1. Analysis of correlation plots for standard and PV-based variables

Questions related to correlation plots (standard and PV schemes) at
www.met.rdg.ac.uk/~ross/DARC/PVcv/Feb09_2007/Correlations.html

1 Why are the correlation plots (for both sets of variables) antisymmetric across the equator?
2 Why do the standard variable correlations have mainly negative (positive) correlations in the NH (SH), yet the PV-based variable correlations have mixed signs?
3 Why do the PV-based variables have a different character across the tropopause (in troposphere it is dominated by small horizontal/large vertical (SH/LV) scales, and in the stratosphere it is dominated by large horizontal/small vertical (LH/SV) scales)? The tropopause is between L22 and L28.
4 Should we be able to show by construction that the PV-based variables will be less correlated than the standard set?

Standard variables

Let $C = \langle \psi' \phi'_u \rangle / \langle \psi'^2 \rangle^{1/2} \langle \phi'^2_u \rangle^{1/2}$ where $\phi'_u = \phi' - \phi'_b$ and $\phi'_b = f\psi'$ (linear balance equation, LBE). There is no 'unbalanced' streamfunction.

$$C = \frac{\langle \psi' (\phi' - \phi'_b) \rangle}{\langle \psi'^2 \rangle^{1/2} \langle \phi'^2_u \rangle^{1/2}}, \tag{1}$$

$$= \frac{\langle \psi' \phi' \rangle - \langle \psi' \phi'_b \rangle}{\langle \psi'^2 \rangle^{1/2} \langle \phi'^2_u \rangle^{1/2}}, \tag{2}$$

$$= \frac{\langle \psi' \phi' \rangle - f \langle \psi' \psi' \rangle}{\langle \psi'^2 \rangle^{1/2} \langle \phi'^2_u \rangle^{1/2}}. \tag{3}$$

The standard results (Q1 and Q2) are consistent with dominance of the second term in (3). Why is this term dominant?

PV-based variables

Let $C = \langle \psi'_b \phi'_u \rangle / \langle \psi'^2_b \rangle^{1/2} \langle \phi'^2_u \rangle^{1/2}$ where $\psi'_b = \psi' - \psi'_u$, $\phi'_u = \phi' - \phi'_b$, $\phi'_b = f\psi'_b$ (LBE) and $gh\nabla^2 \psi'_u = f\phi'_u$ (zero PV, zPV). A possible expression for C is in terms of full and balanced increments

$$C = \frac{\langle \psi'_b (\phi' - \phi'_b) \rangle}{\langle \psi'^2_b \rangle^{1/2} \langle \phi'^2_u \rangle^{1/2}}, \tag{4}$$

$$= \frac{\langle \psi'_b \phi' \rangle - \langle \psi'_b \phi'_b \rangle}{\langle \psi'^2_b \rangle^{1/2} \langle \phi'^2_u \rangle^{1/2}}. \tag{5}$$

At LH/SV scales, $\phi' = \phi'_b$, and so we would expect $C = 0$. We see this in the troposphere, which is dominated by correlations of SH/LV scale. In the stratosphere however the correlations are dominated by LH/SV scales and the correlations are stronger. Could this indicate that the scheme is not working properly there (Q3)?

By the LBE, (5) is

$$C = \frac{\langle \psi'_b \phi' \rangle - f \langle \psi'_b \psi'_b \rangle}{\langle \psi'^2_b \rangle^{1/2} \langle \phi'^2_u \rangle^{1/2}}. \tag{6}$$

If the second term dominates this, then we would see negative correlations in the NH, mirrored by positive correlations in the SH, which are not always there in the PV results (Q2).

Another possible expression for the correlation uses zPV and is developed as follows

$$C = \frac{\langle \psi'_b \phi'_u \rangle}{\langle \psi'^2_b \rangle^{1/2} \langle \phi'^2_u \rangle^{1/2}}, \tag{7}$$

$$= \frac{\langle (\psi' - \psi'_u)\, \phi'_u \rangle}{\langle \psi'^2_b \rangle^{1/2} \langle \phi'^2_u \rangle^{1/2}}, \tag{8}$$

$$= \frac{\langle (\psi' - \nabla^{-2}[f/gh\, \phi'_u])\, \phi'_u \rangle}{\langle \psi'^2_b \rangle^{1/2} \langle \phi'^2_u \rangle^{1/2}}, \tag{9}$$

$$= \frac{\langle \psi' \phi'_u \rangle - \langle \phi'_u \nabla^{-2}[f/gh\, \phi'_u] \rangle}{\langle \psi'^2_b \rangle^{1/2} \langle \phi'^2_u \rangle^{1/2}}. \tag{10}$$

At LH/SV scales, ϕ'_u should vanish, but the ∇^{-2}/h in the second term may compensate for this, as these scales are amplified. In fact it is usually at these scales where the NH correlations are negative, indicating that the second term may even dominate under these circumstances (Q2), particularly above the tropopause (Q3). Again this may an indication that the scheme in not working properly at these scales.

