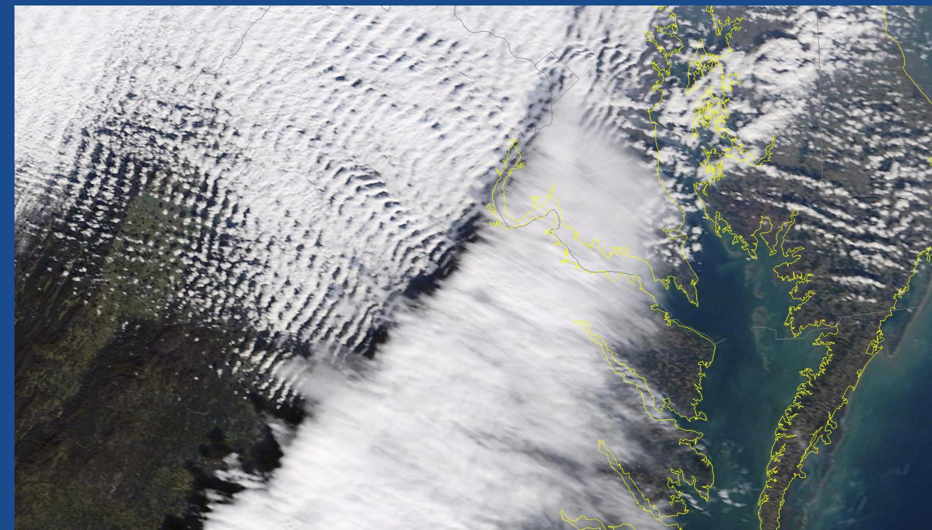
Trapped lee waves

Miguel Teixeira

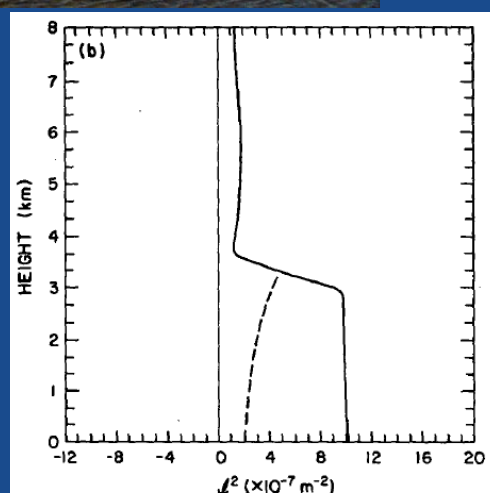
Examples

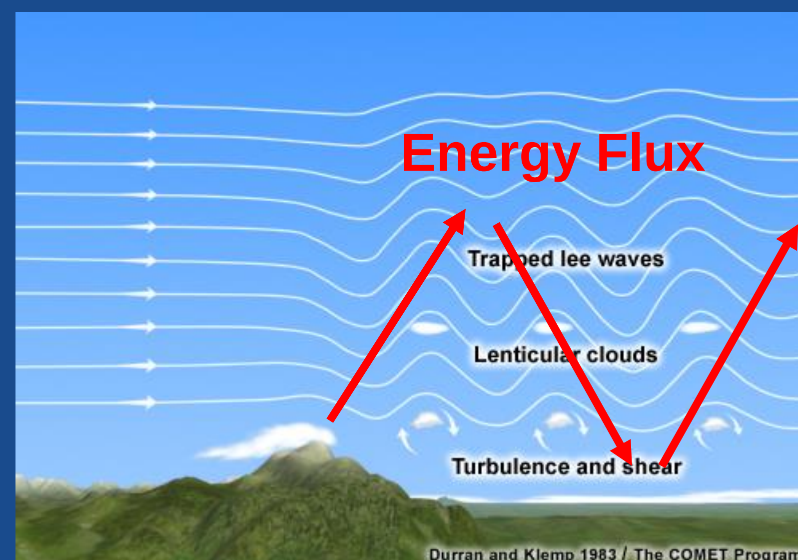


$$N^2(z) = \frac{g}{\theta_0}\frac{d\bar{\theta}}{dz}$$

Scorer parameter

$$l(z) = \left(\frac{N^2}{U^2} - \cancel{\frac{1}{U}\frac{d^2U}{dz^2}}\right)^{1/2}$$





Linear theory

University of Reading

- 2D flow
- Earth's rotation unimportant
- Stationary flow
- Linearization

$$\frac{d^2 w}{dz^2} + (l^2 - k^2) w = 0$$

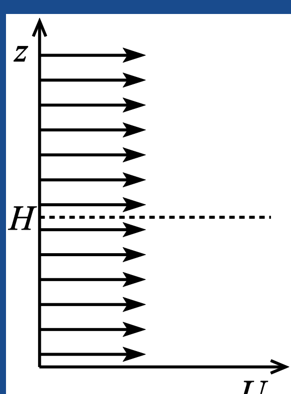
Solutions:

