Simulating the Convective Boundary Layer with a Dynamic Smagorinsky Model

MM Bopape[1,2], RS Plant[1] and O Coceal[1]

[1] *Department of Meteorology, University of Reading, Reading, UK*
[2] *Centre for High Performance Computing, CSIR Meraka Institute, Pretoria, South Africa*

Abstract

A convective boundary layer is simulated with the Smagorinsky model with different variations of the dynamic Smagorinsky model using different grid spacings. One variation of the dynamic model is scale dependent and is thought to be suitable for the grey zones where large eddies and the grid spacing are of similar size. Shortcomings in the Smagorinsky model at lower resolutions are found in all variations of the dynamic model, while at higher resolution all simulations are similar. The results suggest that the choice of the subgrid model is not important when simulating a convective boundary layer because large eddies dominate the flow.

Keywords: grey zone, dynamic model, atmospheric boundary layer, large eddy model

Introduction

The numerical modelling of the atmosphere at different time scales is based on the filtering of the Navier-Stokes equation (Bryan *et al.* (2003)). Scales that are larger than the filter length are simulted explicitly by the model, while the sub-filter scales must be parameterised. Most parameterisations are designed to be applicable for the specific resolution that they will be applied for. When modelling the atmosphere with grid spacings that are larger than 1km, the boundary layer is parameterised fully using column based models (Honnert *et al.* (2011)). Numerical Models used for predicting or projecting the future state of the atmosphere at different timescales, including Numerical Weather Prediction (NWP) have until recently used grid spacings that are much larger than 1km.

The grid spacing used in NWP by a number of meteorological organisations has decreased to around 1km, and below 1km in certain centres. That means the grid spacing is about the same as the size of energy containing large eddies in convective boundary layers (CBLs). The large eddies in CBLs have been found to scale with the boundary layer height which can be thought to be about 1km. That means the column based boundary layer parameterisations that assume that the filter scale is much larger than the large eddies in the boundary layer are no longer applicable for current NWP models (Bryan *et al.* (2003)). On the other end of the scales, large eddy models (LEMs) are run with grid spacing of about 5m to 200m, and they make an assumption that the grid spacing is much smaller than the large eddies. They are therefore expected to simulated large eddies explicitly, and parameterise smaller eddies. They are also not designed to work well with grid spacings where the large eddies and the grid spacings are about the same size. There are therefore no suitable boundary layer parametrizations where the grid spacing and the large eddies are about the same, and as a result this regime has been termed the grey-zone or terra incognita (Bryan *et al.* (2003); Honnert *et al.* (2011)).

The Smagorinsky-Lilly method is a popular classic sub-filter sheme used in many large eddy models (Smagorinsky (1963); Lilly (1965)). The scheme is based on the concepts of eddy viscosity and mixing length, where the mixing length is representative of maximum size of the eddies that are parametrised. A number of enhancements have been introduced to the Smagorinsky-Lilly model, including one that is thought to make the scheme suitable for the greyzones. The particular scheme is called the lagrangian averaged scale dependent (LASD) dynamic Smagorinsky model (Bou-Zeid *et al.* (2004)). In this study we compare simulations made with the original Smagorinsky model to those made with variations of the dynamic model to determine whether or not the dynamic model makes improvements to the simulations of a CBL.

Model and Numerical Simulations

UK Met Office LEM

The UK Met Office LEM (MetLEM) is used in this study. MetLEM solves momentum, continuity and thermodynamic equations given by Equations 1 to 3 respectively. The letters and symbols have their usual meaning. The term on the left of Equation 1 is the total time-derivative of momentum, the first term on the right hand side is the pressure gradient force, the second term, buoyancy, is non-zero in the vertical, the third term is the divergence of the turbulent stress, and the final term is the Coriolis acceleration.

$$\frac{Du_i}{Dt} = -\frac{\partial}{\partial x_i}\left(\frac{p'}{\rho_s}\right) + \delta_{i3}B' + \frac{1}{\rho_s}\frac{\partial \tau_{ij}}{\partial x_i} - 2\epsilon_{ijk}\Omega_j u_k \quad (1)$$

$$\frac{\partial}{\partial x_i}\left(\rho_s u_i\right) = 0 \quad (2)$$

$$\frac{D\theta}{Dt} = \frac{1}{\rho_s}\frac{\partial h_i^\theta}{\partial x_i} + \left(\frac{\partial \theta}{\partial t}\right)_{mphys} + \left(\frac{\partial \theta}{\partial t}\right)_{rad} \quad (3)$$

The left hand side of Equation (3) is the total derivative of potential temperature, while the first term on the right hand side is the divergence of the heat flux. The second and final terms represent the effect of latent heating or cooling due to phase changes, and the effect of radiation, respectively. The last two terms are zero in our study because we use dry simulations and a constant sensible surface heat flux.

