data
assimilation

Exploring coupled 4D-Var data assimilation using an idealised atmosphere-ocean model

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Problem

- Seasonal-decadal forecasting requires initialisation of coupled atmosphere-ocean models
- Current approach uses analyses generated from independent atmosphere and ocean data assimilation systems
    - ➢ ignores interactions between systems
    - ➢ analysis states likely to be unbalanced
    - ➢ inconsistency at interface can lead to imbalance when states are combined for coupled model forecast (initialisation shock)
    - ➢ near surface data not fully utilised, e.g. SST, scatterometer winds

Problem

- Operational forecasting centres want to move towards coupled assimilation systems

What is the best way to do this and is it worth the effort?

Objective

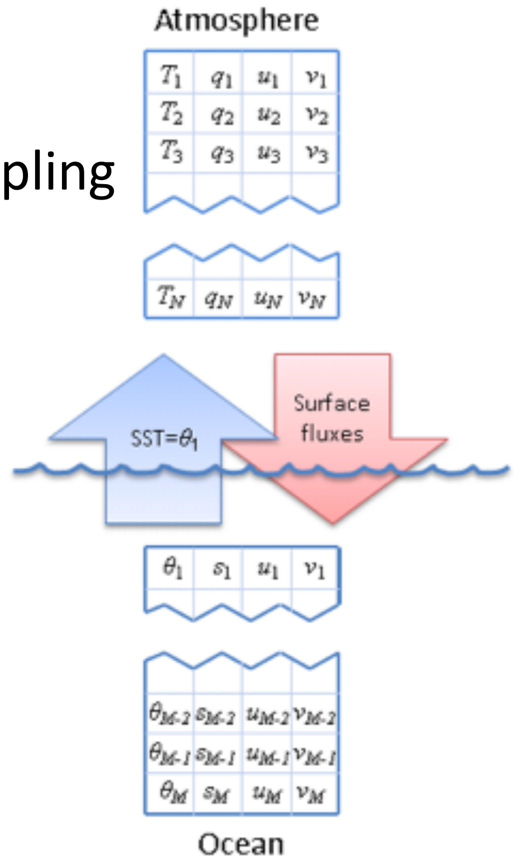
To investigate some of the fundamental questions in the design of coupled atmosphere-ocean data assimilation systems within the context of an idealised strong constraint incremental 4D-Var system:

- avoids issues associated with more complex models
- allows for more sophisticated experiments than in an operational setting
- easier interpretation of results
- guide the design and implementation of coupled methods within full 3D operational scale systems

Idealised system

The system needs to be

- simple and quick to run
- able to represent realistic atmosphere-ocean coupling



Atmosphere

Simplified version of the ECMWF single column model (SCM)

- based on early version of the IFS code
- 4 state variables on 60 model levels
- hybrid (η)coordinate system
- forced by large scale horizontal advection

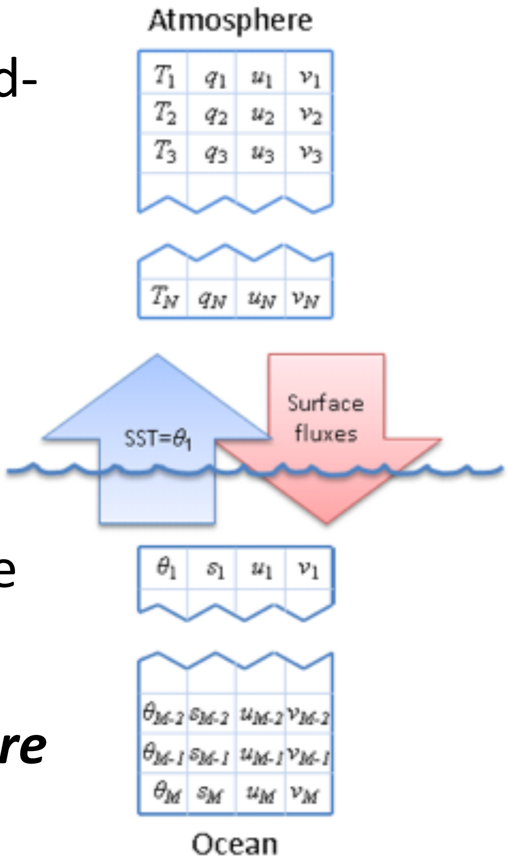
Idealised system

Ocean

Single column K-Profile Parameterisation (KPP) mixed-layer model based on the scheme of *Large et al*[1]

- developed by the NCAS climate group at UoR
- 4 state variables on 35 model levels (increased resolution near to the surface)
- forced by short and long wave radiation at surface

coupled via SST and surface fluxes of heat, moisture and momentum



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[1] *DOI: 10.1029/94RG01872*

Atmosphere model equations

$$\begin{aligned}
\frac{\partial u}{\partial t} + \dot{\eta}\frac{\partial u}{\partial \eta} &= f(v - v_g) + F_u + P_u \, , \\
\frac{\partial v}{\partial t} + \dot{\eta}\frac{\partial v}{\partial \eta} &= -f(u - u_g) + F_v + P_v \, , \\
\frac{\partial T}{\partial t} + \dot{\eta}\frac{\partial T}{\partial \eta} &= \frac{R}{c_p} T \frac{\omega}{p} + F_T + P_T \, , \\
\frac{\partial q}{\partial t} + \dot{\eta}\frac{\partial q}{\partial \eta} &= F_q + P_q \, ,
\end{aligned}$$

horizontal advection term

tendencies due to parameterisation of physics

ω - vertical velocity in pressure co-ordinates

$\dot{\eta}$ - vertical velocity in η co-ordinates

Ocean model equations

$$\begin{aligned}
\frac{\partial \overline{\theta}}{\partial t} &= -\frac{\partial \overline{w'\theta'}}{\partial z} - \frac{\partial Q_n}{\partial z}\,, \\
\frac{\partial \overline{s}}{\partial t} &= -\frac{\partial \overline{w's'}}{\partial z}\,, \\
\frac{\partial \overline{u}_o}{\partial t} &= -\frac{\partial \overline{w'u'_o}}{\partial z} + f v_o\,, \\
\frac{\partial \overline{v}_o}{\partial t} &= -\frac{\partial \overline{w'v'_o}}{\partial z} - f u_o\,.
\end{aligned}$$

w - turbulent vertical velocity

$\overline{w'\phi'}$ - turbulent flux

Q_n - non-turbulent heat flux

Idealised system

Simplifications for 4D-Var development

- stripped down atmosphere code
  - adiabatic component + vertical diffusion
- diffusion scheme computes surface fluxes to pass to ocean model
- perturbations to the diffusion coefficients ignored in tangent linear model
- non-local turbulent mixing term in the KPP-model switched off
- retained option to run coupled model with full physics

4D-Var

$$\min \; J(\mathbf{x}_0) = \frac{1}{2}(\mathbf{x}_0 - \mathbf{x}_b)^T \mathbf{B}^{-1} (\mathbf{x}_0 - \mathbf{x}_b)$$

$$+ \sum_{i=0}^{n} (h_i[\mathbf{x}_i] - \mathbf{y}_i)^T \mathbf{R}_i^{-1} (h_i[\mathbf{x}_i] - \mathbf{y}_i)$$

subject to

$$\mathbf{x}_{i+1} = m_i(t_{i+1}, t_i, \mathbf{x}_i), \quad i = 0, \ldots, n-1$$

$\mathbf{x}_b$ - *a priori* (background) state

$\mathbf{y}_i$ - Observations

h_i - Observation operator

$\mathbf{B}$ - Background error covariance matrix

$\mathbf{R}_i$ - Observation error covariance matrix

Incremental 4D-Var

Solve iteratively

set $\mathbf{x}_0^{(0)} = \mathbf{x}_b$

outer loop: for k = 0, … , Nouter

compute $\mathbf{d}_i^{(k)} = \mathbf{y}_i - h(\mathbf{x}_i^{(k)})$, where $\mathbf{x}_i^{(k)} = m(t_i, t_0, \mathbf{x}_0^{(k)})$

