Effects of vertical wind shear on mountain wave drag and wave momentum flux

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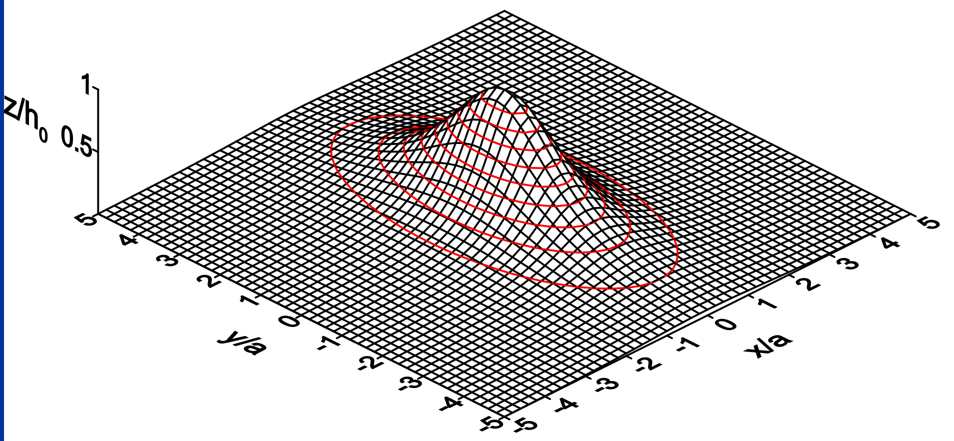
Theoretical framework

(collaboration with Pedro Miranda (Univ. Lisbon) and Jose Argain (Univ. Algarve) (Portugal)

Linear theory: assumptions

- Boussinesq approximation (may be relaxed)
- Inviscid, adiabatic, stationary, non-rotating flow
- Hydrostatic flow $\dfrac{Na}{U} >> 1$ All waves propagate vertically
- Linearization of equations $\dfrac{Nh}{U} << 1$

Flow over an isolated elliptical mountain



$$h(x,y) = h_0 f\left[(x/a)^2 + (y/b)^2\right]$$

$\gamma = a/b$ horizontal aspect ratio

Taylor-Goldstein equation:

$$\frac{\partial^2 \hat{w}}{\partial z^2} + \left[\frac{N^2(k^2+l^2)}{(Uk+Vl)^2} - \frac{U''k+V''l}{Uk+Vl} \right] \hat{w} = 0$$

$N^2 = \frac{g}{\theta_0}\frac{d\bar{\theta}}{dz}$ assumed to be constant (may be relaxed)

- Slowly varying but generic ***U***(z),***V***(z): WKB approximation (wave refraction)

$$\hat{w} = \hat{w}(z=0)\exp\left\{ i\int_0^z \left[m_0(\varepsilon z') + \varepsilon m_1(\varepsilon z') + \varepsilon^2 m_2(\varepsilon z') + \varepsilon^3 m_3(\varepsilon z') \right] dz' \right\}$$

Calculation of:

Surface wave drag:

$$\vec{D} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} p(z=0)\vec{\nabla}_H h\, dxdy$$

Wave momentum flux:

$$\vec{M}(z) = \rho_0 \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \vec{v}_H w\, dxdy$$

$$\vec{D} = -\vec{M}(z=0)$$

- Calculations carried out in a frame of reference aligned with mountain
- Momentum flux expressed as $\vec{M}(z) = \mathbf{A}(\gamma, z)\vec{D}_0$
- **A**(γ,z) does not depend on form of the orography except via $\gamma = a/b$

Orographic GWD parameterization

- Wave drag is a substantial part of total orographic drag

$$(\tau_1, \tau_2) = \rho_H U_H N_H (H_{\text{eff}}^2/4)(\sigma/\mu) G\{B \cos^2 \psi_H + C \sin^2 \psi_H, (B - C) \sin \psi_H \cos \psi_H\}$$

- Surface drag formula above assumes constant wind and static stability
- Way momentum flux is distributed vertically also open to improvement

Missing effects (included in our work)

- Impact of vertical wind shear (slow variations of N can also be incorporated) on surface gravity wave drag.
- Wave momentum flux changes with height due to linear processes: either absorbed totally at critical levels (at hjgh Ri) or filtered (at low Ri)