I cannot find an answer to Q4.

2. Discussion of pressure pseudo observation tests for standard and PV-based variables

Comments related to analysis increments (standard and PV schemes) at
www.met.rdg.ac.uk/~ross/DARC/PVcv/Feb09_2007/PressVsWind.html
(upper frame) after the assimilation of five pseudo observations of pressure around the globe at level 20. We now (unlike in Sec. 1) refer to pressure, p', rather than height, ϕ'. Note that since there are no wind observations, the wind analysis is unconstrained by observations.

1 By design there are no unbalanced wind increments in the standard scheme.
2 Pressure increments from the standard and PV schemes are similar at all locations.
3 In the PV scheme, p'_b is similar to p' near the poles (where unbalanced pressure is small), but the two are different near the equator (where the unbalanced pressure is larger). L/L_R (lengthscales are large compared to the Rossby radius) is largest near the poles, where it is expected that p_u vanishes.
4 The balanced winds from the PV scheme are similar to total winds of the standard scheme. This indicates that the wind response in the standard scheme is largely balanced at all locations.
5 The winds that are in anti-balance with p'_u dominate the wind analysis. They have LH/SV scales since the anti-balance amplifies these scales. These are the scales at which wind is unbalanced. This suggests that the p'_u is too large at these scales.
6 ψ'_b should have smaller horizontal scale and larger vertical scale than ψ'. There is mixed evidence of this.
7 χ' has an increment in the standard scheme. I'm not sure why this is at present (is there a pressure contribution due to χ' in the control variable transforms?).

3. Discussion of wind pseudo observation tests for standard and PV-based variables

Comments related to analysis increments (standard and PV schemes) at
www.met.rdg.ac.uk/~ross/DARC/PVcv/Feb09_2007/PressVsWind.html
(lower frame) after the assimilation of five pseudo observations of zonal wind and five pseudo observations of meridional wind around the globe at level 20. Note that since there are no pressure observations, the pressure analysis is unconstrained by observations.

1 By design, the unbalanced pressure field cannot be modified by wind observations in the standard scheme. The unbalanced pressure field can be modified by wind observations in the PV scheme due to the presence of unbalanced wind. This is seen in the results.
2 There are strong p' increments in the standard scheme. These are weak in the PV scheme, but the +/- patterns the similar in each case. The p' increments in the PV scheme are dominated by the balanced variable.
3 The wind in the PV scheme is dominated by LH/SV scales (due to strong sensitivity to p'_u at these scales). This does not match the correct structures exhibited by the standard scheme.
4 The balanced wind response in the PV scheme is very weak.
5 The PV scheme's weak p' increments (point 2) and weak balanced wind increments (point 4) are explained by the strong sensitivity mentioned in point 3 - see (11) below. A small, but non-zero p'_u has a large wind response via the antibalance operator (AB). This then contributes to most of the ψ' increment to fit the pseudo observations of wind. ψ'_b then only has to be small (point 4) to fit the wind observations.

$$\begin{pmatrix} \psi' \\ \chi' \\ p' \end{pmatrix} = \begin{pmatrix} 1 & 0 & AB \\ 0 & 1 & 0 \\ LBE & 0 & 1 \end{pmatrix} \begin{pmatrix} \psi'_b \\ \chi' \\ p'_u \end{pmatrix} \tag{11}$$

Possible solutions

The following may need to be explored.

- Use of a different solver the calibration phase which has smaller residuals than the GCR.
- Filter the unbalanced pressure at LH/SV scales when performing the calibration. This will make the p'_u variances smaller and so limit the size that p'_u can be at those scales.
- Filter the unbalanced pressure (or the unbalanced streamfunction) at LH/SV scales when doing the assimilation.

Ross Bannister, 20th March 2007.