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Evaluation of a tensor eddy-diffusivity model for the terra-incognita using DNS data

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Introduction

In high-resolution mesoscale modelling or in coarse-resolution LES turbulence is partially resolved and traditional subfilter-scale models are no longer appropriate. A new approach proposed by Wyngaard (2004) generalises the scalar eddy diffusivity in the standard Smagorinsky sub-filter model to a tensor by considering the effect of extra production terms in the sub-filter scale conservation equations that are usually neglected. We diagnose the importance of these extra terms by direct computation using a direct numerical simulation (DNS) dataset and evaluate the resulting model for the sub-filter scale fluxes proposed by Wyngaard (2004) for a flow typical of urban and engineering CFD.

Tensor models of SFS flux and stress

The filtered equation for a conserved scalar c is given by $\frac{\partial \bar{c}}{\partial t} + \bar{u}_i \frac{\partial \bar{c}}{\partial x_i} + \frac{\partial f_i}{\partial x_i} = 0,$

where the SFS scalar flux is $f_i = \overline{cu_i} - \bar{c}\,\bar{u}_i.$

Wyngaard (2004) proposed the following model for the SFS scalar flux:

$$\frac{\partial f_i}{\partial t} + f_j \frac{\partial \bar{u}_i}{\partial x_j} + R_{ij} \frac{\partial \bar{c}}{\partial x_j} = -\frac{f_i}{T}.$$

where $-R_{ij} = -\overline{u_i u_j} + \bar{u}_i \bar{u}_j$ $\quad T = C\Delta/e^{1/2}.$

The solution of this equation can be formally written as $f_i = -K_{ij} \frac{\partial \bar{c}}{\partial x_j},$

where K_{ij} is a second-rank tensor that generalises the usual scalar eddy-diffusivity.

Hatlee & Wyngaard (2007) give a corresponding model for the SFS stress τ_{ij}:

$$\frac{\partial \tau_{ij}}{\partial t} = \frac{2}{3} e \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) - \left[\tau_{ik} \frac{\partial \bar{u}_j}{\partial x_k} + \tau_{jk} \frac{\partial \bar{u}_i}{\partial x_k} - \frac{1}{3} \delta_{ij} \tau_{kl} \left(\frac{\partial \bar{u}_k}{\partial x_l} + \frac{\partial \bar{u}_l}{\partial x_k} \right) \right] - \frac{\tau_{ij}}{T},$$

$$T = C_\tau \Delta/e^{1/2}, \qquad \tau_{ij} = -(\overline{u_i u_j} - \bar{u}_i \bar{u}_j) + \frac{2}{3}\delta_{ij} e = -\left(R_{ij} - \frac{2}{3}\delta_{ij} e \right)$$

Data and filtering procedure

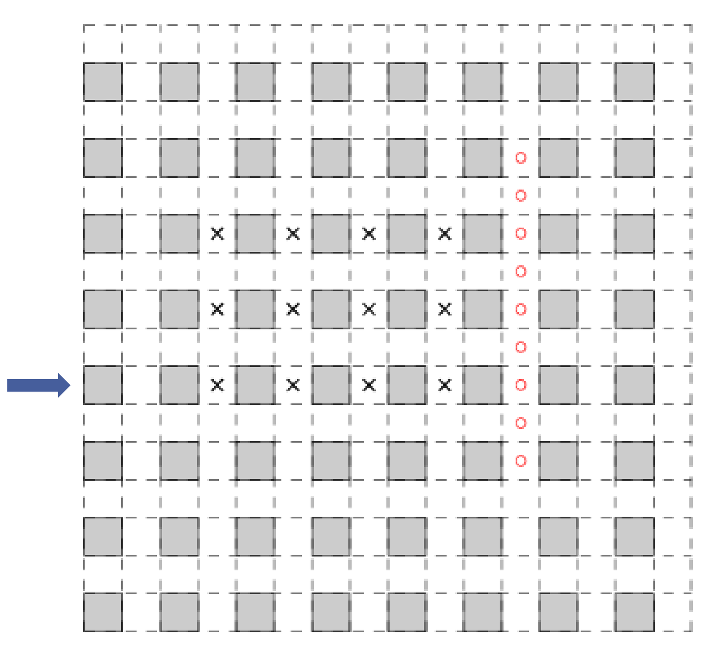

Figure 1 Plan view of DNS domain showing an array of cubes each of height H. Domain size is 16Hx16Hx8H. Crosses denote sources of scalar. Circles denote time-series data locations.

