DARC Seminar
12 November 2013

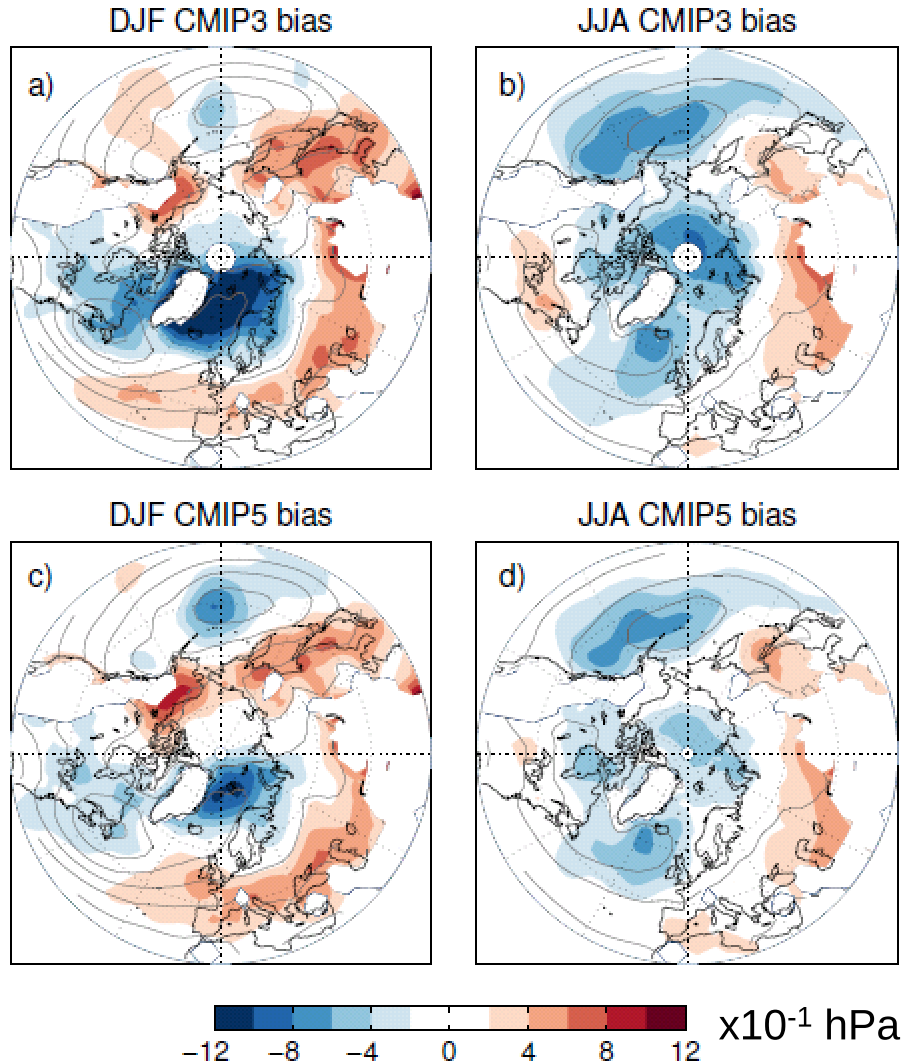
University of Reading

Implications of model error for numerical weather and climate prediction

Oscar Martínez-Alvarado

Motivation

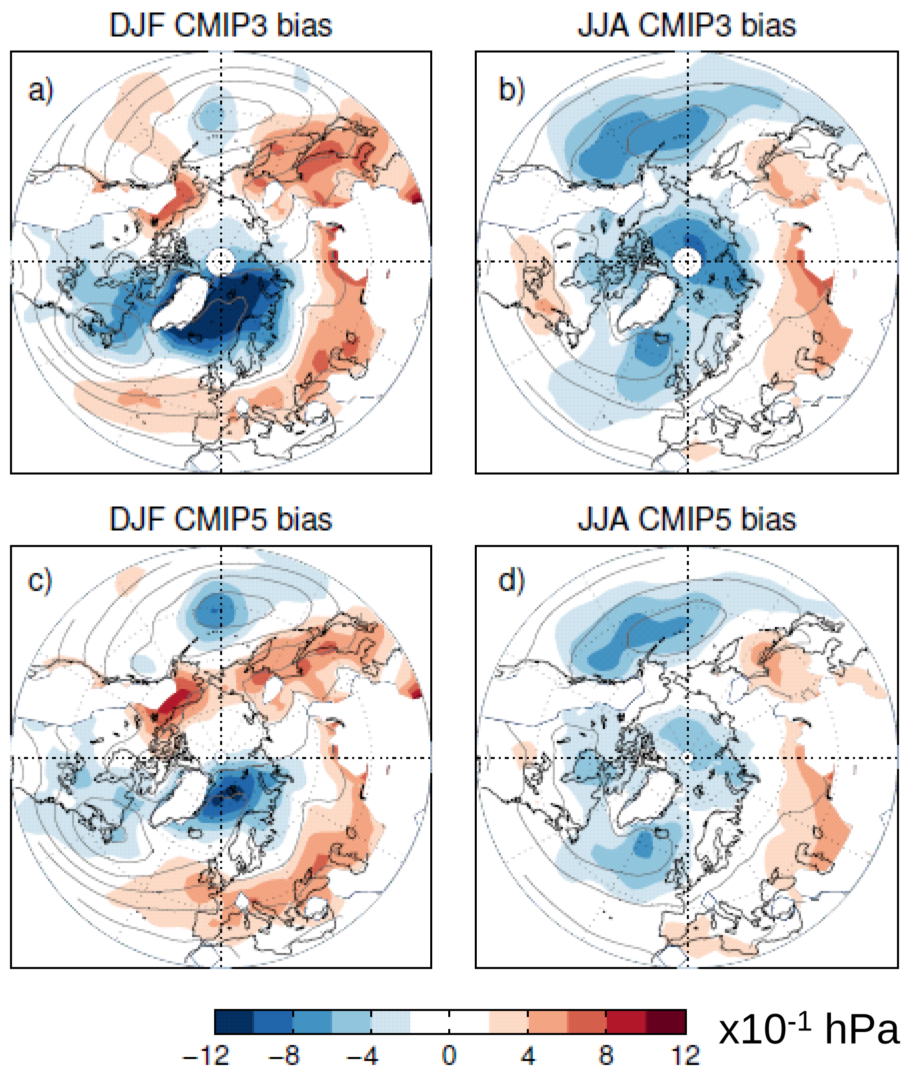
- Comprehensive numerical climate models are the best tool available to understand climate change and to tackle the problems of mitigation and adaptation under future climate scenarios
- However, these models show biases with respect to current climate conditions
    - In the figure, biases in storm track position in CMIP3 and CMIP5 models w.r.t. ERA-Interim



Zappa *et al.* (J. Clim. 2013)

Motivation

- The origin and mechanisms that give rise to these biases are not yet well understood
- Can we find a key to understand these mechanisms in the interaction weather–climate?
- A theory to link these two timescales has not been developed yet



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Suitability of climate models

The suitability of numerical climate models to study the climate is based in two assumptions:

1. A climate attractor exists
2. The solutions provided by a numerical climate model lie on the climate attractor

A third assumption is required if climate models are to be used in climate change studies:

3. The model climate attractor responds in the same way as the actual climate attractor under changing forcing conditions

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Methodology

Lorenz system (Lorenz 1963)

$$\dot{x} = \sigma(y - x)$$

$$\dot{y} = rx - y - xz$$

$$\dot{z} = xy - bz$$

(x, y, z) are dynamical variables (phase space variables)

σ, b and r are constant parameters

The behaviour of the system depends on the values of these parameters

The types of behaviour in the Lorenz system includes chaotic behaviour

This system has been widely used as a prototype of atmospheric and climate behaviour

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Let us define our **prototype system** by the parameter values

$$\sigma = 10, b = \frac{8}{3}, r = 28$$

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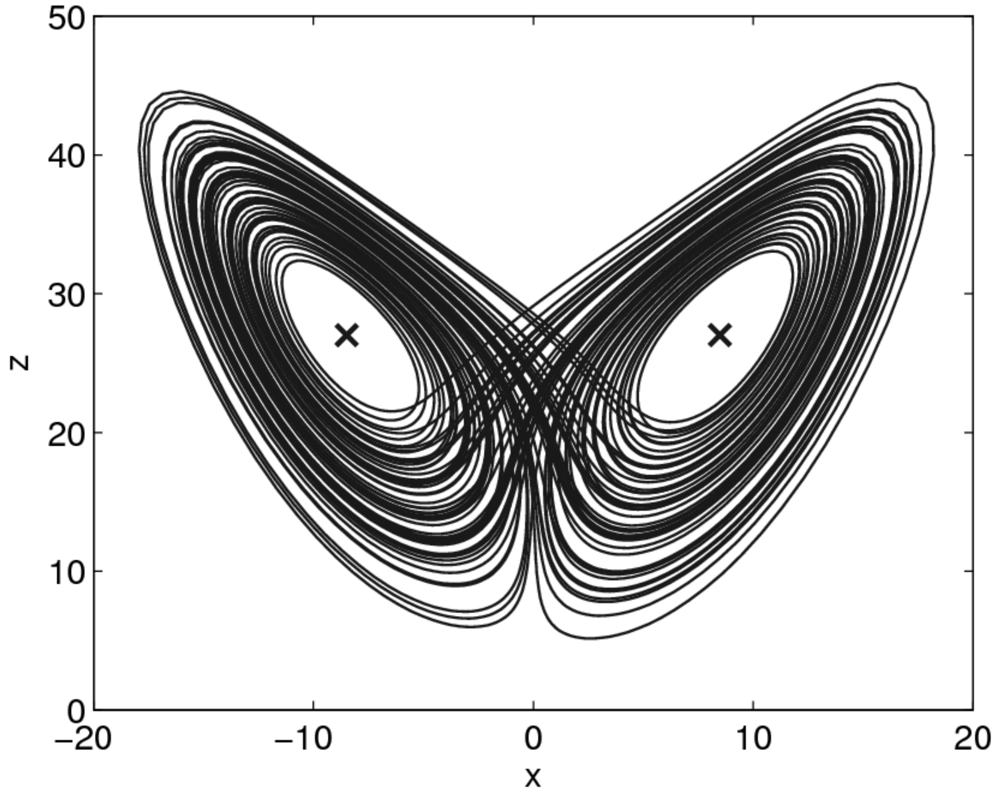