Smagorinsky Model

The turbulent stress and sub-filter-scale heat flux in Equations 1 and 3 and parameterised using the Smagorinsky-Lilly scheme and are given by the equations 4 and 5 below, respectively.

$$\tau_{ij} = \rho_s \nu S_{ij} \quad (4)$$

$$h_i^\theta = -\rho_s \nu_h \frac{\partial \theta}{\partial x_i} \quad (5)$$

$$S_{ij} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \quad (6)$$

$$\nu = \lambda^2 S f_m\left(Ri_p\right) \quad (7)$$

$$\nu_h = \lambda^2 S f_h\left(Ri_p\right) \quad (8)$$

$$S = \left(\frac{1}{2}\sum_{i,j=1,3} S_{ij}^2\right)^{1/2} \quad (9)$$

The rate of the strain tensor is defined by Equation 6, while the eddy viscosity and diffusivity are prescribed as in equations 7 and 8 and they are functions of the Richardson number. S is the modulus given by Equation 9.

$$\frac{1}{\lambda^2} = \frac{1}{\lambda_0^2} + \frac{1}{\left[k\left(z+z_0\right)\right]^2} \quad (10)$$

$$\lambda_0 = c_s \triangle \quad (11)$$

The mixing length is given by equation 10, where the first term on the right determines the basic mixing length given in equation 11. The second term calculates the mixing length as a function of height and the roughness length. In the original Smagorinsky-Lilly model, c_s is a constant and was used as 0.23 in our study. A number of studies have shown that the suitability of the Smagorinsky constant depends on the flow and suitable values suggested for different stratifications include 0.1, 0.17, 0.2 and 0.23.

Dynamic Smagorinsky Model

The dynamic model was introduced by Germano (1991) and its aim is to determine a suitable value of c_s using the flow, by employing the grid scale filter (Equation 12), and a second test scale, usually with α=2 (Equation 13).

$$\tau_{ij} = -2c_{s,\Delta}^2 \Delta^2 |\tilde{S}|\tilde{S_{ij}} \quad (12)$$

$$T_{ij} = -2c_{s,\alpha\Delta}^2 \left(\alpha\Delta\right)^2 |\overline{S}|\overline{S_{ij}} \quad (13)$$

$$L_{ij} = \overline{\tilde{u}_i\tilde{u_j}} - \overline{u}_i\overline{u}_j \quad (14)$$

$$L_{ij} - \left(\overline{T_{ij}} - \overline{\tilde{\tau_{ij}}}\right) = e = L_{ij} - c_{s,\Delta}^2 M_{ij} \quad (15)$$

$$M_{ij} = -2\Delta^2\left(\overline{|\tilde{S}|\tilde{S_{ij}}} - \alpha^2\beta\overline{|\overline{S}|\overline{S_{ij}}}\right) \quad (16)$$

$$\beta = c_{s,\alpha\Delta}^2 / c_{s,\Delta}^2 \quad (17)$$

Equation 14 uses the smallest resolved scale, and by taking its difference from the parametrized flow, we are able to get the value of the Smagorinsky coefficient. The calculated coefficient is averaged along a plane and is therefore suitable only for horizontally homogeneous flows over a plane. Meneveau et al. (1996) used a similar method to Germano, however used lagrangian averaging which makes the scheme suitable for inhomogeneous flows and complex geometries. Both the Germano and

Meneaveu scheme assume that the Smagorinsky coefficient is independent, that is β in equation 17 is 1. The Germano and Meneauveu models will be referred to as the Plane-Averaged Scale Invariant (PASI) and the Lagrangian-Averaged Scale Invariant (LASI), respectively.