Momentum flux satisfies extension of Eliassen-Palm theorem to 3D flow:

$$\vec{U} \cdot \frac{d\vec{M}}{dz} = 0$$

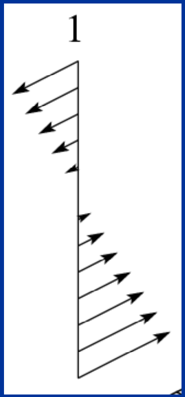
Following results: examples for axisymmetric mountain, constant

$$Ri = \frac{N^2}{U'^2 + V'^2}$$

Surface drag

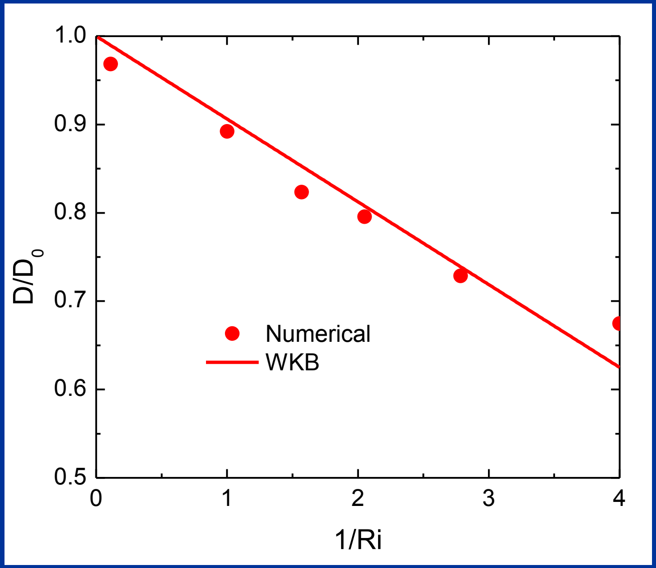
- Axisymmetric mountain
- Linear unidirectional wind profile 1

$$U = U_0 - \alpha z, V = 0$$



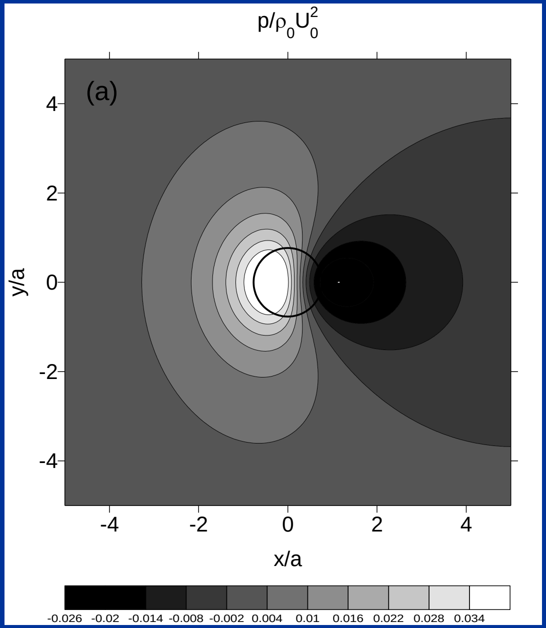
Drag as a function of Ri

$$\frac{D_x}{D_0} = 1 - \frac{3}{32Ri}, D_y = 0$$

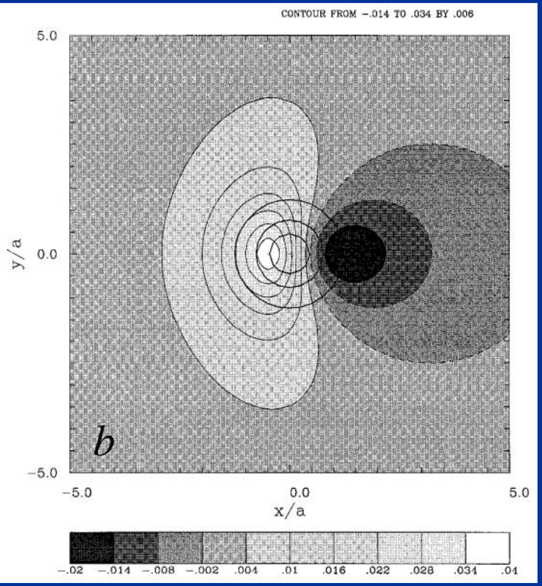


from Teixeira et al., 2004, JAS

Surface pressure perturbation for Ri=0.5



Analytical (from Teixeira et al., 2004, JAS)

