The uncertainty of a forecast is extremely important in data assimilation. Present day variational data assimilation systems are designed for large-scale flow characterised by:

- predominately geostrophically balanced flow,
- perfectly hydrostatically balanced flow.

Both balances diminish as the scale of the flow reduces. What should be done if the flow includes small scale flow?

Questions:

- What are geostrophic and hydrostatic balances in the atmosphere?
- How do present day variational data assimilation systems impose these balances?
- What is the evidence that these balances are less effective at small scales?
- How can hydrostatic balance be relaxed at smaller scales?
- How can geostrophic balance be relaxed at smaller scales?

**Data Assimilation for Multi-Scale Atmospheric Flow**

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**Introduction**

Data assimilation is needed to correct today’s forecast by blending it with latest observations. Present day variational data assimilation systems are designed for large scale flow characterised by:

- predominately geostrophically balanced flow,
- perfectly hydrostatically balanced flow.

Both balances diminish as the scale of the flow reduces. What should be done if the flow includes small scale flow?

What are geostrophic and hydrostatic balances?

**Geostrophic balance**

\[
\frac{\partial u}{\partial x} - f \frac{\partial v}{\partial y} = \frac{\partial p}{\partial x},
\]

\[
\frac{\partial v}{\partial x} + f \frac{\partial u}{\partial y} = \frac{\partial p}{\partial y},
\]

\[
\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = \frac{\partial}{\partial z} \left( \frac{1}{\rho} \frac{\partial \rho}{\partial z} \right),
\]

Where \( u \) and \( v \) are the zonal and meridional components of the flow, \( f \) is the Coriolis parameter, \( p \) is pressure, and \( \rho \) is density.

**Hydrostatic balance**

\[
\rho \frac{\partial \mathbf{v}}{\partial t} + \rho \mathbf{u} \cdot \nabla \mathbf{v} = -\nabla p + \rho \mathbf{f} \times \mathbf{v} + \nabla \cdot \tau,
\]

Where \( \mathbf{v} \) is the velocity, \( \mathbf{f} \) is the Coriolis force, and \( \tau \) is the shear stress.

**Questions:**

- How do present day variational data assimilation systems impose these balances?
- What is the evidence that these balances are less effective at small scales?
- How can hydrostatic balance be relaxed at smaller scales?
- How can geostrophic balance be relaxed at smaller scales?

**What is the evidence that these balances are less effective at small scales?**

Rain rate (kg/m²/s)

182.26h July 2007

**How do present day variational data assimilation systems impose these balances?**

A variational data assimilation system makes corrections \( \Delta p \) to a forecast, \( p \), which is:

- consistent with the ‘uncertainty bubble’ associated with the forecast state,
- consistent with the latest observations,
- consistent with dynamical balance relationships.

The bubble’s shape is difficult to determine, but can be modelled with dynamical balance relationships.

In the Met Office variational data assimilation scheme, the above are performed irrespective of scale.

**How can the imposition of geostrophic balance be relaxed at small scales for data assimilation?**

The imposition of geostrophic balance may be made scale-dependent in a prescribed way.

**How can the imposition of hydrostatic balance be relaxed at small scales for data assimilation?**

There are a number of ways that this could be done depending upon the approximations that one is willing to be made.

A. Assume that \( p \) is a hydrostatically balanced variable (i.e. that \( p \) can be used to calculate hydrostatic \( T \) with the hydrostatic balance relation), but allow for the possibility of a non-hydrostatic \( T \) component.

B. Allow for the possibility of non-hydrostatic \( T \) and non-hydrostatic \( p \), but assume that another form of balance is appropriate (e.g. anelastic balance).

This is important for convective-scale data assimilation because hydrostatic balance is disturbed in regions of strong convection, such as at a thunderstorm.

**Summary**

- The uncertainty of a forecast is extremely important in data assimilation.
- Most contemporary data assimilation systems model this error bubble by assuming that balance relations that are valid for large-scale flow (geostrophic and hydrostatic balance) are universal.
- Weather prediction models are more capable of dealing with fine-scale processes. In order for data assimilation to be useful for these models, these balance assumptions need to be relaxed.

**Associated research questions**

- How closely is 3D model error observed at the convective scale?
- How can model error to forecasts to improve the variability of forecast error?
- How easily and efficiently can the above proposed schemes be implemented in an operational data assimilation system?