$k^2 < l^2$ $\quad w(x,z) = a\cos(kx + mz)$ waves propagate vertically

$k^2 > l^2$ $\quad w(x,z) = a e^{-nz}\cos(kx)$ waves are evanescent

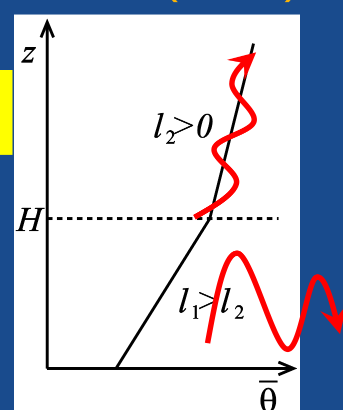
If $\boldsymbol{l}$ decreases with height, waves with wavenumber k my propagate at low levels but not at high levels $\Rightarrow$ Trapped lee waves

Examples: 2-layer atmospheres



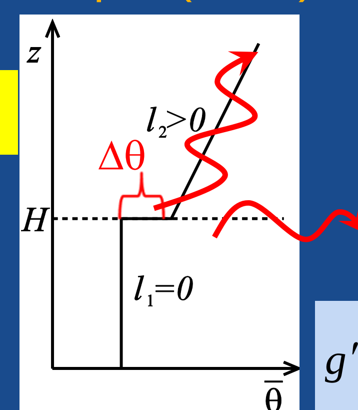
Scorer (1949)

Case 1



Vosper (2004)