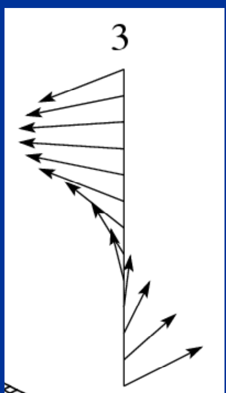


Numerical (from Grubisic & Smolarkiewicz 1997, JAS)

Surface drag

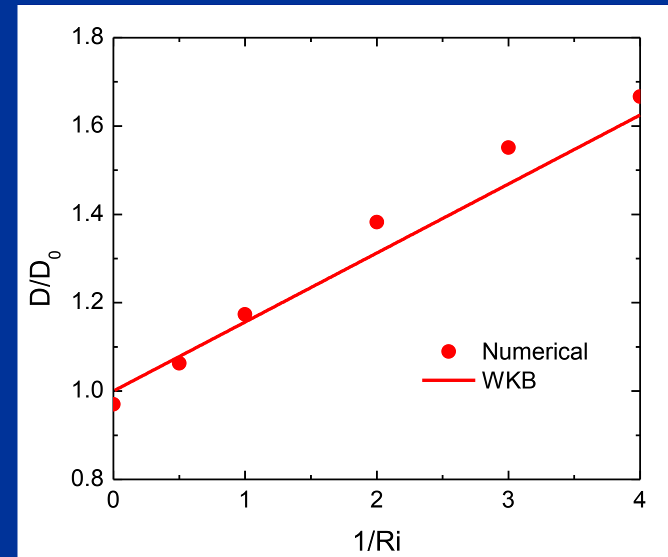
- Axisymmetric mountain
- Wind with directional shear 3

$$U = U_0 \cos(\beta z),\ V = U_0 \sin(\beta z)$$



Drag as a function of Ri

$$\frac{D_x}{D_0} = 1 + \frac{5}{32Ri},\ D_y = 0$$



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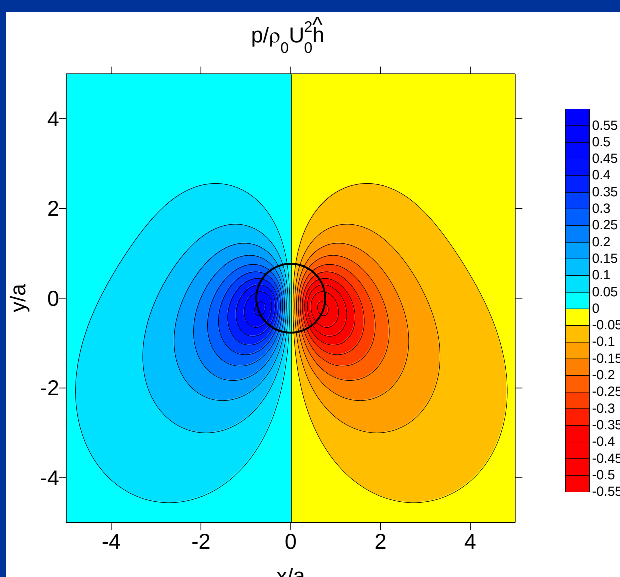
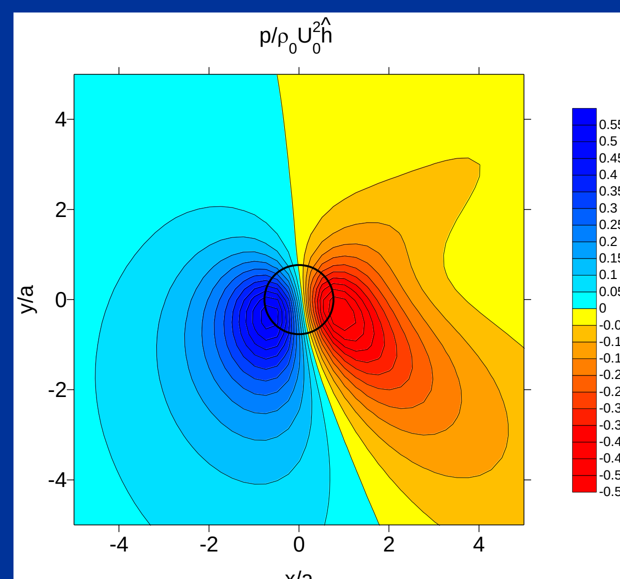




Numerical

Key finding:
Model explains why drag decreases for profile 1 but increases for profile 3