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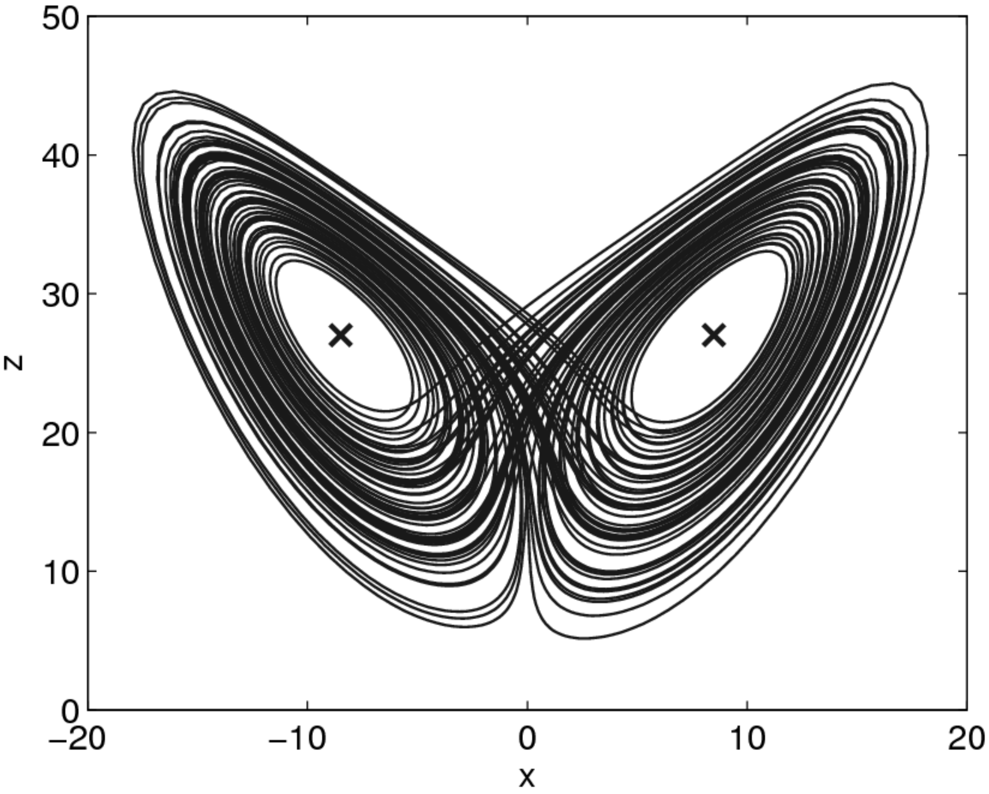
Let us define our **imperfect model** by the parameter values

$$\sigma = 10, b = \frac{8}{3}, r = 25$$



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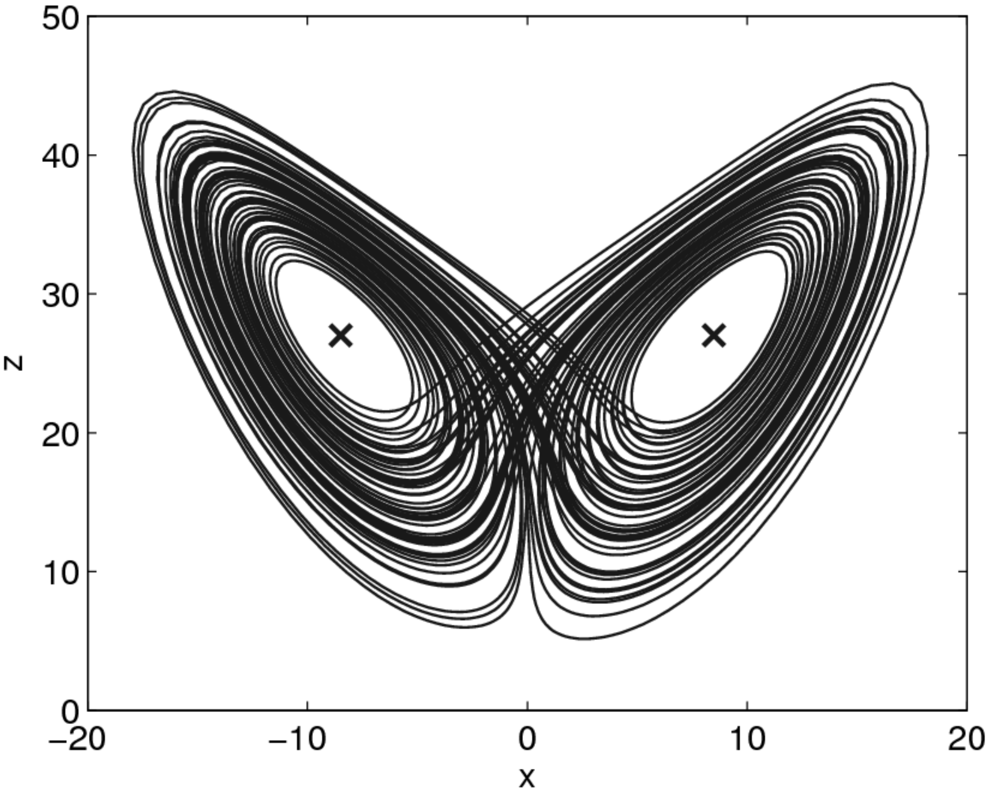

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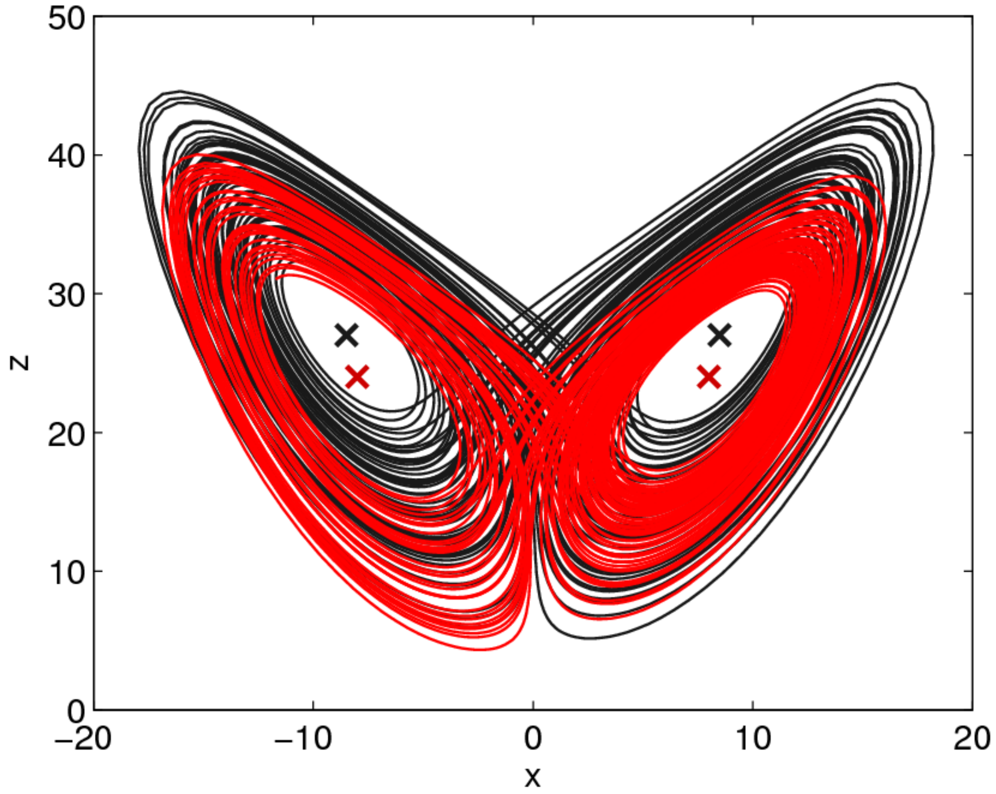

Model error in the prototype system/imperfect model combination

- Let us assume that we observe the prototype system at regular intervals (e.g. every 1 time unit)
- These observations are perfect
- Then let us use the imperfect model to make forecasts of the prototype system from these perfect initial conditions
- Let us assume that we have access to only one of the three phase space variables (*e.g. x*)

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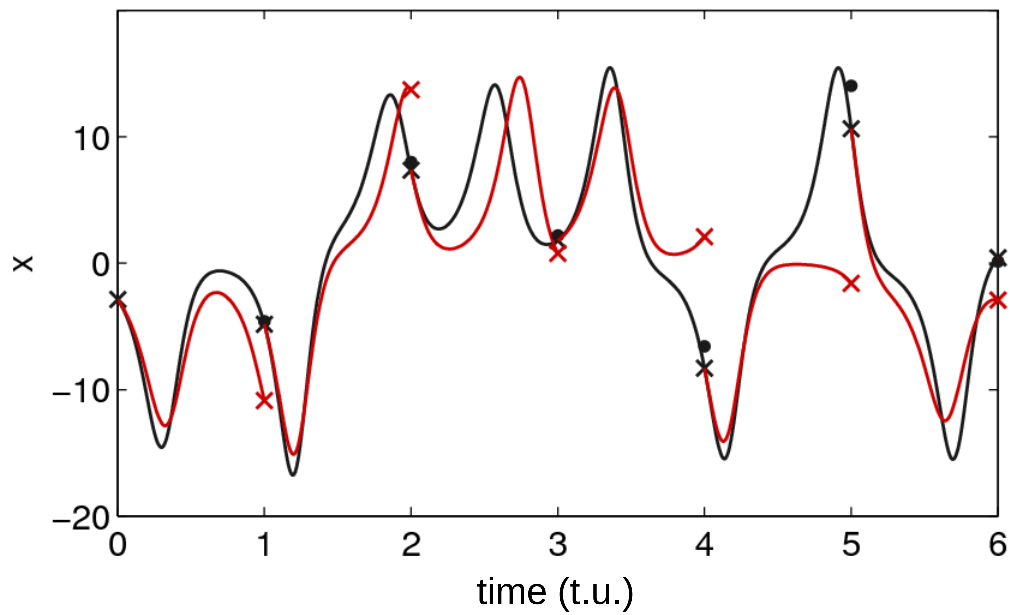


Black line – True orbit

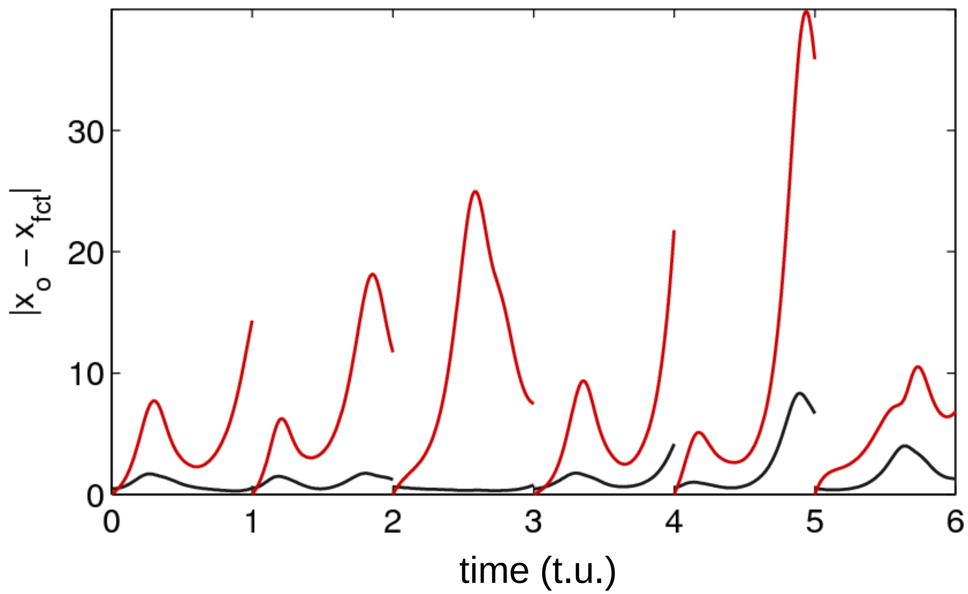
Red line – Model orbit initialised every 1 t.u. from perfect initial conditions (dots)

x – Forecast state after 1 t.u.

