Limitations of Equilibrium
Or:
What if $\tau_{LS} \gg \tau_{adj}$?

Bob Plant, Laura Davies

Department of Meteorology, University of Reading, UK

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Outline

Deliberately depart from a key quasi-equilibrium assumption

- The timescale separation issue
- A simple model with memory
- CRM results with variable forcing timescale
- Memory and its origins
- Conclusions / implications
Timescale Separation
Consider an “adjustment timescale” $\tau_{\text{adj}}$

Describes rate at which a pre-existing convective instability would be removed by convective activity in the absence of forcing.

Key assumption of quasi-equilibrium thinking is that $\tau_{LS} \gg \tau_{\text{adj}}$

Parameterizations often include some such adjustment timescale in computing the closure.

Their behaviour is sensitive to the timescale used.
Adjustment Timescale

- May be related to cumulus lifetime?
- Or related to time taken for gravity waves to travel between clouds?
  - i.e., to communicate local temperature perturbations from clouds throughout environment
- For a step change in forcing, adjustment of mass flux is proportional to cloud spacing in unadjusted state (Cohen and Craig 2004)
- Implies that it depends on magnitude of forcing
Timescale Separation

- Arakawa and Schubert (1974), p691

Usually the large-scale forcing is changing in time and therefore the cumulus ensemble will not reach an exact equilibrium. The properties of the cumulus ensemble ... depend on the past history of the large-scale forcing, but this dependency should be significant only within the time scale of the adjustment time.

If $\tau_{LS} \gg \tau_{adj}$, past history of forcing is effectively encoded in the current state of the atmosphere.
Timescale Separation

Arakawa and Schubert, continued...

We assume that this is the case for the cumulus ensembles we wish to parameterize. We call this assumption “the quasi-equilibrium assumption”. It is also an assumption on parameterizability, if by parameterization we mean a relation between the properties of the cumulus ensemble and the large-scale variables at some instant.
Separation Issues

But...

- What forcing timescale can be considered “large enough”?
- What happens as quasi-equilibrium starts to break down?
Simple Model for Convective Memory
Simple Model

Two-layer model of atmosphere and surface

\[
\frac{dT}{dt} = \text{COOL} + Q_1
\]

where \(Q_1\) adjusts to a rate \(R\) with a memory timescale \(t_{\text{mem}}\),

\[
\frac{dQ_1}{dt} = \frac{R - Q_1}{t_{\text{mem}}}
\]

and the rate \(R\) is such as to produce a neutral temperature with a “closure” or adjustment timescale \(t_{\text{close}}\)

\[
R = \text{MAX} \left( \frac{T_{\text{surf}} - T}{t_{\text{close}}}, 0 \right)
\]
Simple Model Forcing

Surface temperature prescribed, and is characterized by a timescale $\tau$

Half of a sine wave during “day”, constant at “night”
Regime Diagram

Qualitative diagram for $t_{\text{close}} = 1\text{hr}$

Choose $\tau = 24\text{hr}$ and look at regimes for increasing $t_{\text{mem}}$

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Regime Type E

Type E: $t_{\text{mem}} \ll \tau$, repetitive response, dominated by evolution of forcing

$Q_1$ timeseries for $t_{\text{mem}} = 1\text{hr}$
Regime Type D

Type D: a transitional regime, with variability and skipped cycles

$Q_1$ timeseries for $t_{\text{mem}} = 5\text{hr}$
Regime Type C

Type C: the heating over each forcing cycle is variable

$Q_1$ timeseries for $t_{\text{mem}} = 12\text{hr}$
Regime Types B and A

Type B: heating oscillates about a mean response

\[ Q_1 \text{ timeseries for } t_{\text{mem}} = 24\text{hr} \]

Type A: \( t_{\text{mem}} \gg \tau \), very slow response, not adjusted after 20+ cycles

