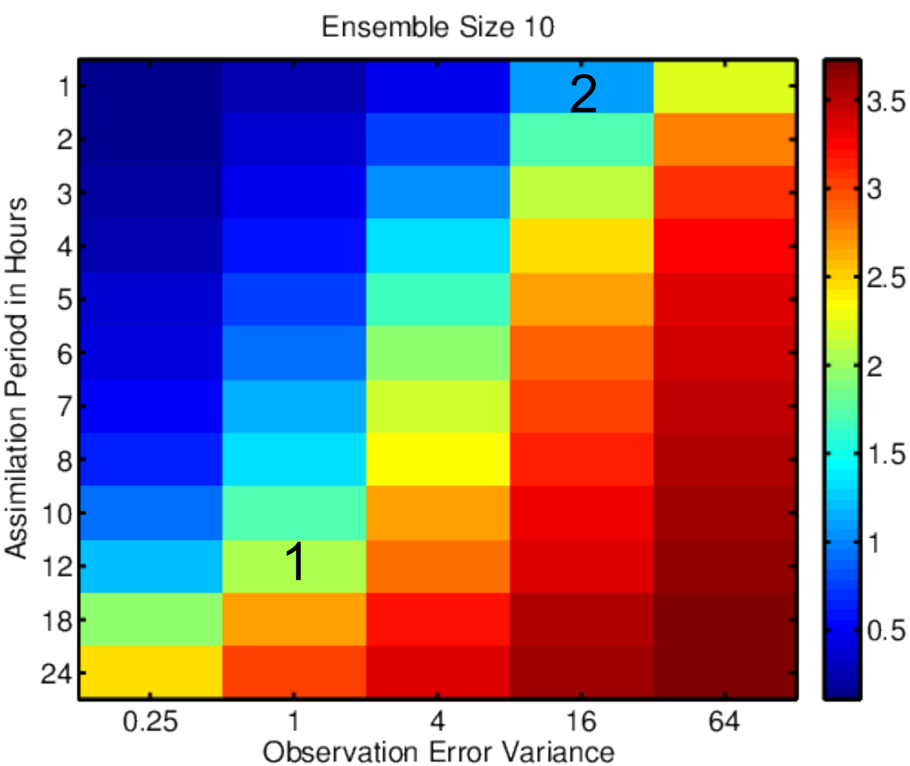


CER RMSE / Best GC RMSE: Post

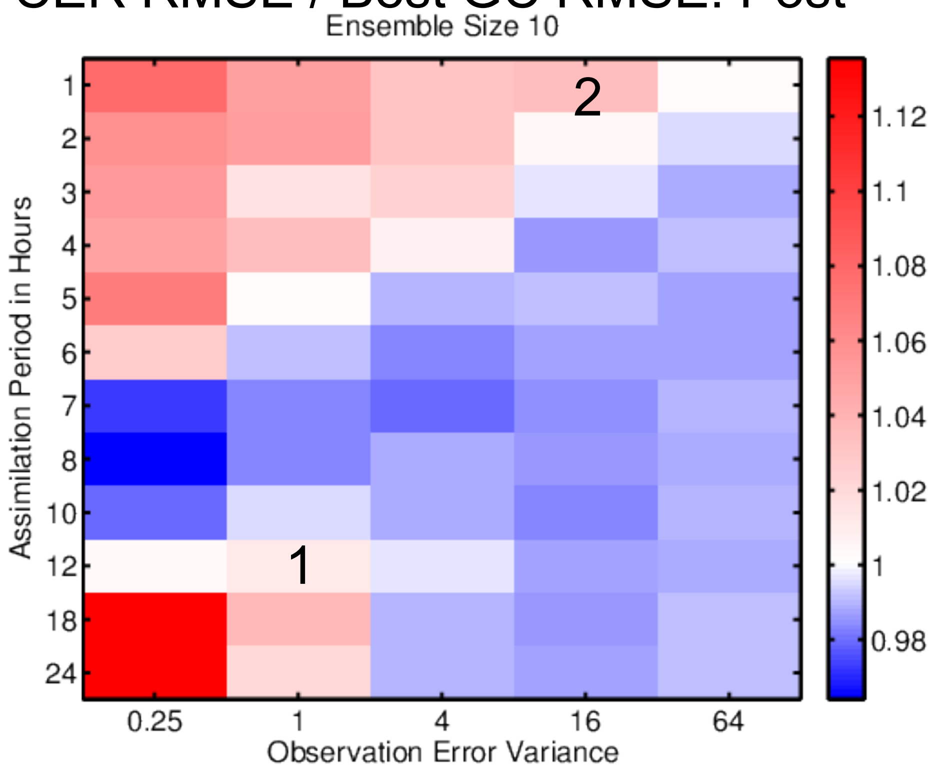


Lorenz96 Identity Observations Summary (N=10)

RMSE for Best GC

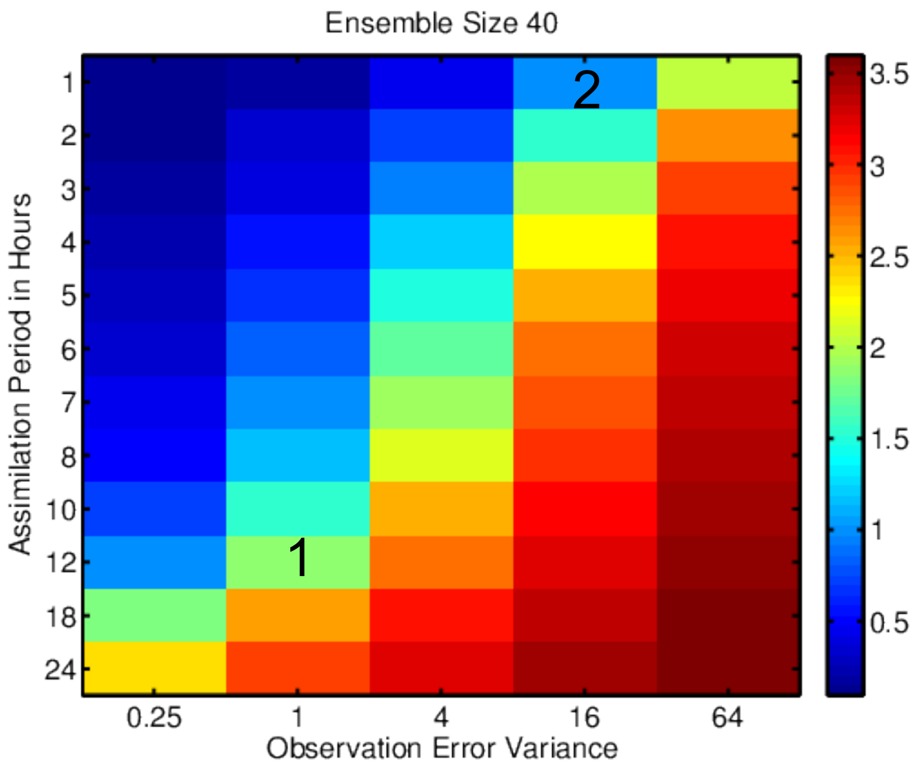


CER RMSE / Best GC RMSE: Post

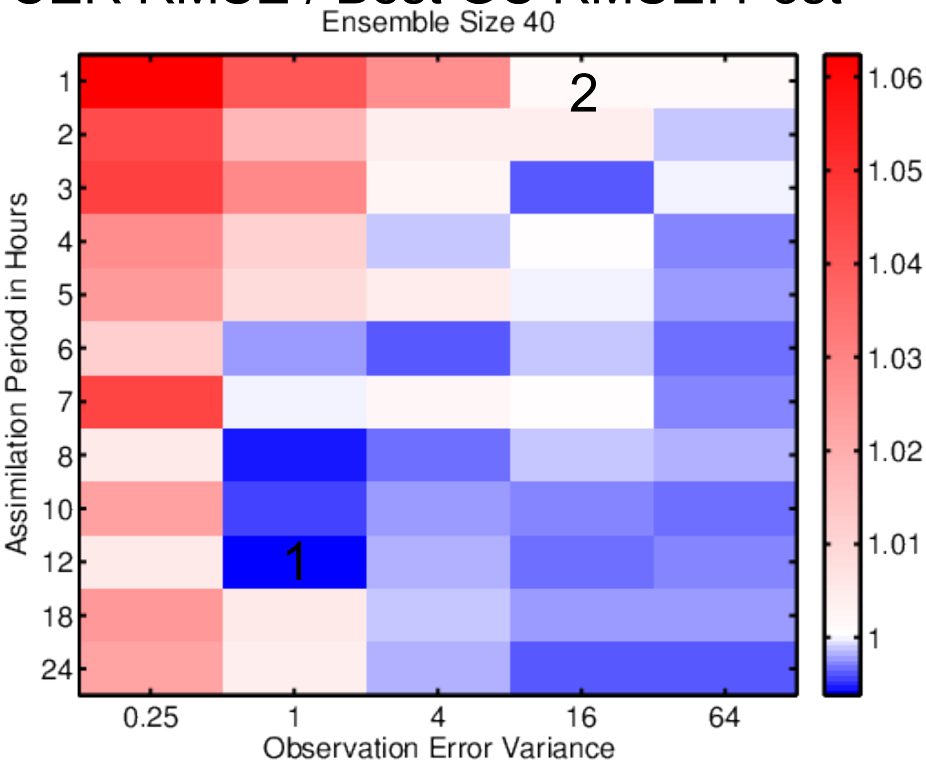


Lorenz96 Identity Observations Summary (N=40)

RMSE for Best GC

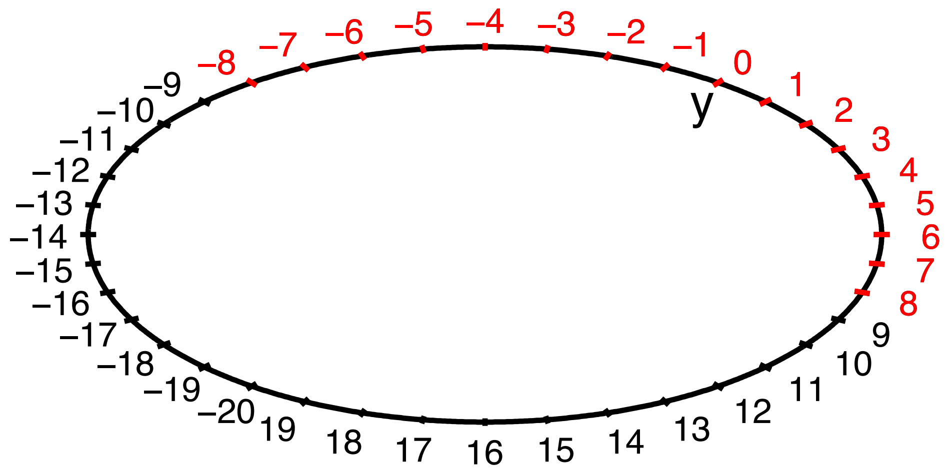


CER RMSE / Best GC RMSE: Post



L96 Case 3: Integral Observations

Each observation is average of grid point plus its nearest 8 neighbors on both side; total of 17 points. (Something like a radiance observation.)



Case 3: Integral Observations

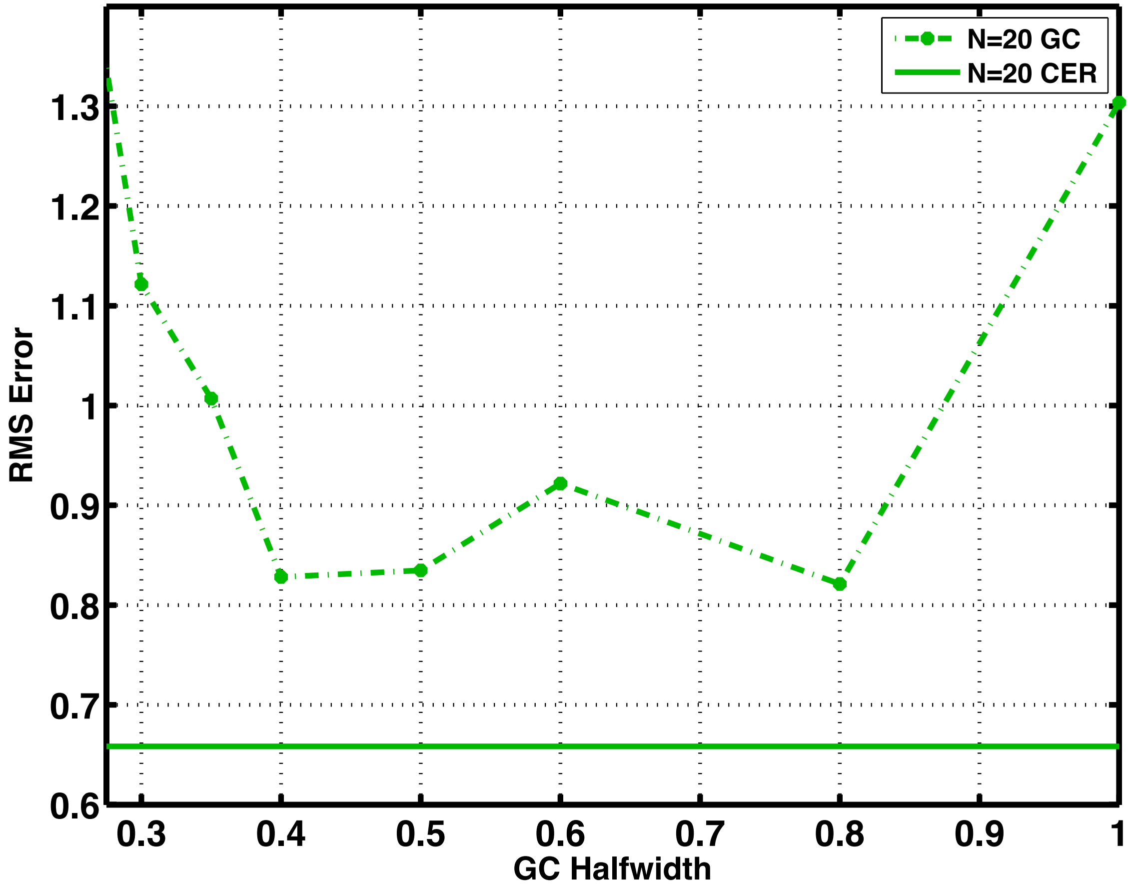
Each observation is average of grid point plus its nearest 8 neighbors on both side; total of 17 points.
(Something like a radiance observation.)

Error variance 0.0625.

Assimilate every standard model timestep.

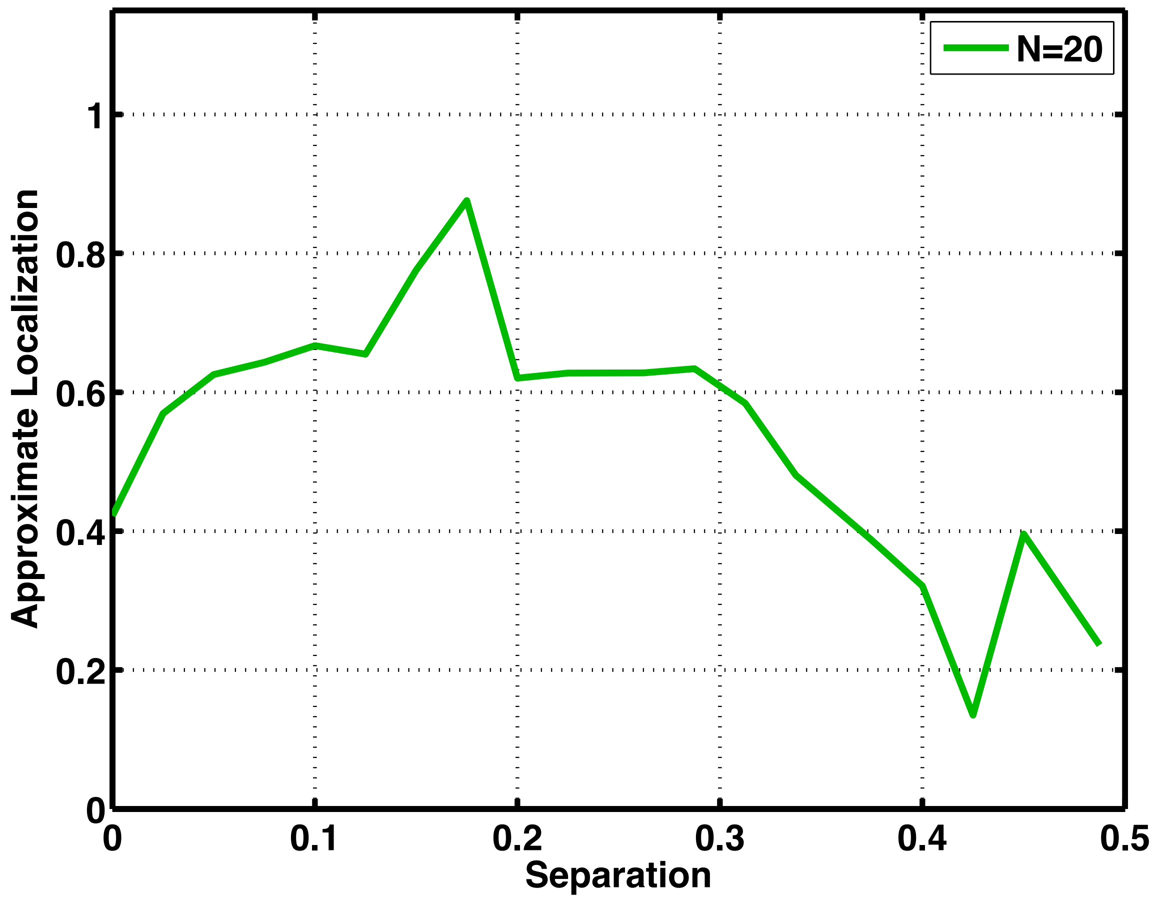
Case 3: Integral Observations

Compare Correlation Error Reduction to Gaspari Cohn Localization
Ensemble Size 20