x – Forecast state after 1 t.u. using a perfect model initialised with imperfect initial conditions

Model error in the prototype system/imperfect model combination

- Let us assume that we observe the prototype system at regular intervals (e.g. every 1 time unit)
- These observations are perfect
- Then let us use the imperfect model to make forecasts of the prototype system from these perfect initial conditions
- Let us now assume that we have access to all three phase space variables (*i.e. x, y, z*)



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Red line – Forecast error (imperfect model)

Black line – Forecast error (perfect model initialised with imperfect initial conditions)

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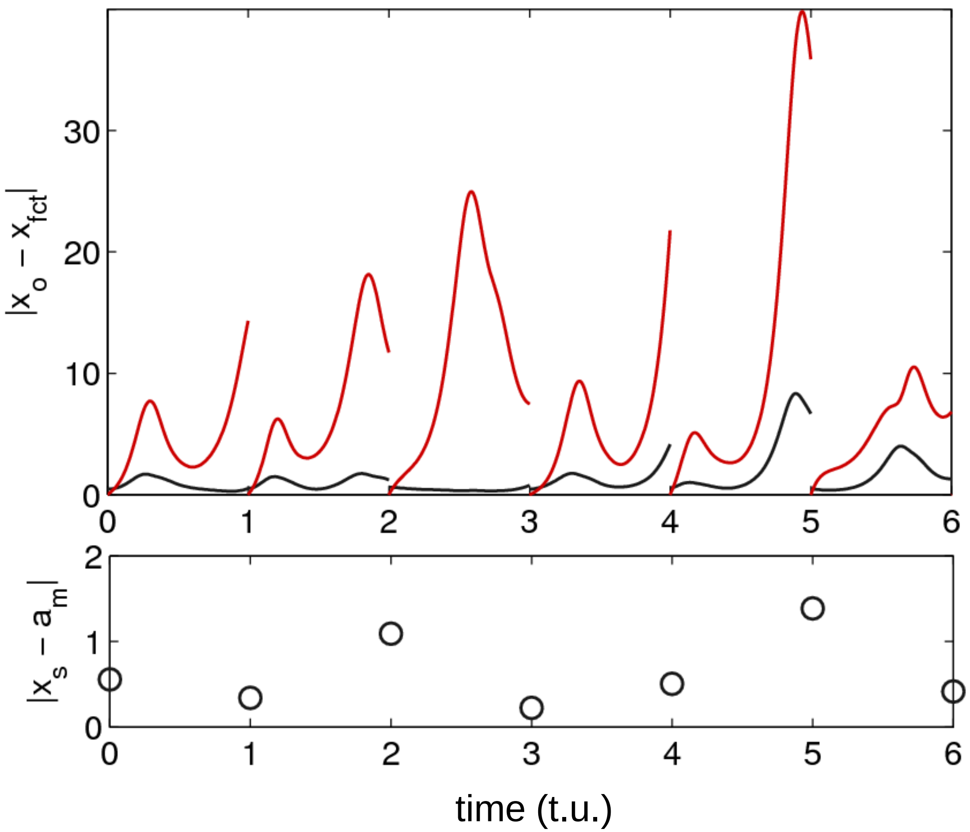
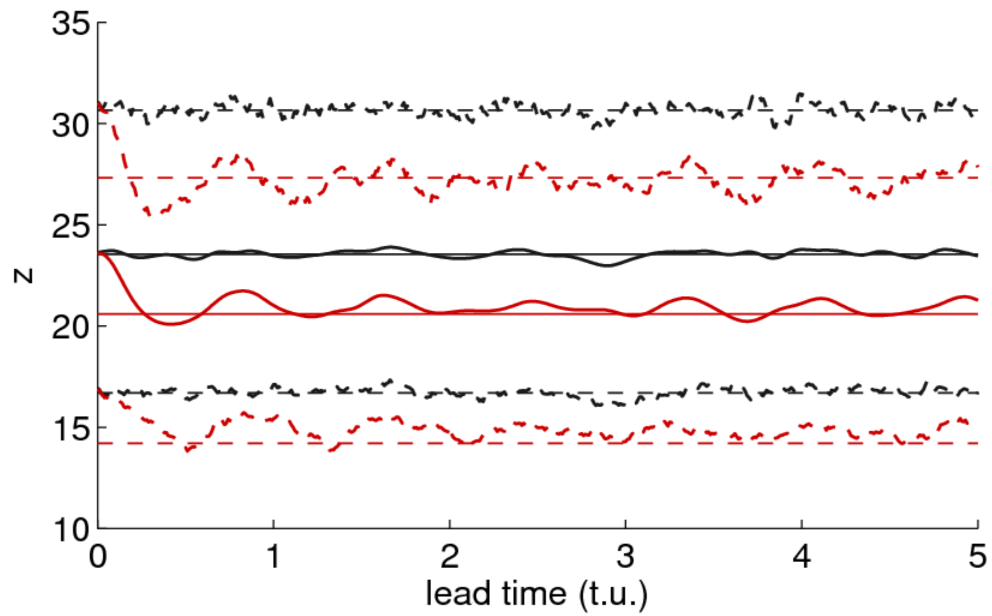


Circles – Distance between the perfect initial conditions and the imperfect model attractor

Model error in the prototype system/imperfect model combination

- Let us generate a climatology from the collection of analyses and let us call this climatology at lead time $t_L = 0$
- Similarly let us generate climatologies from the collection of perfect and imperfect forecasts for a given lead time $t_L > 0$



Black – Perfect model with imperfect initial conditions

Red – Imperfect model with perfect initial conditions

Model error in the prototype system/imperfect model combination

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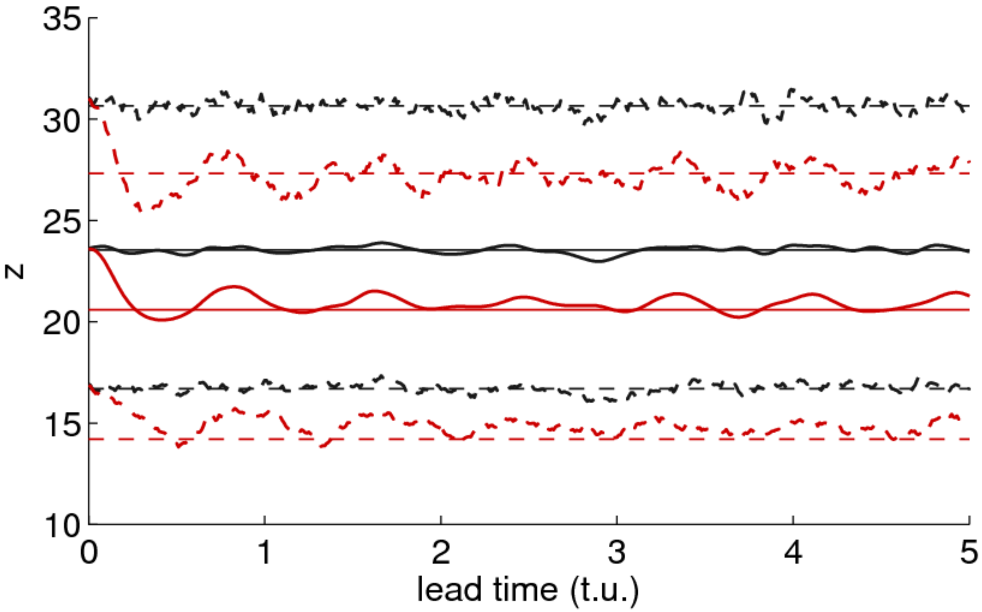
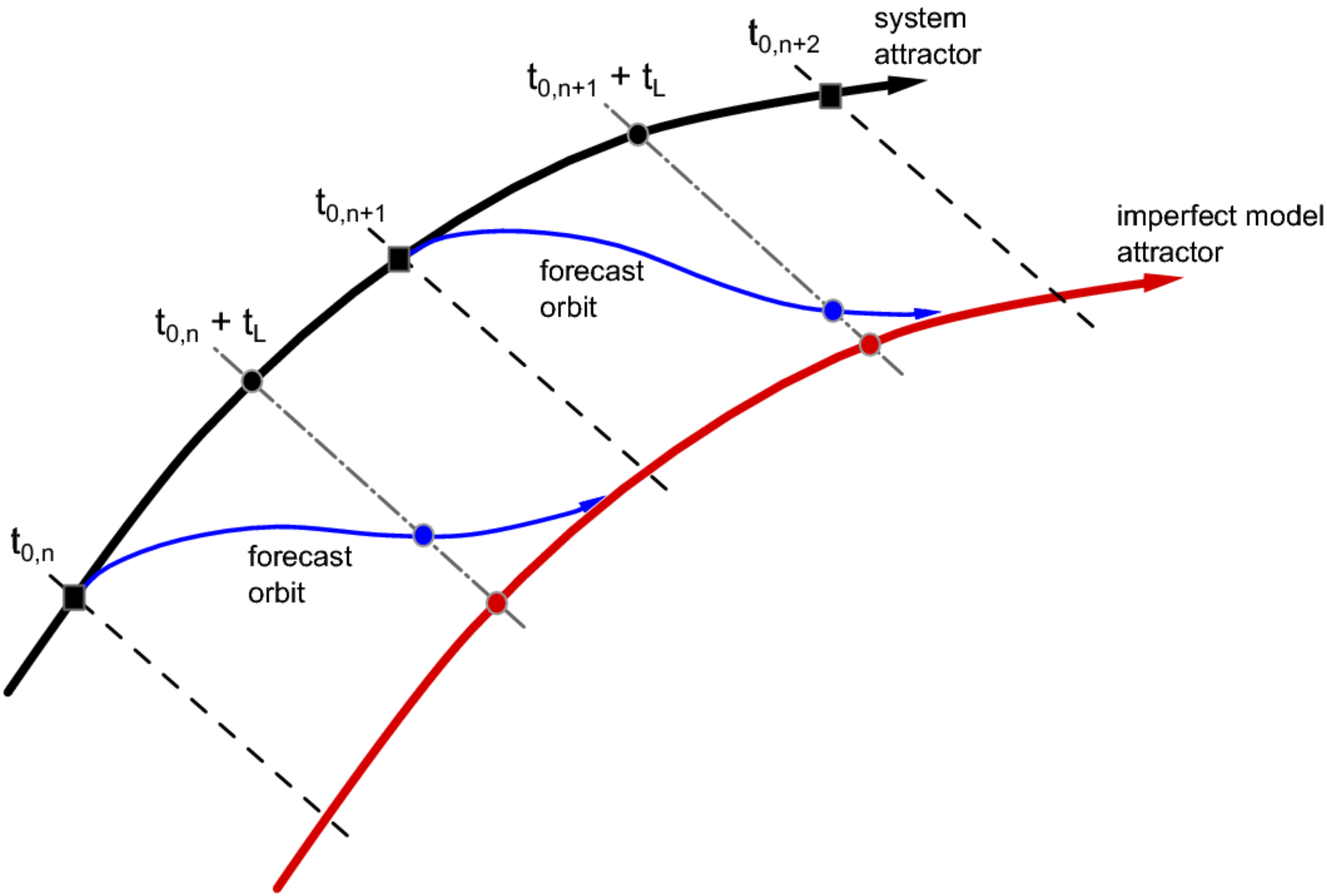


If the model was perfect, the reconstructed climate attractor would be invariant to lead time. In contrast, an imperfect model will tend towards the imperfect model attractor.