What if \( \tau_{LS} \gg \tau_{\text{adj}} \)?
In summary

- As $t_{\text{mem}}$ increases to $\sim \tau$ the response is not directly related to current forcing.
- Implies feedback, with current heating dependant on the time-history of the convection.
- Are there such regimes for atmospheric convection?
- Hard to vary $t_{\text{mem}}$ (!), but we can vary $\tau$.....
CRM Results for Variable $\tau_{LS}$
The CRM Experiments

Using Met Office LEM

- 1km horizontal resolution on 64x64km² domain
- Prescribed radiative cooling of troposphere, constant with height
- No wind shear imposed, $f = 0$
- Run to radiative-convective equilibrium with prescribed sensible and latent heat fluxes
- Then vary surface fluxes: half sine wave during “day” and switched off at “night”
- Not a diurnal cycle simulation: e.g., no shallow convection phase
Equilibrium: “Weak” Definition

If $\tau$ is long (e.g., for constant forcing), equilibrium state easily defined...

What if $\tau_{LS} \gg \tau_{adj}$? – p.19/34
Equilibrium: “Weak” Definition

If $\tau$ is finite...

- A strict definition is to compare the instantaneous convective heating to the instantaneous forcing.
- Not satisfied by convection due to adjustment. Does not respond instantaneously: eg, when forcing first switches on.
- Here we consider a “weak” definition of an equilibrium: compare the total convective heating to the total forcing, integrated over the forcing cycle.
- A balance between these is a necessary condition for AS’s quasi-equilibrium.
Timeseries of Mass Flux

For $\tau = 24\text{hr}$, with similarities to regime E

Type E: repetitive response, dominated by evolution of forcing
Timeseries of Mass Flux

For $\tau = 3$hr, with similarities to regime C

Type C: the heating over each forcing cycle is variable
Timeseries of Mass Flux

For $\tau = 1$ hr, with similarities to regime B

Type B: heating oscillates about a mean response
Variations in Convection Per Cycle

As measured by cycle-integrated cloud base mass flux,

Enhanced variability at $\tau \lesssim 12$ hr
Variations in Convection Per Cycle

As measured by cycle-integrated precipitation,

Enhanced variability at $\tau \lesssim 12$ hr
Feedback from one cycle to the next?

For $\tau = 36\text{h (pink)}, 24\text{h (dark blue)}, 18\text{h (light blue)}$

No correlation between convection on successive cycles for $\tau \gtrsim 18\text{hr}$
Feedback from one cycle to the next?

For $\tau = 12\text{h}$ (green), $6\text{h}$ (red), $3\text{h}$ (black)

$R^2 \sim -0.5$ for $3 \lessapprox \tau \lessapprox 12\text{hr}$
Physical Origins of Memory
Mean State

Is the cycle-to-cycle variability explained by differences in domain-mean profiles?

- Domain-mean profiles of $\theta$ and $q_v$ at start of forcing conditioned on the convection occurring in the following cycle:
  - integrated mass flux exceeds mean (strong cycles)
  - integrated mass flux exceeds mean (weak cycles)
Mean State

Solid line and dark shading for strong cycles; dashed line and light shading for weak cycles.

Strong/weak difference < variations between strong cycles
Spatial Water Vapour Anomaly

Snapshots during active convection, for $\tau = 3$ h (left) and $\tau = 24$ h (right)

Convection develops coherent structures on scales of $\sim 10$–$20$ km
Spatial Water Vapour Anomaly

Snapshots just before onset of convection, for $\tau = 3\text{h}$ (left) and $= 24\text{h}$ (right)

Note different scales used.
For longer $\tau$, the power at intermediate scales is lost during the break in convective activity.
Conclusions

- Memory appears to be carried by spatial structures in the moisture field, not in the domain-mean state.
- Memory effects found for timescales $\lesssim 12\text{hr}$.
- For $u \sim 10\text{ms}^{-1}$, this translates to a spatial scale of $500\text{km}$.
- Suggests that for forcing mechanisms with scales $\lesssim 500\text{km}$, convection may not be in quasi-equilibrium.
- Convection in upstream grid boxes may be relevant.
- The perfect parameterization might have non-local aspects, and not be purely 1D?

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