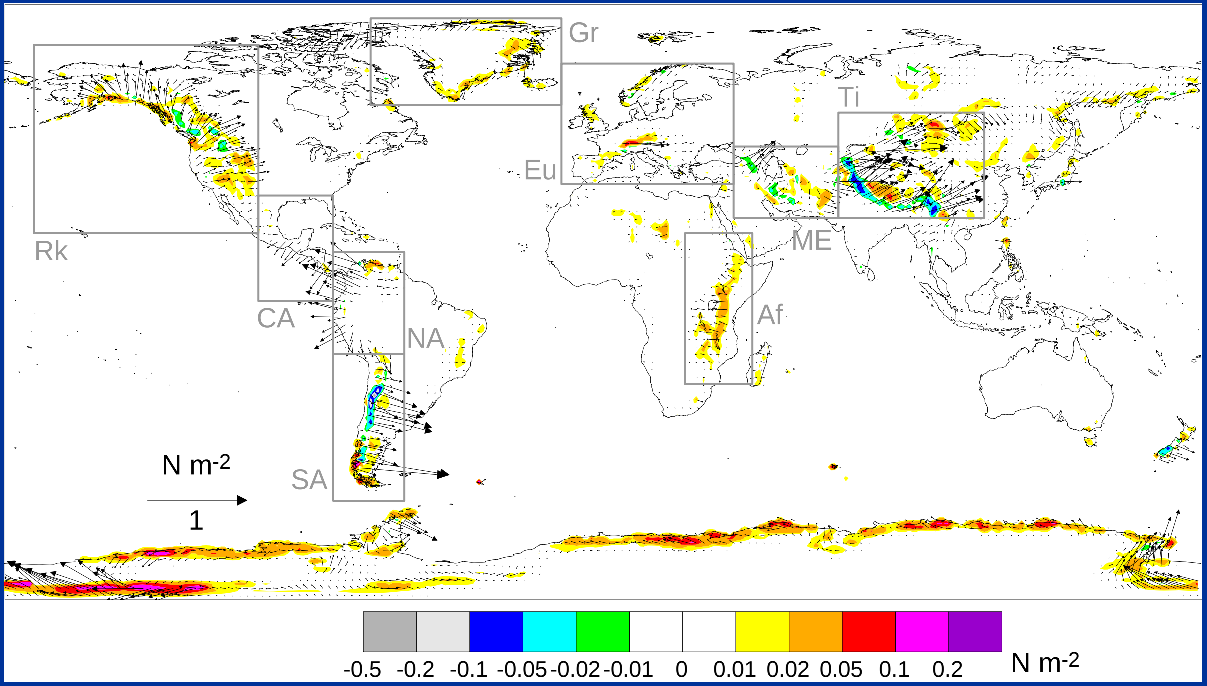
Analytical

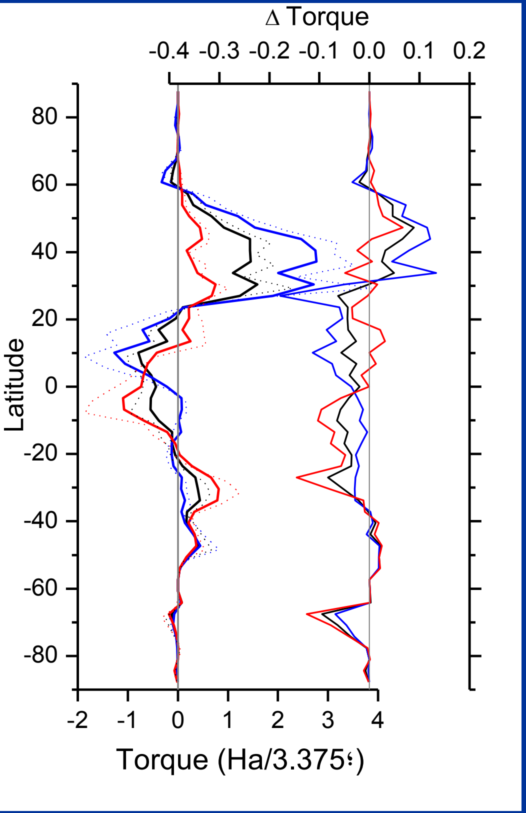
Surface drag – global impact

Question: What is impact of these effects for real orography and atmospheric profiles?