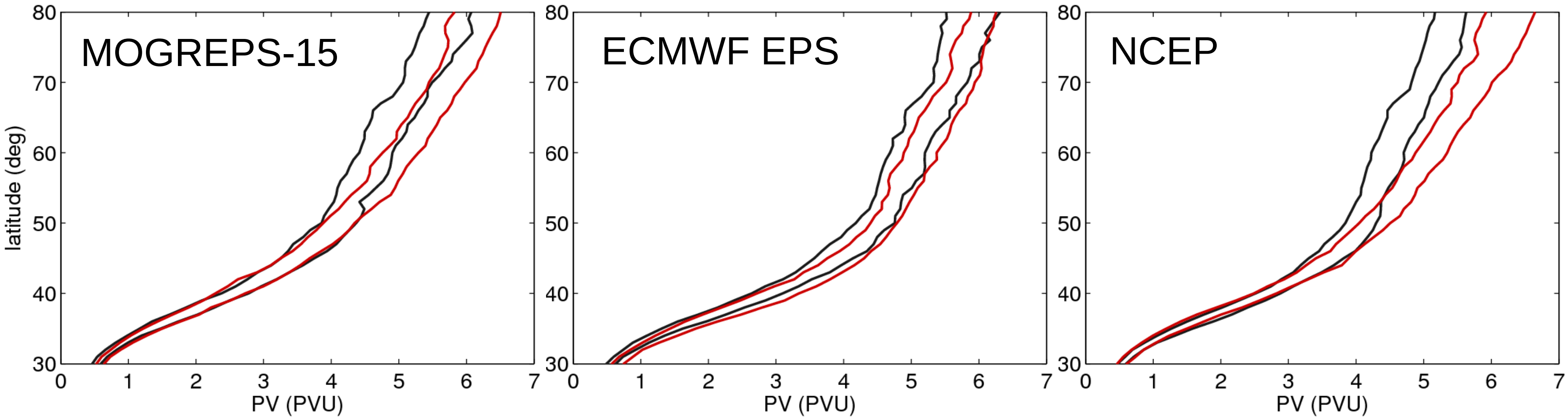
Black – Perfect model with imperfect initial conditions

Red – Imperfect model with perfect initial conditions

Generation of biases in the prototype system/imperfect model combination

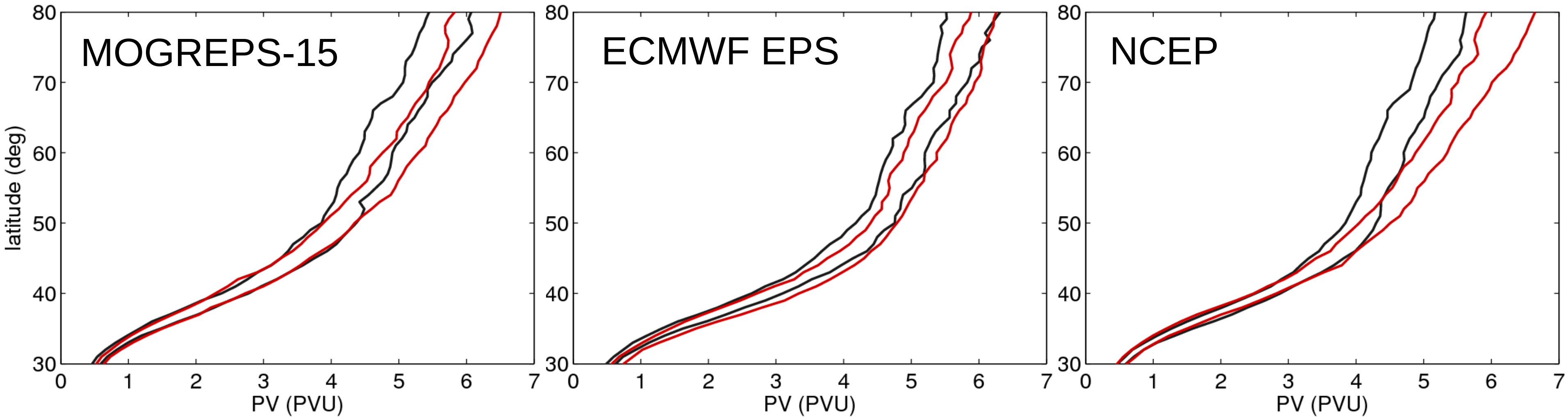


Generation of biases in the actual climate/numerical models combination



First and third quartiles of daily zonally averaged 320-K PV in analyses (T+0, **black**) and T+15d (control member, **red**) for the season from December 2009 to February 2010

Generation of biases in the actual climate/numerical models combination

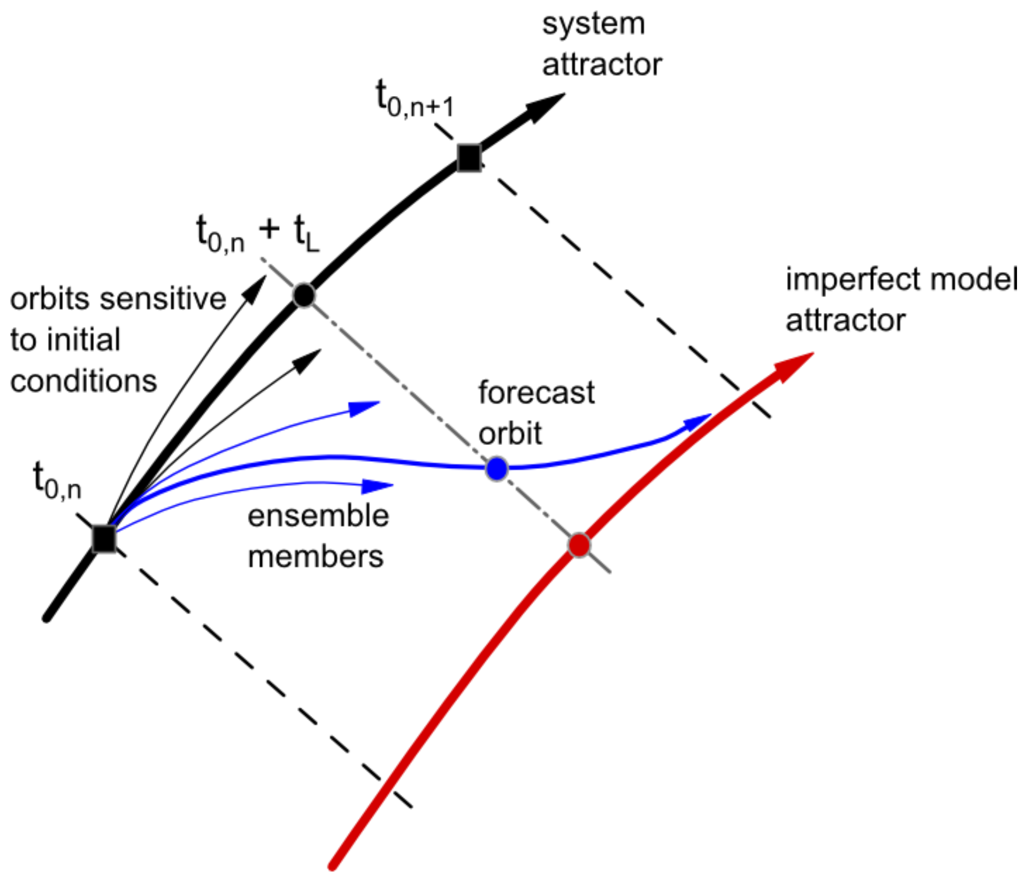


First and third quartiles of daily zonally averaged 320-K PV in analyses (T+0, **black**) and T+15d (control member, **red**) for the season from December 2009 to February 2010

- The displacement from the black profile to the red profile indicate a similar behaviour to that described for the prototype system/imperfect model combination
- The same qualitative behaviour is found in other seasons and in the Southern hemisphere
- However, only control ensemble members are being considered here!