The two schemes were designed for the inertial subranges, i.e. large eddy regimes and are therefore not designed for the grey-zones. Bou-Zeid et al. (2005) used lagrangian averaging similar to Meneavu, however, he introduced scale dependence by using a second test scale which in our study is taken as 4* grid scale to determine a suitable value of β. The Bou-Zeid model is referred to as the Lagrangian-Averaged Scale-Dependent (LASD) Model.

Simulations

Simulations of a convective boundary layer were made using different grid spacings of 25m, 50m, 100m, 200m and 400m with the original Smagorinsky model as well the different variations of the dynamic model. The 25m resolution simulations is considered as the " truth run" because previous studies have shown that 25m is sufficient for simulating a CBL. The grid spacing in the vertical is take as 0.4* horizontal grid scale. A constant surface sensible heat flux of $241 Wm^{-2}$ and weak geostrophic winds of $(U_g , V_g) = (1,0)\ ms^{-1}$ are used following Sullivan and Patton (2011). The horizontal domain size for all the simulations is taken as $(9.6\ km)^2$, while the model top is taken at a height of 2km.

Results and Discussion

The Smagorinsky coefficient calculated by all the dynamic models is comparable to all the Smagorinsky constant throughout the whole domain (Fig 1). PASI and LASD calculate larger values than the constant 0.23 of the original Smagorinsky model. LASI simulates the smallest values of the coefficient. c_s is below 0.23 above the boundary layer height. The calculated coefficients do not show an obvious reliance on grid spacing, so the 50m, 100m and 200m grid spacing lines almost fall on top of another.

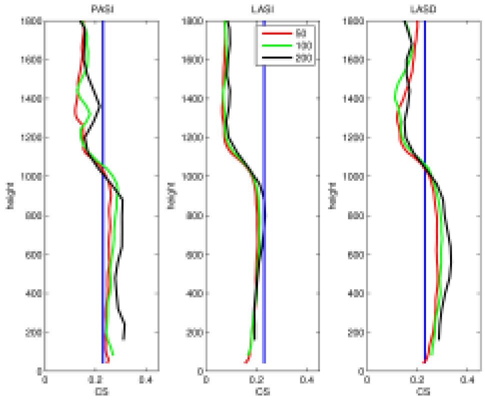

Figure 1: The vertical profile of averaged values of c_s for 50m, 100m and 200m grid spacing, and the line of the constant c_s for the original Smagorinsky model. The three columns are for different version of the dynamic Smagorinsky model.

Quadrant analysis was applied to the temperature flux of the original Smagorinsky-Lilly model simulations at different resolutions to determine what goes wrong when the grid spacing is increased to target what the dynamic models can possibly improve. Partitioning of a combination of vertical velocity perturbation (w') and potential temperature perturbation (ϴ') was performed according to their signs as follows: quadrant 1 is given by w'>0; ϴ'>0 which represents warm air rising, quadrant 2 is given by w'>0; ϴ'<0 which represent cold air descending, quadrant 3 is given by w'<0; ϴ'<0 which gives cold air descending, and quadrant 4 is given by w'<0; ϴ'>0 which is warm air descending. For all the simulations the lower part of the boundary layer is dominated by thermals (i.e quadrant 1). In the upper troposphere, close to the inversion layer, the contribution of warm air descending is also significant which represent entrainment of warm air from the inversion layer into the boundary layer. The largest contribution is however from quadrant 2, which is cold air descending. The contribution of quadrant 2 in the vicinity of the boundary layer height increases with increased grid spacing (Figure 2).

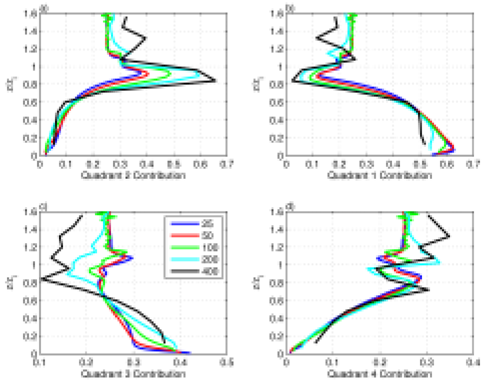

Figure 2: The contribution towards the total temperature flux by different quadrant using different grid spacings with the Smagorinsky Lilly model, using a Smagorinsky constant of 0.23. The quadrants are ordered according to their position on the cartesian place.