Approximate Localization: Observing Average of 17 States

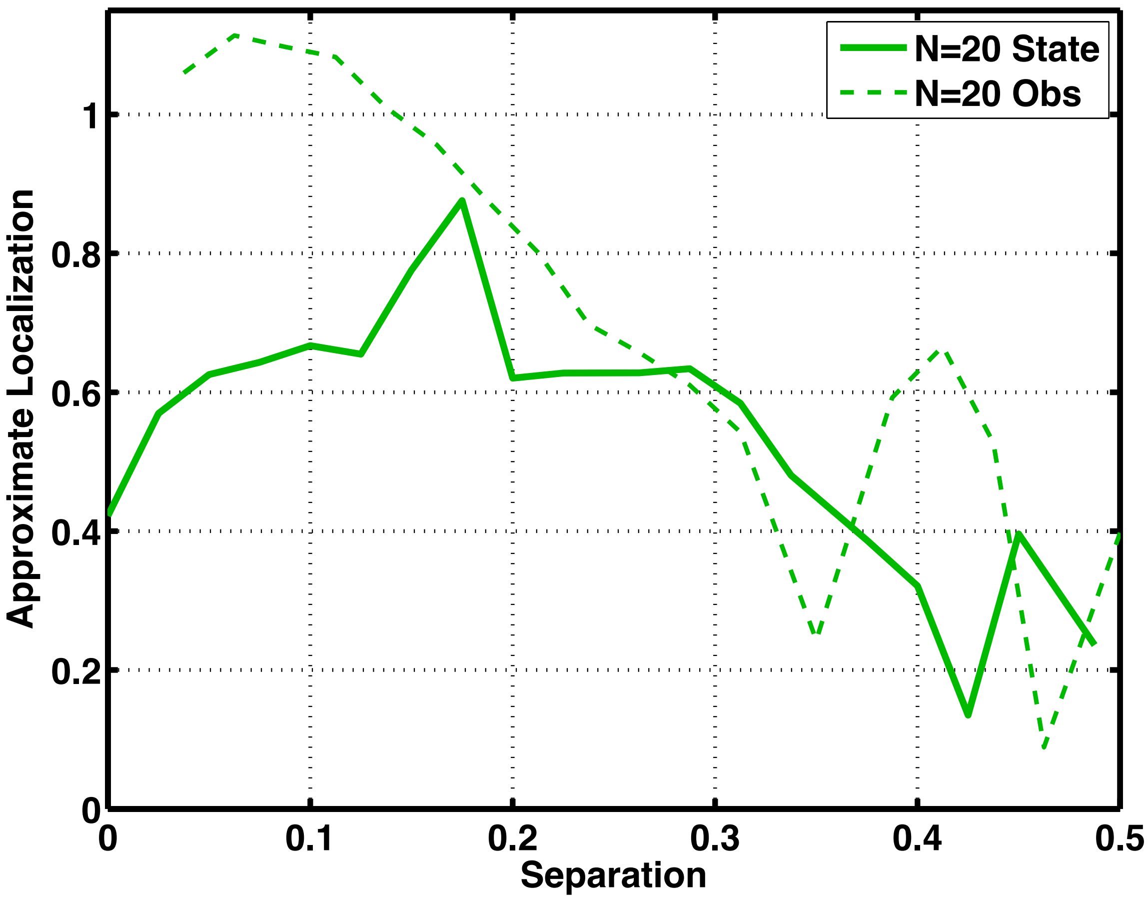
Ensemble Size 20



Approximate Localization: Observing Average of 17 States

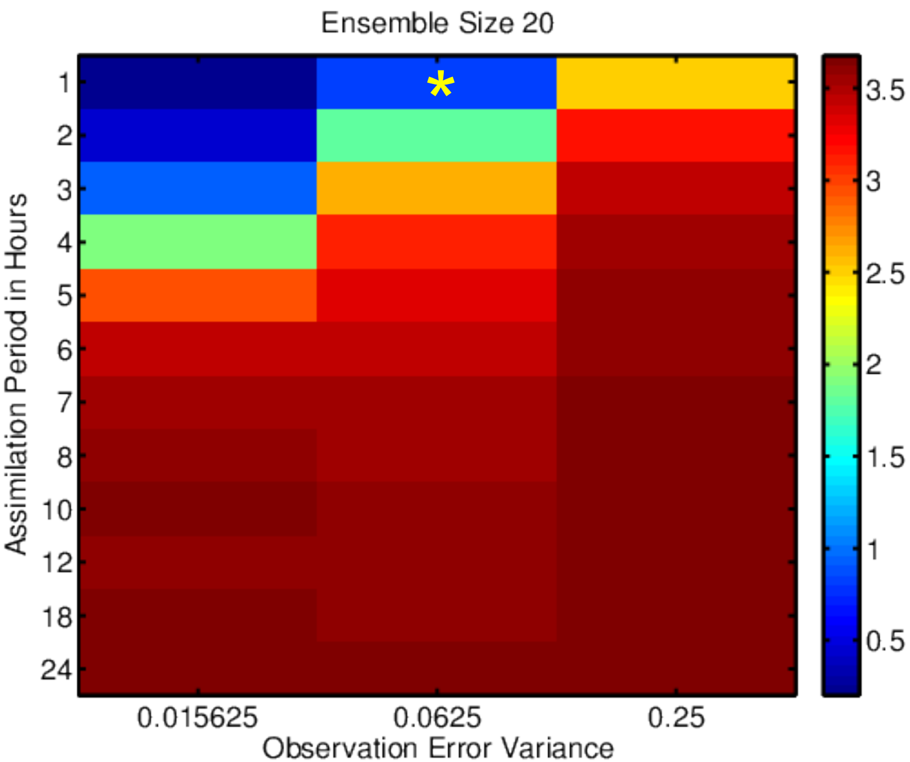
Ensemble Size 20

Plus localization for observations

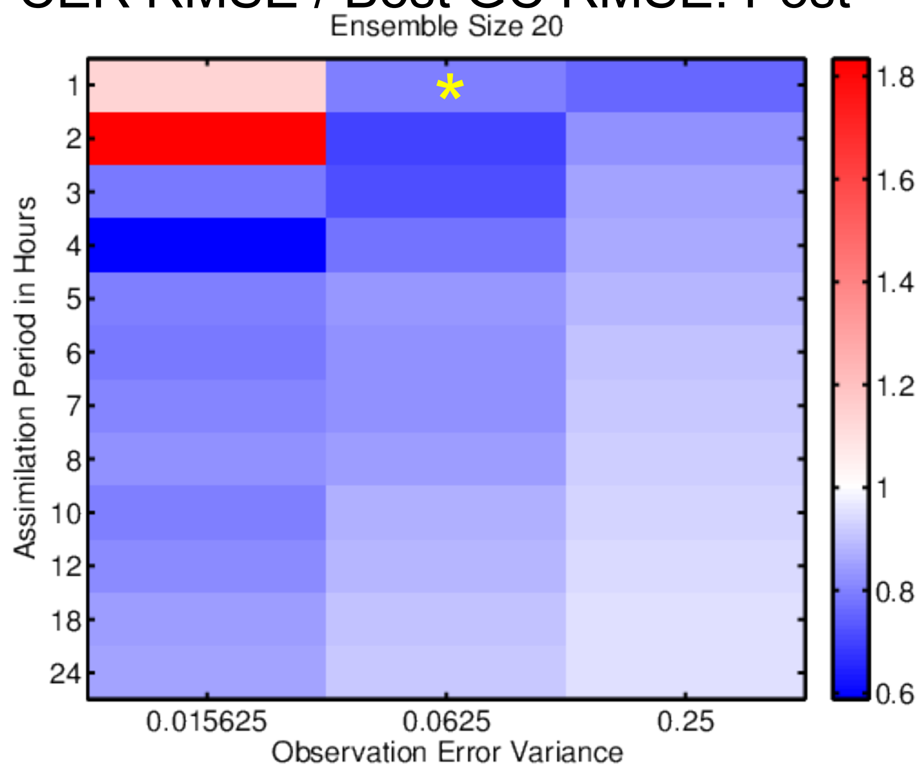


Lorenz96 Integral Observations Summary (N=20)

RMSE for Best GC



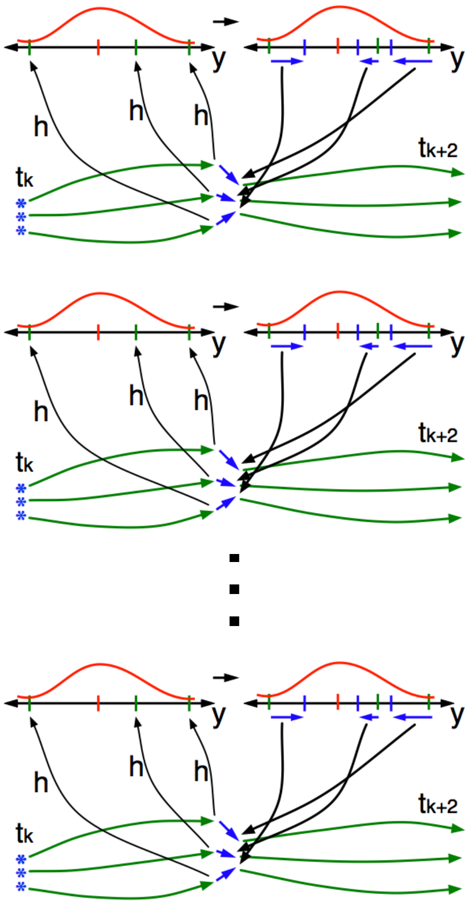
CER RMSE / Best GC RMSE: Post



Localization Method 3: Optimized

- Get optimal localization:
- Minimize RMSE in OSSE *a posteriori*.
- Initial guess is best Gaspari Cohn.
- Tricky optimization problem:
  Expensive (many long OSSEs),
  Noisy,
  Possible multiple minima.

Localization Method 4: Global Group Filter



$\hat{b}_1$

$\hat{b}_2$

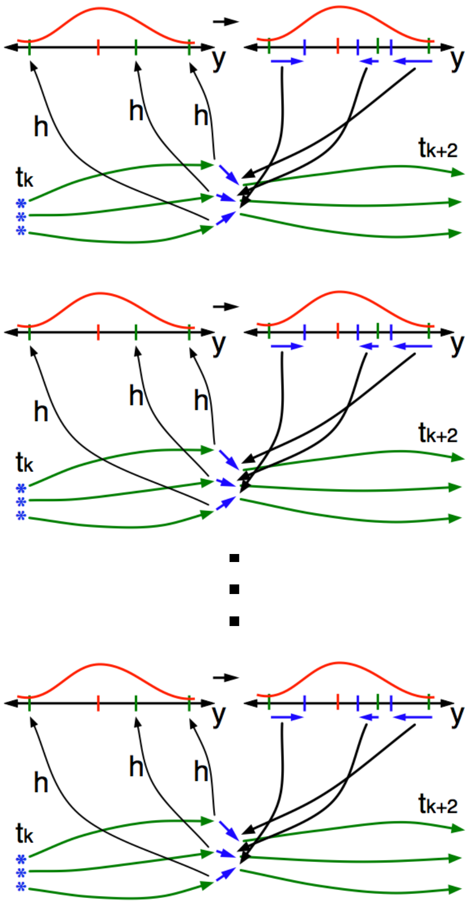
$\vdots$

$\hat{b}_M$

Compute $\overline{b}$ and σ_b .

Compute optimal localization α for this y and x_i.

Localization Method 4: Global Group Filter



$\hat{b}_1$

$\hat{b}_2$

$\vdots$

$\hat{b}_M$

Run 200 groups for 3000 steps.

Do least squares fit for localization for each separation subset.

Resulting localization for each separation used with single ensemble.

Expensive to initialize.

Localization Methods Not in this Talk

5. Empirical Localization Function (ELF):

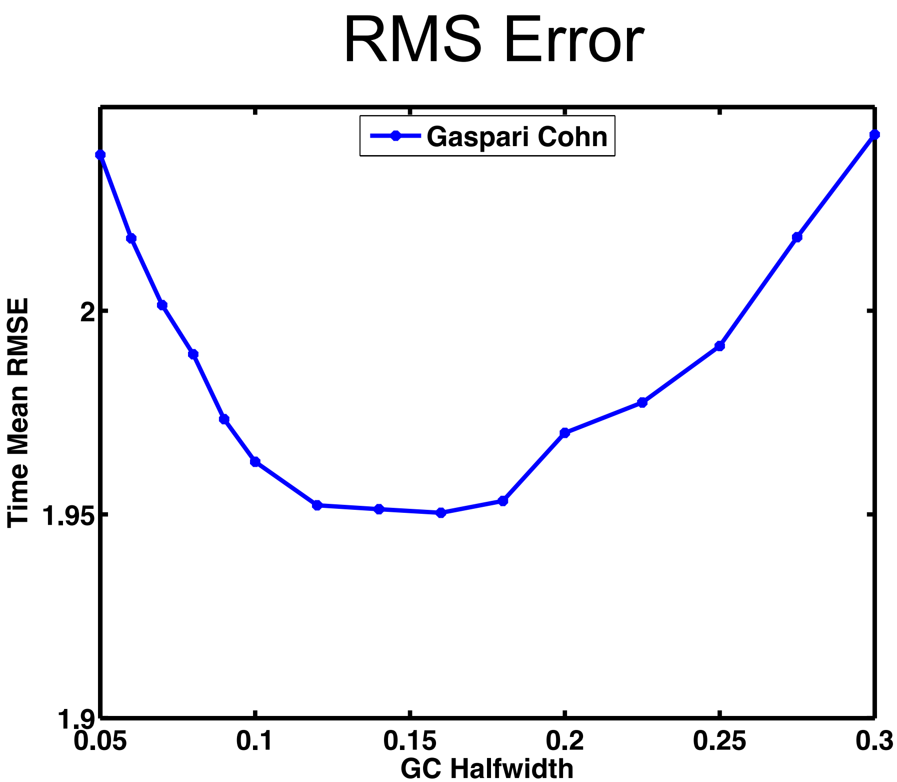
Work with Lili Lei now with Jeff Whitaker at NOAA,

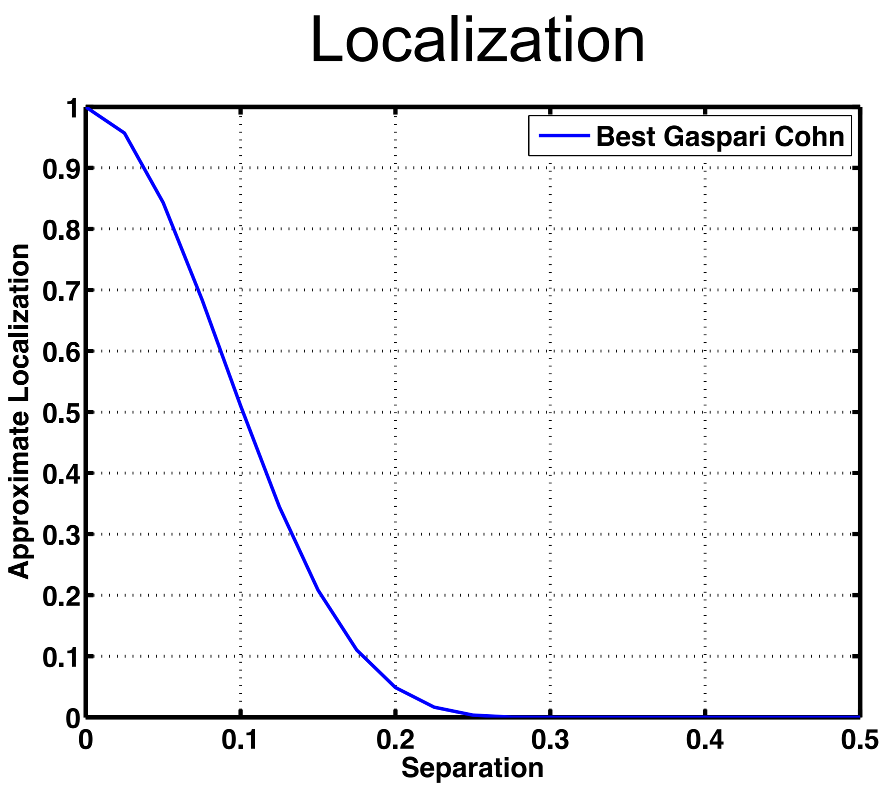
Find localization that gives least error compared to known true state in OSSE.

6. Sampling Error Correction:

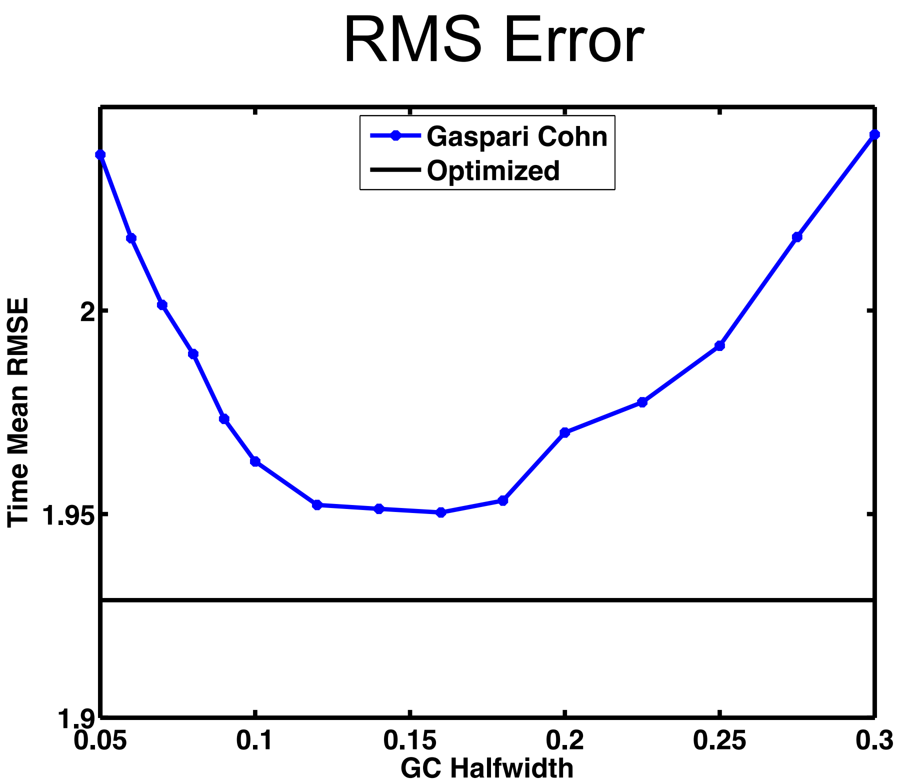
Function of sample correlation only.