Representation of model error variability in numerical models

Ideally, the ensemble members of an ensemble prediction system should represent not only variability due to sensitivity to initial conditions but also variability due to model error (*i.e.* the **blue ensemble members** should resemble the **black orbits**)



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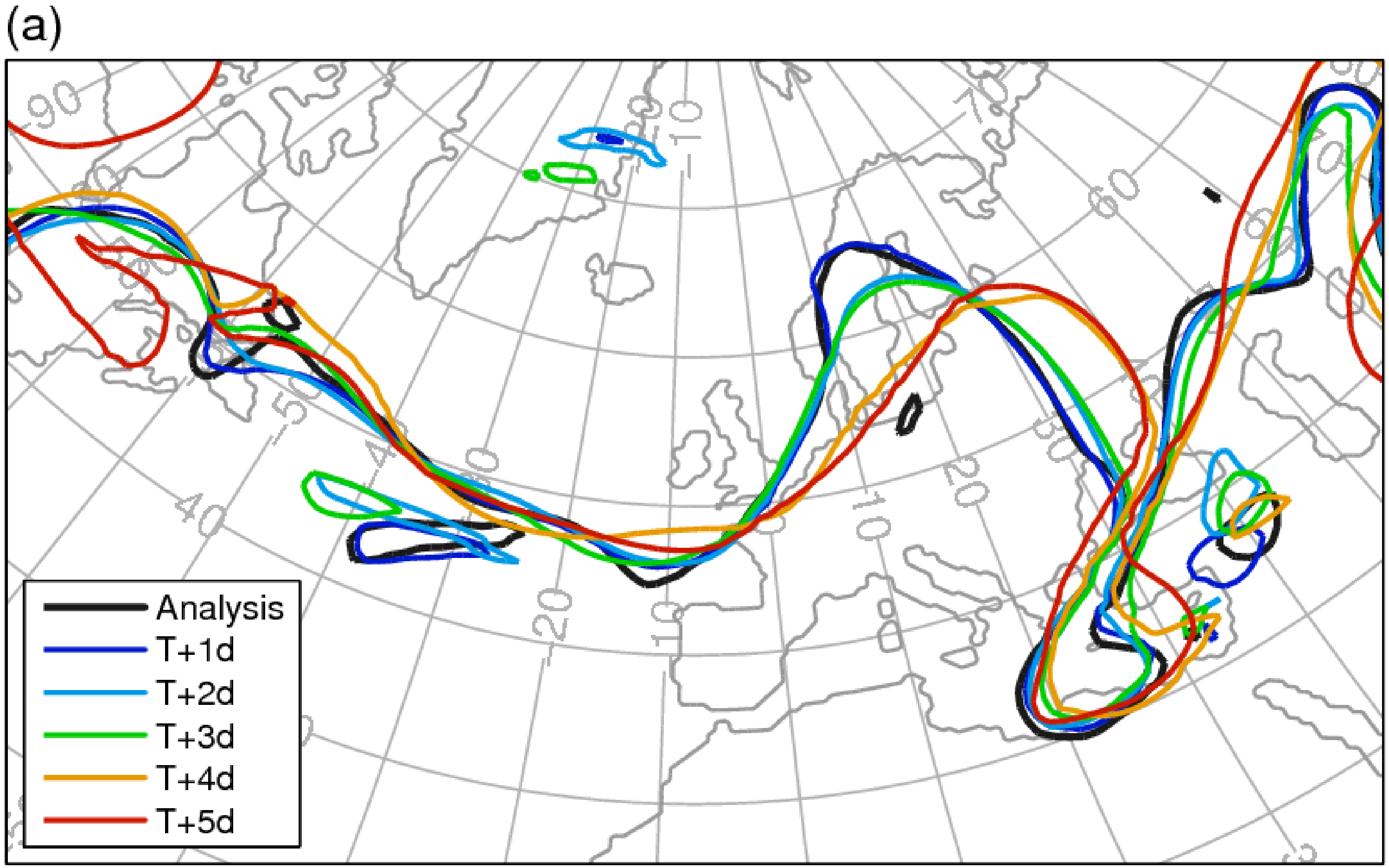
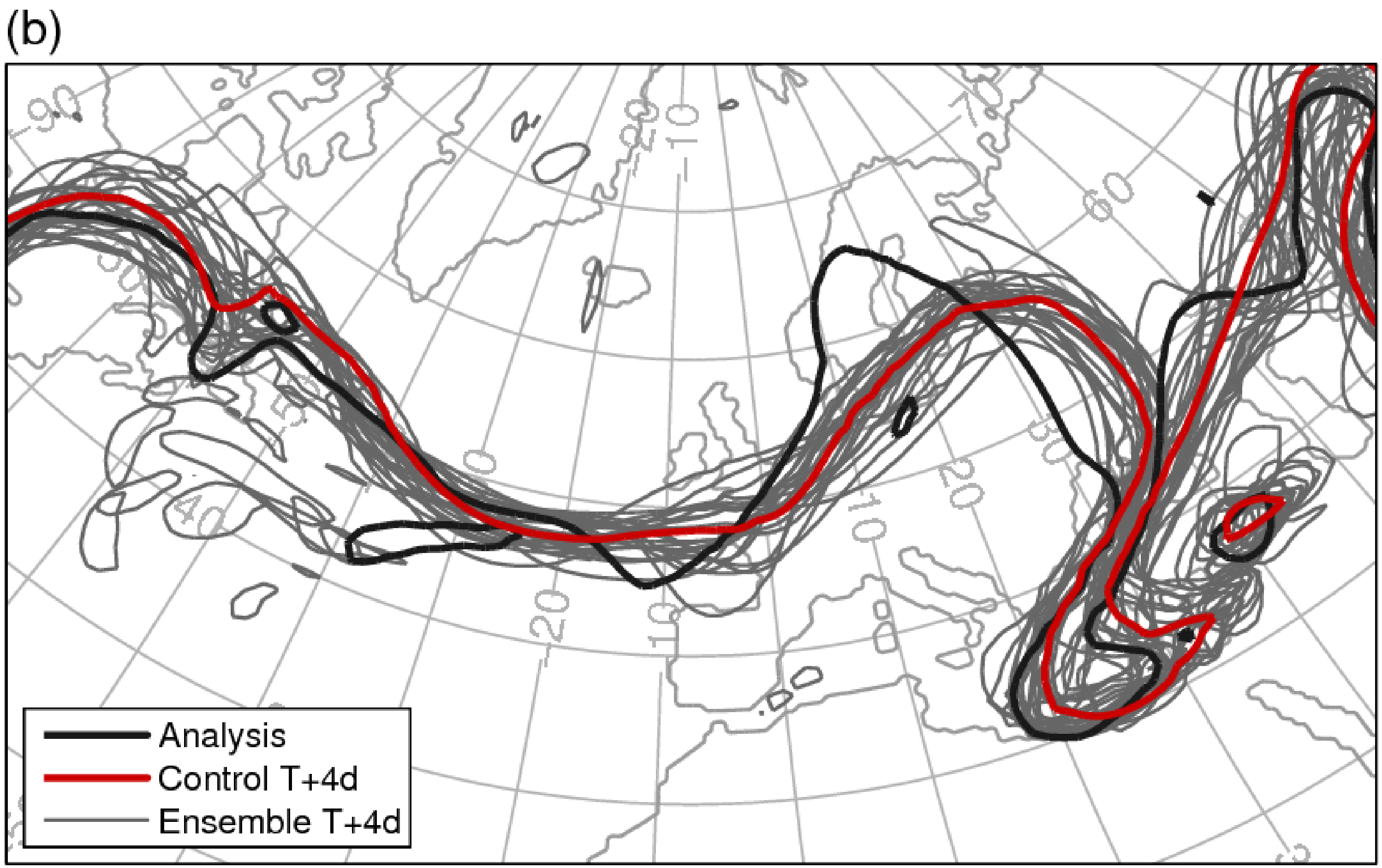


Sideri (2013)

Representation of model error variability in numerical models



Sideri (2013)

Implications of model error for numerical weather and climate prediction

1. An imperfect model will show a drift from analysis to its own attractor.
2. A climatology constructed by taking forecasts at different lead times show this drift until the model attractor has been reached
3. If the model was perfect, the reconstructed climate attractor would be invariant to lead time

The appearance of the terms ‘**reconstructed climate attractor**’ and ‘**lead time**’ provide the link between weather and climate

Lorenz '96 system

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Prototype system

$$\frac{dX_k}{dt} = -X_{k-1}(X_{k-2} - X_{k+1}) - X_k + F - U_k(Y)$$

$$\frac{dY_j}{dt} = -cbY_{j+1}(Y_{j+2} - Y_{j-1}) - cY_j + \frac{hc}{b} X_{\mathrm{int}[(j-1)/J]+1}$$