Quadrant analysis was performed for different grid spacings using the Smagorinsky model, and different variations of the dynamic model using coarse grained 25m grid spacing simulation as the target simulation. At higher resolution, the lines with the different grid spacing fall on top of one another, which shows that the use of different subgrid models has little effect on the simulations (not shown).

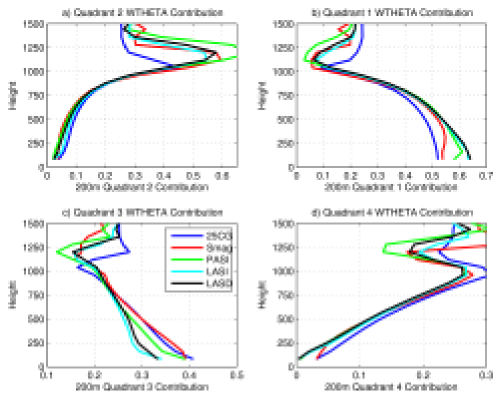

Figure 3: The contribution towards the total temperature flux by different quadrants using three versions of the dynamic model, the original Smagorinsky model, as well the coarse grained 25m grid spacing simulations with a 200m grid spacing.

With lower resolutions the lines start to diverge but the issues associated with the original Smagorinsky model, are still found with all dynamic models. All the schemes underestimate entrainment of warm air into the boundary layer, and overestimate the contribution of cool air ascending close to the boundary layer height. The LASDcode, and advising on the implementation in the MetLEM.

Conclusions

Simulations of a convective boundary layer were performed using the Smagorinsky-Lilly model, and three different variations of the dynamic model using grid spacings of 25m, 50m, 100m, 200m and 400m. The dynamic models did not show a major improvement compared to the original Smagorinsky Lilly model. The Lagrangian averaged scale dependent model which is thought to be suitable for the greyzones also showed similar issues to the Smagorinsky model. This result is thought to be associated with the fact that large eddies dominate the flow in a convective boundary layer. Previous studies have shown an improvement by dynamic models when stable and neutral stratifications are simulated, but the same result is not found for a convective boundary layer. More tests are planned with the different dynamic models for different stratification, complex geometries, and for transitioning boundary layers where improvements are expected with the use of dynamic models.

Acknowledgements

This study was funded by a NERC grant (contract no.\ NE/K011502/1) under the GREYBLS project. O. C. gratefully acknowledges funding from NCAS (NERC contract no.\ R8/H12/83/002). We acknowledge use of the MONSooN system, a collaborative facility supplied under the Joint Weather and Climate Research Programme, which is a strategic partnership between the Met Office and the Natural Environment Research Council. The authors thank Bob Beare and George Efstathiou for useful discussions and for kindly helping with the setup of the convective boundary layer case. Elie Bouzeid is acknowledged for providing his LASD

code, and advising on the implementation in the MetLEM.

References

Bou-Zeid, E., Meneveau, C. and Parlange M. (2005). A scale-dependent Lagrangian dynamic model for large eddy simulation of complex turbulent flows. *Physics of Fluids.* 17: 025105.

Bryan, G.H., Wyngaard, J.C. and Fritsh J.M. (2003). Resolution requirements for the simulation of deep moist convection. *Monthly Weather Review*. 131: 2394-2416.

Germano, M., Piomelli, U., Moin, P. and Cabot, W.H. (1991). A dynamic subgrid-scale eddy viscosity model. *Physics Fluids* : 3, 1760-1765.

Honnert, R., Masson, V., and Couvreux F. (2011). A diagnostic for evaluating the representation of turbulence in atmospheric models at the kilometric scale. *Journal of Atmospheric Science*. 68: 3112-3131.

Lilly, D.K. (1965). On the computational stability of numerical solutions of time-dependent non-linear geophysical fluid dynamics problems. *Monthly Weather Review*. 93: 11–25.

Meneveau, C., Lund, T. and Cabot, W. (2000). A Lagrangian dynamc subgrid-scae model of turbulence. *Annual Review of Fluid Mechanics.* 32: 1-32.

Smagorinsky, J. (1963). General circulation experiments with the primitive equations. *Monthly Weather Review*. 91: 99–164.

Sullivan, P.P. and Patton, E.G. (2011). The Effect of Mesh Resolution on Convective Boundary Layer Statistics and Structures generated by Large-Eddy Simulation. *Journal of the Atmospheric Sciences.* 68: 2395-2419.