Case 2

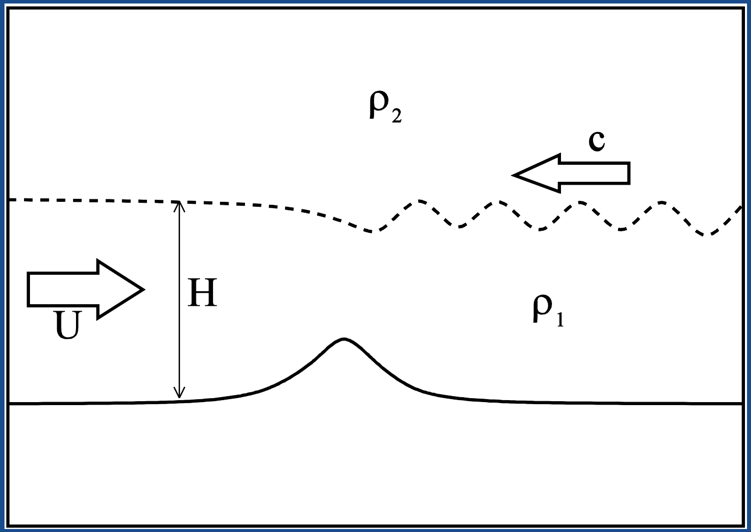


$$g' = g\frac{\Delta\theta}{\theta_0}$$

Simplest case

University of Reading

$l_2=0$



Surface waves at density interface

Phase speed: $c=\sqrt{\dfrac{g'\tanh(kH)}{k[1+\tanh(kH)]}}$

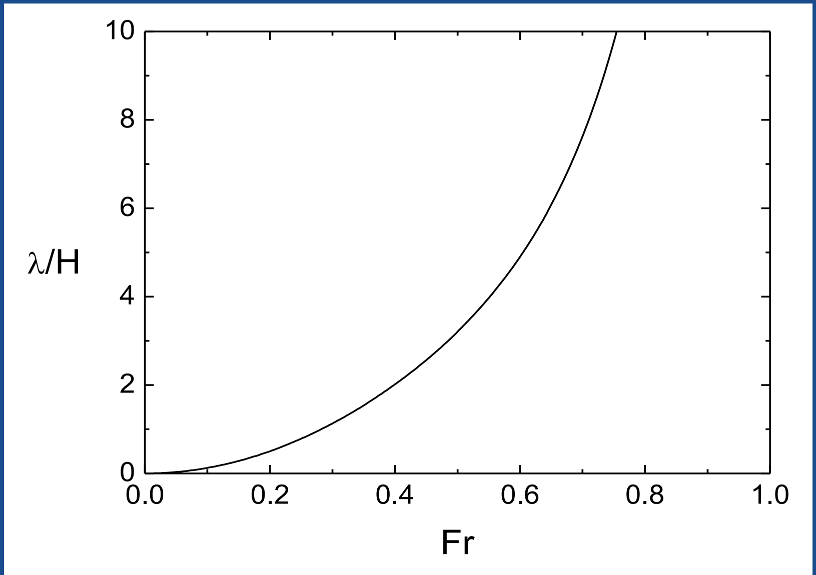
Resonance condition: $c=U$

$$\tanh(k_L H)=\frac{k_L H}{Fr^{-2}-k_L H}$$

$$Fr=\frac{U}{\sqrt{g'H}}$$

$k_L H$ is only a function of **Fr**

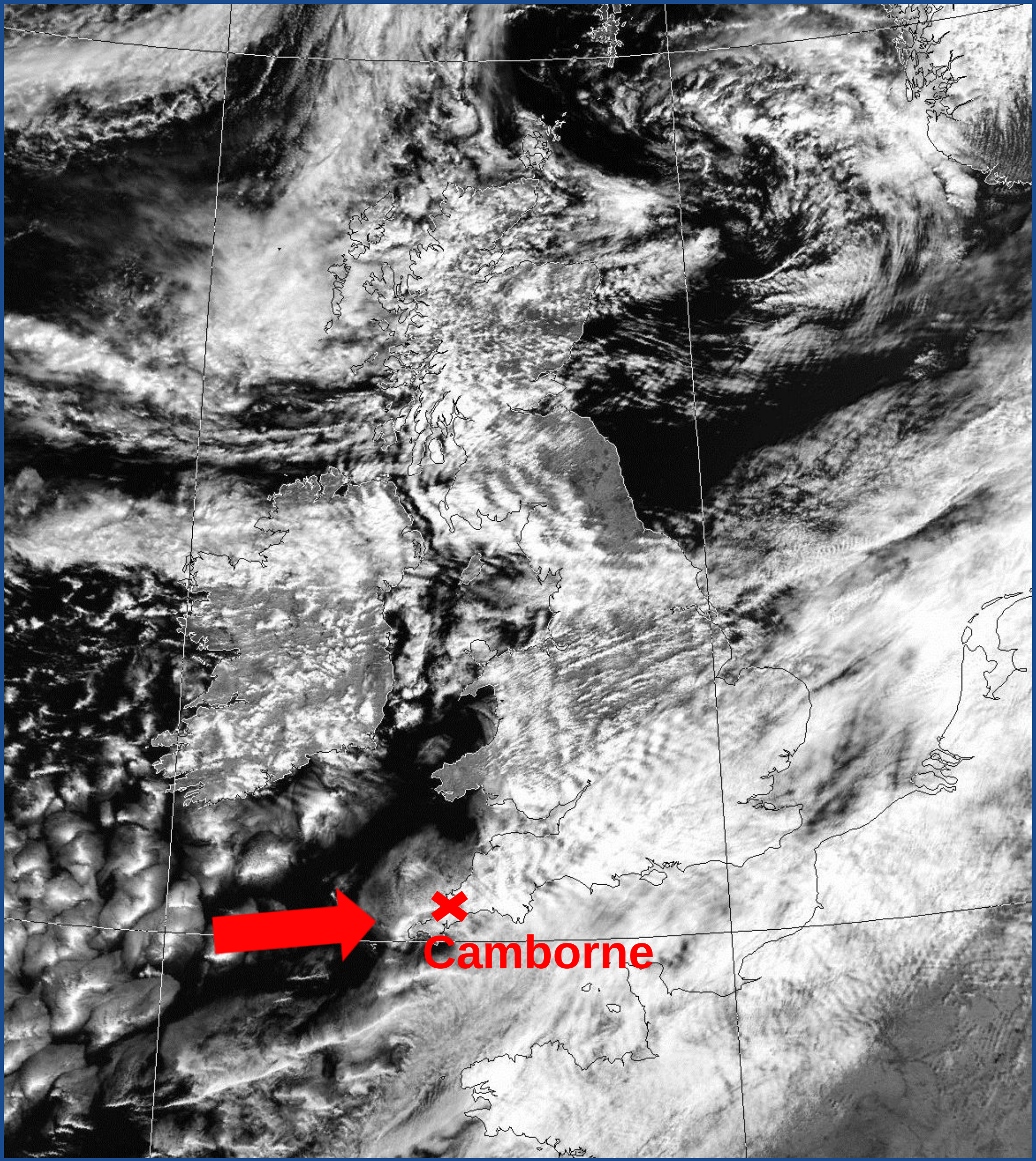
Wavelength of trapped lee waves: $\lambda_L=\dfrac{2\pi}{k_L}$