Model considering elliptical mountains and generic wind profiles used to evaluate impact on drag parametrization (Miranda et al., 2009, QJRMS)

Meteorological data from Reanalysis (1992-2001)





Key findings:

- Drag generally increases because of vertical wind shear
- Effect important in Antarctica, where easterly torque strengthened

Wave momentum flux

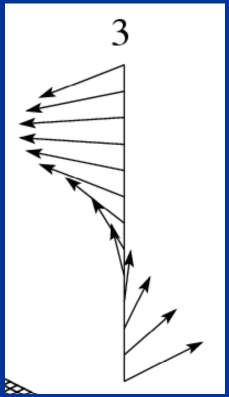
Teixeira & Miranda. 2009, JAS

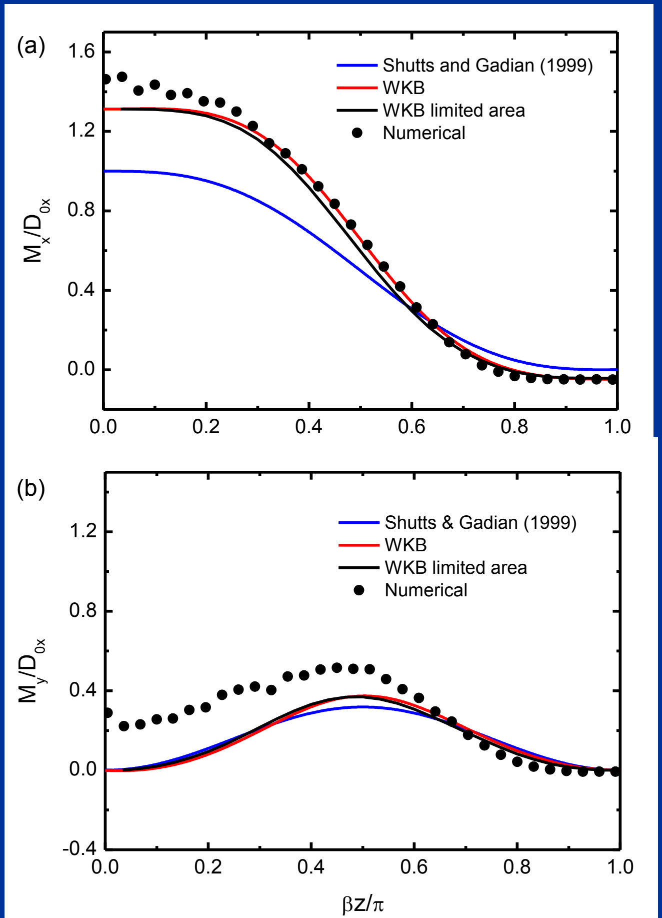
- Axisymmetric mountain
- Directional shear flow 3

$$U = U_0 \cos(\beta z), V = U_0 \sin(\beta z)$$

Surface drag: $\dfrac{D_x}{D_{0x}} = 1 + \dfrac{5}{32 Ri}$ $D_y = 0$

Momentum flux depends on height and on Ri





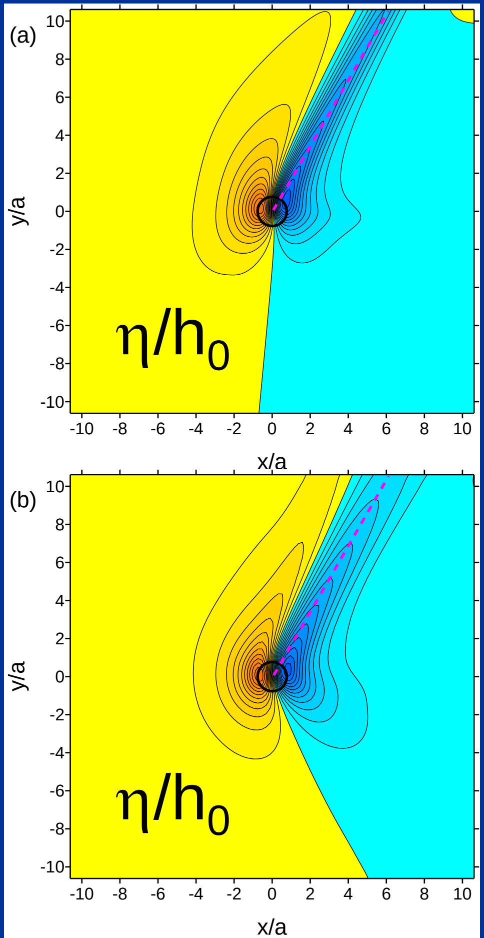
Momentum flux profiles for Ri=0.5

$\beta z/\pi=1/3$

$Ri = 0.5$

WKB

Numerical

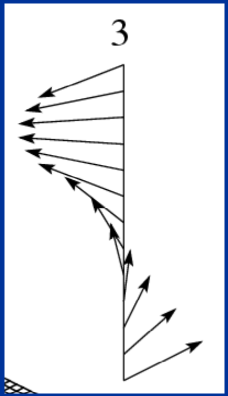


Wave momentum flux

Calculations of critical level absorption of (M_x,M_y) for generic directional shear flows: extend Booker & Bretherton (1967) and Grubisic & Smolarkiewicz (1997)