Data generated by a DNS of turbulent flow over an array of large cubical roughness elements (Branford et al., 2011).

Periodic boundary conditions applied in the horizontal; flow driven by a constant body force.

A passive scalar was released continuously at several locations within the array .

Time series were stored at the centre of each HxHxH block in the domain.

Following the procedure of Horst et al. (2004), an array of nine sampling locations is chosen along the cross-wind direction to compute filtered and SFS variables. A top-hat filter over five points is used in the cross-wind direction and filtering in time is done, using Taylor's hypothesis to convert to streamwise filtering over an equivalent filter scale Δ. This allows the direct computation of the SFS fluxes and stresses f_i and τ_{ij}. To compute f_i and τ_{ij} from the tensor models, their evolution equations are solved by advancing in time. The calculation is repeated for different vertical locations of the sampling points, corresponding to different values of the ratio l/Δ, where l is the integral length scale of the turbulence, and for different array locations downstream.

Results from DNS

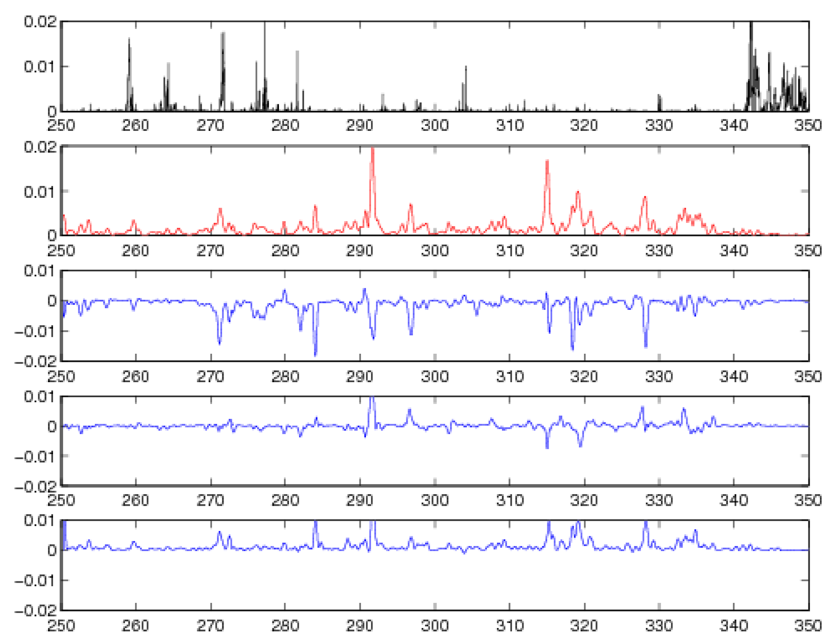

Figure 2 (a) Raw time-series of scalar; (b) Filtered time-series; (c) to (e) subfilter scalar fluxes f_1, f_2 and f_3 respectively.