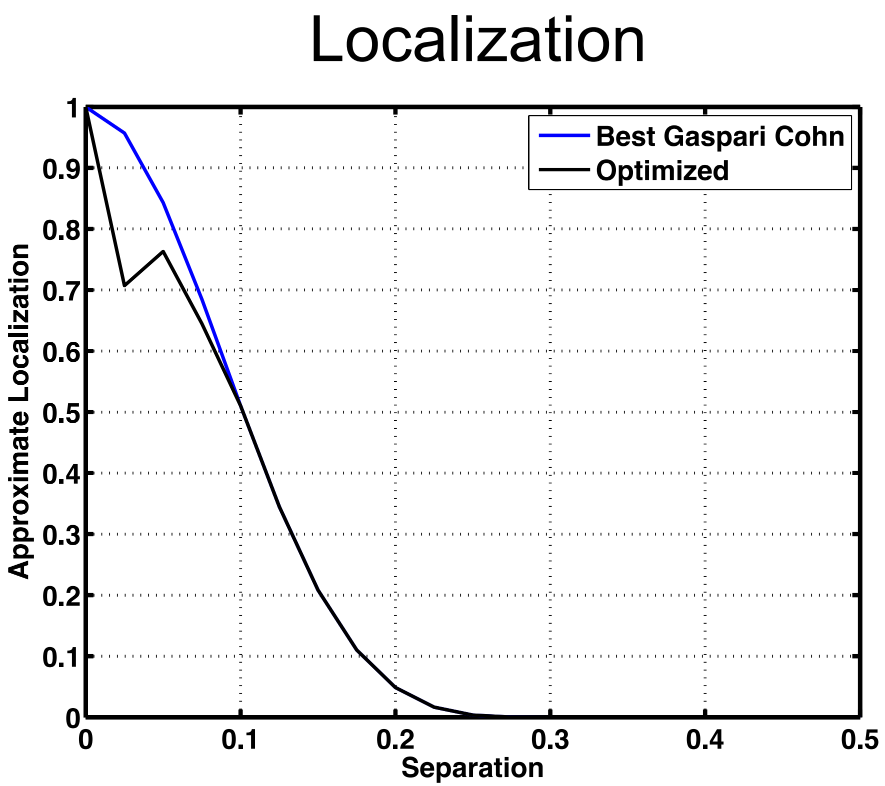
Observing all State every 12 Hours, Error Variance 1, N=20



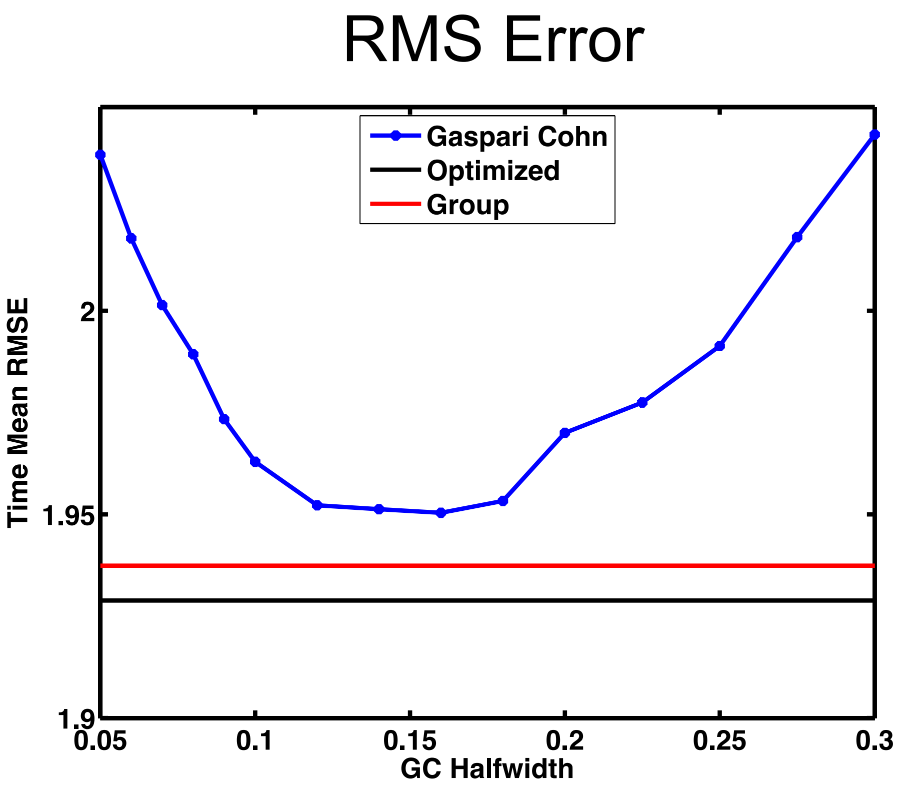


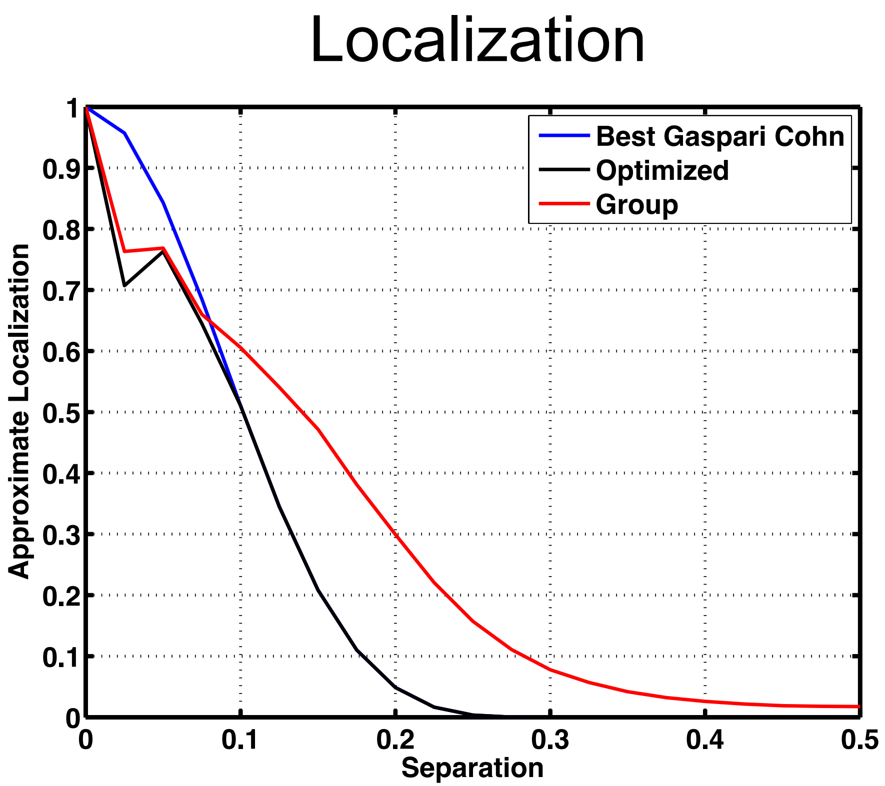
Observing all State every 12 Hours, Error Variance 1, N=20



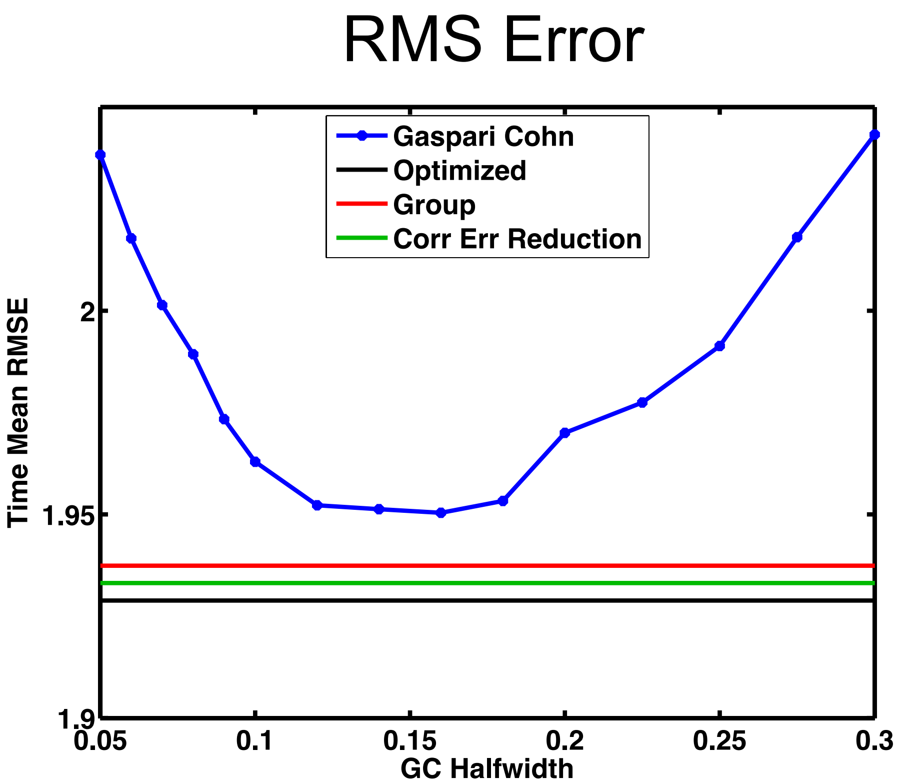


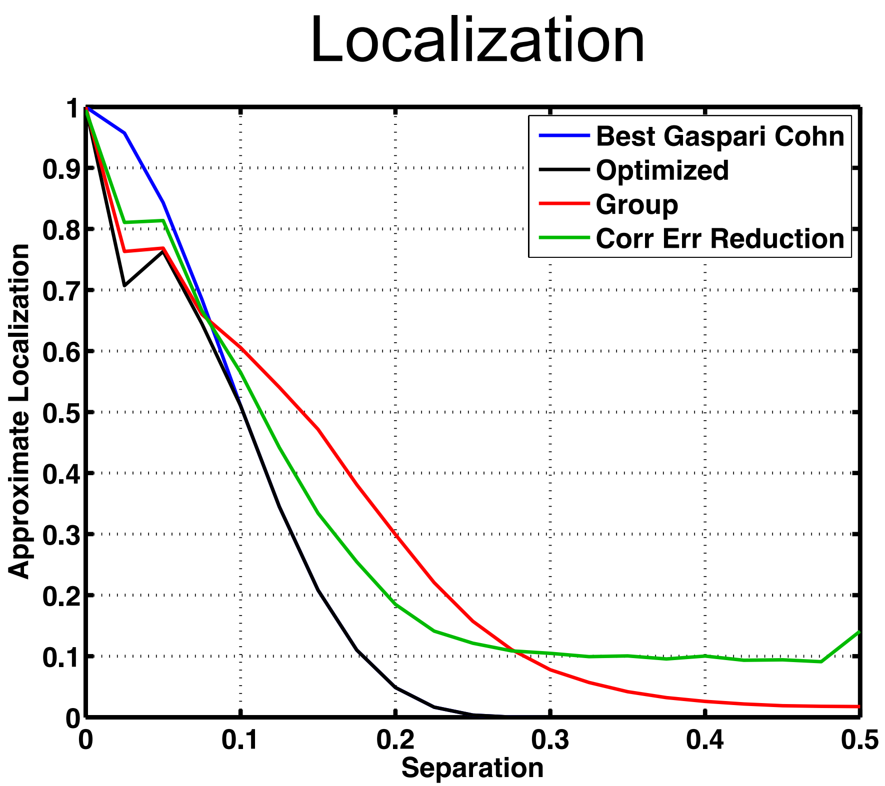
Observing all State every 12 Hours, Error Variance 1, N=20



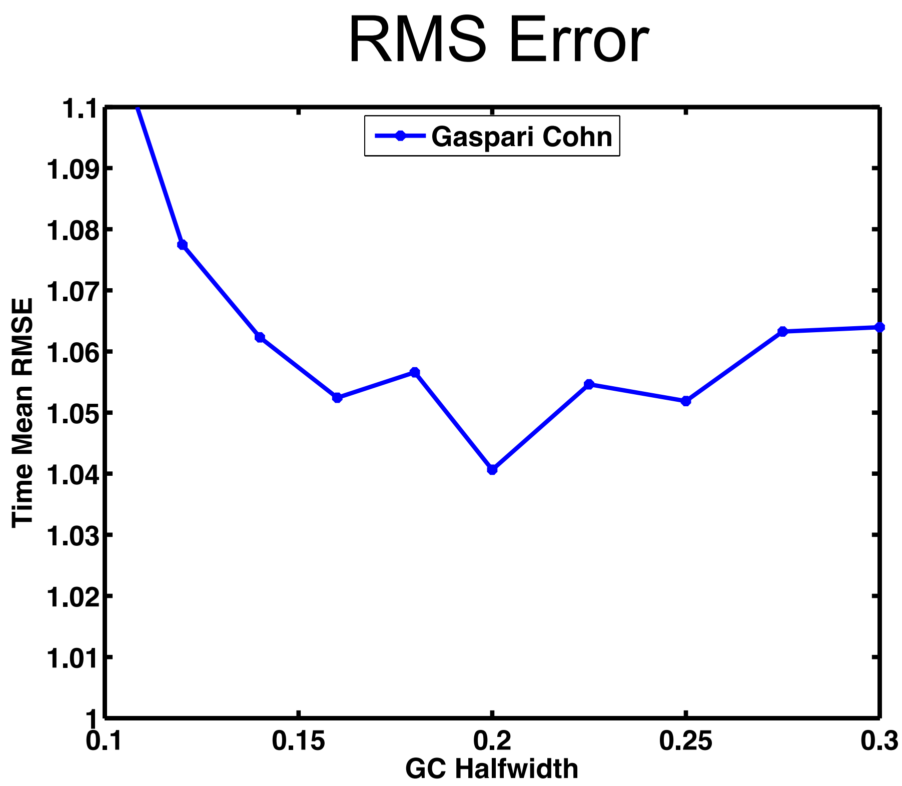


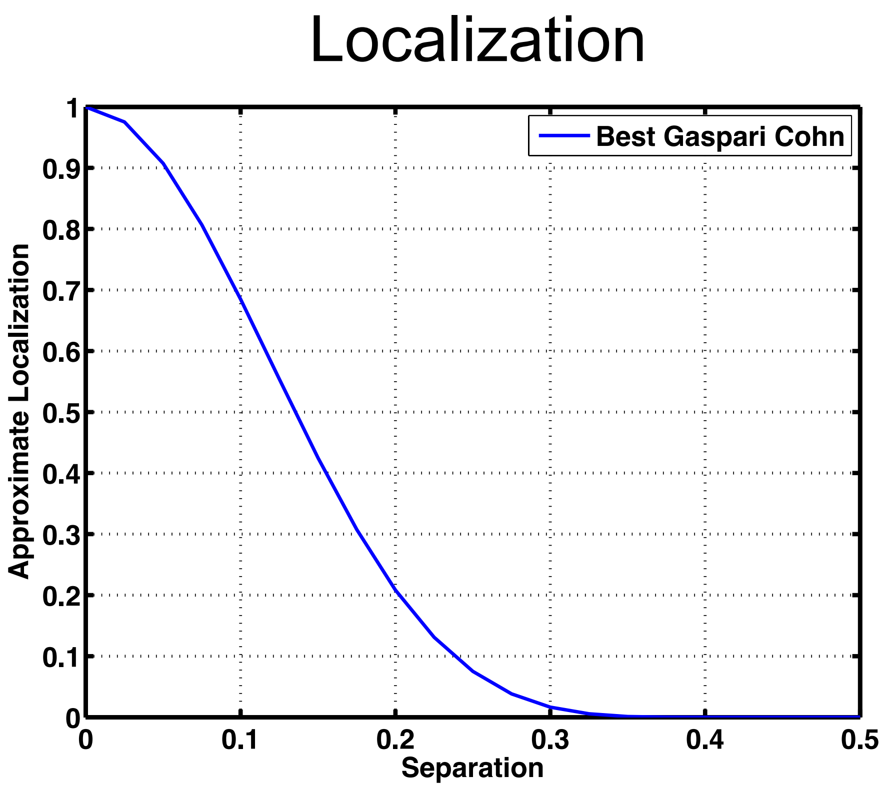
Observing all State every 12 Hours, Error Variance 1, N=20