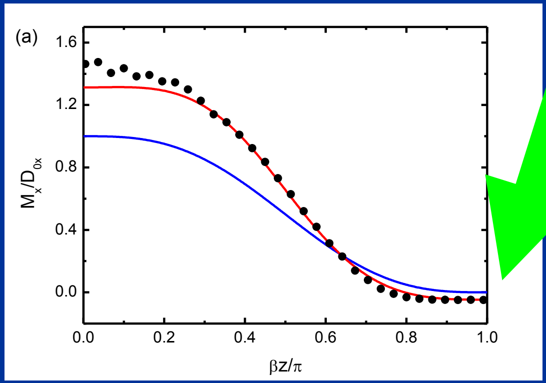
- Axisymmetric mountain
- Wind that turns with height 3

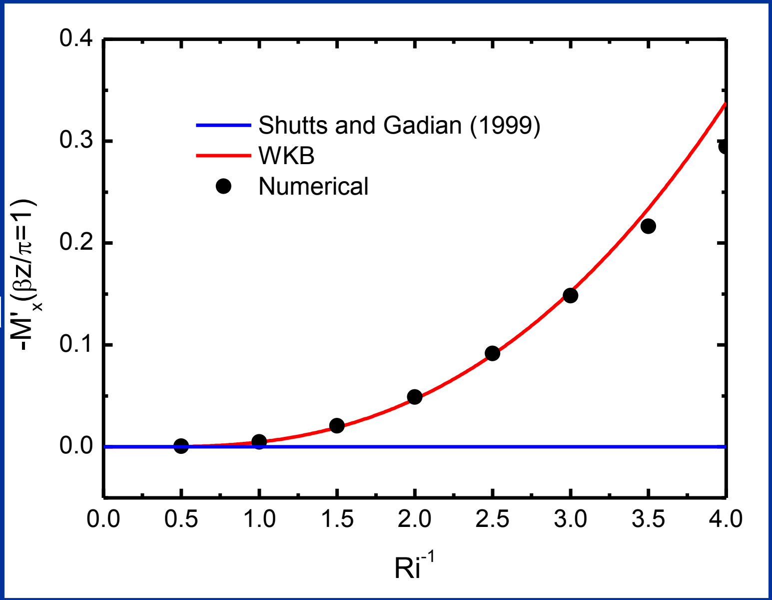
$$\frac{M_x}{D_{0x}}\left(\frac{\beta z}{\pi}=1\right)=-\left(1+\frac{5}{32Ri}\right)\exp\left[-2\pi Ri^{1/2}\left(1-\frac{1}{8Ri}\right)\right]$$



Height where all wavenumbers in wave spectrum have passed through critical levels

Tests how accurate absorption/filtering by critical levels is





from Teixeira and Miranda, 2009, JAS

Key finding: model explains attenuation of momentum flux as waves pass through critical levels ⇒ may be calculated for **any** wind profile and low Ri

Summary

- Present calculations are hydrostatic but incorporate effects of vertical wind shear on surface drag and wave momentum flux.
- Surface drag may either decrease or increase, and wave momentum flux is filtered by critical levels. Effects of multiple critical levels may be treated.
- Generic, slowly varying wind profiles may be considered. In practice, results are shown to be good in linear conditions for Ri=O(1). Important to consider 3rd order WKB approximation to account for effect of shear on drag, and partial wave energy absorption at critical levels.
- Corrections to drag and momentum flux are formulated for elliptical mountains and are independent of the detailed shape of orography.
- Wave momentum flux expressed as 1D integral over azimuthal angle, but momentum flux divergence has closed analytical form.
- Calculations must be done in frame of reference aligned with mountain.
- Corrections to drag or momentum flux depend on local flow: necessary to determine height at which to calculate wind derivatives for surface drag.