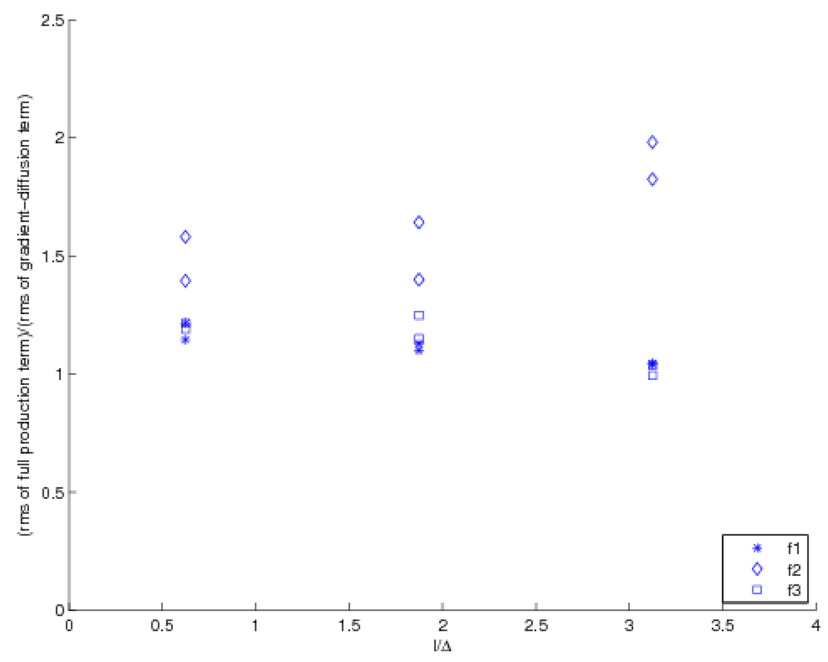

Figure 3: Ratio of rms value of full production rate of SFS scalar flux to rms value of gradient-diffusion term.

The r.m.s. values of computed SFS fluxes cannot be accounted for solely by a gradient-diffusion flux term

Hence, the other production terms need to be taken into account in turbulence models

Evaluation of models with DNS results

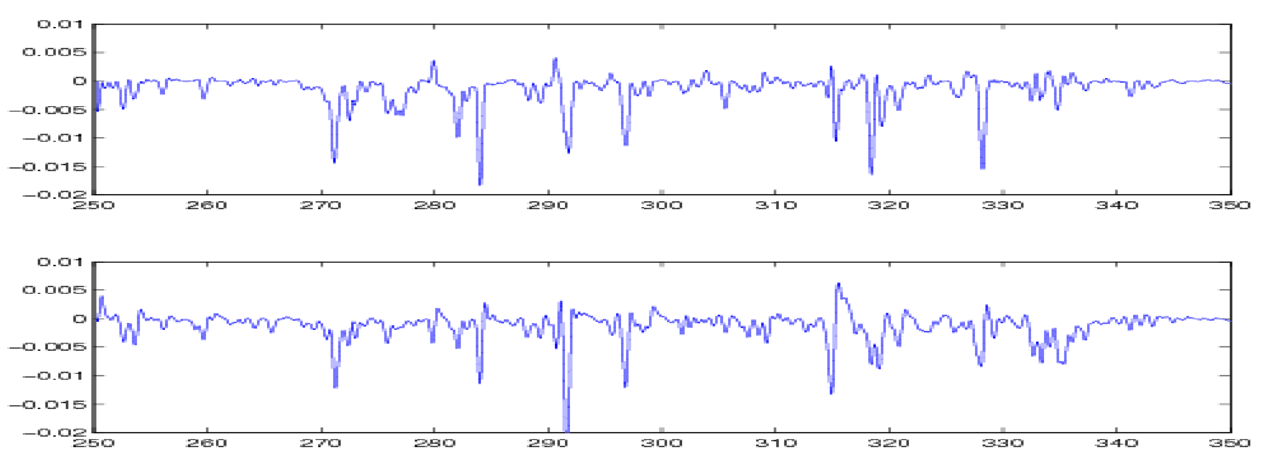

Figure 4 Comparison of time-series of f_1 computed directly from DNS (top) and tensor model (bottom)