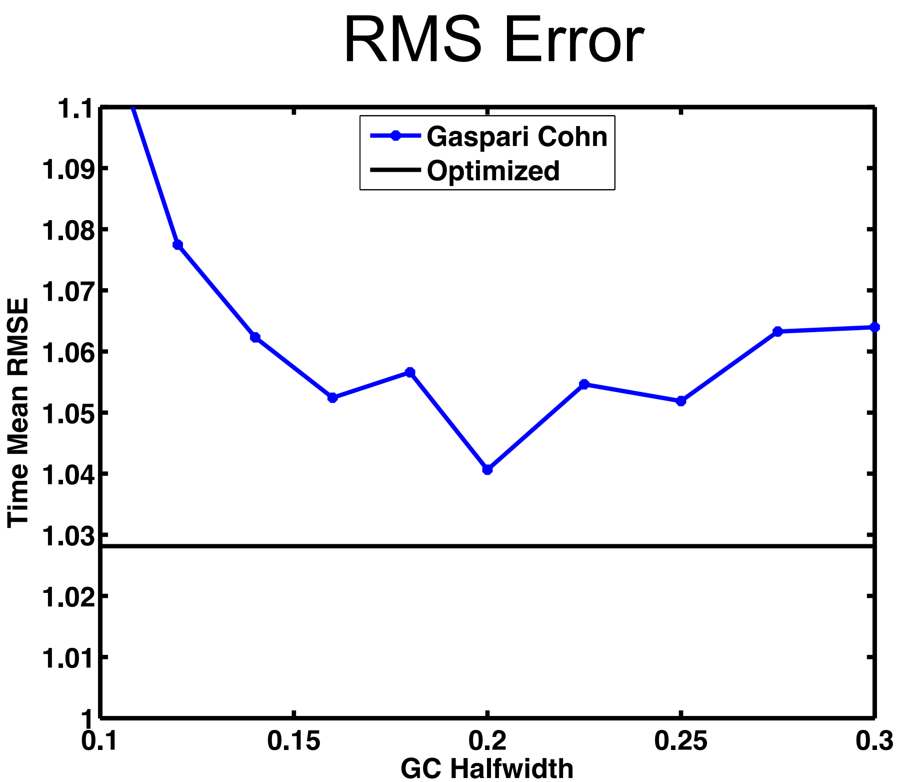


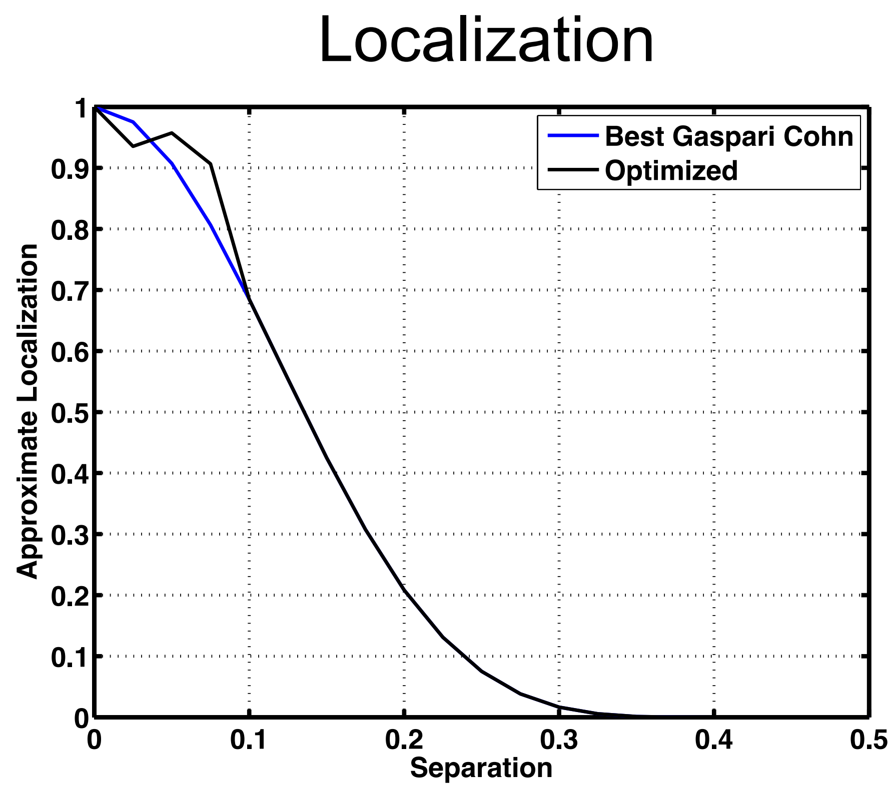
Observing all State every Hour, Error Variance 16, N=20



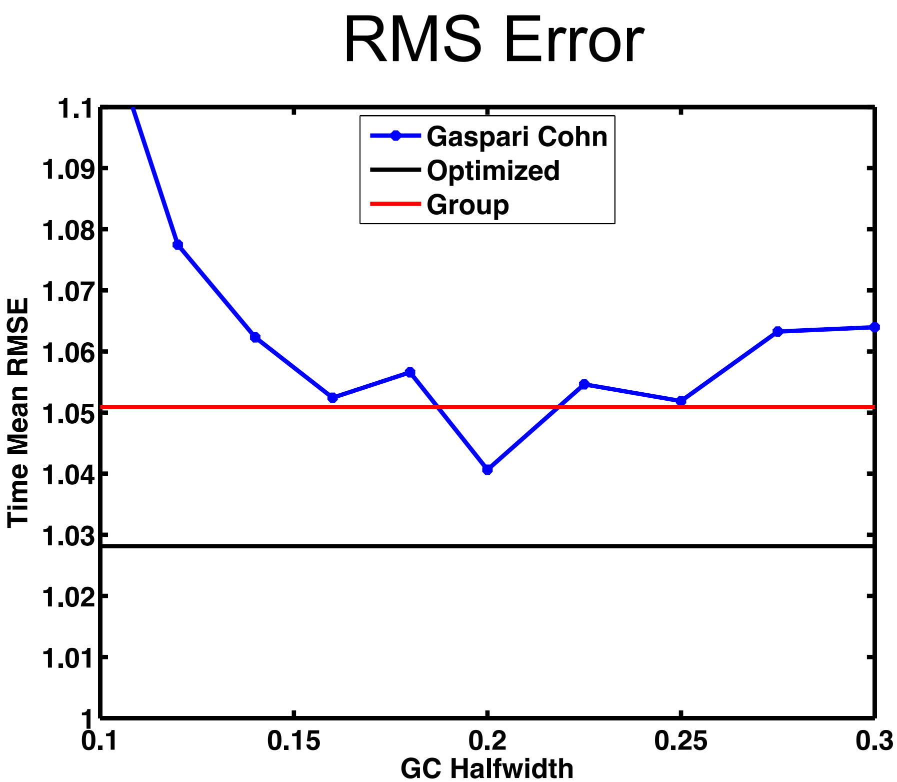


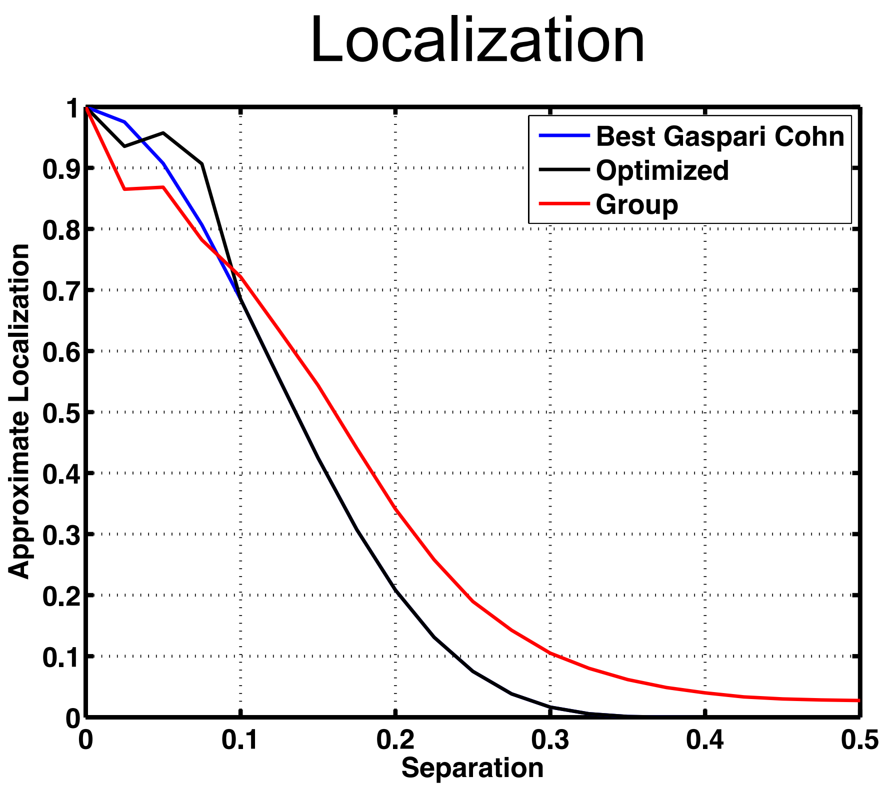
Observing all State every Hour, Error Variance 16, N=20



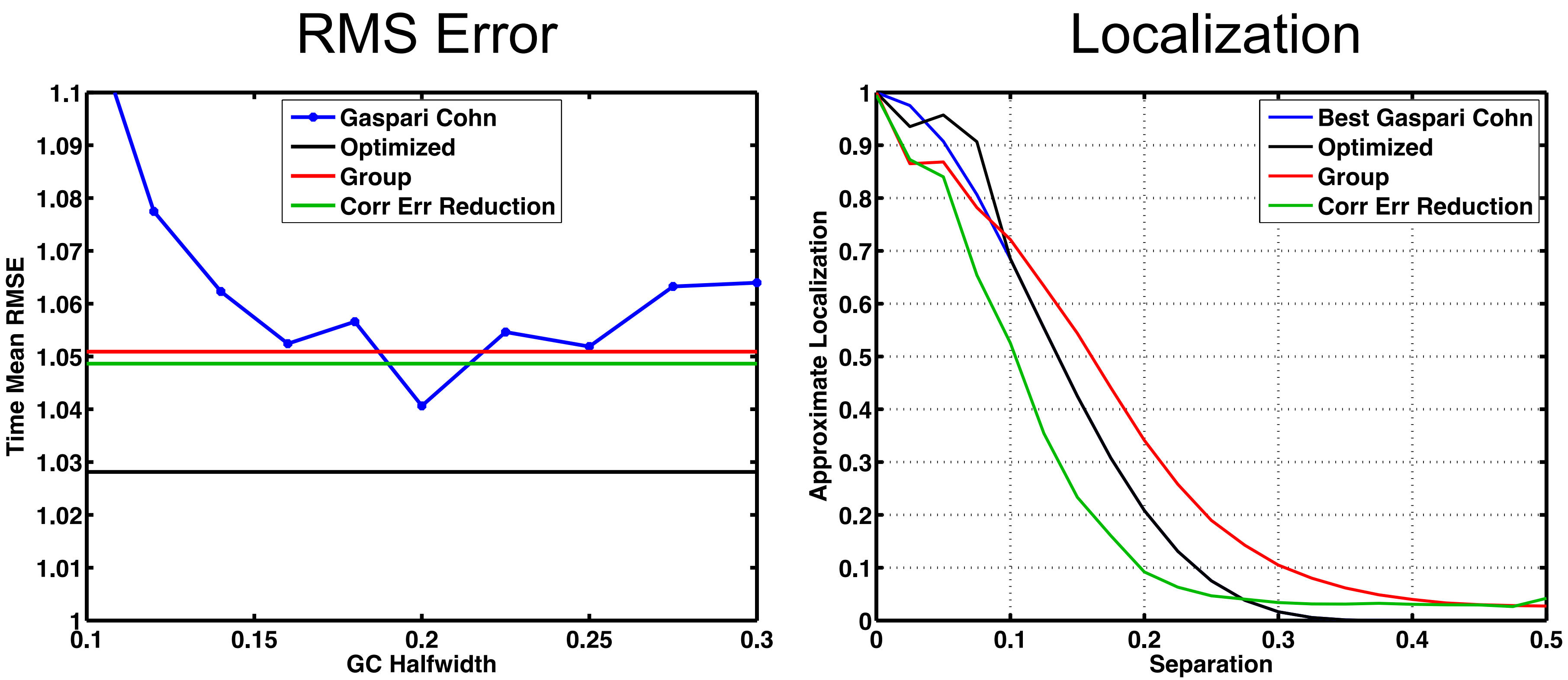


Observing all State every Hour, Error Variance 16, N=20

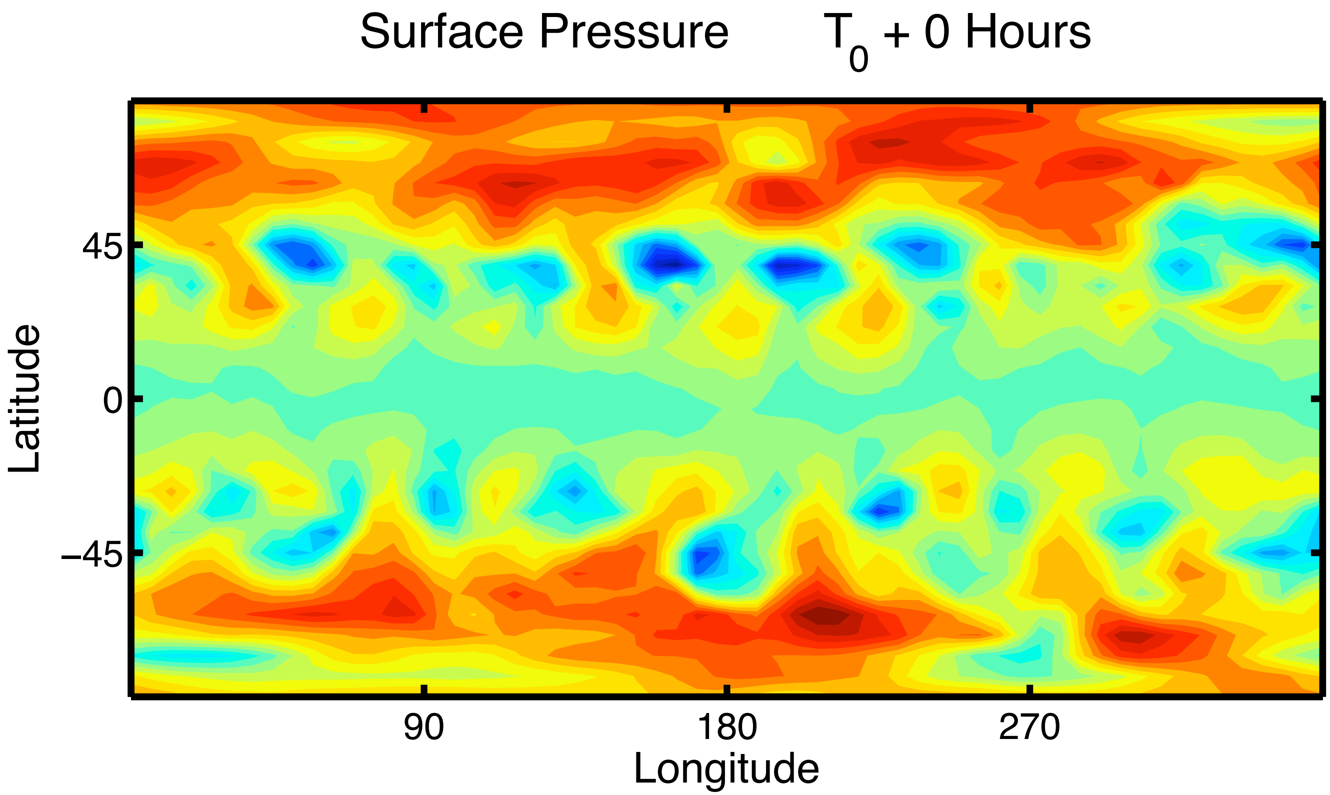




Observing all State every Hour, Error Variance 16, N=20

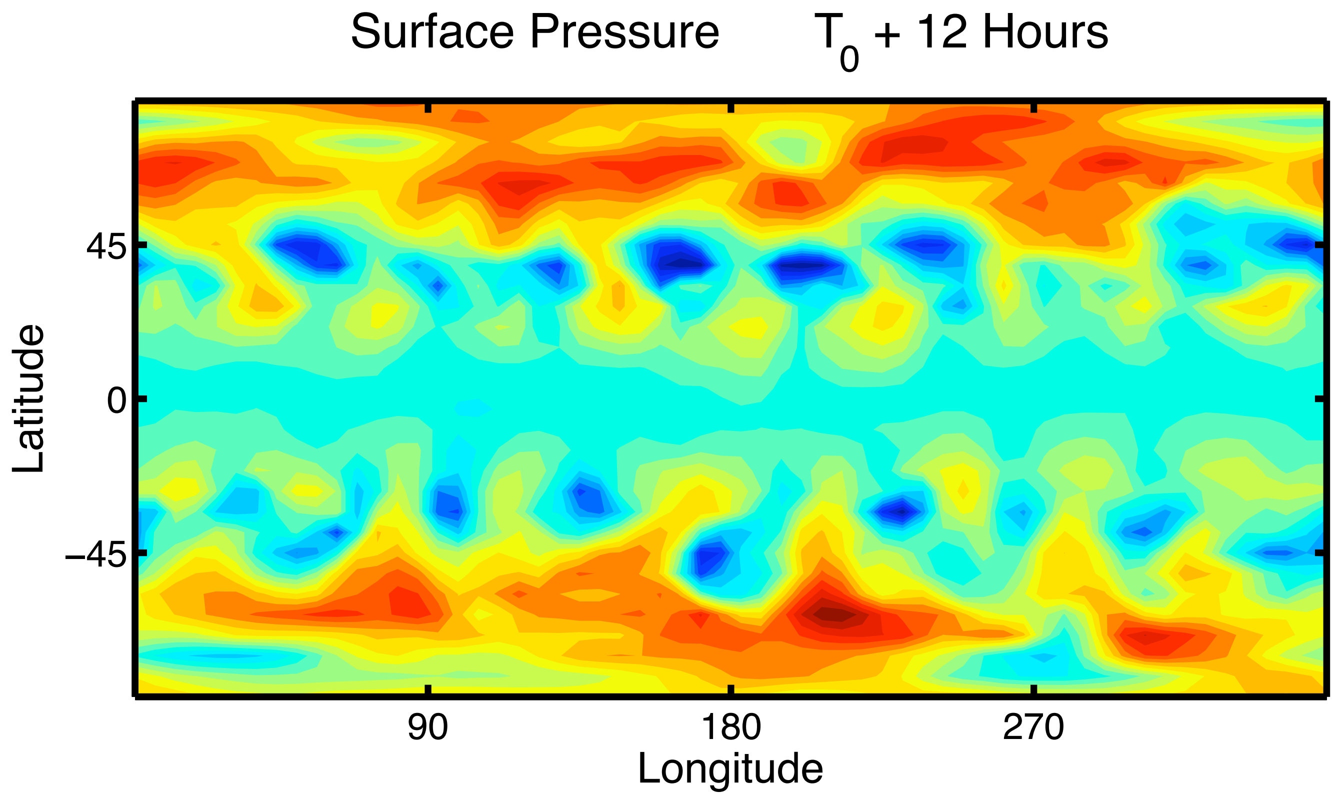


Low-Order Dry Dynamical Core



Evolution of surface pressure field every 12 hours.
Has baroclinic instability: storms move east in midlatitudes.