Last Tuesday

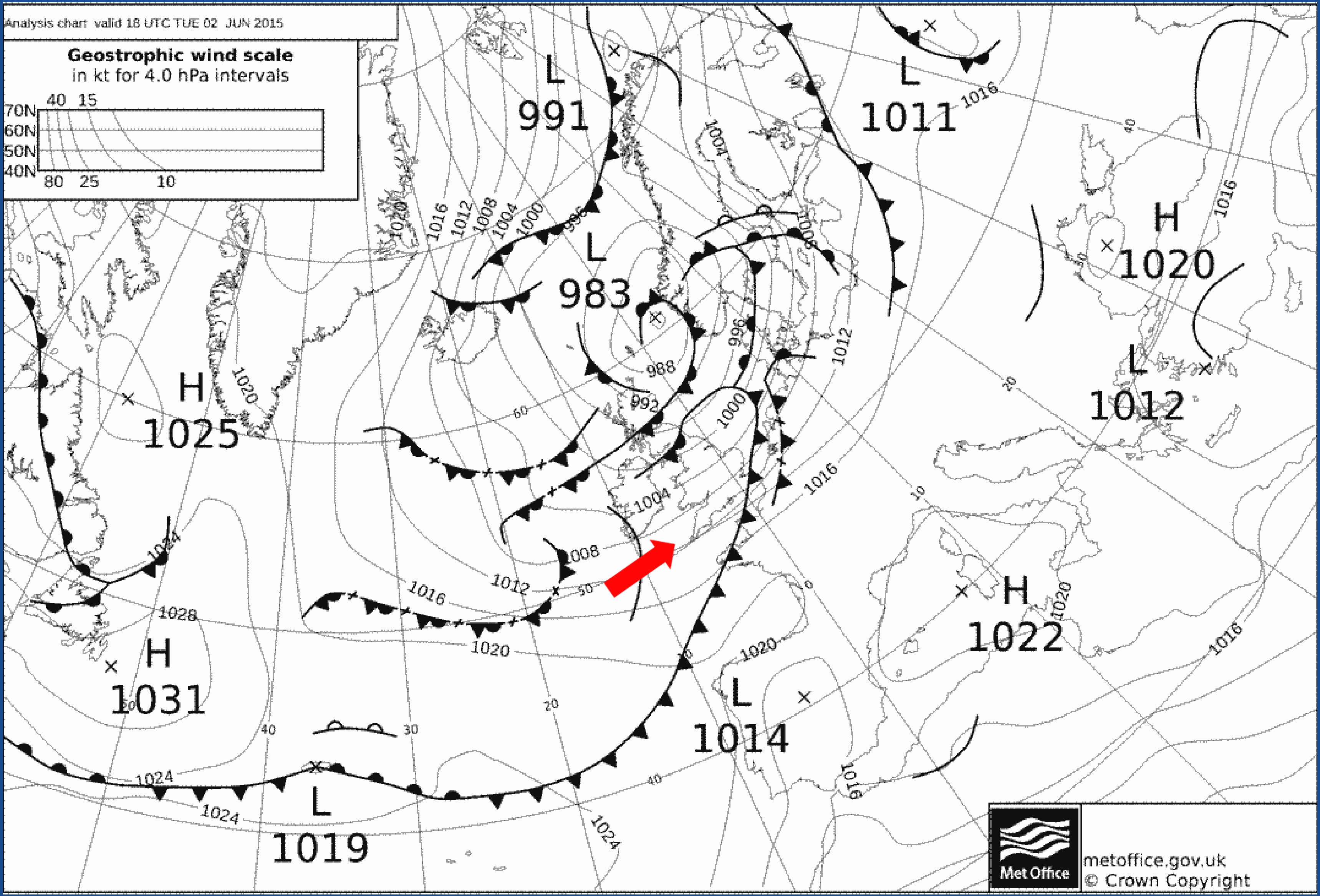
Visible, 2/6/2015, 10.35 UTC

University of Reading

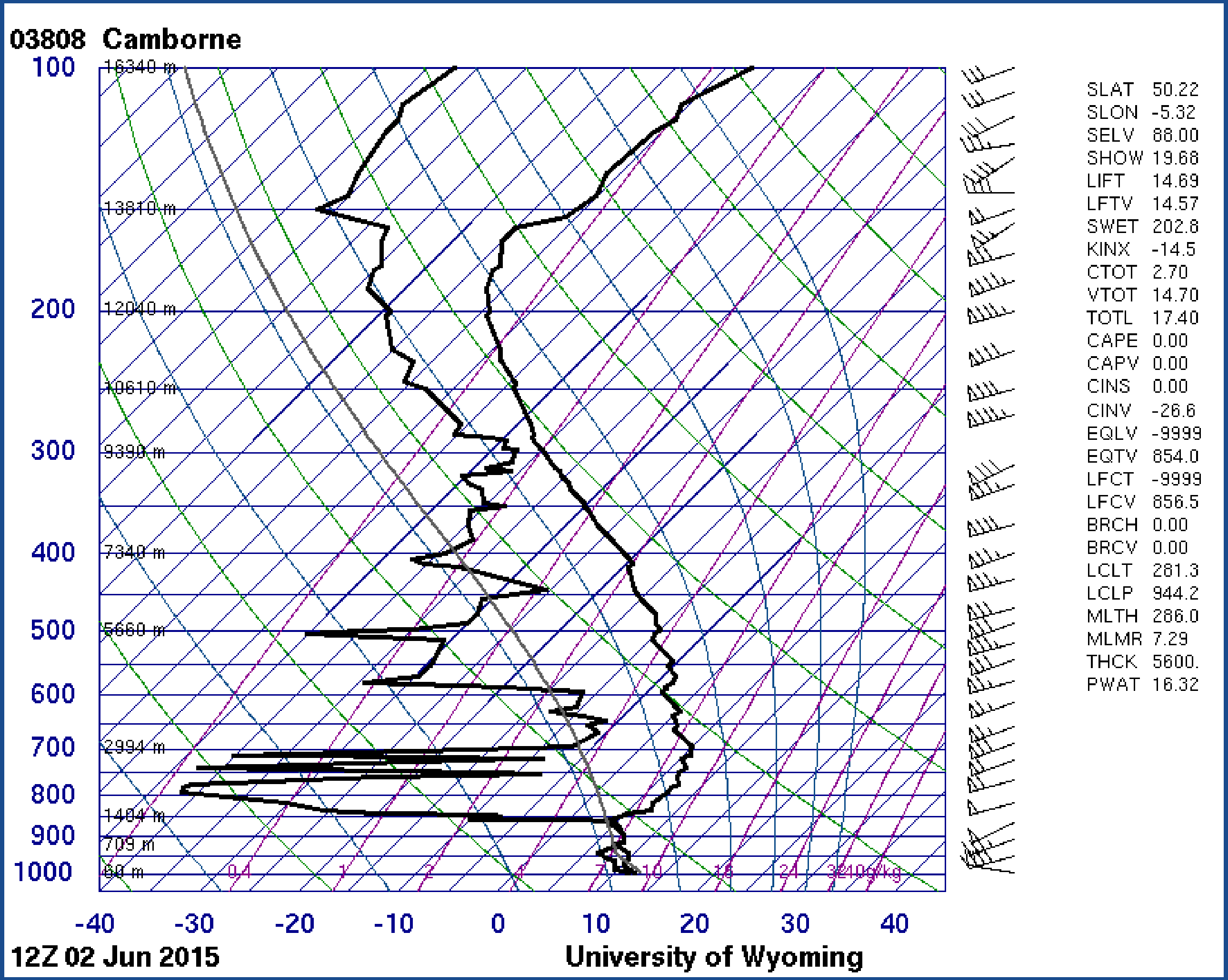


2/6/2015, 18 UTC

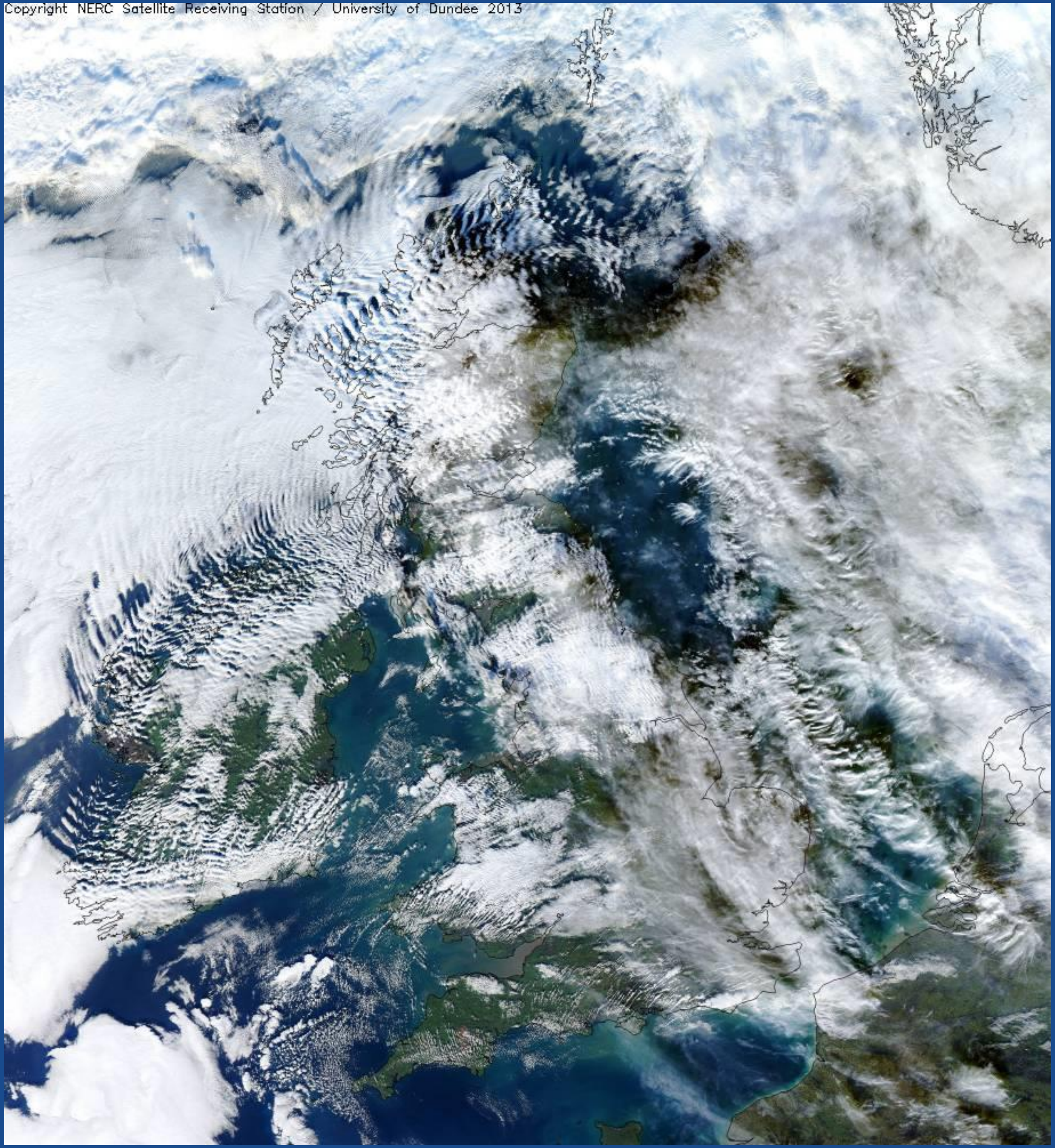
University of Reading



2/6/2015, 12 UTC