$$U_k(Y) = \frac{hc}{b} \sum_{j=J(k-1)+1}^{kJ} Y_j$$

X and Y are cyclic

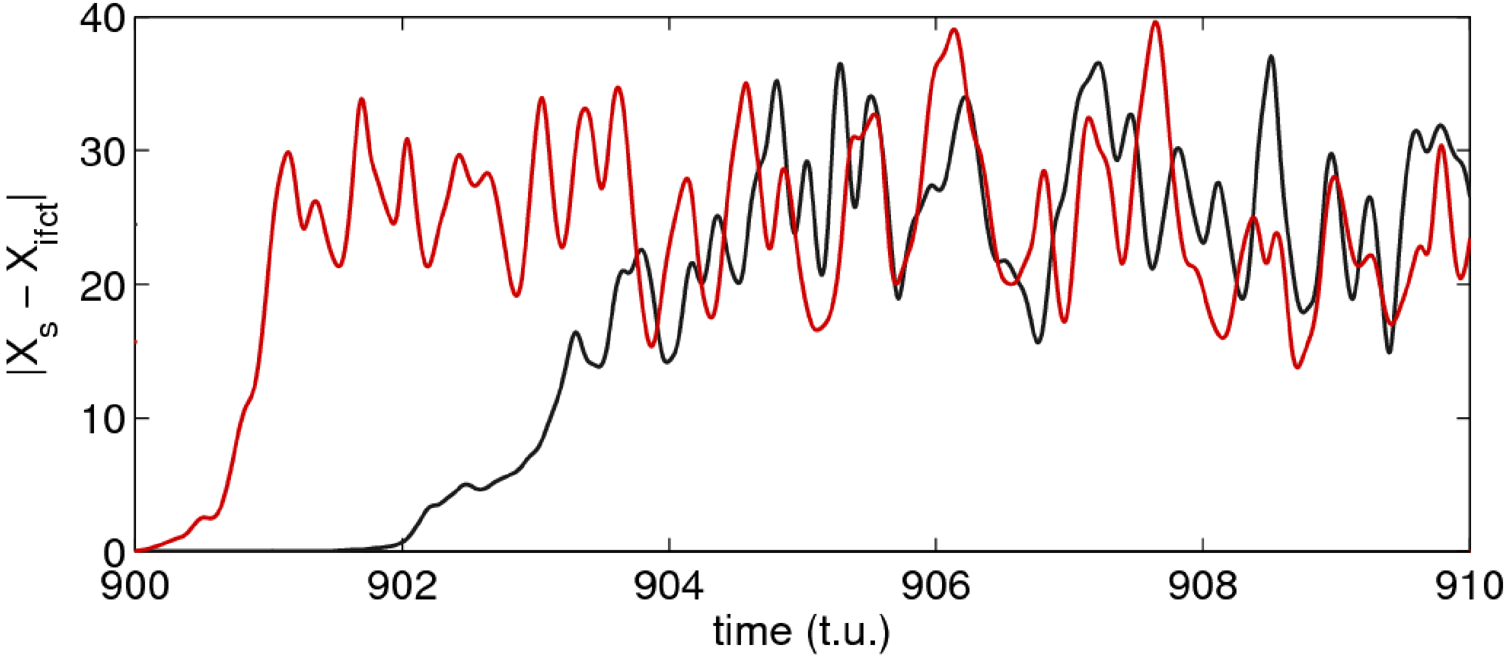
Imperfect model

$$\frac{dX_k}{dt} = -X_{k-1}(X_{k-2} - X_{k+1}) - X_k + F - U_p(X)$$

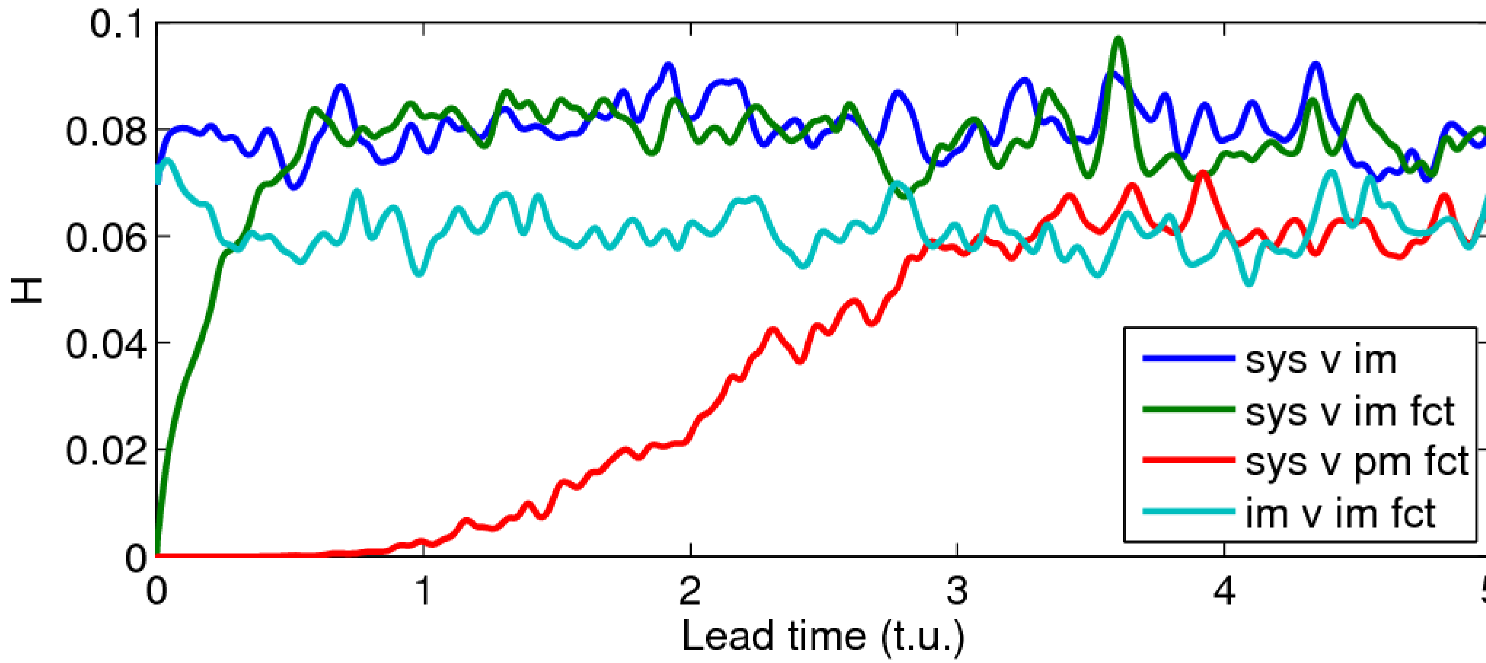
$$U_p(X) = U_{\det}(X) + e(t)$$

Parametrisation following Arnold et al. (2013)

Model error in the prototype system/imperfect model combination



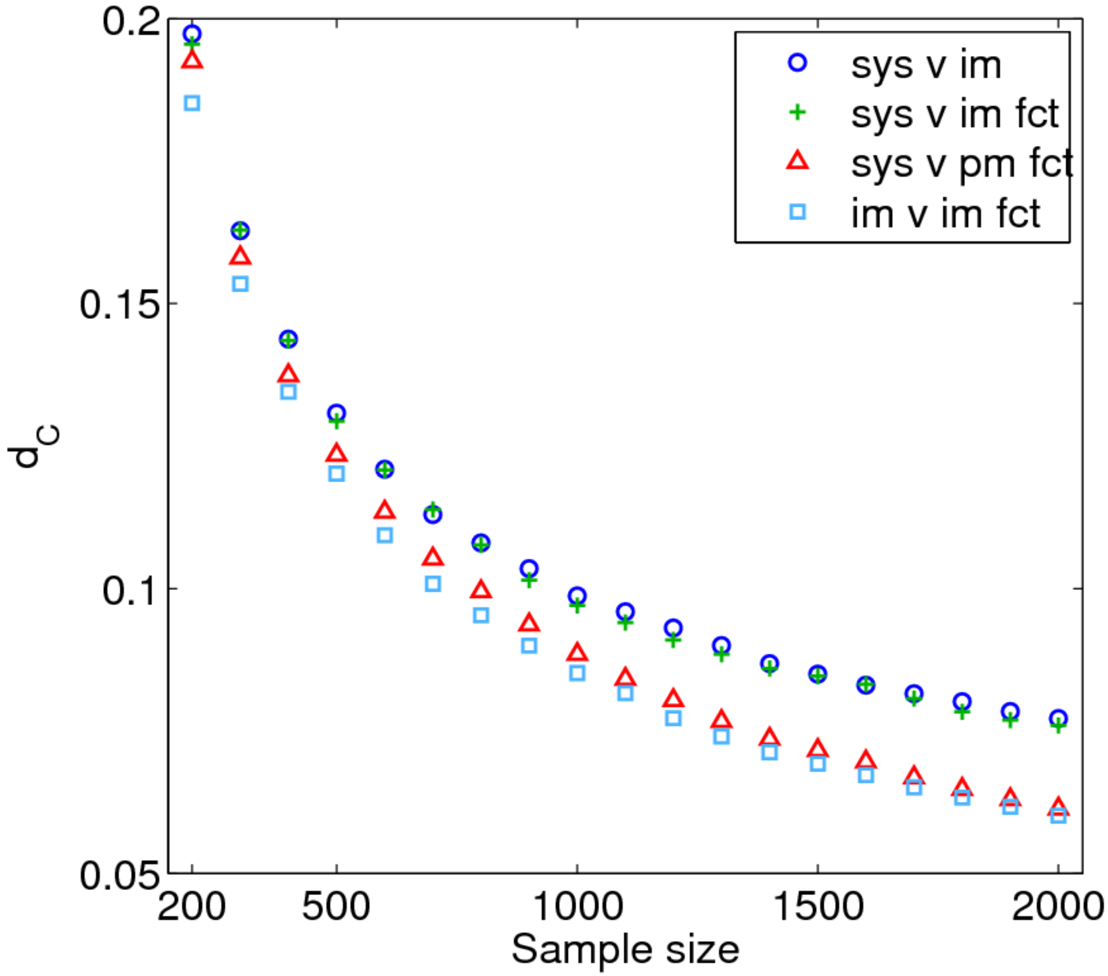
Distance between climatologies as functions of lead time



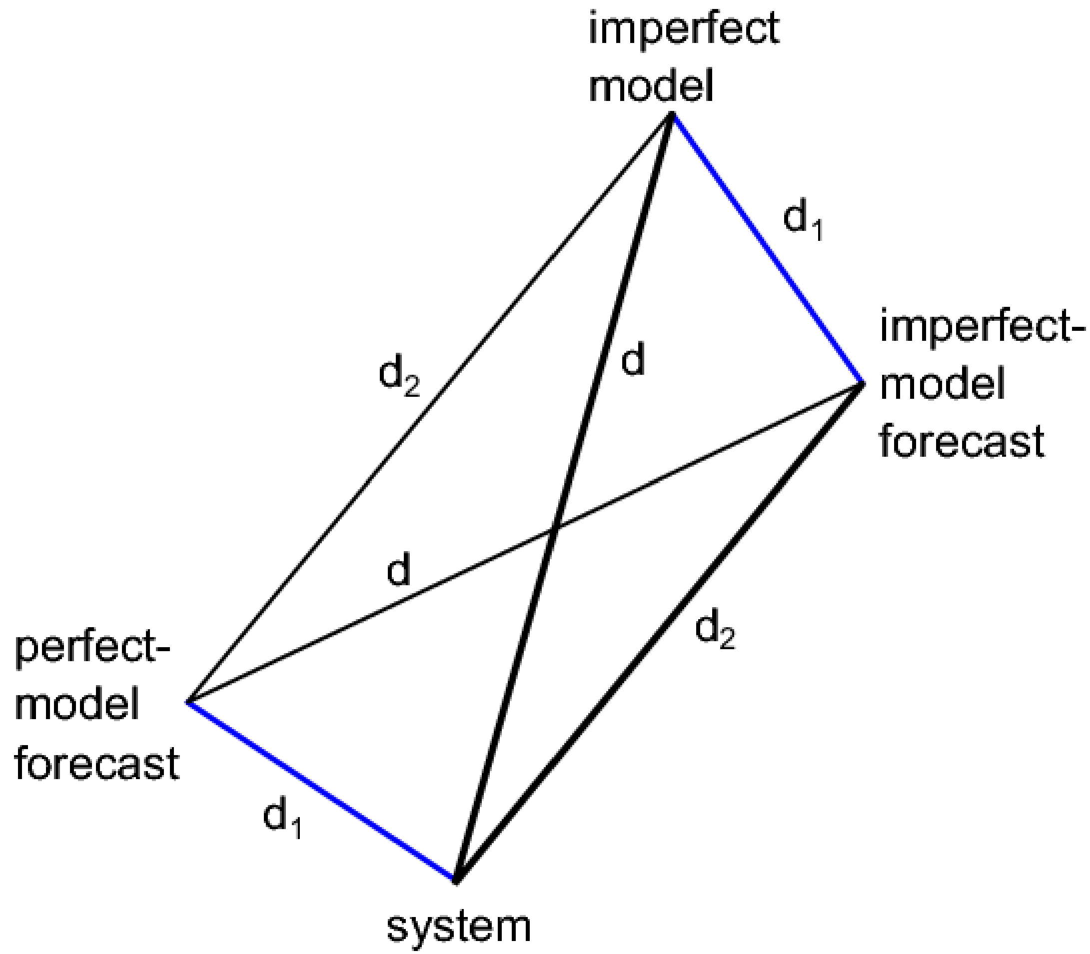
Distance between climatologies given by the Hellinger distance (following Arnold et al. (2013))