Low-Order Dry Dynamical Core



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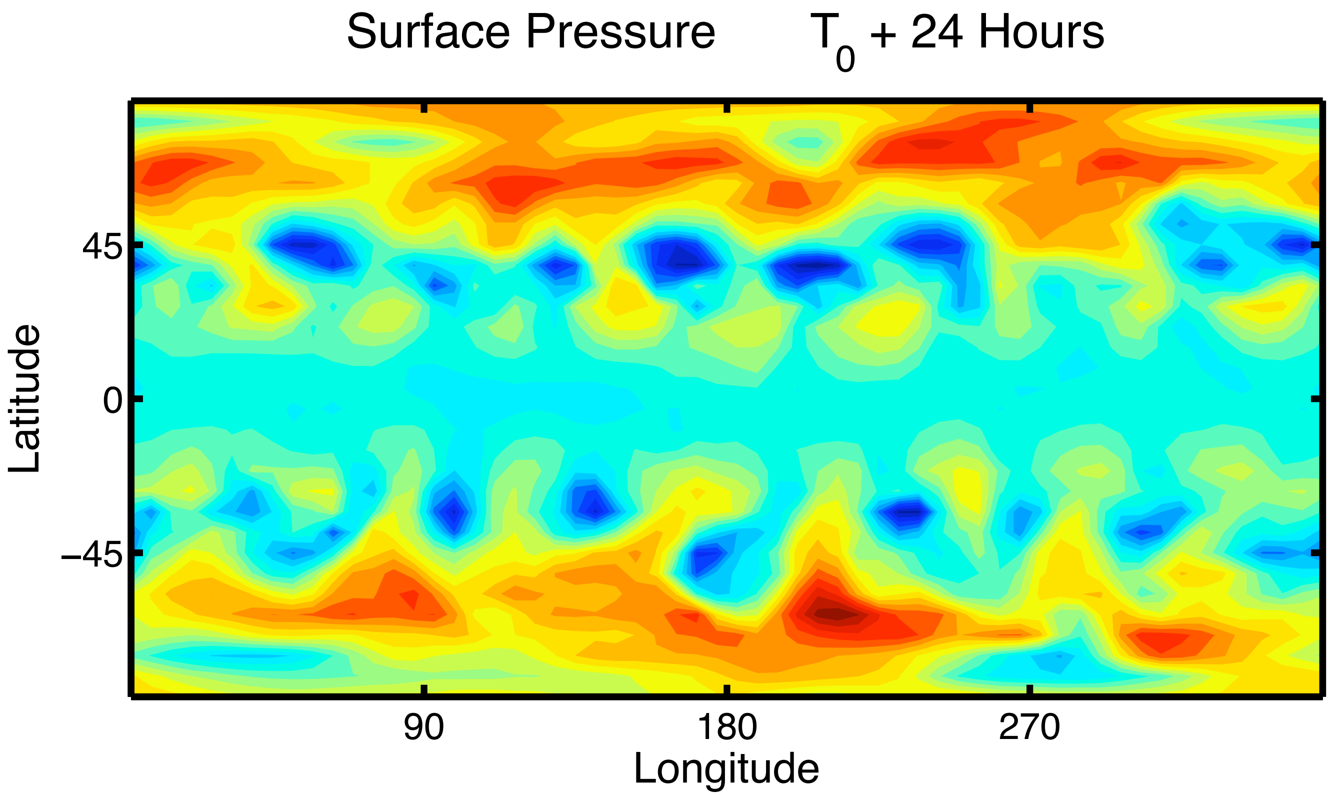

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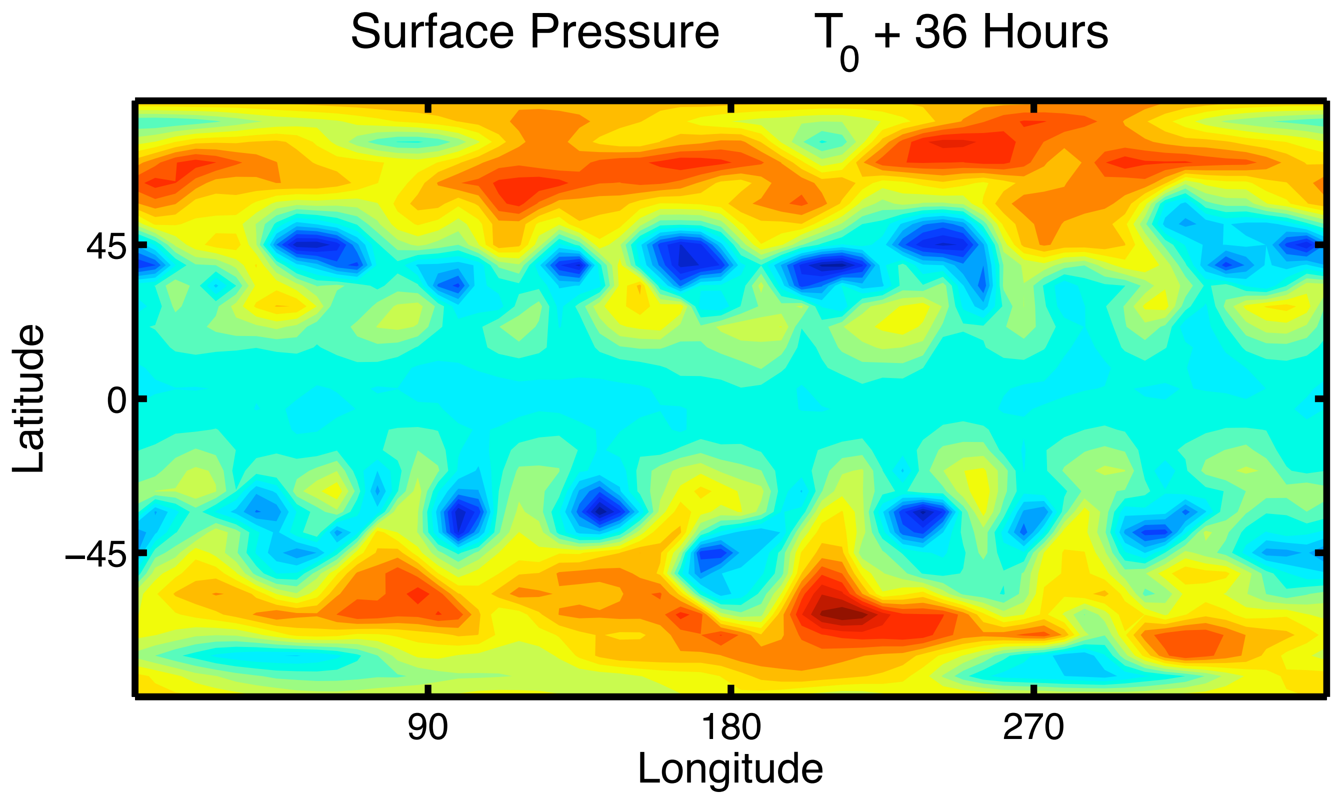
Low-Order Dry Dynamical Core



Evolution of surface pressure field every 12 hours.
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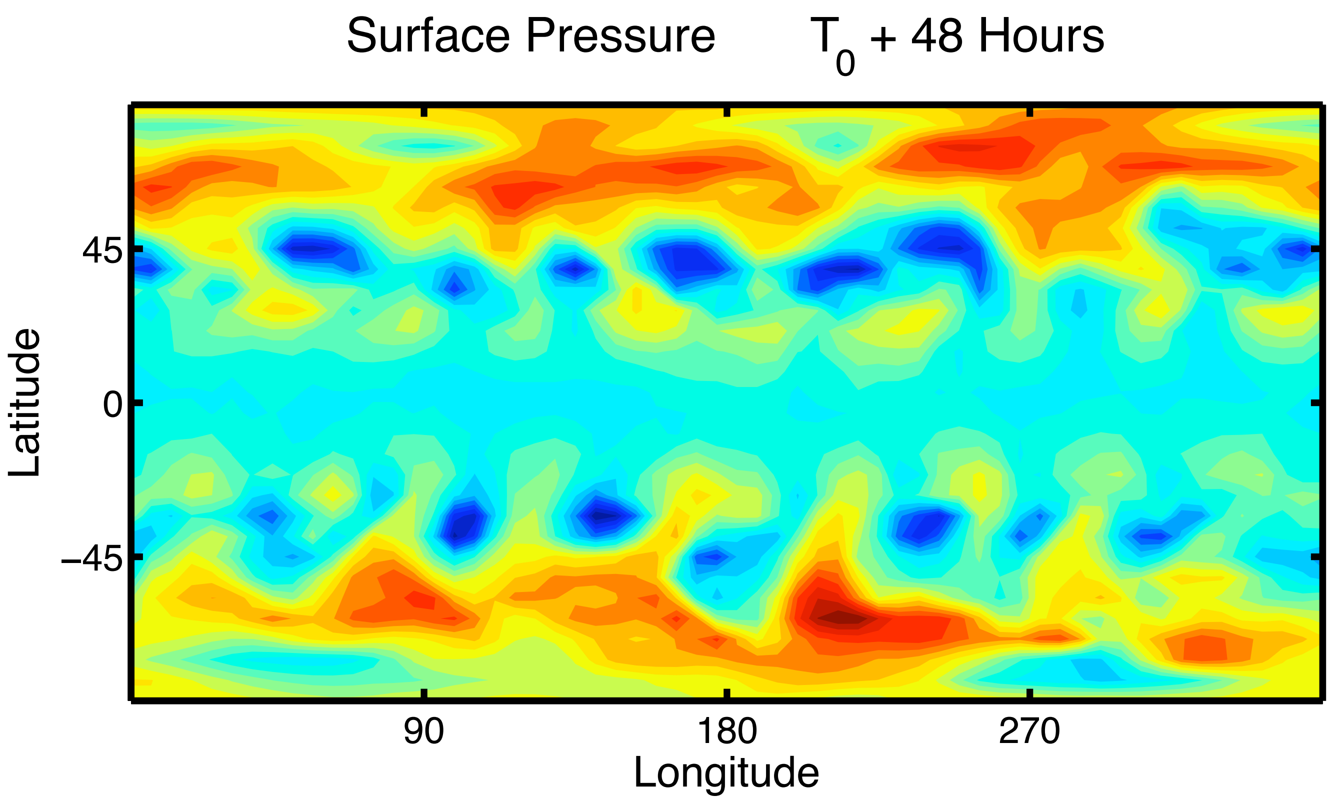
Low-Order Dry Dynamical Core



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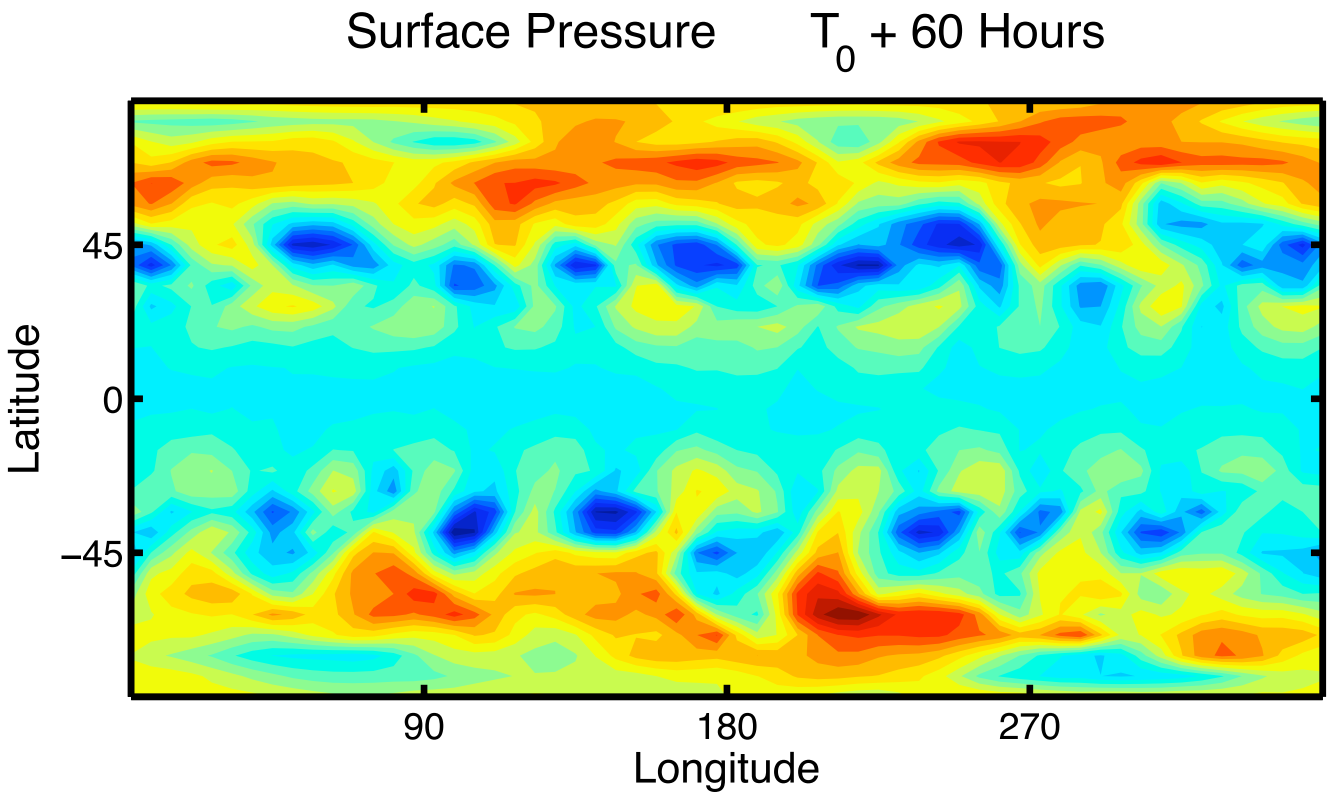
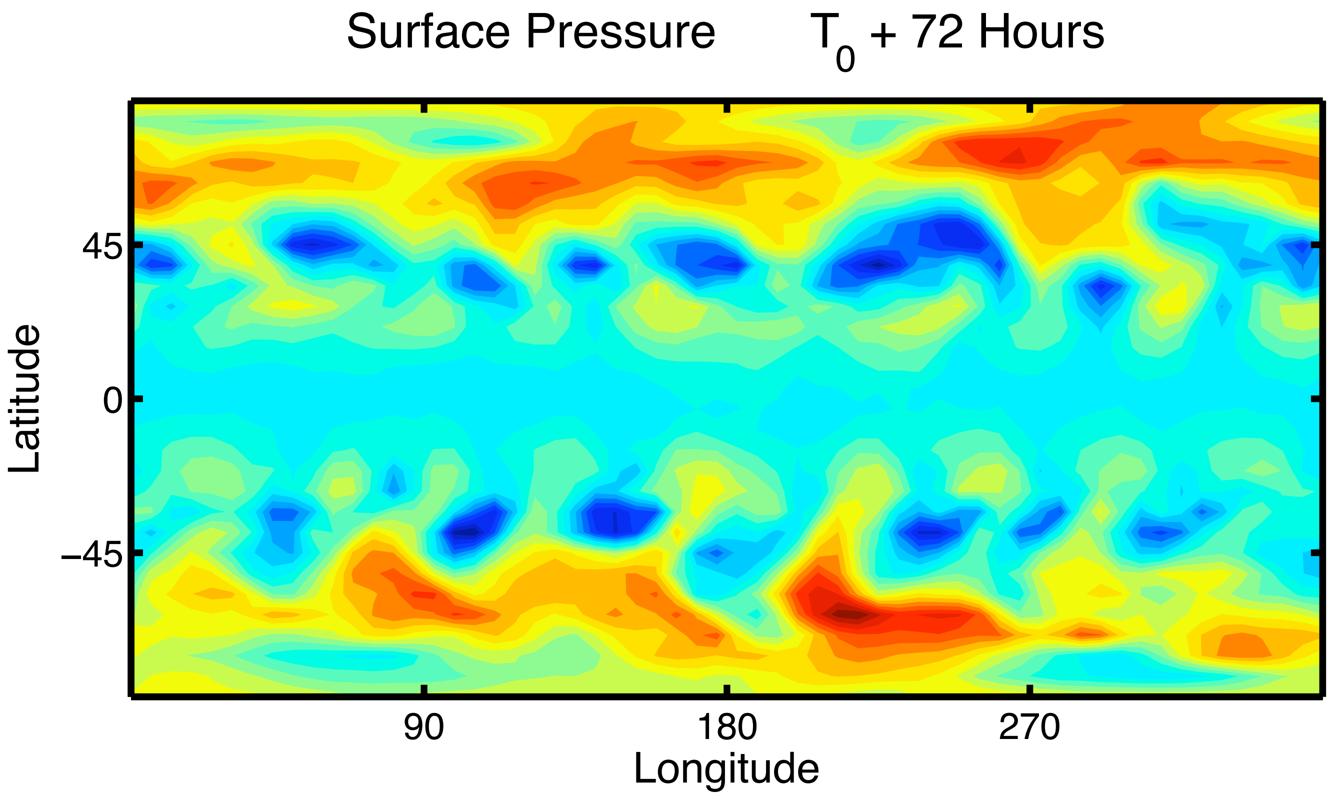
Low-Order Dry Dynamical Core



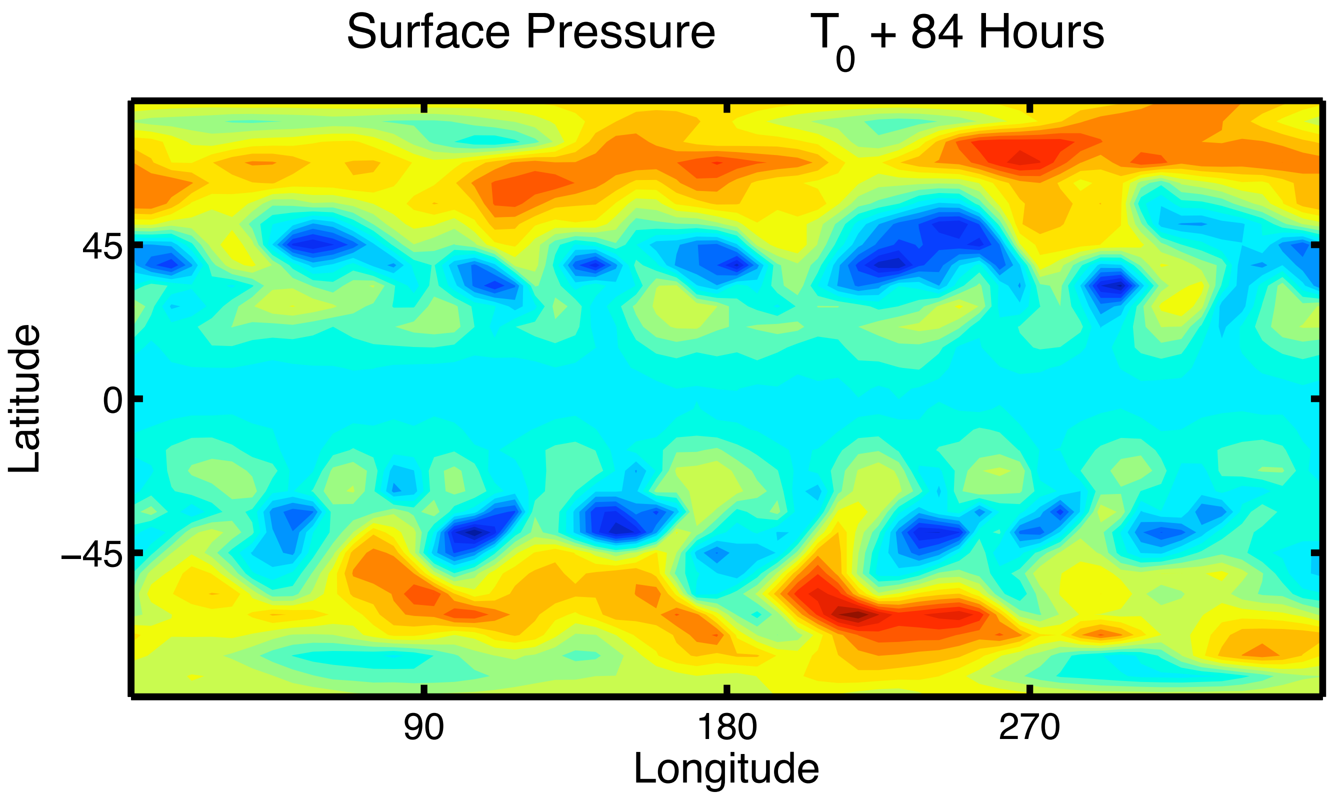
Evolution of surface pressure field every 12 hours.
Has baroclinic instability: storms move east in midlatitudes.