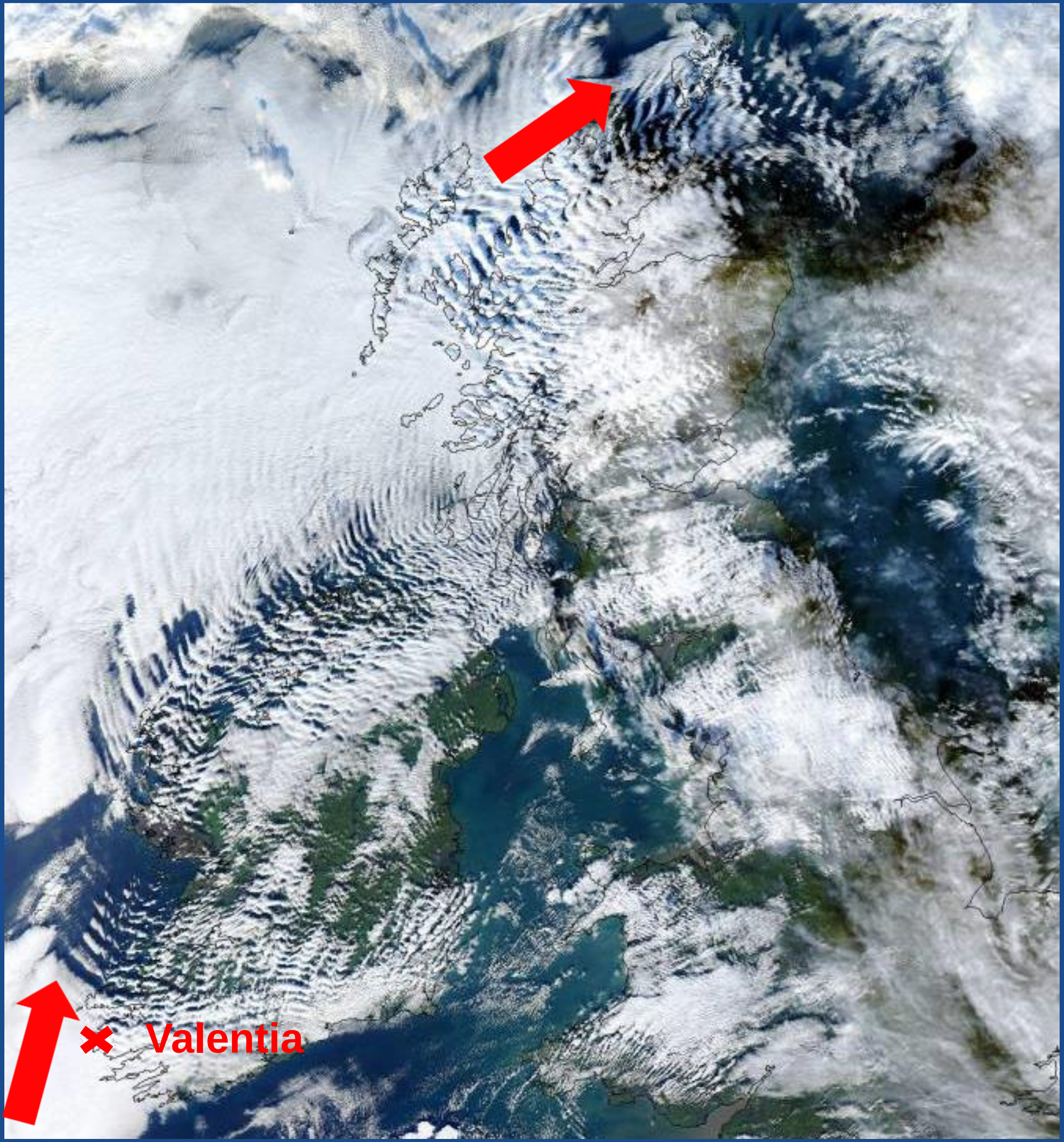
9/12/2013, ~12 UTC



University of Reading

A good example

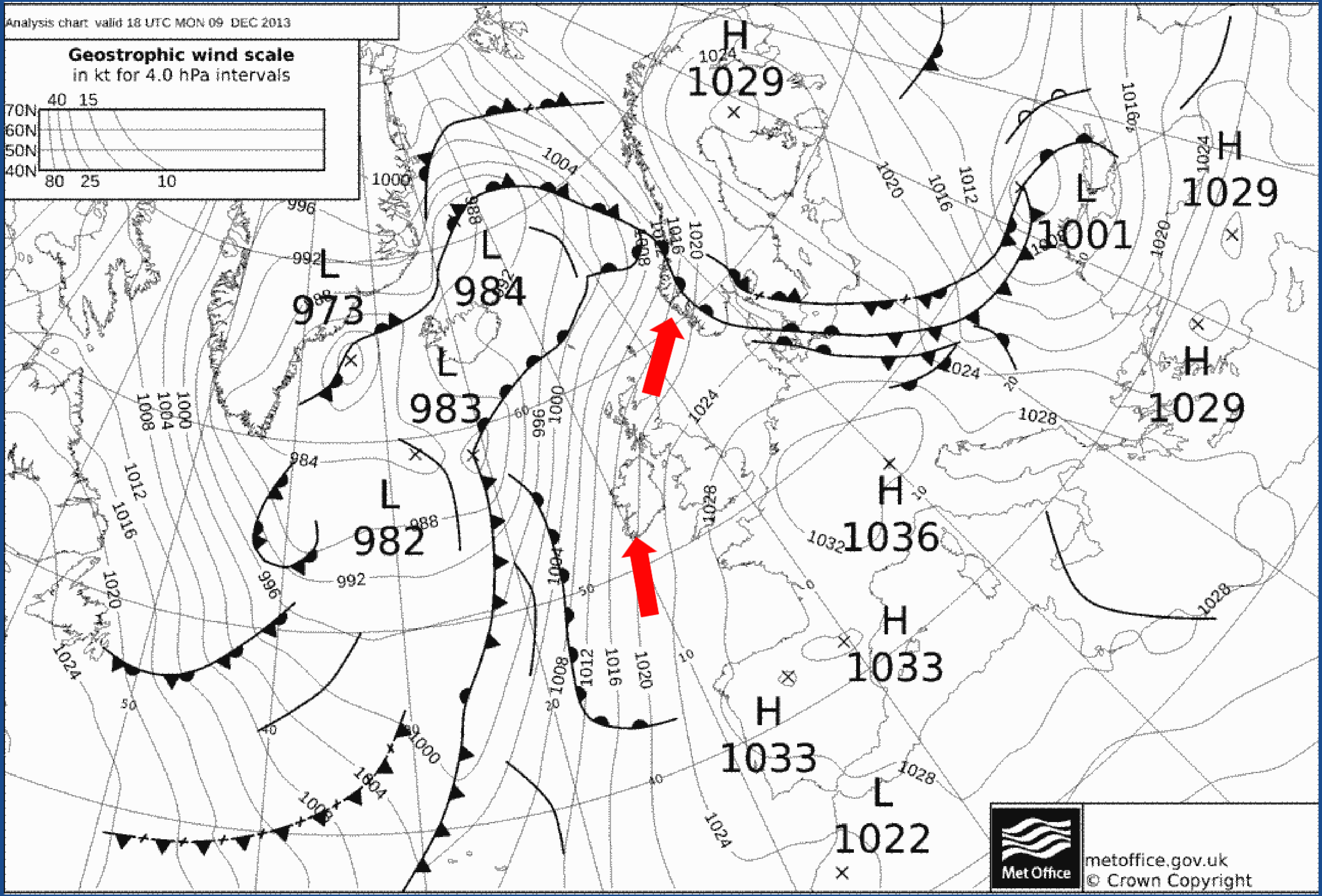
9/12/2013, ~12 UTC



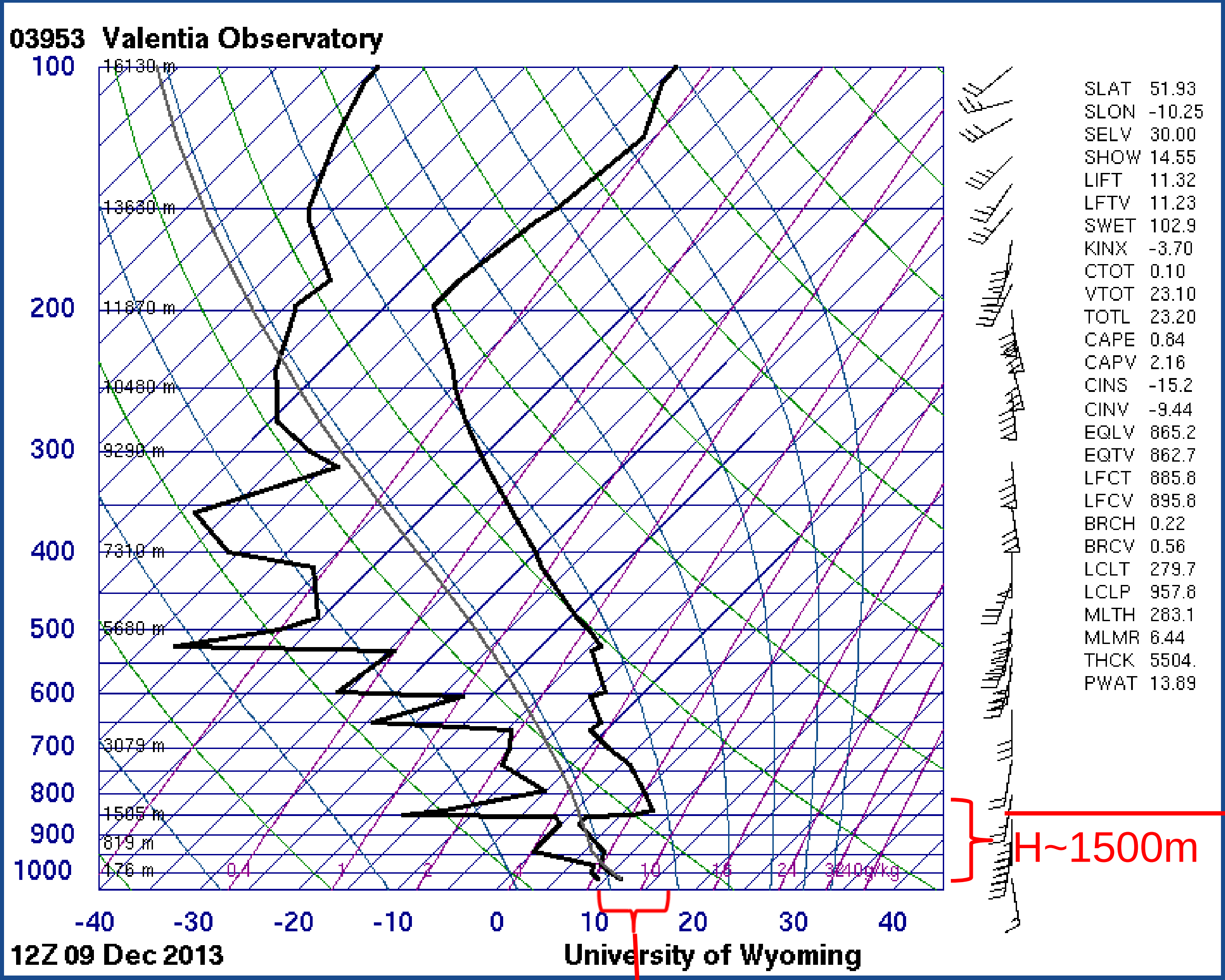
University of Reading

9/12/2013, ~18 UTC

University of Reading