$$H^2(f,g) = \frac{1}{2}\int \left(\sqrt{f(x)} - \sqrt{g(x)}\right)^2 dx$$

Distance between climatologies as a function of sample size

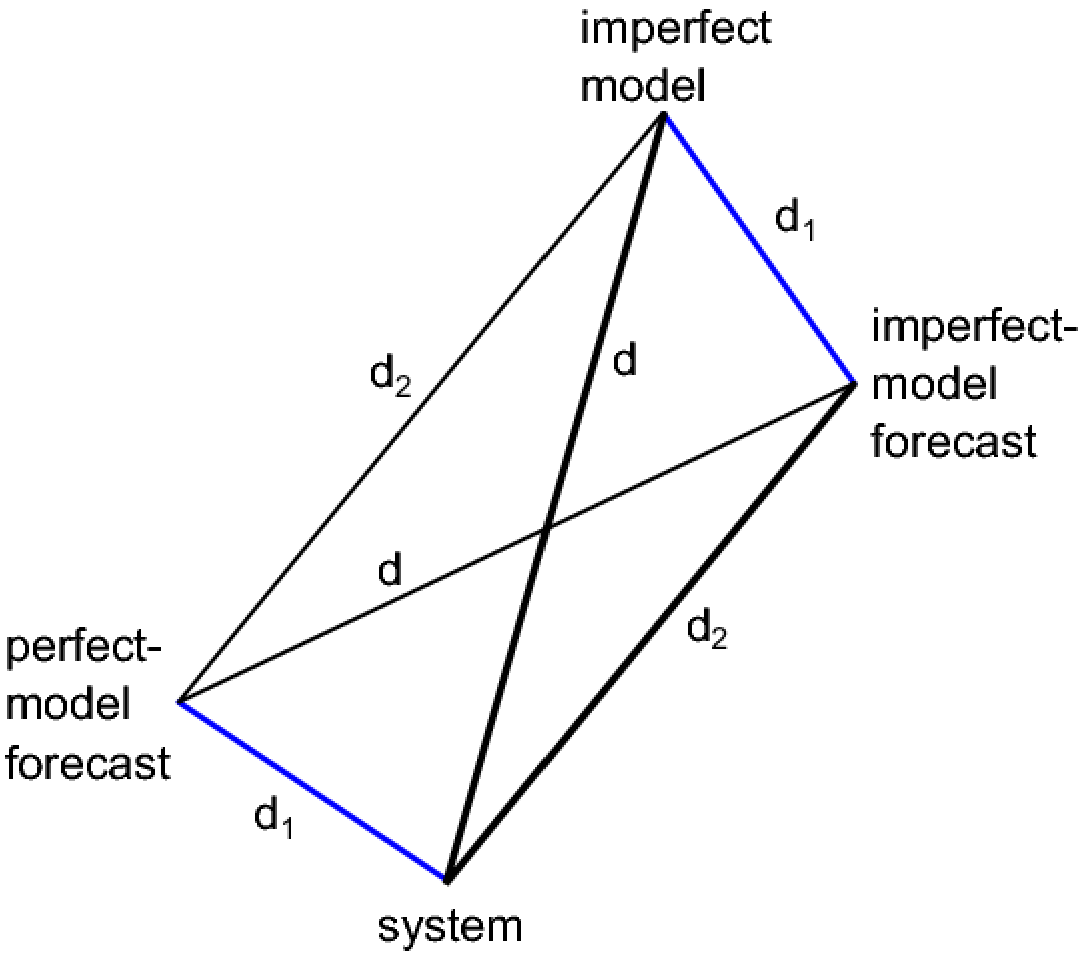


Schematic of the distance between climatologies



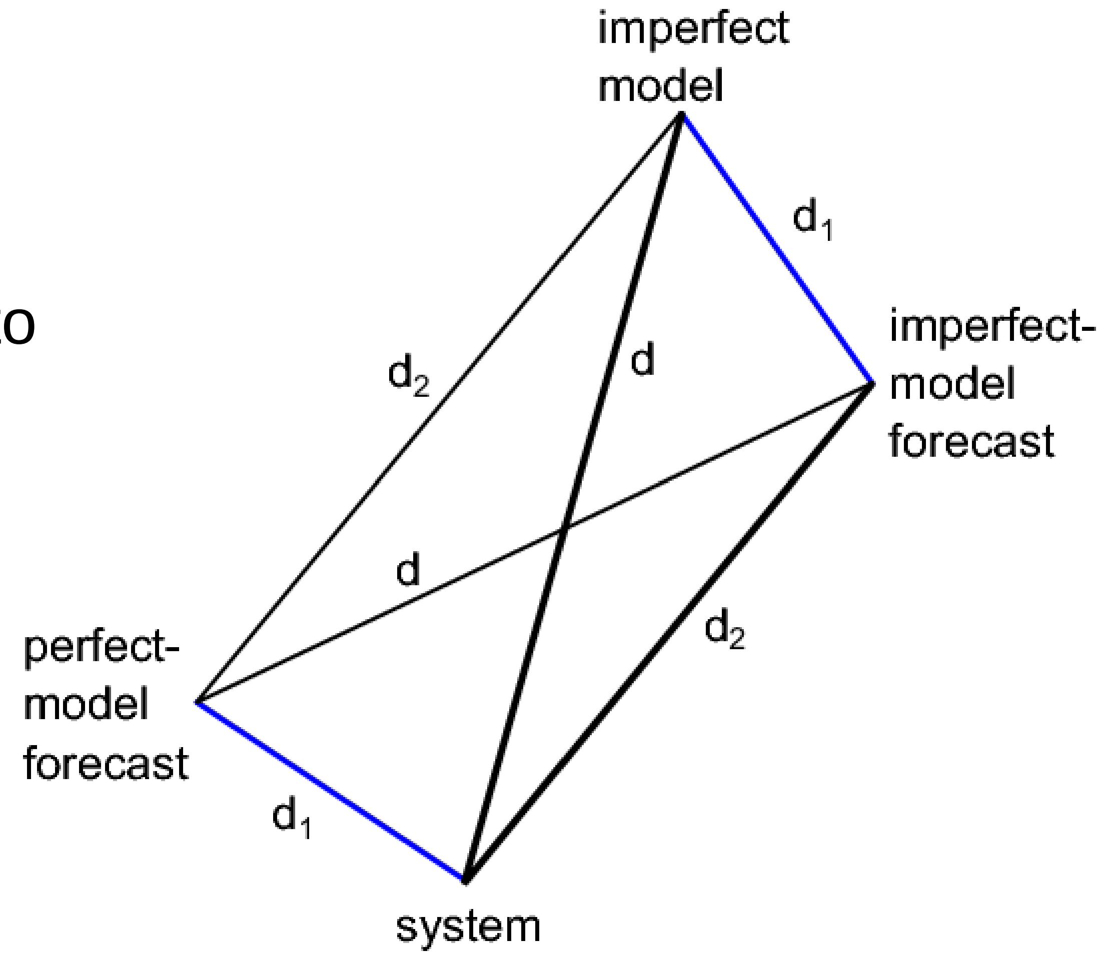
Schematic of the distance between climatologies

- For the cases shown
  d2 ≈ d



Schematic of the distance between climatologies

- For the cases shown $d2 \approx d$
- Improvement of the models should lead to $d \rightarrow 0$ and $d_2 \rightarrow d_1$.



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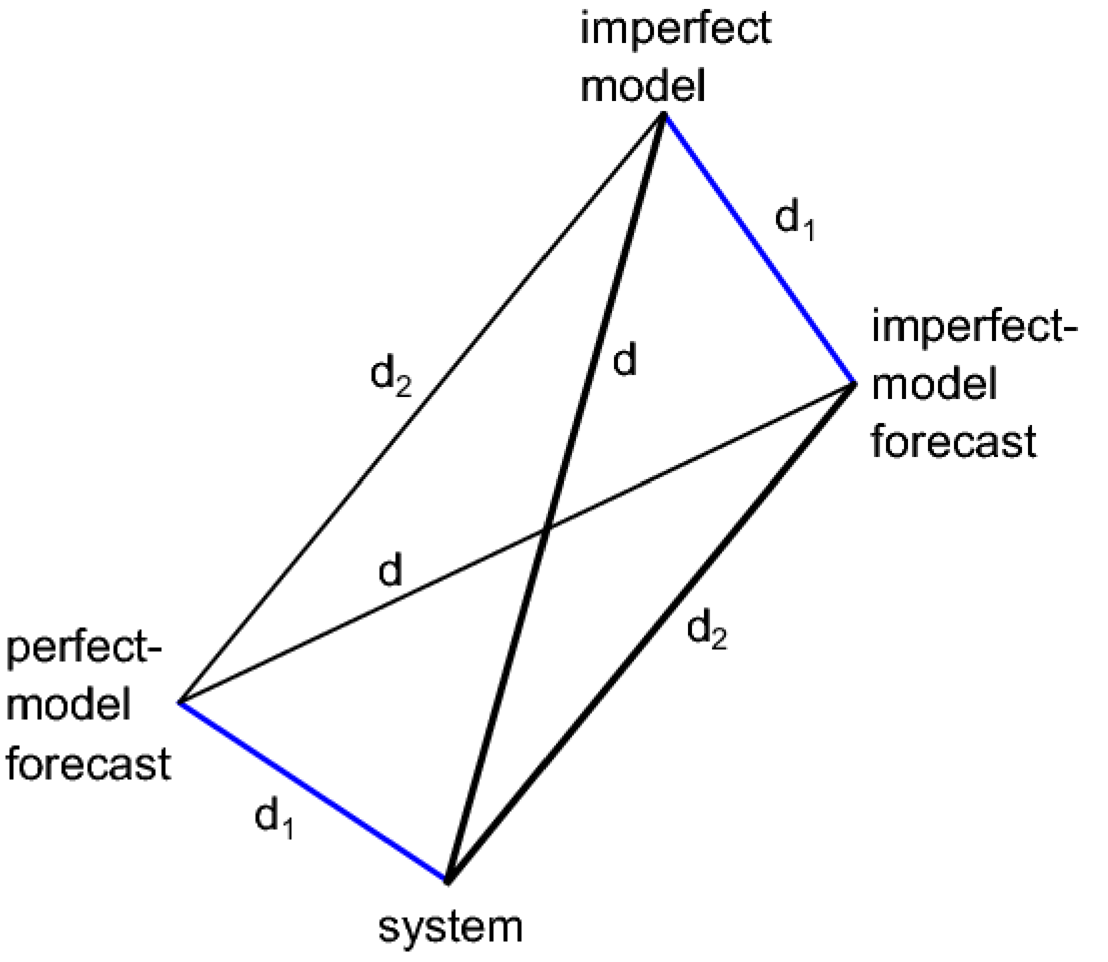
Where are we now?