Low-Order Dry Dynamical Core



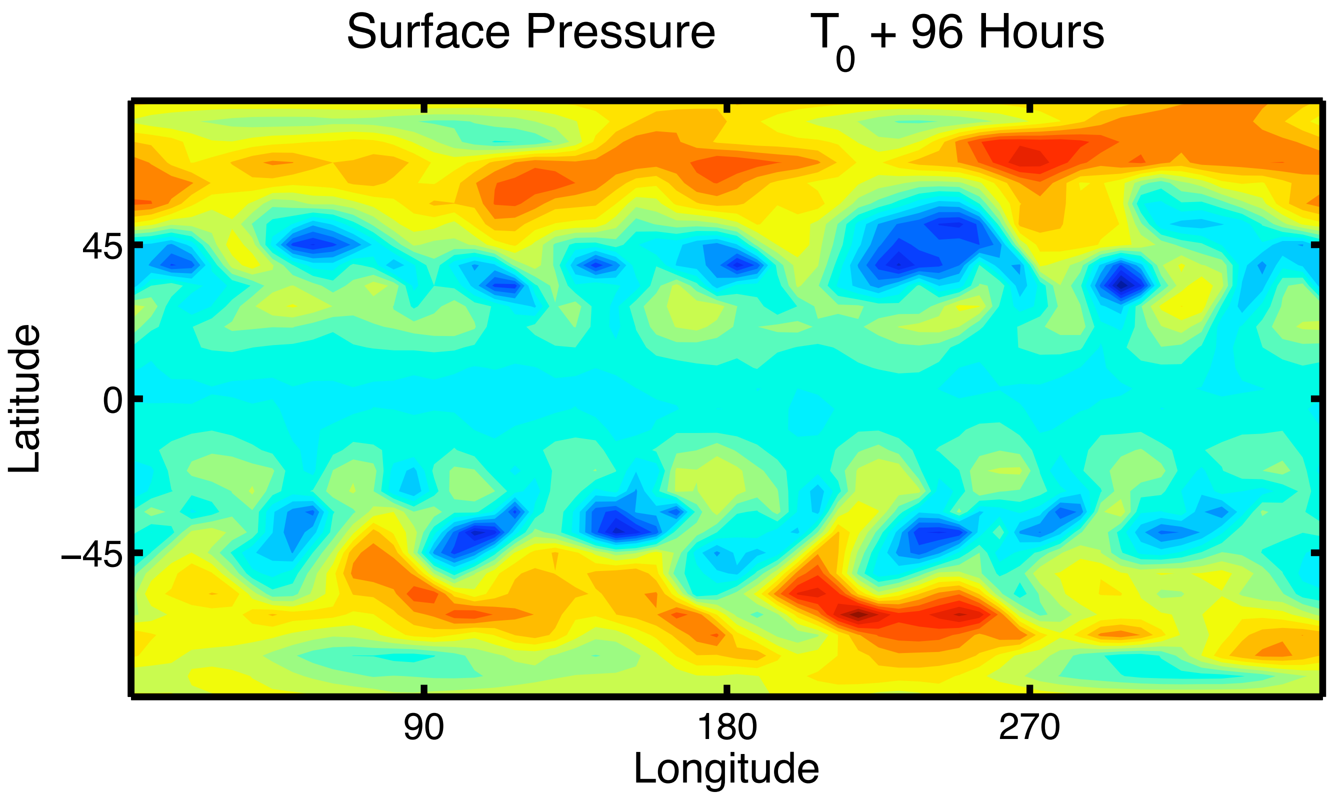
Evolution of surface pressure field every 12 hours.
Has baroclinic instability: storms move east in midlatitudes.

Low-Order Dry Dynamical Core



Evolution of surface pressure field every 12 hours.
Has baroclinic instability: storms move east in midlatitudes.

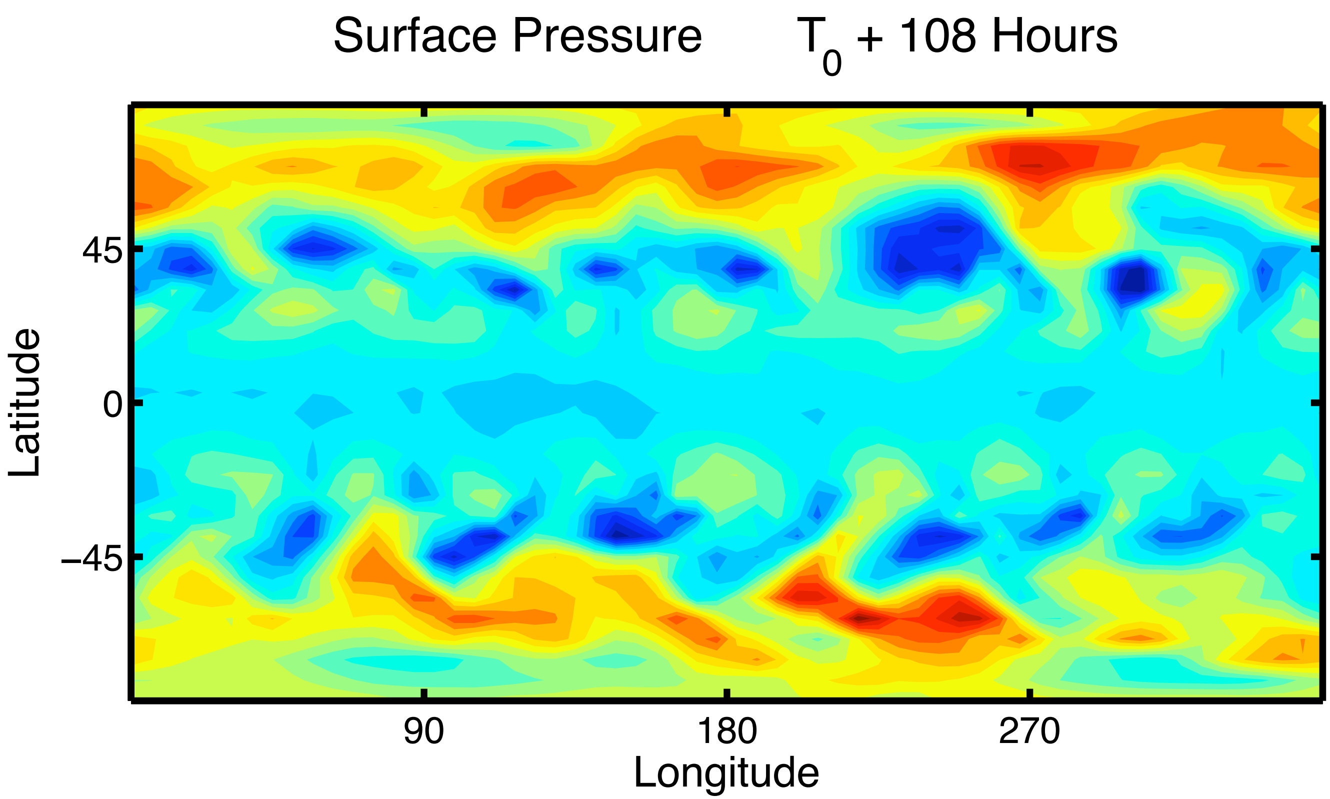
Low-Order Dry Dynamical Core



Evolution of surface pressure field every 12 hours.

Has baroclinic instability: storms move east in midlatitudes.

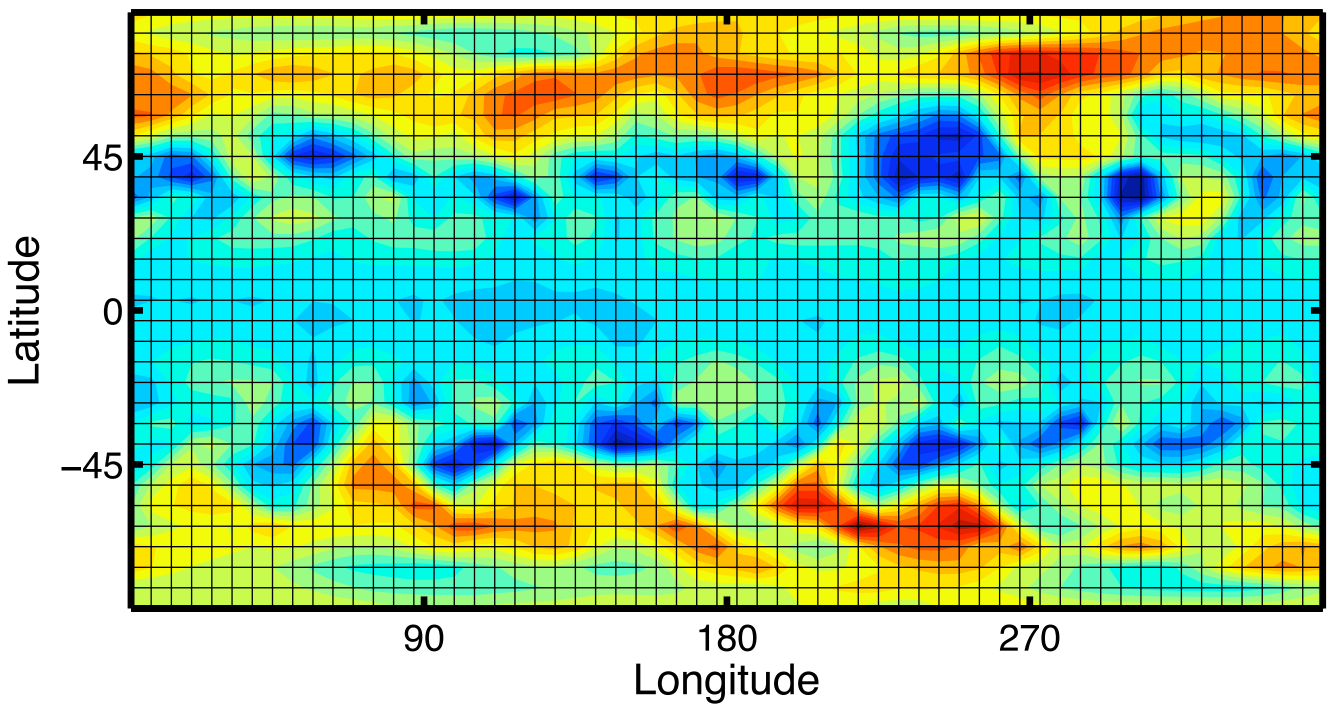
Low-Order Dry Dynamical Core



Evolution of surface pressure field every 12 hours.
Has baroclinic instability: storms move east in midlatitudes.

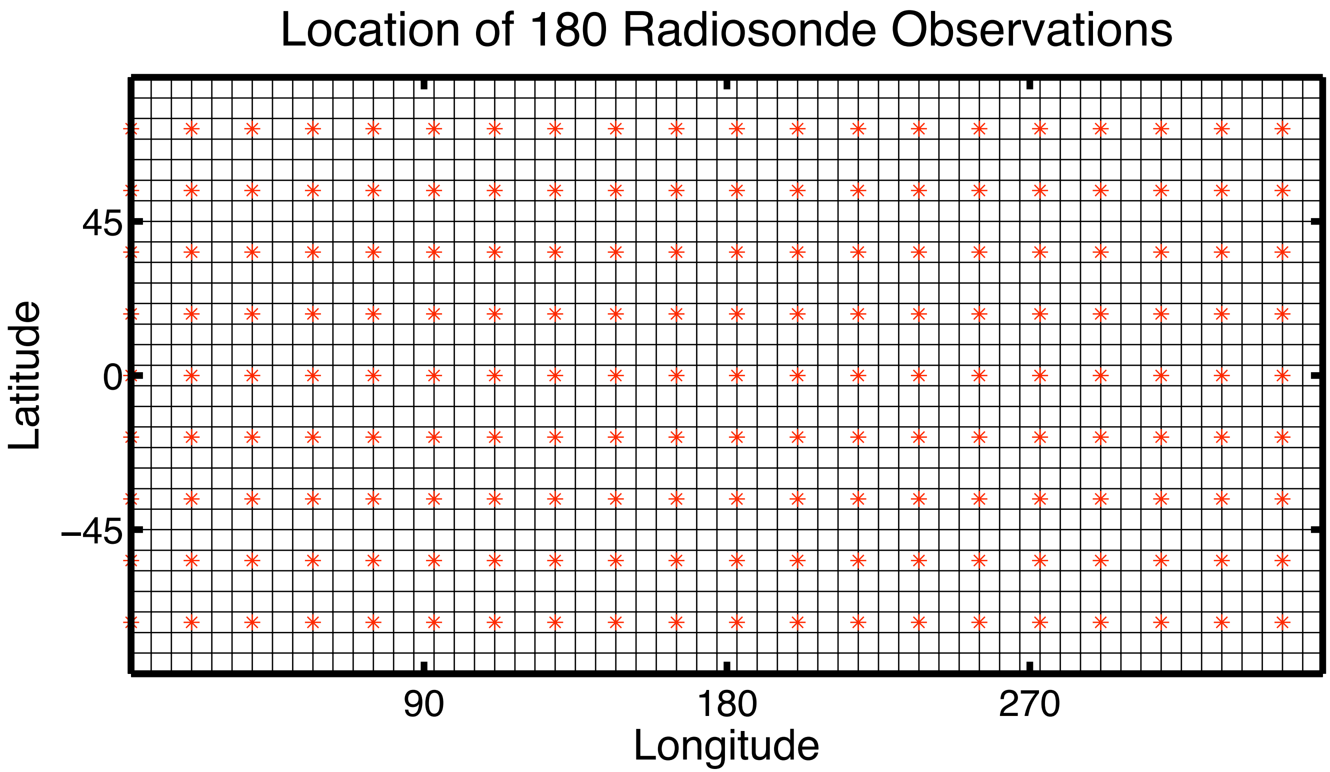
Low-Order Dry Dynamical Core: Grid

Location of 30 x 60 Model Grid



30x60 horizontal grid, 5 levels.
Surface pressure, temperature, wind components.
28,800 variables.

Low-Order Dry Dynamical Core: Observations



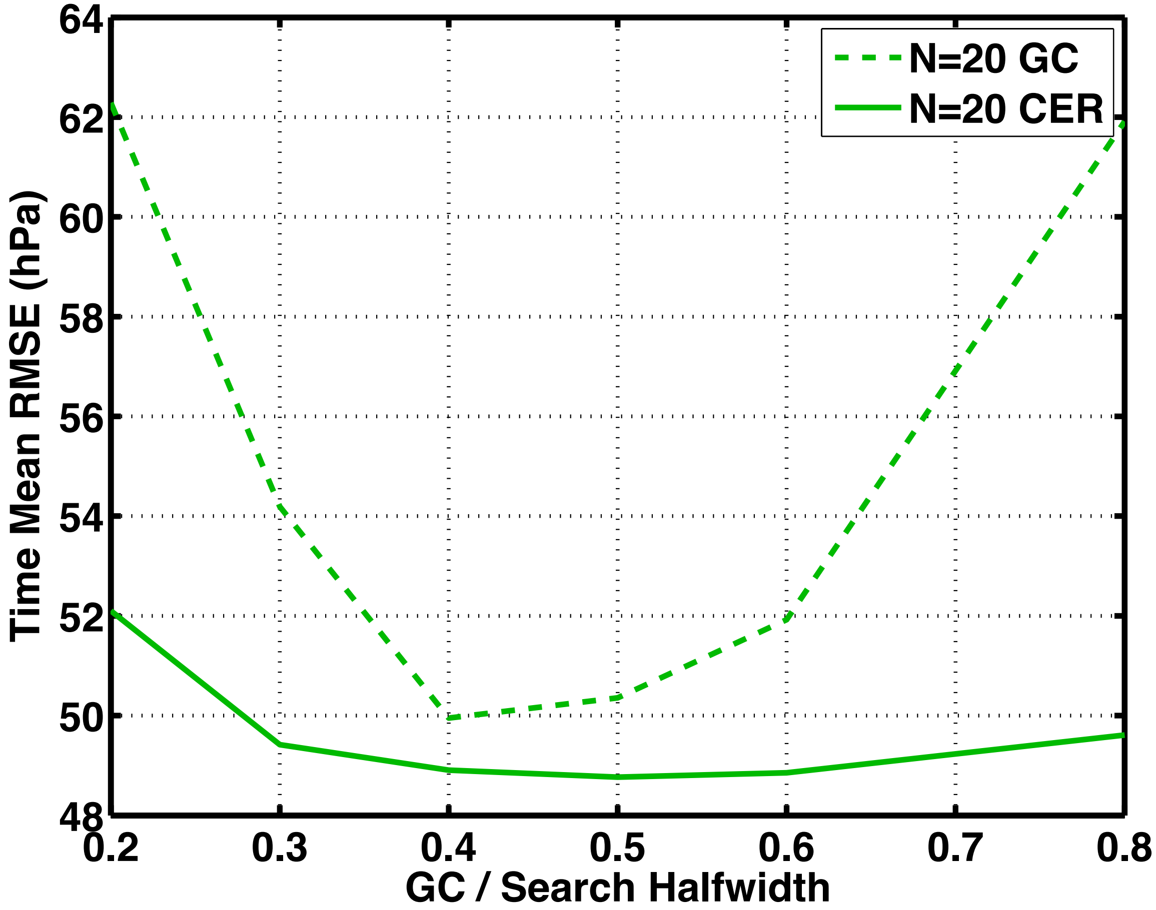
Observe every 12 hours for 200 days.
Observe all 16 variables in 180 columns shown.
Error SD: Ps=2hPa, T=3K, winds=3m/s.