inner loop: minimise

$$\mathrm{J}^{(k)}\left(\delta\mathbf{x}_0^{(k)}\right) = \frac{1}{2}\left(\delta\mathbf{x}_0^{(k)} - \left(\mathbf{x}_b - \mathbf{x}_0^{(k)}\right)\right)^T \mathbf{B}^{-1}\left(\delta\mathbf{x}_0^{(k)} - (\mathbf{x}_b - \mathbf{x}_0^{(k)})\right) + \frac{1}{2}\sum_{i=0}^{n}\left(\mathbf{H}_i\delta\mathbf{x}_i^{(k)} - \mathbf{d}_i^{(k)}\right)^T \mathbf{R}_i^{-1}\left(\mathbf{H}_i\delta\mathbf{x}_i^{(k)} - \mathbf{d}_i^{(k)}\right)$$

subject to $\delta\mathbf{x}_i^{(k)} = \mathbf{M}(t_i, t_0, \mathbf{x}^{(k)})\delta\mathbf{x}_0^{(k)}$

update $\mathbf{x}_0^{(k+1)} = \mathbf{x}_0^{(k)} + \delta\mathbf{x}_0^{(k)}$

Uncoupled incremental 4D-Var

first guess $\mathbf{x}_{0,atmos}^{(0)} = \mathbf{x}_{b,atmos}$

outer loop (k)

non-linear trajectory computed using **atmosphere** model

$\mathbf{x}_{i,atmos}^{(k)} = m_{atmos}(t_i, t_0, \mathbf{x}_{0,atmos}^{(k)})$, prescribed SST

innovations $\mathbf{d}_i^{(k)} = \mathbf{y}_i - h(\mathbf{x}_{i,atmos}^{(k)})$

perturbation first guess $\delta\mathbf{x}_{i,atmos}^{(k)} = 0$

inner loop

TL of **atmosphere** model: $J_{atmos}^{(k)}$

ADJ of **atmosphere** model: $\nabla J_{atmos}^{(k)}$

update $\mathbf{x}_{0,atmos}^{(k+1)} = \mathbf{x}_{0,atmos}^{(k)} + \delta\mathbf{x}_{0,atmos}^{(k)}$

first guess $\mathbf{x}_{0,ocean}^{(0)} = \mathbf{x}_{b,ocean}$

outer loop (k)

non-linear trajectory computed using **ocean** model

$\mathbf{x}_{i,ocean}^{(k)} = m_{ocean}(t_i, t_0, \mathbf{x}_{0,ocean}^{(k)})$, prescribed surface fluxes

innovations $\mathbf{d}_i^{(k)} = \mathbf{y}_i - h(\mathbf{x}_{i,ocean}^{(k)})$

perturbation first guess $\delta\mathbf{x}_{i,ocean}^{(k)} = 0$

inner loop

TL of **ocean** model: $J_{ocean}^{(k)}$

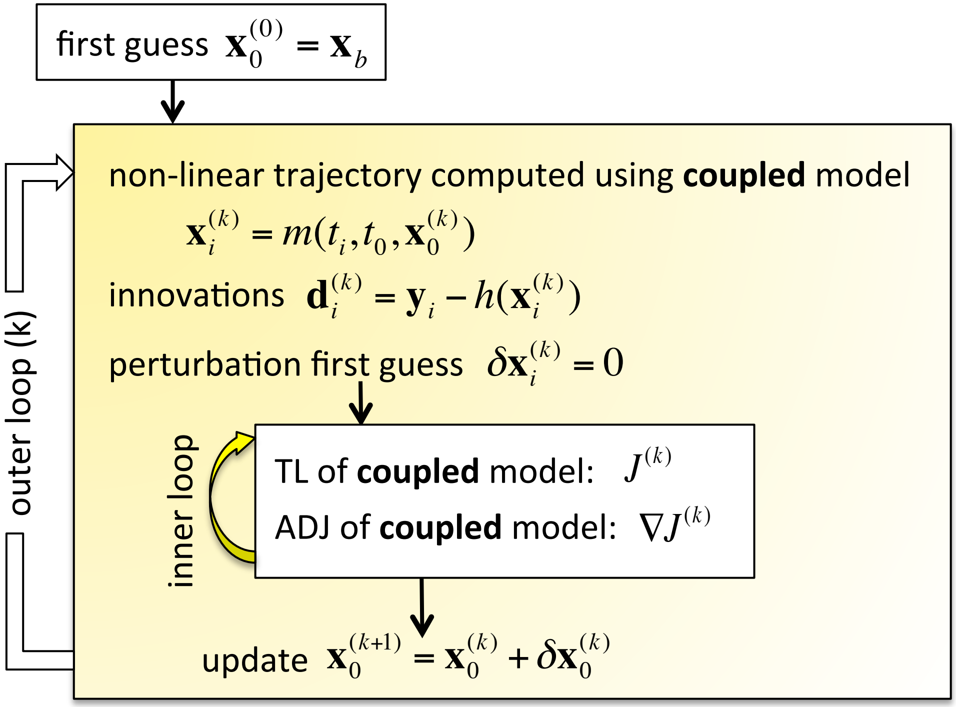
ADJ of **ocean** model: $\nabla J_{ocean}^{(k)}$

update $\mathbf{x}_{0,ocean}^{(k+1)} = \mathbf{x}_{0,ocean}^{(k)} + \delta\mathbf{x}_{0,ocean}^{(k)}$

- allows for different assimilation window lengths and schemes in atmosphere and ocean
- avoids new technical development

- atmosphere and ocean analysis dynamically inconsistent - can lead to imbalance in forecast
- observations not used to full potential

Fully coupled incremental 4D-Var



first guess $\mathbf{x}_0^{(0)} = \mathbf{x}_b$

non-linear trajectory computed using **coupled** model

$\mathbf{x}_i^{(k)} = m(t_i, t_0, \mathbf{x}_0^{(k)})$

innovations $\mathbf{d}_i^{(k)} = \mathbf{y}_i - h(\mathbf{x}_i^{(k)})$

perturbation first guess $\delta\mathbf{x}_i^{(k)} = 0$

TL of **coupled** model: $J^{(k)}$

ADJ of **coupled** model: $\nabla J^{(k)}$

update $\mathbf{x}_0^{(k+1)} = \mathbf{x}_0^{(k)} + \delta\mathbf{x}_0^{(k)}$

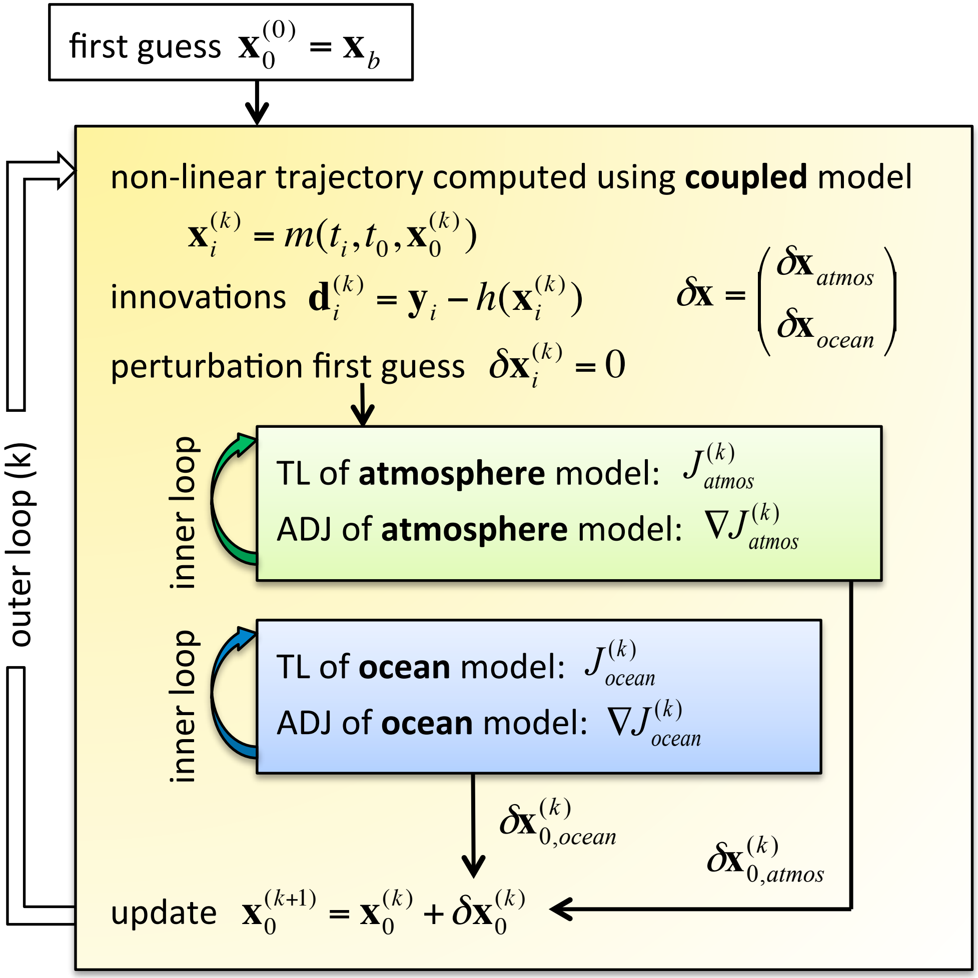
single minimisation process:

- allows for cross-covariances between atmosphere and ocean

$$B = \begin{pmatrix} B_a & B_{ao} \\ B_{ao} & B_o \end{pmatrix}$$

- atmosphere observations can influence ocean analysis and vice versa
- requires same window length in atmosphere and ocean
- technically very challenging

Weakly coupled incremental 4D-Var



first guess $\mathbf{x}_0^{(0)} = \mathbf{x}_b$

non-linear trajectory computed using **coupled** model

$$\mathbf{x}_i^{(k)} = m(t_i, t_0, \mathbf{x}_0^{(k)})$$

innovations $\mathbf{d}_i^{(k)} = \mathbf{y}_i - h(\mathbf{x}_i^{(k)})$

$$\delta\mathbf{x} = \begin{pmatrix} \delta\mathbf{x}_{atmos} \\ \delta\mathbf{x}_{ocean} \end{pmatrix}$$

perturbation first guess $\delta\mathbf{x}_i^{(k)} = 0$

TL of **atmosphere** model: $J^{(k)}_{atmos}$

ADJ of **atmosphere** model: $\nabla J^{(k)}_{atmos}$

TL of **ocean** model: $J^{(k)}_{ocean}$

ADJ of **ocean** model: $\nabla J^{(k)}_{ocean}$

$\delta\mathbf{x}^{(k)}_{0,ocean}$

$\delta\mathbf{x}^{(k)}_{0,atmos}$

update $\mathbf{x}_0^{(k+1)} = \mathbf{x}_0^{(k)} + \delta\mathbf{x}_0^{(k)}$

separate minimisation for atmosphere and ocean:

- new technical development limited
- allows for different assimilation windows and schemes in ocean and atmosphere
- no explicit cross-covariances between atmosphere and ocean
- balance?

Identical twin experiments

comparison of uncoupled, weakly coupled and fully coupled systems

- 12 hour assimilation window, 3 outer loops
- data for June 2013, 188.75°E, 25°N (Pacific Ocean)
- *'true'* initial state is coupled non-linear forecast valid at 00:00 UTC on 3rd June, with initial atmosphere state from ERA Interim and initial ocean state from Mercator Ocean
- initial background state is a perturbed non-linear model forecast valid at same time
- uncoupled atmosphere assimilation - SST from ERA interim
- uncoupled ocean assimilation - surface fluxes from ERA interim

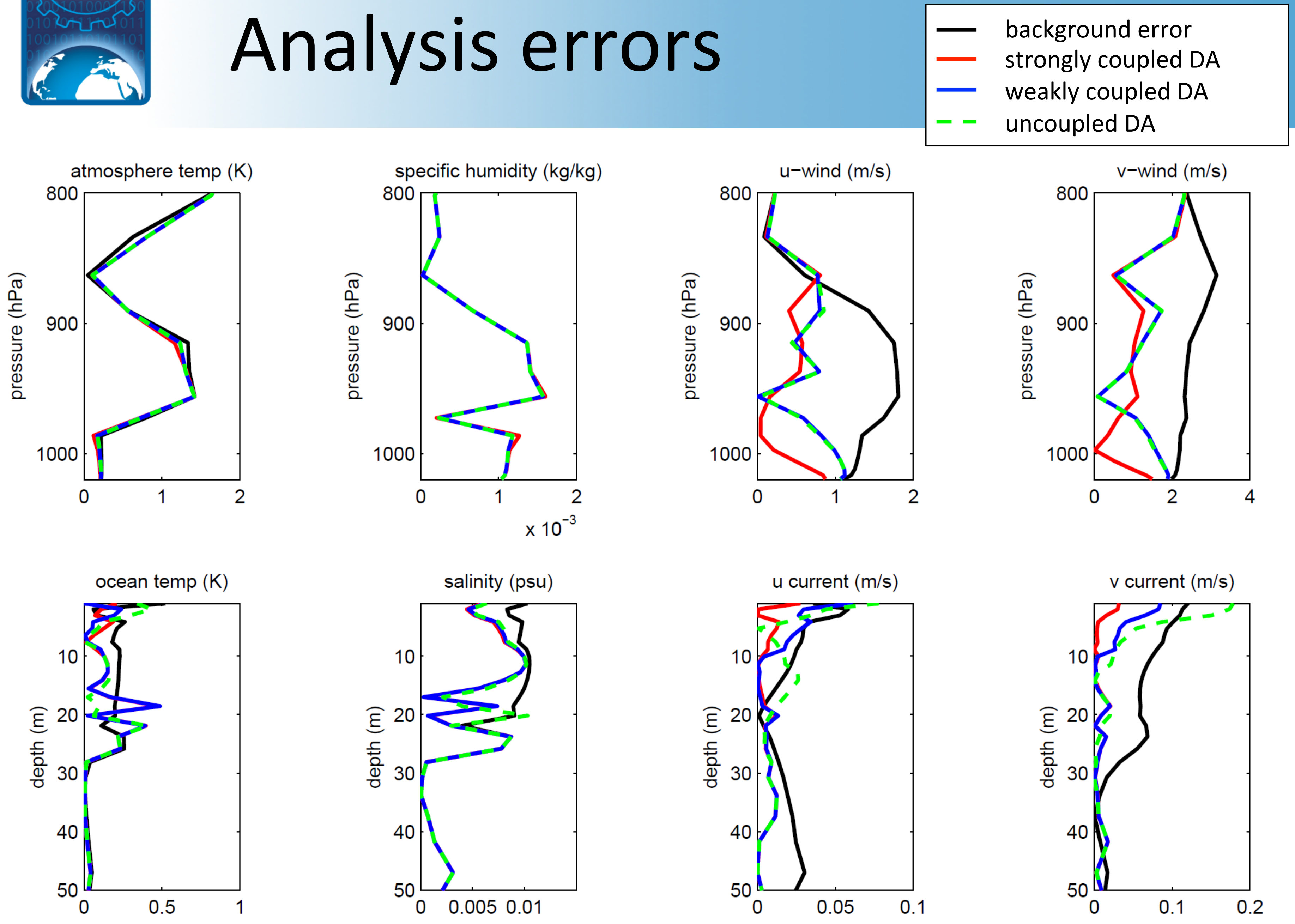
Identical twin experiments

- observations are generated by adding random Gaussian noise to true solution => operator h is linear