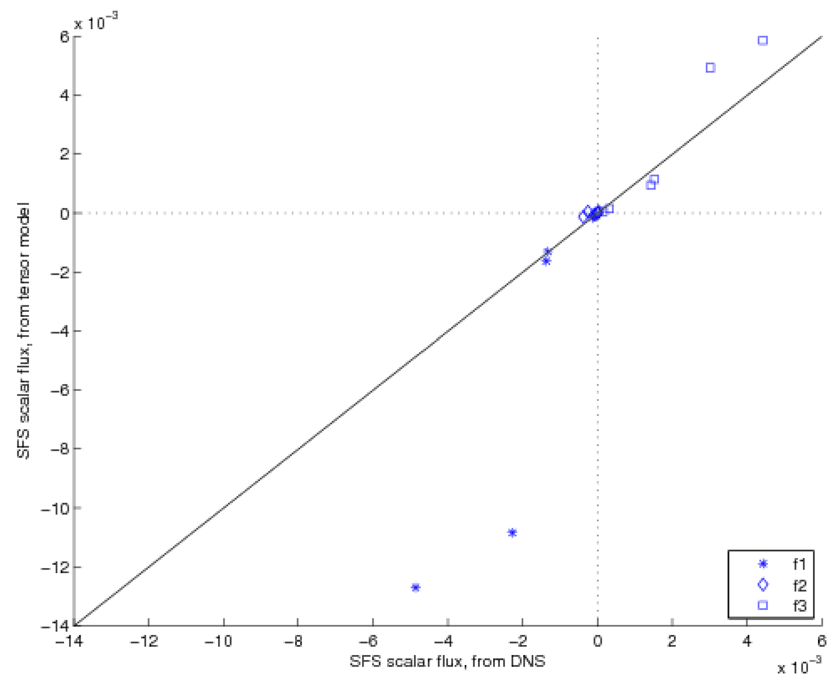

Figure 5: Mean modelled SFS fluxes f_1, f_2 and f_3 compared with DNS-computed values using tensor model

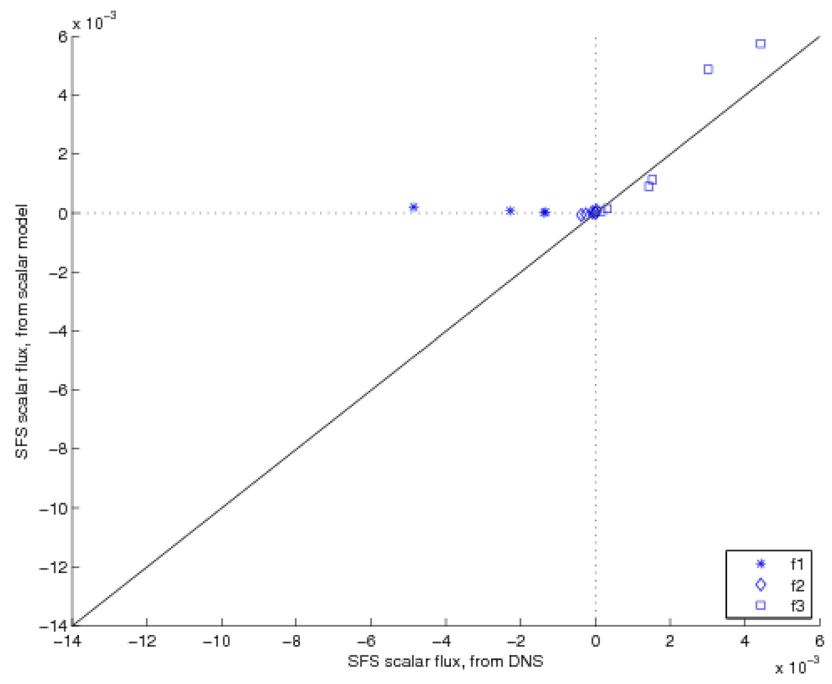

Figure 6: Mean modelled SFS fluxes f_1, f_2 and f_3 compared with DNS-computed values using scalar model

SFS fluxes from tensor model agree better with values computed directly from DNS data

Supports the findings of Wyngaard (2004) using observational data

Conclusions

Computations from DNS data show extra production terms to be significant. SFS fluxes computed from the tensor eddy-diffusivity models agree well with values computed directly from the DNS data and perform better than the standard Smagorinsky model. These results support the potential use of the tensor-based models in the *terra incognita*.

References

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3. Branford S, Coceal O, Thomas TG, Belcher SE (2011), *Boundary-Layer Meteorol, 139*: 367-394.
4. Horst TW, et al. (2004), *J Atmos Sci, 61*: 1566-1581.

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