Low-Order Dry Dynamical Core

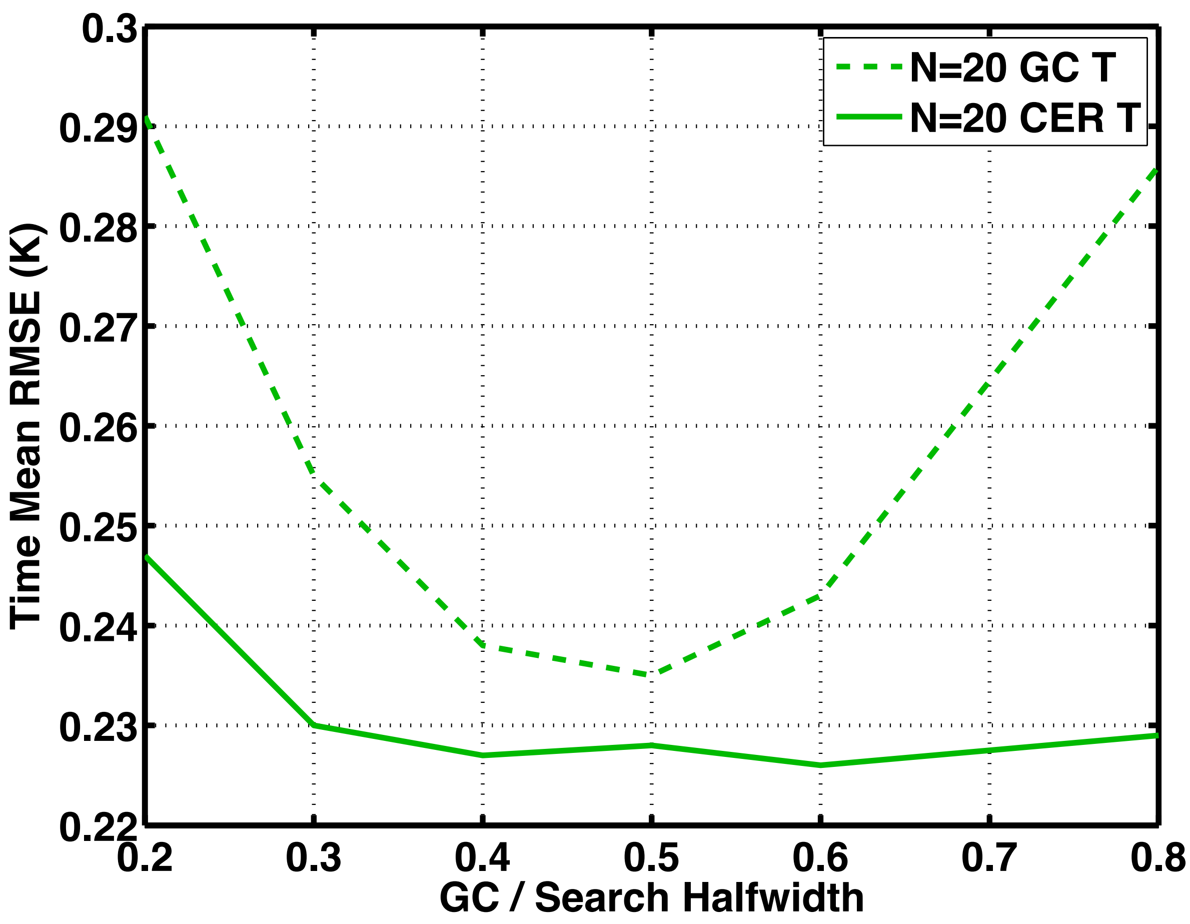
Limit observation impacts to a given halfwidth:

- Reduces computational costs.
- Limits number of very small correlation pairs.

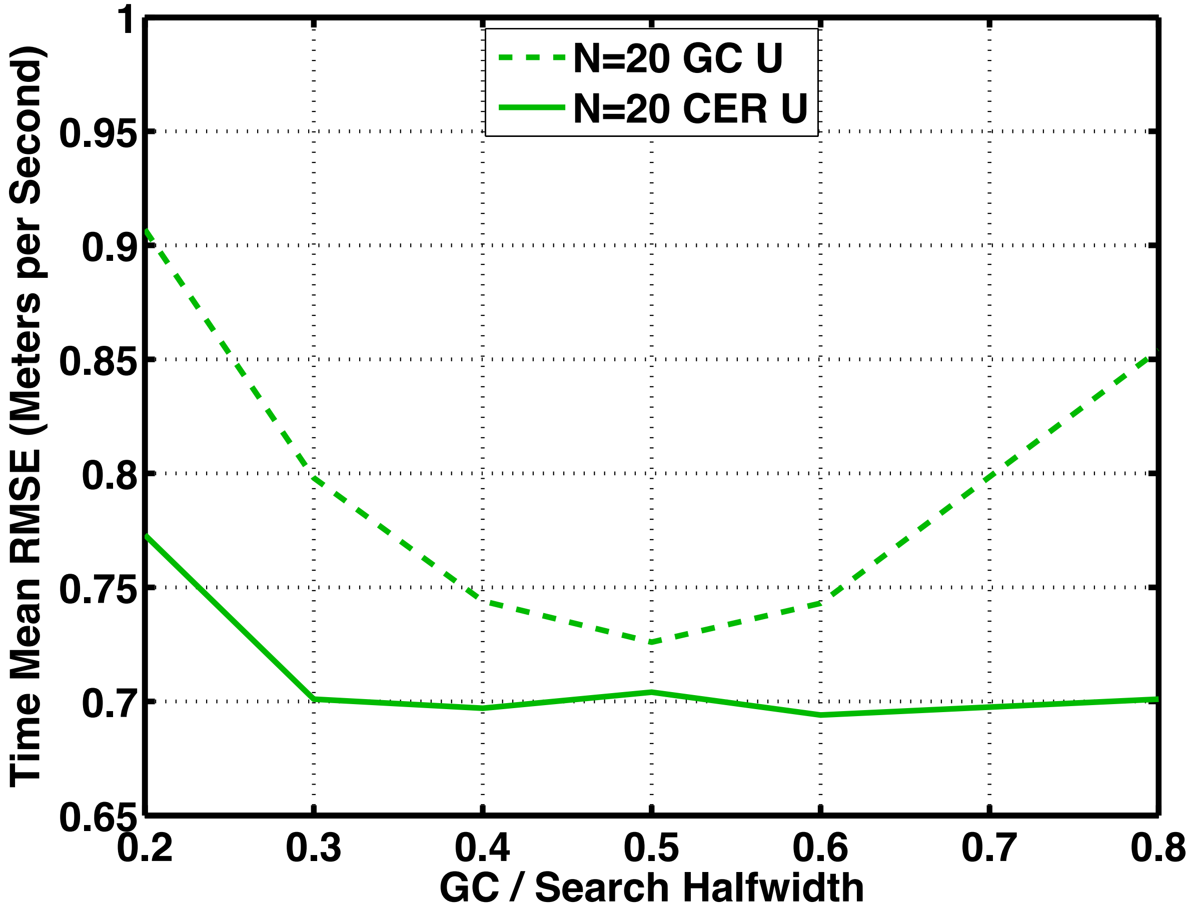
Prior RMSE for Surface Pressure, 20 Member Ensemble