T	u wind	v wind	θ	salinity	u current	v current
1.0	1.5	1.5	0.01	0.003	0.01	0.01

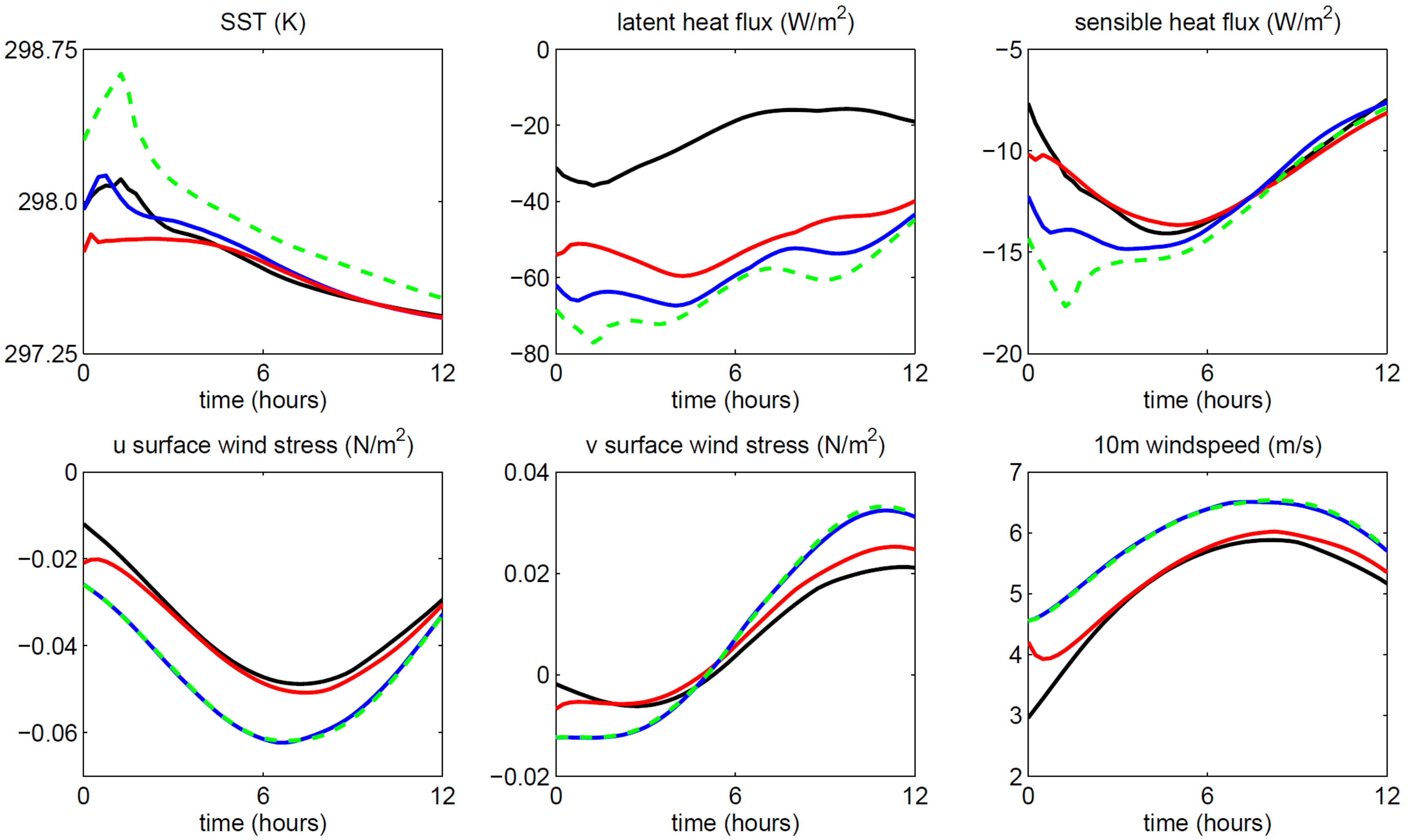
- atmosphere: 3 hourly observations of temperature, *u* and *v* wind components taken at 17 of 60 levels
- ocean: 6 hourly observations of temperature, salinity, *u* and *v* currents taken at 23 of 35 levels
- no observations at initial time
- error covariance matrices $\mathbf{B}$ and $\mathbf{R}$ are diagonal
- simple preconditioning of cost function using $\mathbf{B}^{1/2}$

Analysis errors

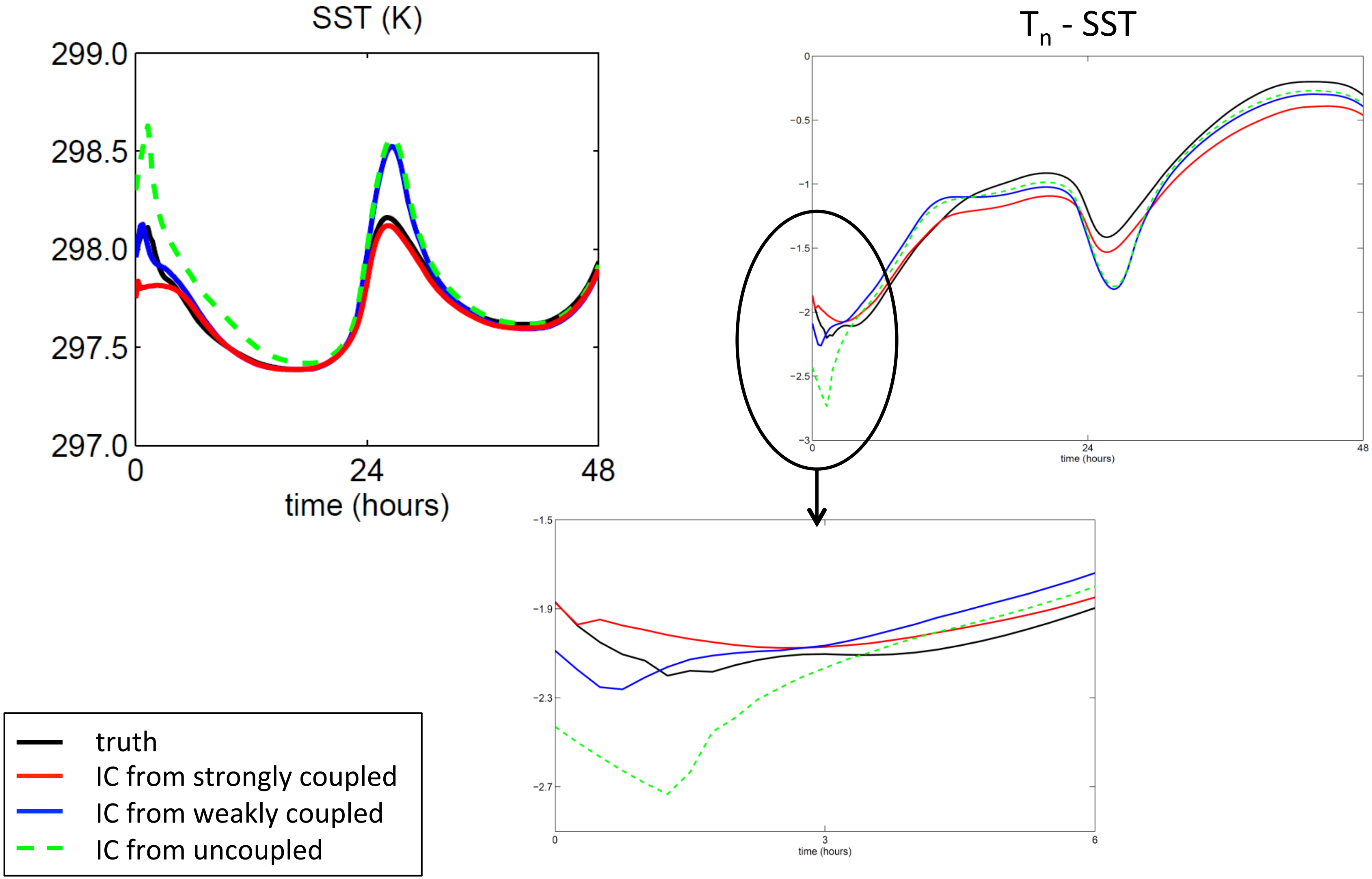


Can coupled data assimilation reduce or eliminate initialisation shock?