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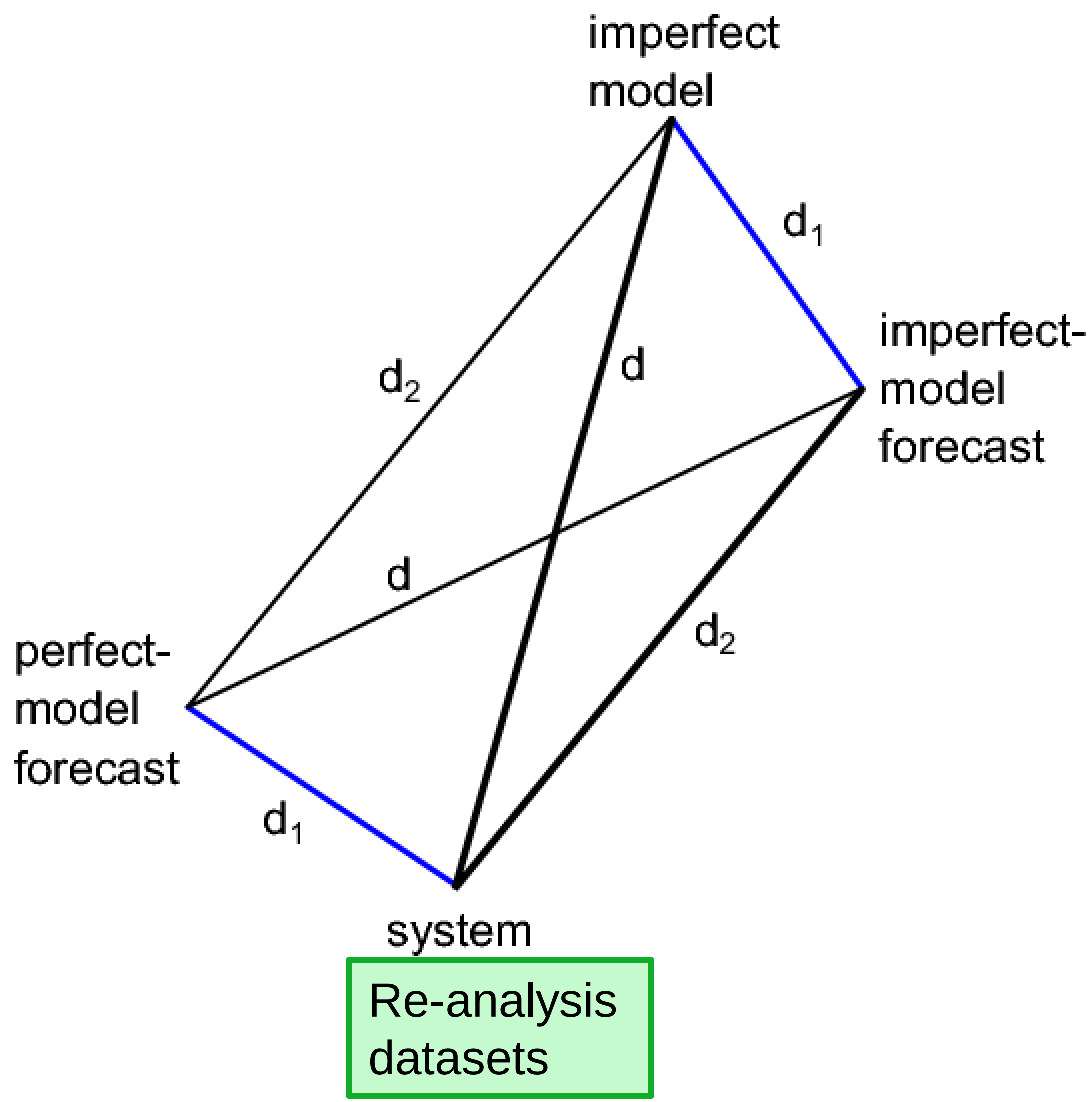

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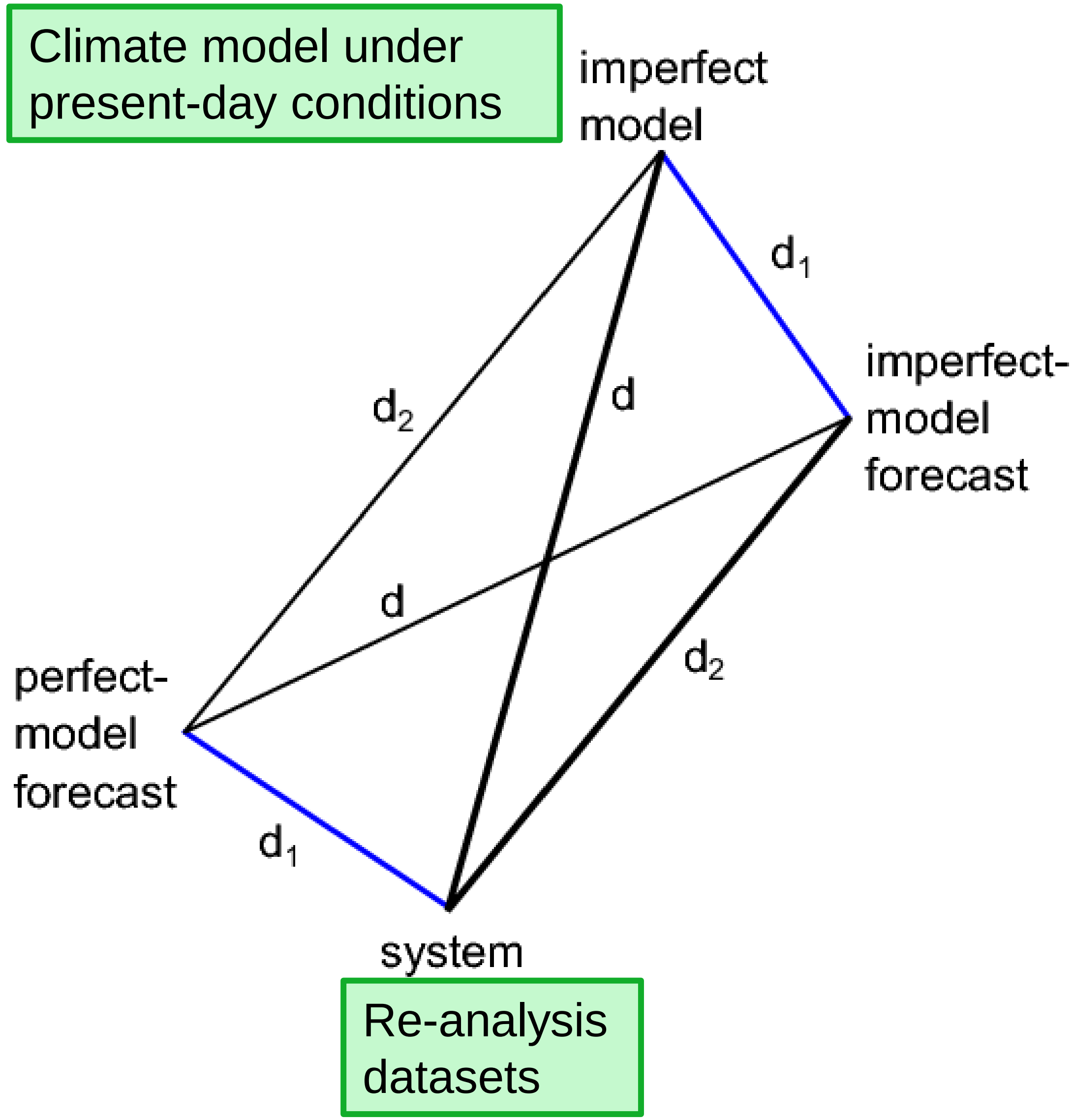

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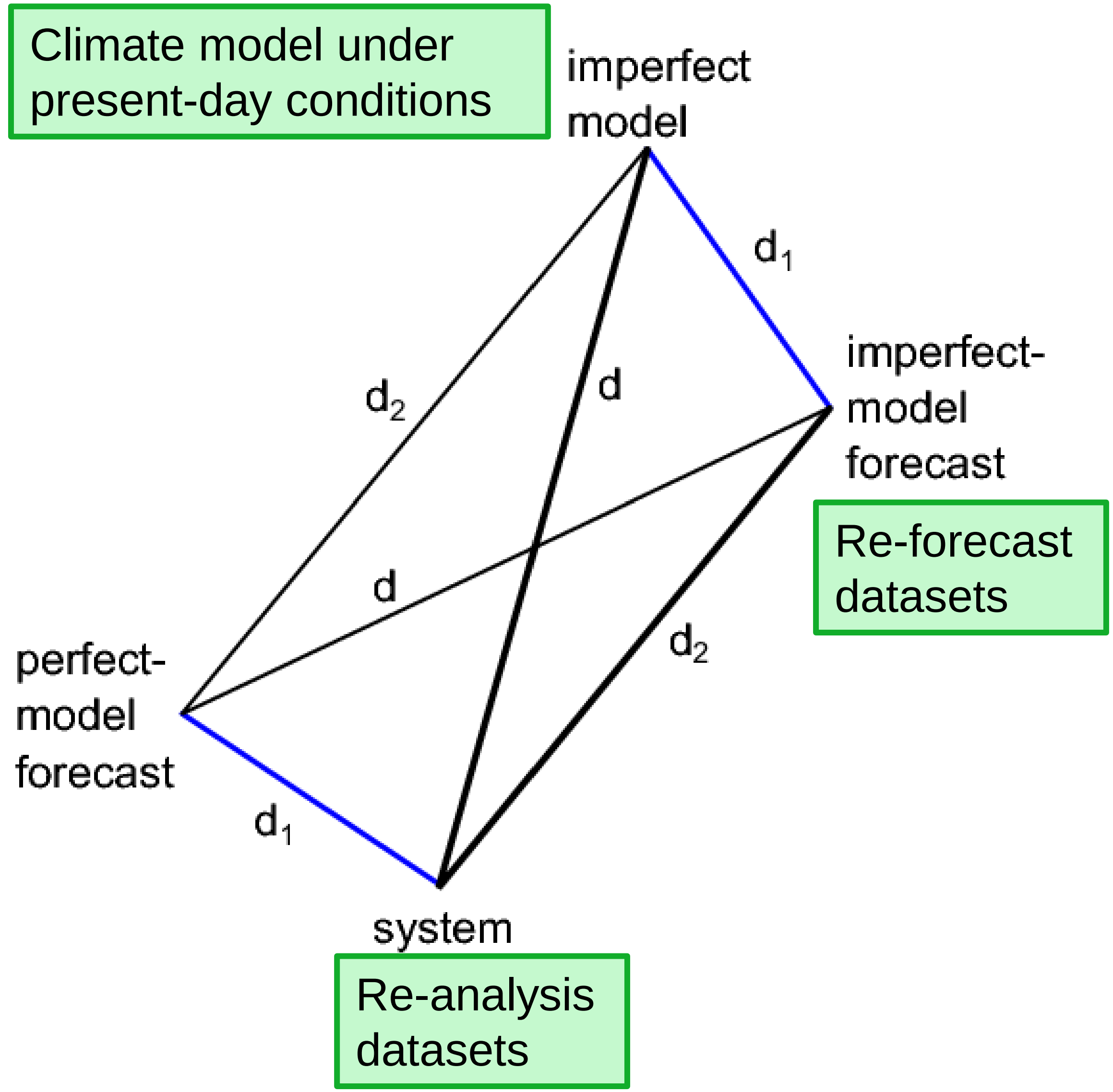

Schematic of the distance between climatologies

Where are we now?

Unavailable?