Prior RMSE for Level 3 Temperature, 20 Member Ensemble

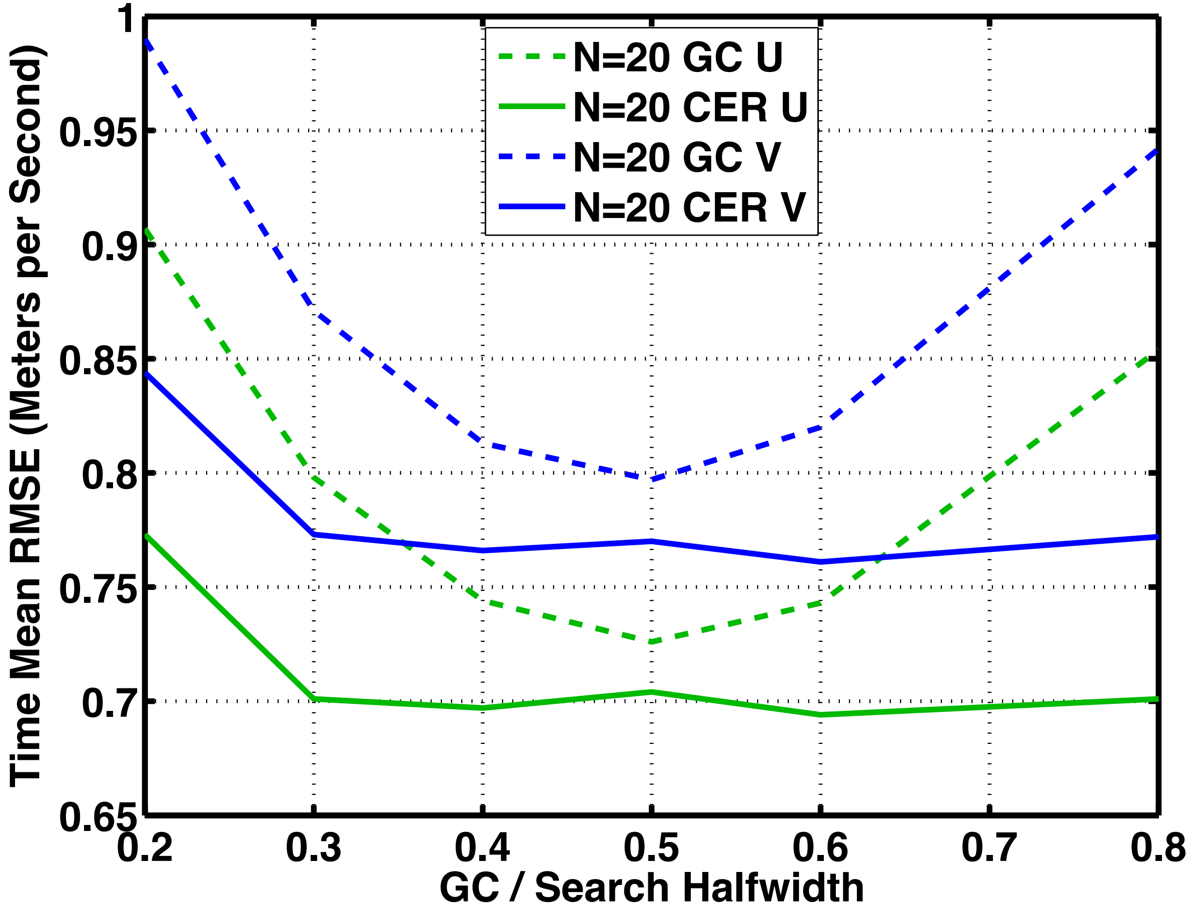


Prior RMSE for Level 3 Wind, 20 Member Ensemble



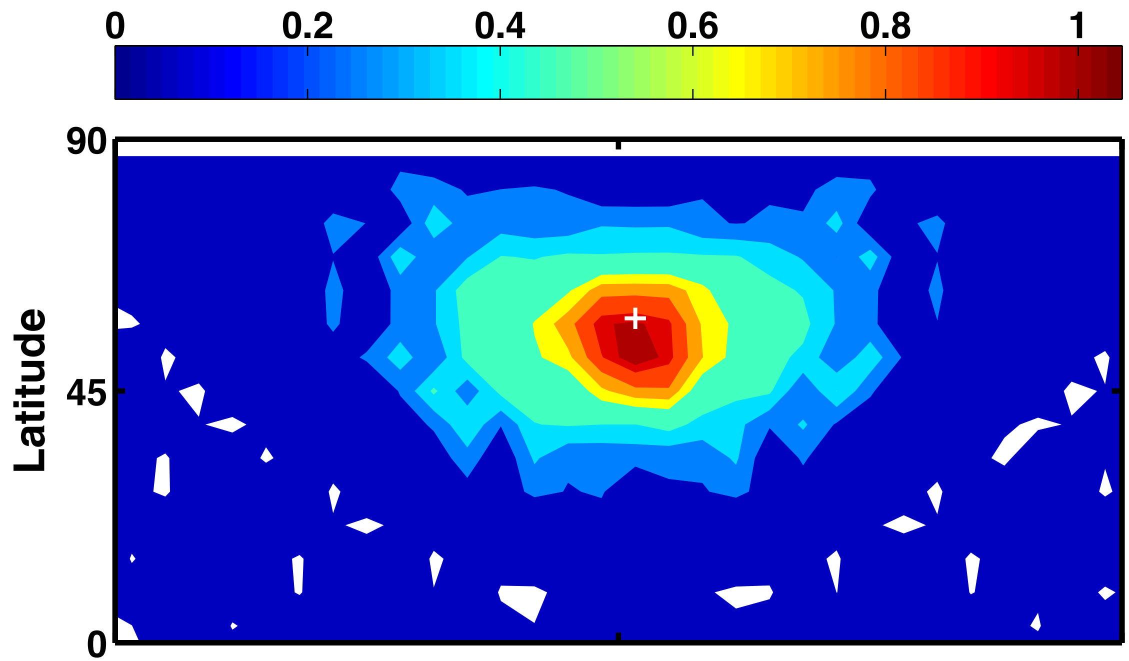
Low-Order Dry Dynamical Core

Prior RMSE for Level 3 Wind, 20 Member Ensemble

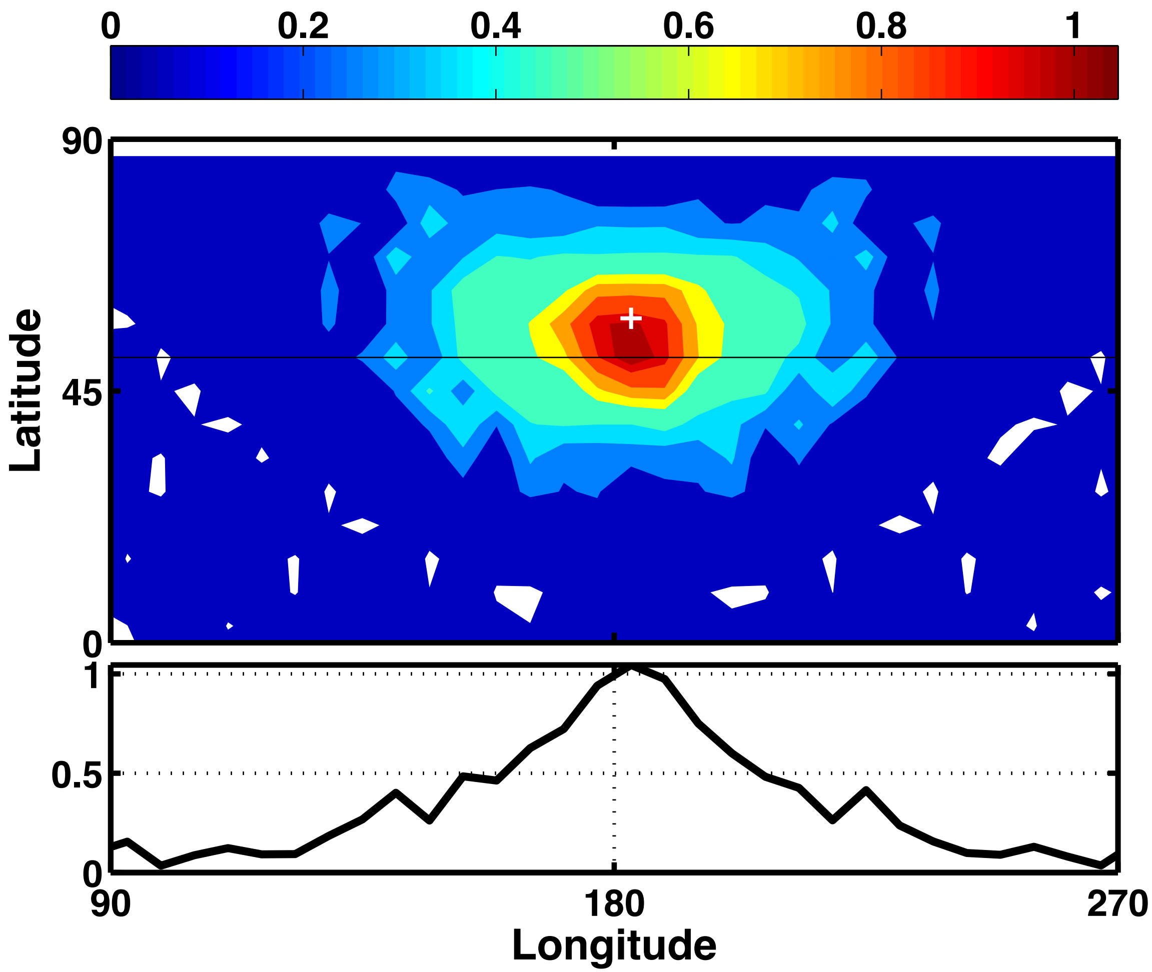


Low-Order Dry Dynamical Core

Localization of Level 3 T Observation on Level 3 T State

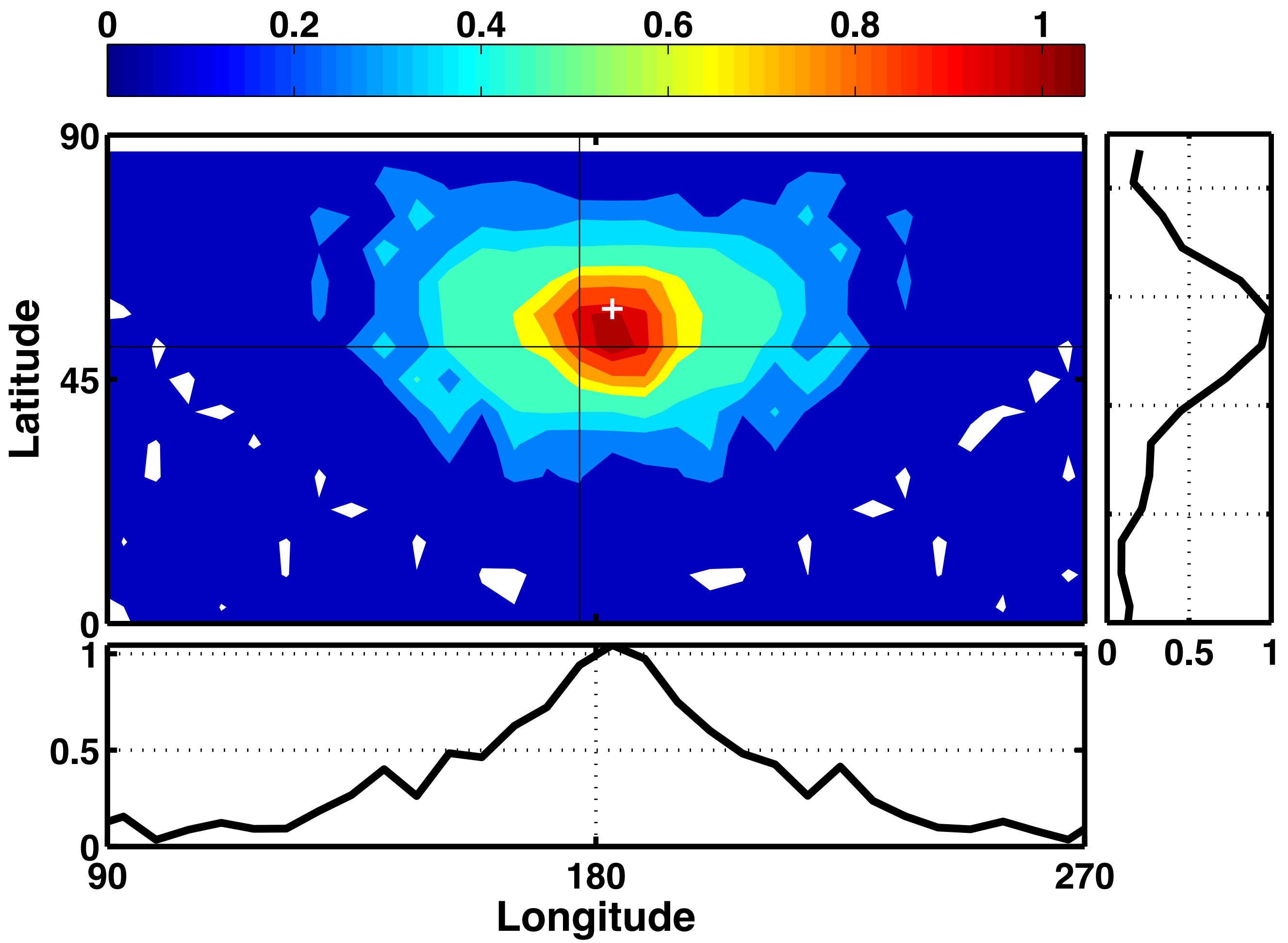


Localization of Level 3 T Observation on Level 3 T State

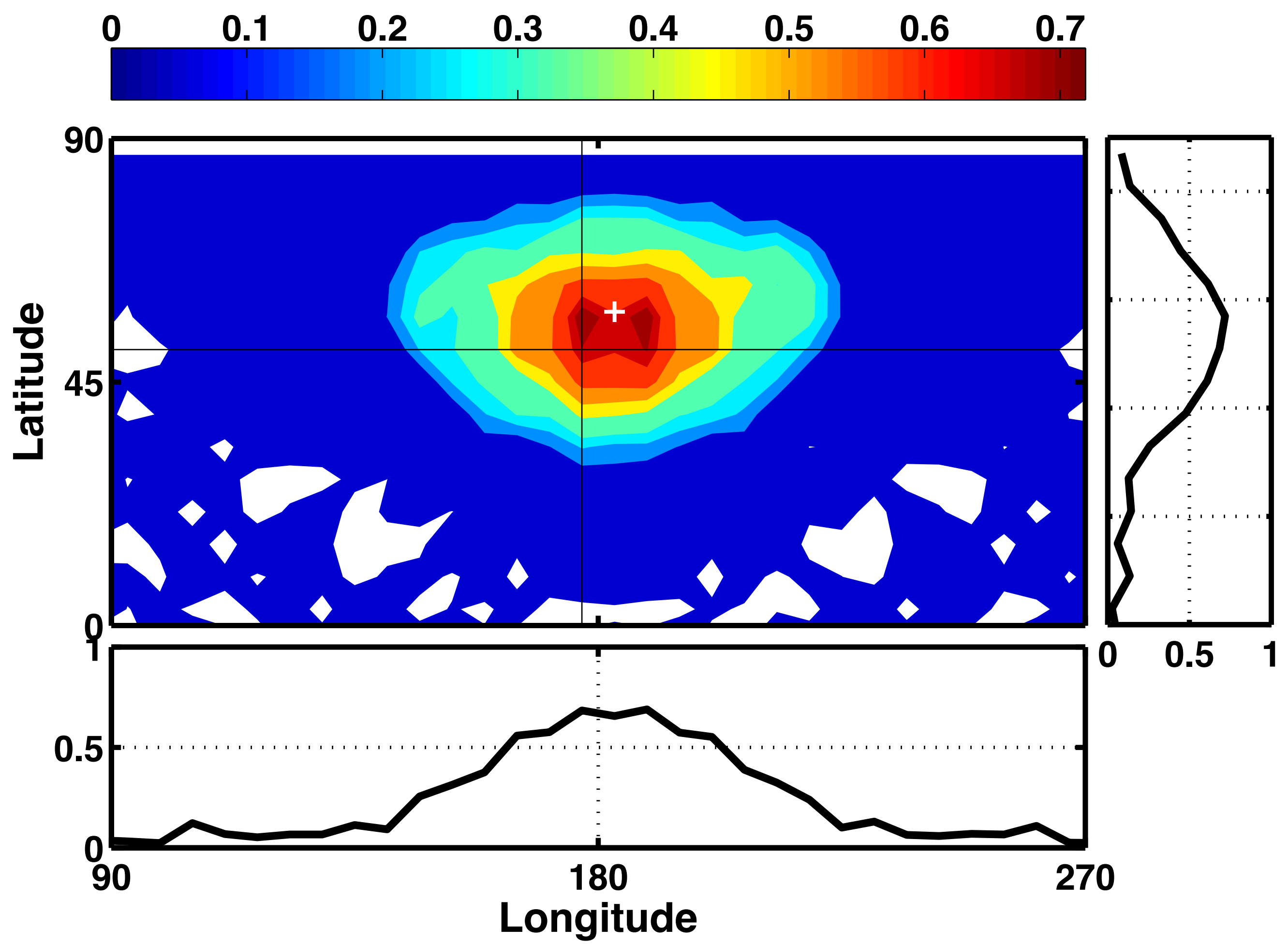


Low-Order Dry Dynamical Core

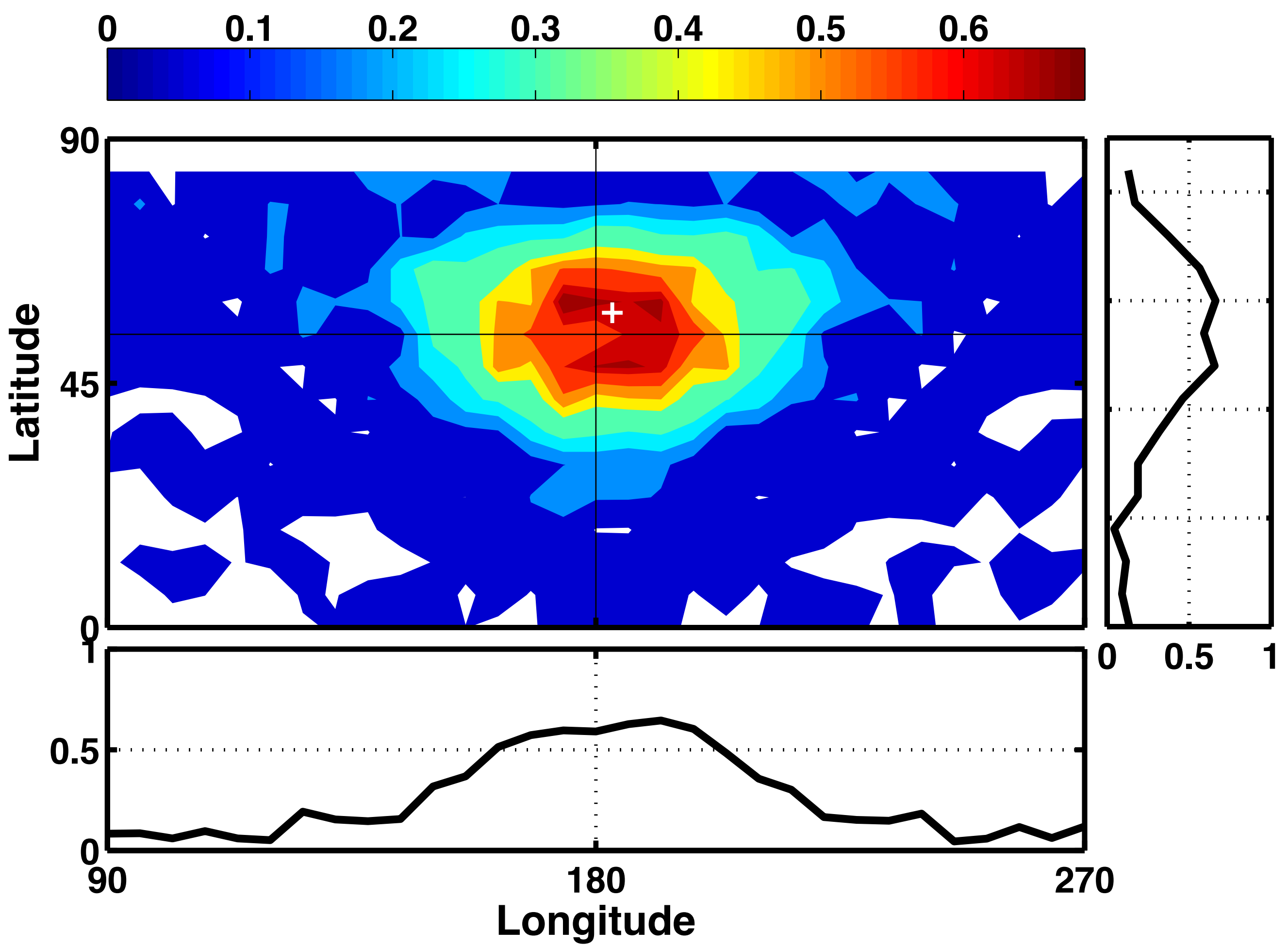
Localization of Level 3 T Observation on Level 3 T State



Localization of Level 3 U Observation on PS State

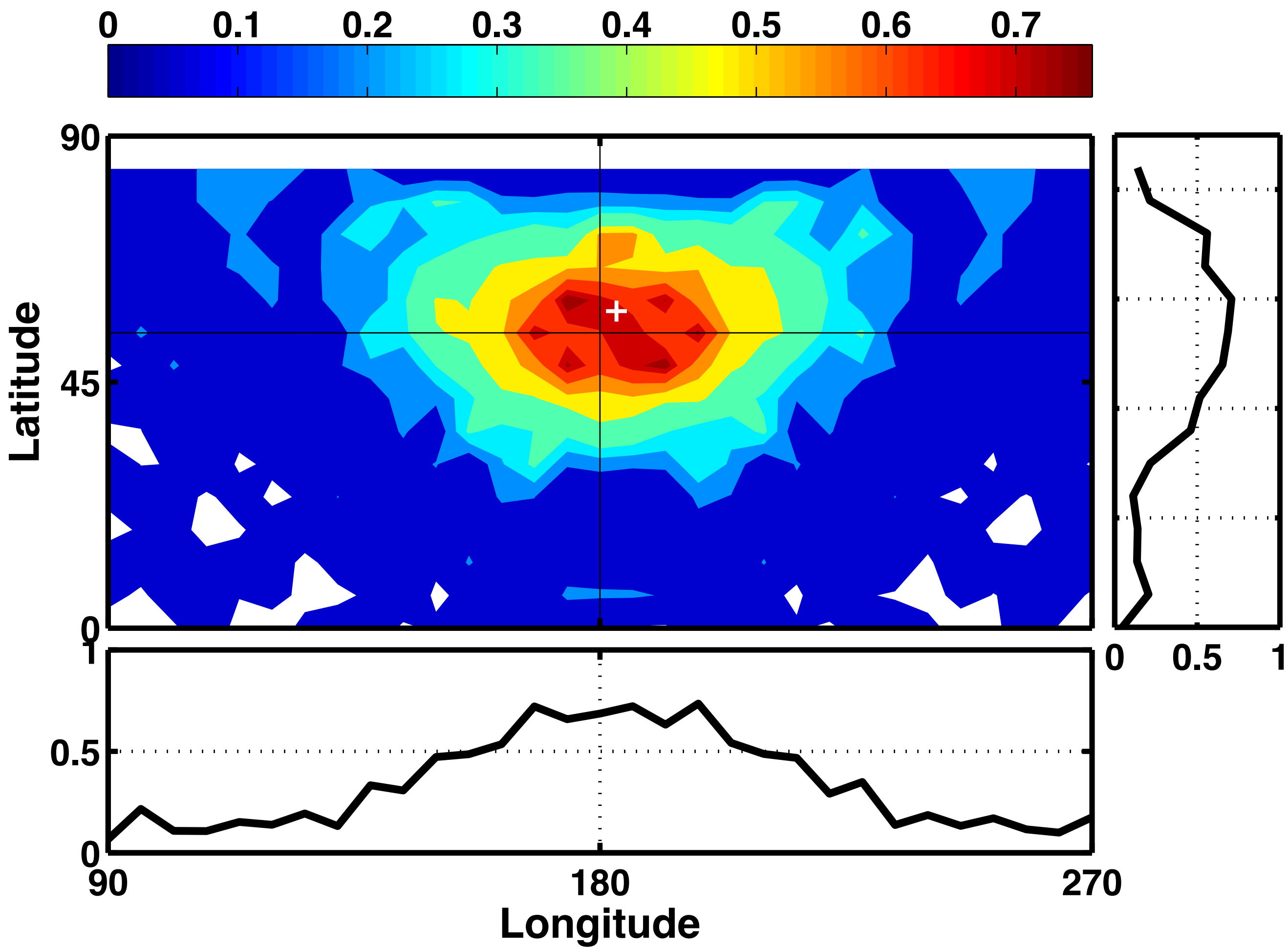


Localization of PS Observation on Level 3 U State



Low-Order Dry Dynamical Core

Localization of PS Observation on Level 3 V State



Low-Order Dry Dynamical Core

- These equivalent localizations are similar to those from the global group filter.
- Can't do optimized for this problem, too costly.

Conclusions

- Localization can greatly enhance small ensemble performance.
- Need affordable methods to find good localization.
- Assuming sampling error is dominant can lead to good estimates in some cases.
- Correlation Error Reduction described here is cheap.
- Need general method for picking subsets.
- Need better theory of need for localization.

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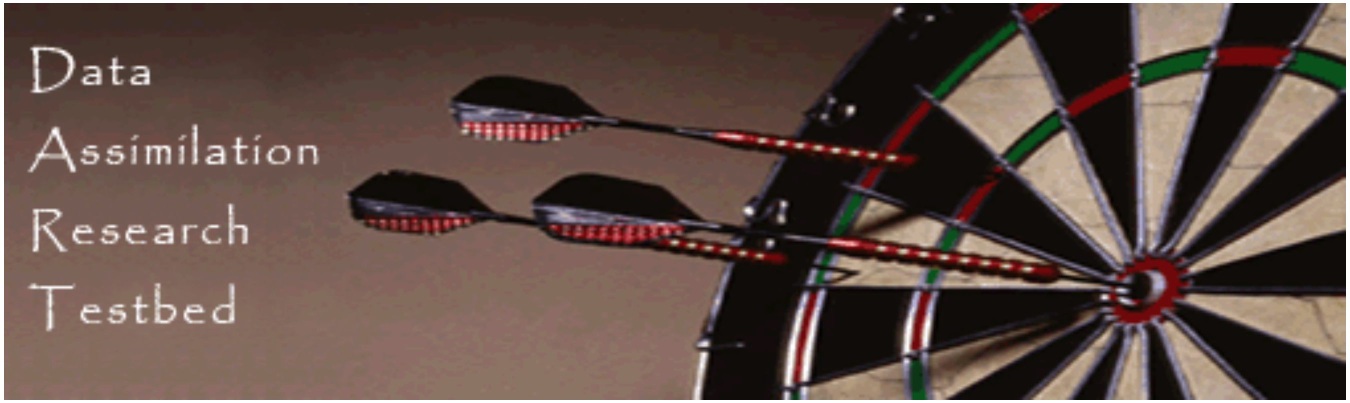

www.image.ucar.edu/DAReS/DART

Anderson, J., Hoar, T., Raeder, K., Liu, H., Collins, N., Torn, R., Arellano, A., 2009: *The Data Assimilation Research Testbed: A community facility.* BAMS, **90**, 1283—1296, doi: 10.1175/2009BAMS2618.1