Initialisation shock

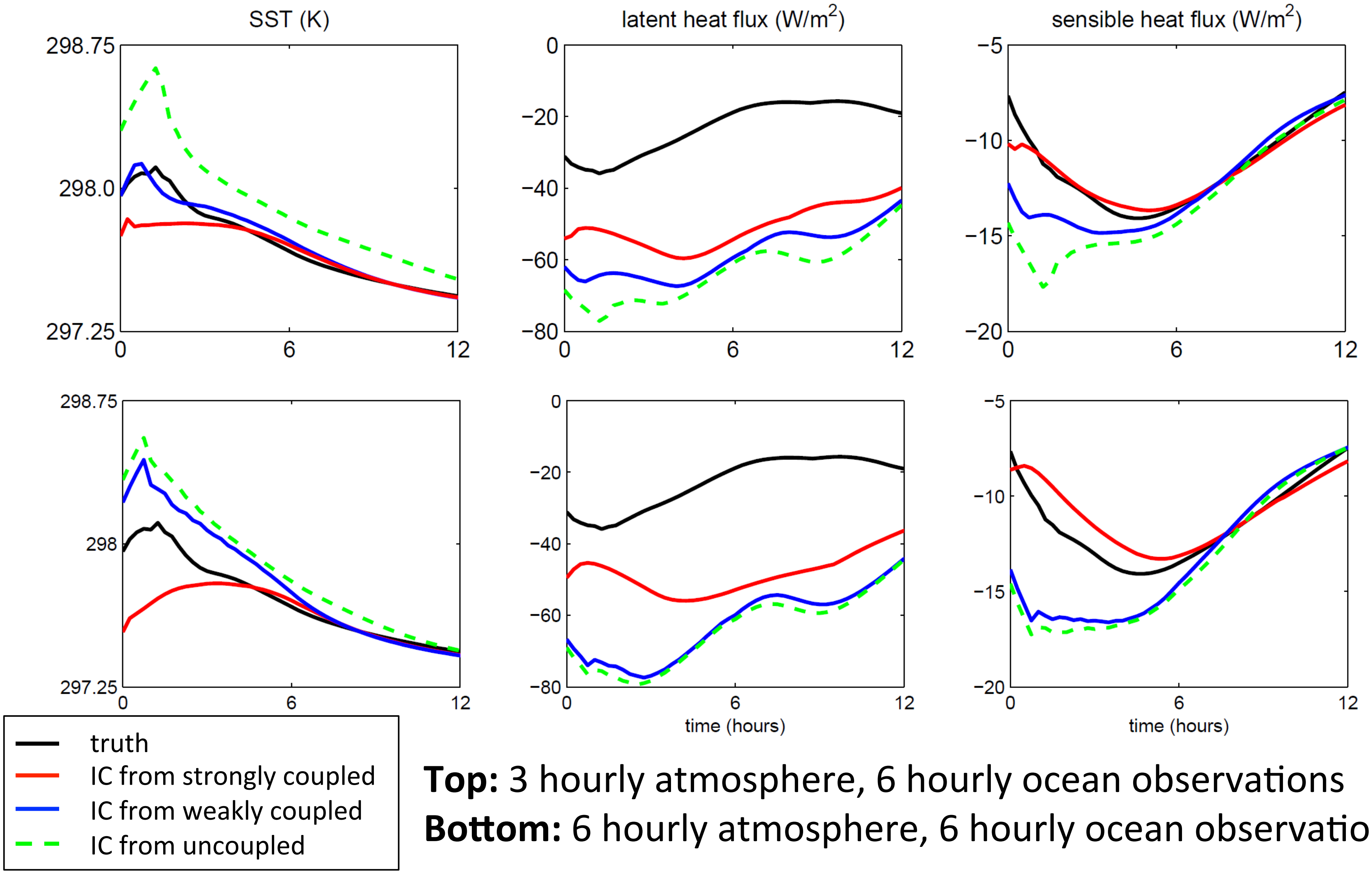


Initialisation shock



How does the frequency of the observations affect the results?

Reduced frequency of obs

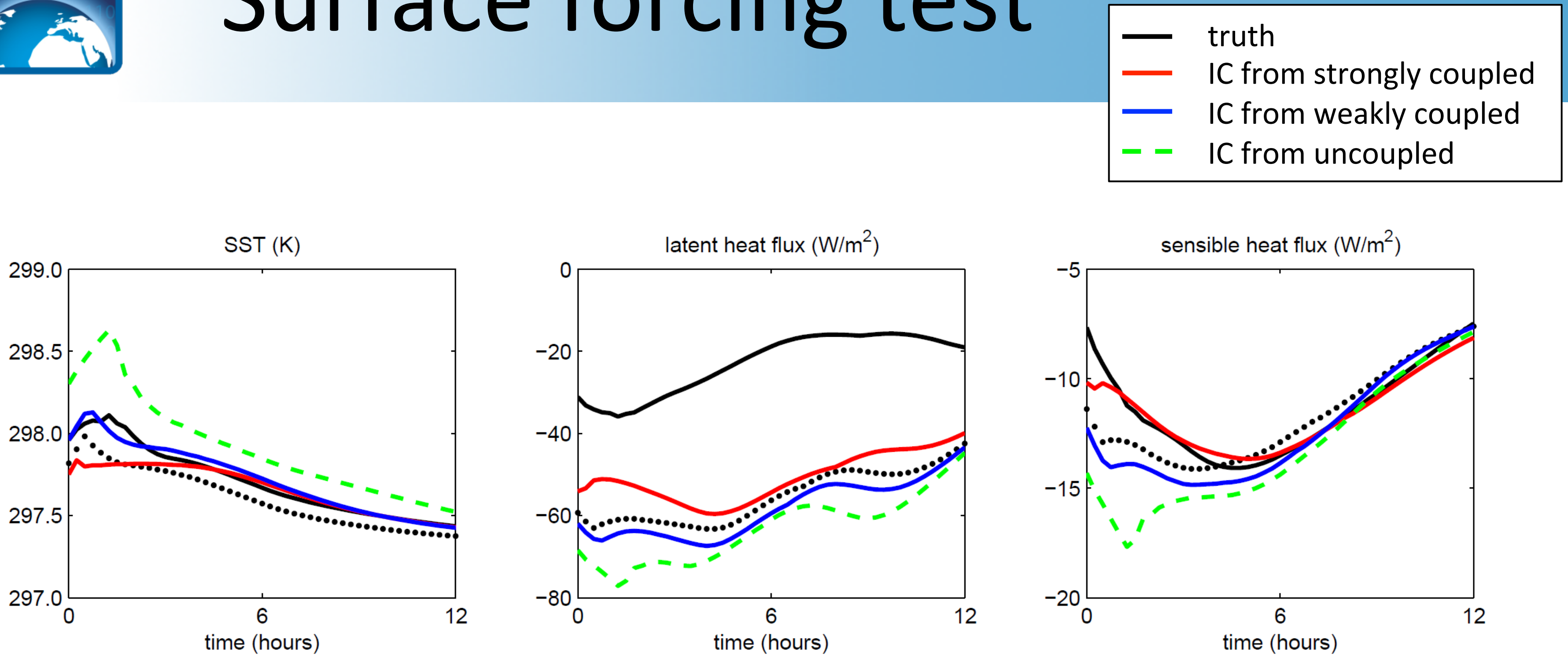


Top: 3 hourly atmosphere, 6 hourly ocean observations

Bottom: 6 hourly atmosphere, 6 hourly ocean observations

How does the accuracy of the prescribed SST and surface fluxes affect the results of the uncoupled assimilation?

