Putting back large-scale pressure into the PV scheme

R.N.B., April 2nd 2009, modified April 20th 2009

The current version of the PV-based transforms do not constrain the level-by-level global mean pressure.

The U-transform

The issue concerns mainly the fix in the PV-based **U**-transform. The basic PV-based **U**-transform is (for streamfunction and pressure)

$$\begin{pmatrix}\delta\psi\\ \delta p\end{pmatrix} = \begin{pmatrix}\mathbf{I} & \bar{\mathbf{H}}\\ \mathbf{H} & \mathbf{I}\end{pmatrix}\begin{pmatrix}\mathbf{U}^{\mathrm{v}}_{\delta\psi_b} & 0\\ 0 & \mathbf{U}^{\mathrm{v}}_{\delta p_u}\end{pmatrix}\begin{pmatrix}\mathbf{F}_h^{-1} & 0\\ 0 & \mathbf{F}_h^{-1}\end{pmatrix}\begin{pmatrix}\Lambda^{1/2}_{\delta\psi_b} & 0\\ 0 & \Lambda^{1/2}_{\delta p_u}\end{pmatrix}\begin{pmatrix}\delta\psi_b\\ \delta p_u\end{pmatrix}. \tag{1}$$

$\mathbf{U}^{\mathrm{v}}_P$ is the vertical **U**-transform for parameter P, $\mathbf{F}_h^{-1}$ is the inverse horizontal spectral transform, $\Lambda^{1/2}_P$ is the horizontal spectrum for parameter P, $\mathbf{H}$ is the linear balance operator (input $\delta\psi_b$ in model space, output δp_b in model space) and $\bar{\mathbf{H}}$ is the anti-balance operator (input δp_u in model space, output $\delta\psi_u$ in model space).

(1) has problems associated with the nature of $\bar{\mathbf{H}}$, which amplifies components of δp_u at large horizontal and small vertical scales. Currently a fix is included in the scheme to filter-out these scales from δp_u as follows

$$\begin{pmatrix}\delta\psi\\ \delta p\end{pmatrix} = \begin{pmatrix}\mathbf{I} & \bar{\mathbf{H}}\\ \mathbf{H} & \mathbf{I}\end{pmatrix}\begin{pmatrix}\mathbf{U}^{\mathrm{v}}_{\delta\psi_b} & 0\\ 0 & \mathbf{U}^{\mathrm{v}}_{\delta p_u}\end{pmatrix}\begin{pmatrix}\mathbf{F}_h^{-1} & 0\\ 0 & \mathbf{F}_h^{-1}\end{pmatrix}\begin{pmatrix}\Lambda^{1/2}_{\delta\psi_b} & 0\\ 0 & \Lambda^{1/2}_{\delta p_u}\mathbf{Z}\end{pmatrix}\begin{pmatrix}\delta\psi_b\\ \delta p_u\end{pmatrix}, \tag{2}$$

where the filter that removes these scales is $\mathbf{Z}$. This highlights the first problem - that a significant part (if not all) of the mean pressure is removed. A possible solution to this problem is to filter δp_u only when used with the $\bar{\mathbf{H}}$-operator. This may be expressed as the following

$$\begin{pmatrix}\delta\psi\\ \delta p\end{pmatrix} = \begin{pmatrix}\mathbf{I} & 0 & \bar{\mathbf{H}}\\ \mathbf{H} & \mathbf{I} & 0\end{pmatrix}\begin{pmatrix}\mathbf{U}^{\mathrm{v}}_{\delta\psi_b} & 0 & 0\\ 0 & \mathbf{U}^{\mathrm{v}}_{\delta p_u} & 0\\ 0 & 0 & \mathbf{U}^{\mathrm{v}}_{\delta p_u}\end{pmatrix}\begin{pmatrix}\mathbf{F}_h^{-1} & 0 & 0\\ 0 & \mathbf{F}_h^{-1} & 0\\ 0 & 0 & \mathbf{F}_h^{-1}\end{pmatrix}\begin{pmatrix}\Lambda^{1/2}_{\delta\psi_b} & 0\\ 0 & \Lambda^{1/2}_{\delta p_u}\\ 0 & \Lambda^{1/2}_{\delta p_u}\mathbf{Z}\end{pmatrix}\begin{pmatrix}\delta\psi_b\\ \delta p_u\end{pmatrix}. \tag{3}$$

Here, two copies of the unbalanced variable exist - one is filtered with $\mathbf{Z}$, and the other is not. Only the one that is filtered is used with $\bar{\mathbf{H}}$ and the other is added to balanced pressure unfiltered. This means that the total pressure may have a global mean component.

The T-transform

The level-by-level mean pressure is not included in the PV inversion, as the balanced pressure, $\mathbf{H}\delta\psi_b$ has no mean value due to the ∇_h^{-2} operator in $\mathbf{H}$. Mean pressure is however included in the unbalanced pressure calculation (although it should really be labelled as balanced).

PTO

Additional thoughts to T-transform, April 20th 2009

- The solution of the linear balance equation has no global mean pressure.
- All of the global mean pressure should therefore appear in the unbalanced pressure field.
- The unbalanced pressure is computed from the unbalanced streamfunction by the anti-balance equations ($PV = 0$).
- However, there is no reason to suppose that this unbalanced pressure has the same global mean as the total pressure perturbation.
- Therefore, add-on a level-by-level constant to the unbalanced pressure so that it has the same global mean as the total pressure perturbation.

This fix will have to be used with the changes to the U-transform in the first part of this document.