9/12/2013, 12 UTC



At inversion height

U~13 m/s

Δθ~7 K

Evaluate Froude number

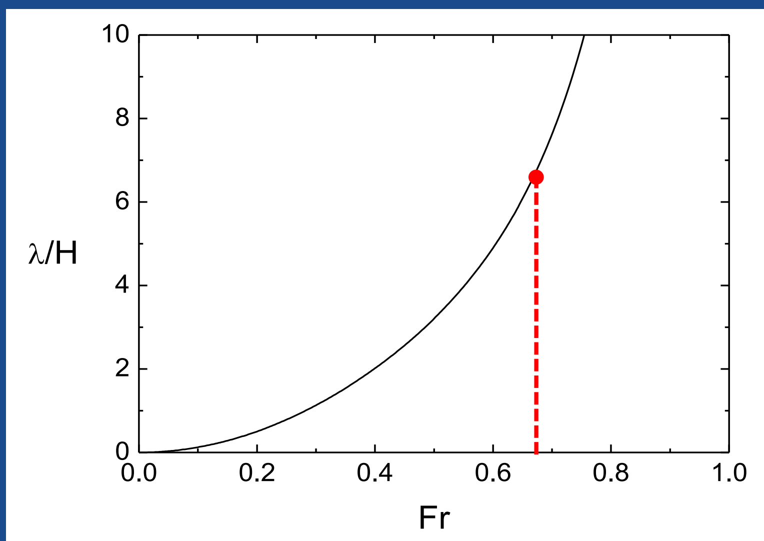
University of Reading

$\theta \sim 290$ K $\Rightarrow g' = g\,\Delta\theta/\,\theta \sim 0.24$ m/s^2

$H \sim 1.5$ km

$U \sim 13$ m/s

$\Rightarrow \mathrm{Fr} = \mathrm{U}/(g'H)^{1/2} \sim 0.68$



$\lambda_R/H \sim 6.5 \Rightarrow \lambda_R \sim 10$ km

Correct?

At least right order of magnitude!

Summary

- It is easy to estimate the wavelength of trapped lee waves
- Simplest case is when these waves propagate at temperature inversion
- Then wavelength over inversion height only function of Fr

Estimates may fail for many reasons:

- Non-stationary flow
- Inversion not sharp enough
- Effects of stratification above or below inversion
- Variation of wind speed with height