Surface forcing test

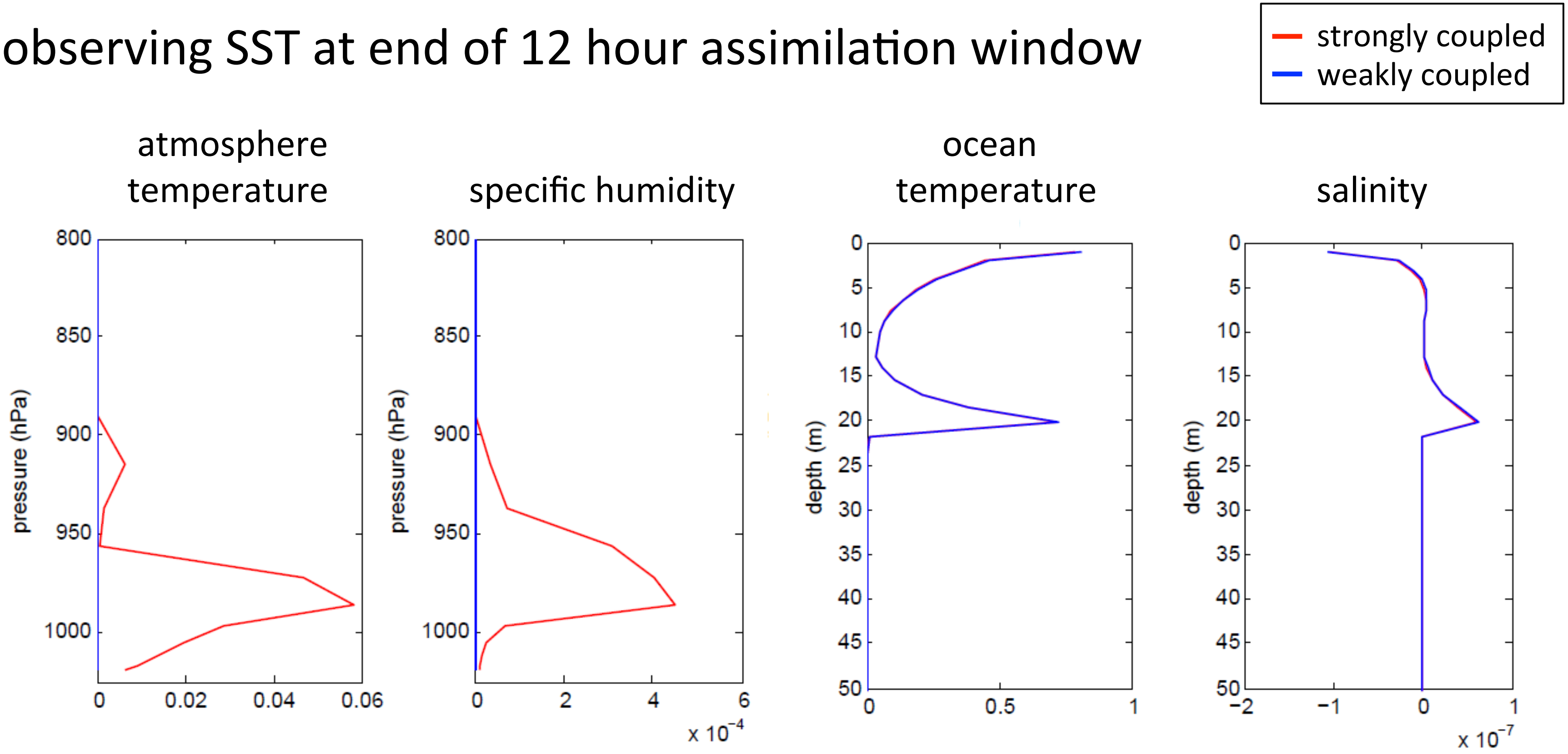


- black circles show forecast from initialised with analysis from uncoupled assimilations that were forced using true SST (uncoupled atmosphere) and surface fluxes (uncoupled ocean) from truth trajectory
- demonstrates best we may expect from uncoupled system

Can coupled data assimilation make greater use of near-surface observations?

Single observation tests

observing SST at end of 12 hour assimilation window



atmosphere temperature

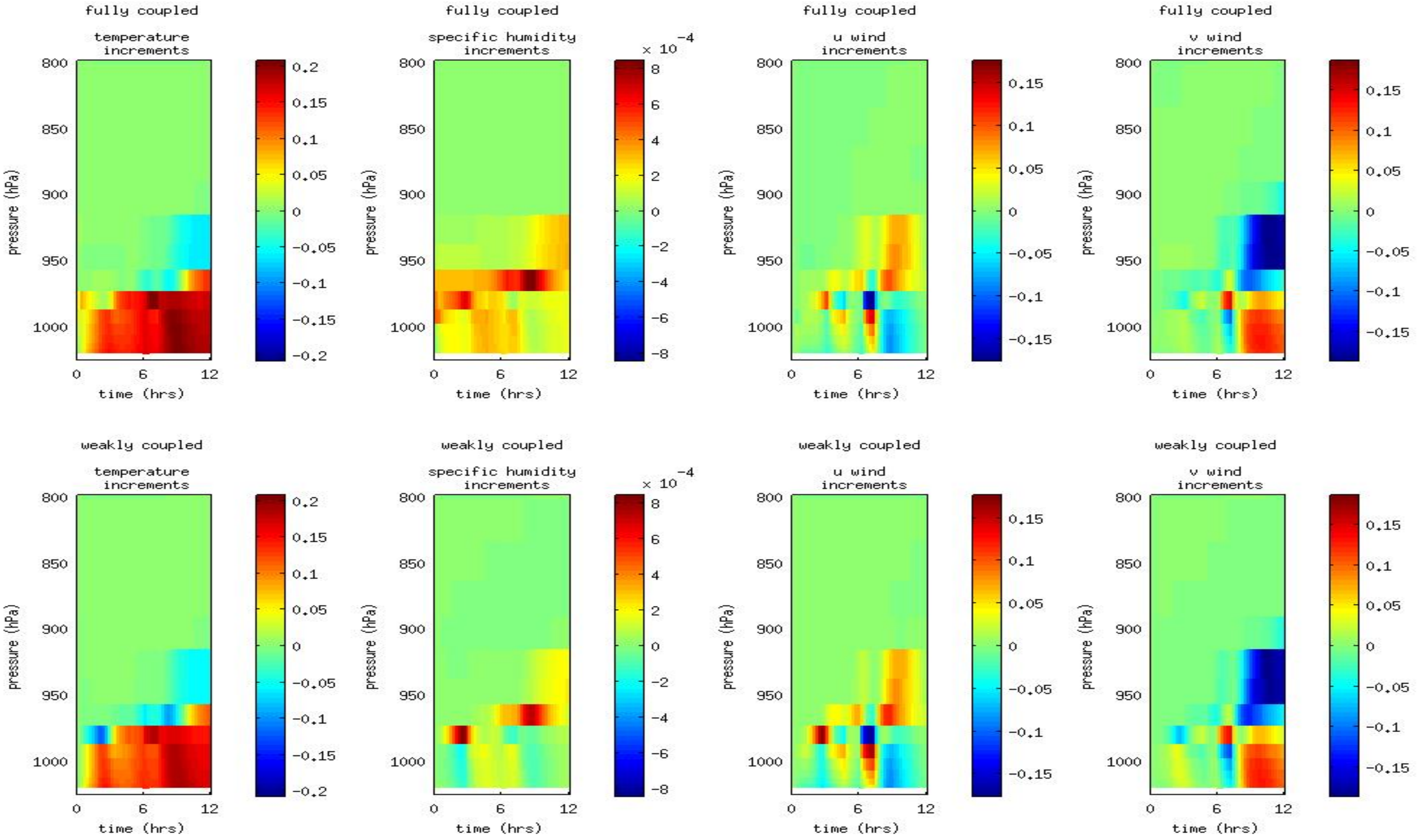
specific humidity

ocean temperature

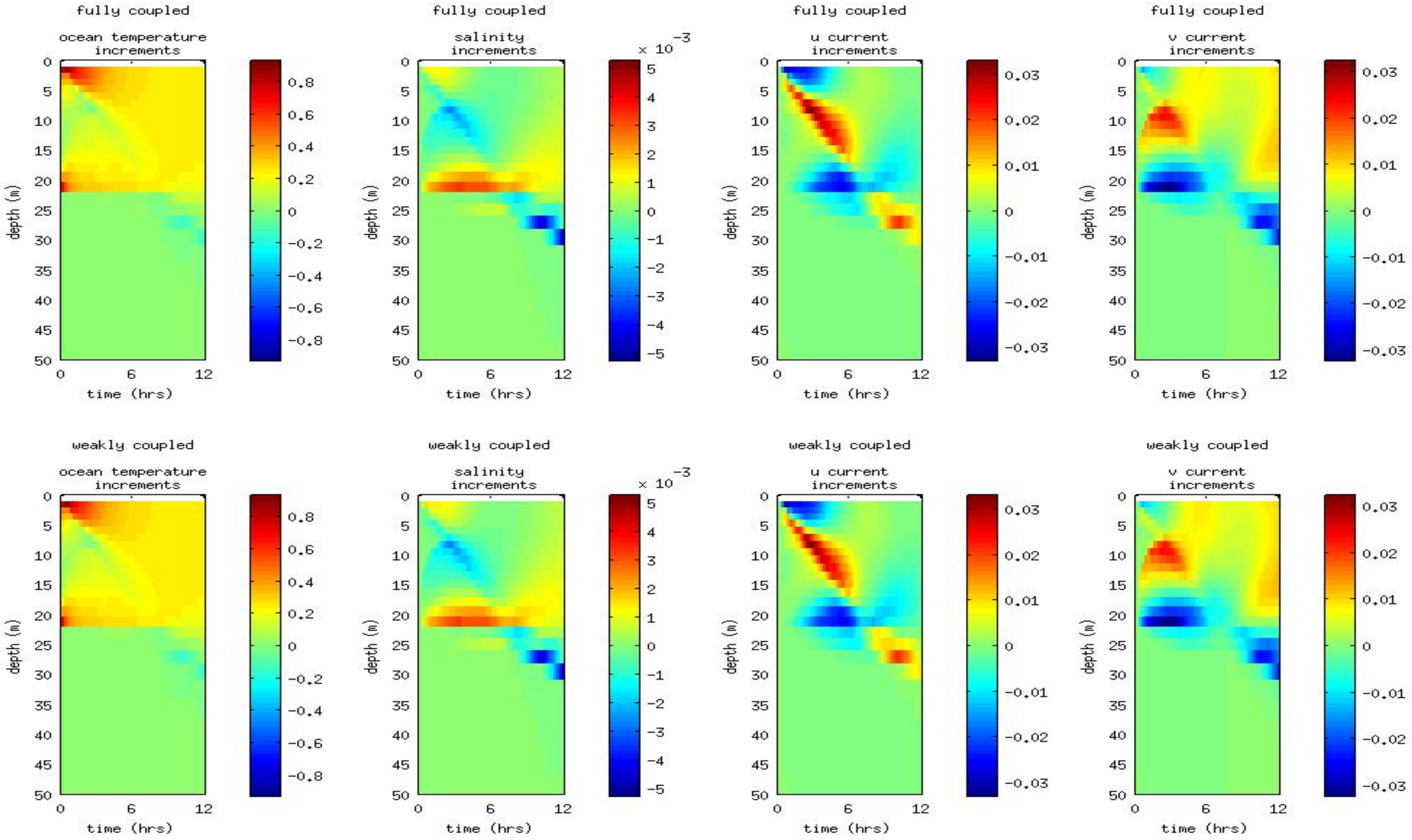
salinity

(x_b - x_a) increments at t_0

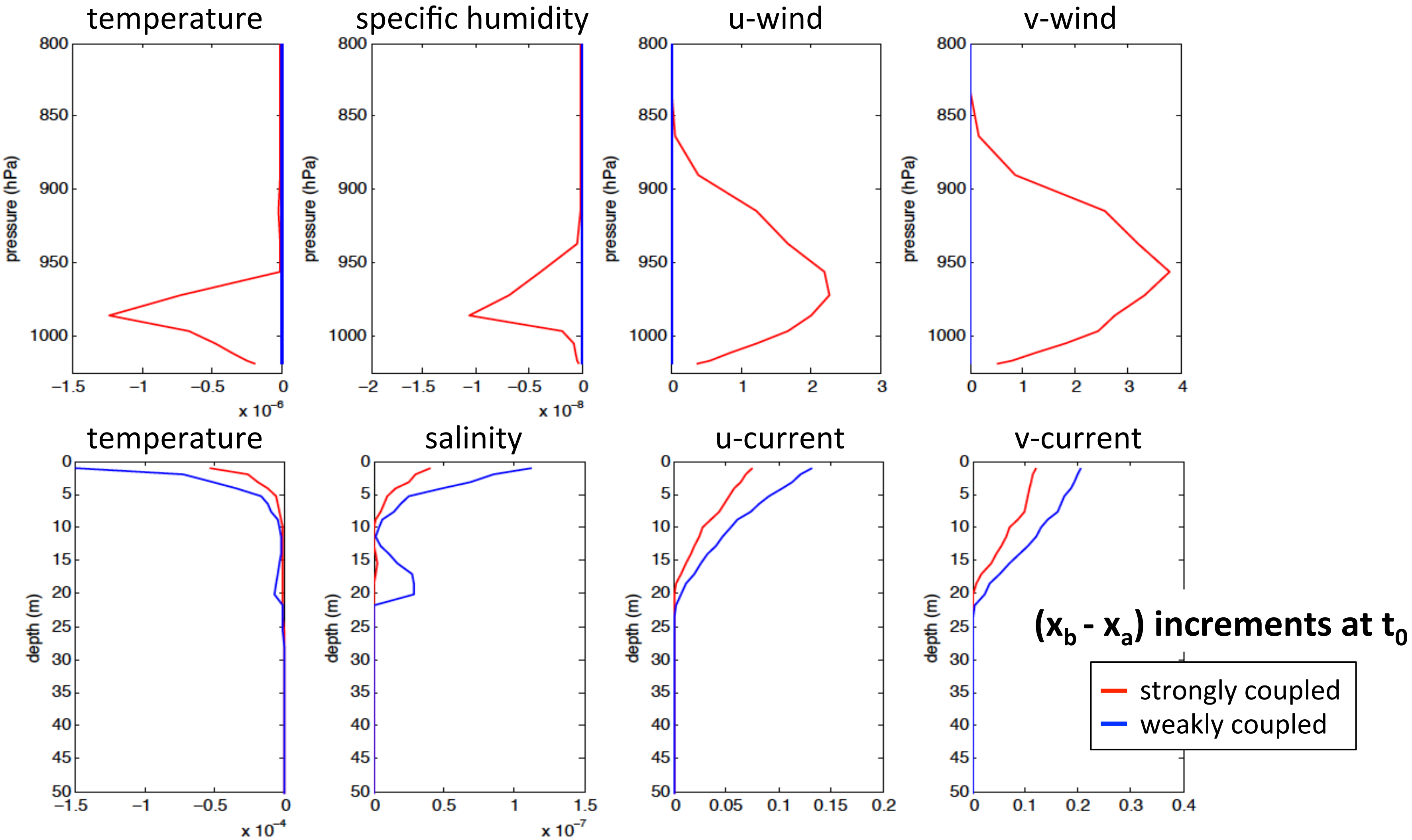
Analysis increments



Analysis increments



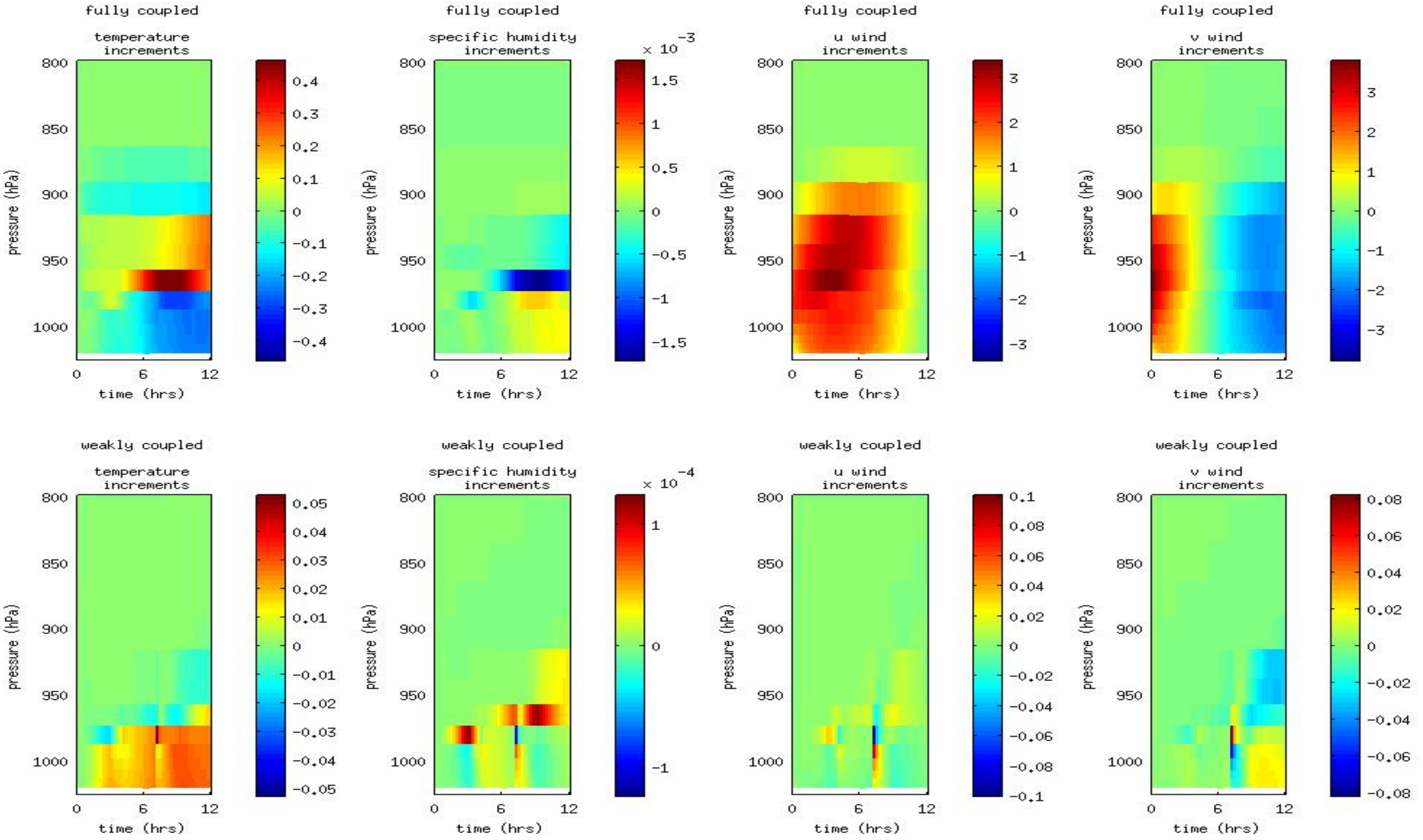
Single observation tests



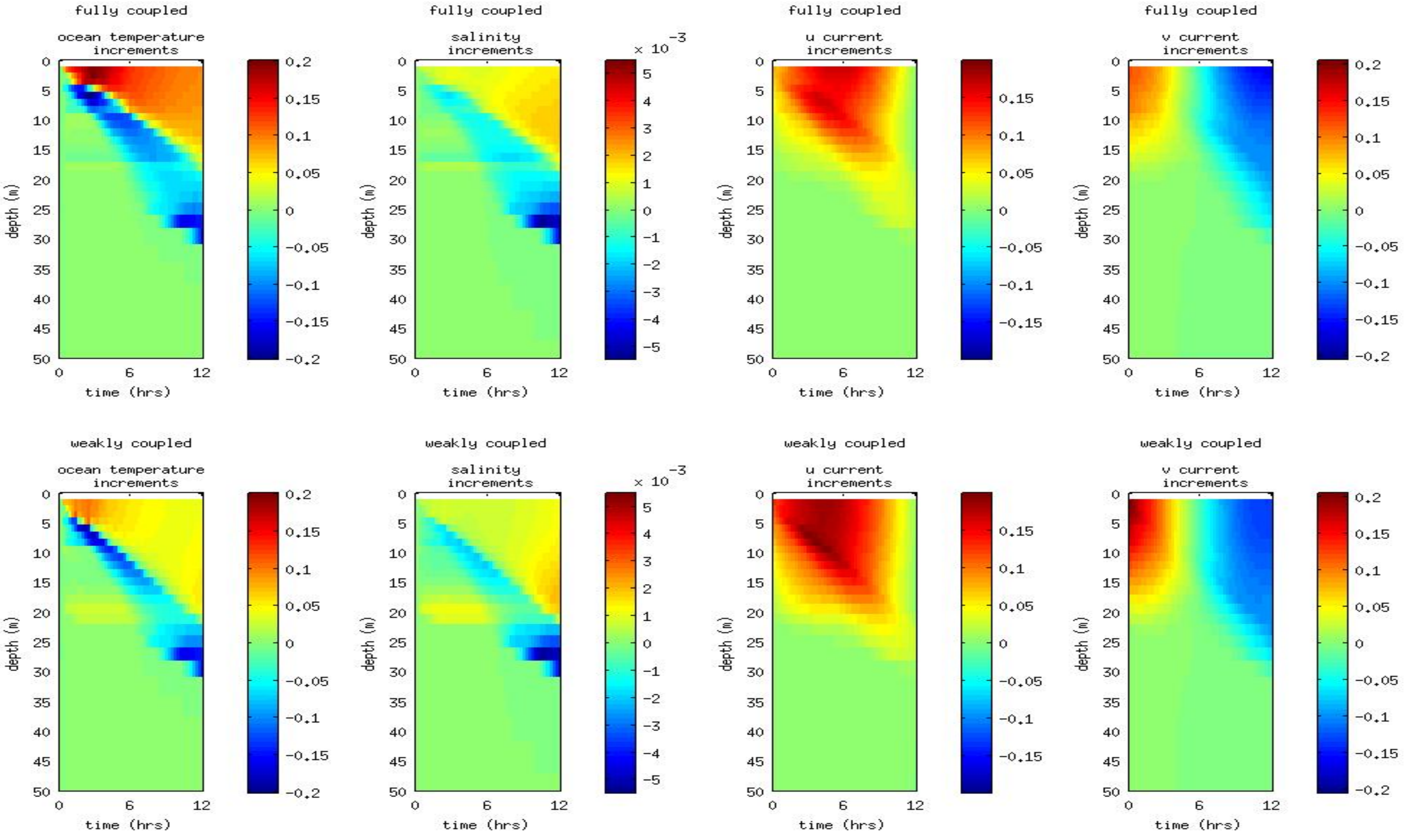
$(x_b - x_a)$ increments at t_0

observing ocean velocity at top level of ocean model, at end of 12hr window

Analysis increments



Analysis increments



Summary

Demonstrated potential benefits of moving towards coupled data assimilation systems:

- initialisation from coupled analysis has positive impact on coupled model forecast, especially in upper ocean.
- coupled data assimilation is able to reduce initialisation shock.
- coupled assimilation systems enable greater use of near-surface data through generation of cross covariance information.
- strongly coupled system generally outperforms the weakly and uncoupled systems.
- weakly coupled system is sensitive to input parameters of the assimilation but still offers benefits over uncoupled system.
- current efforts of operational centres are a step in the right direction.

Extras

A paper is in preparation and will be submitted very soon …

Acknowledgments

Many thanks to Keith Haines, colleagues at the ECMWF and UK Met Office for useful discussions, and to Irina Sandu and Glenn Carver of the ECMWF for proving source code and technical support.