Pressure drag for shallow cumulus clouds -- from thermals to the cloud ensemble

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Parameterization of vertical velocity in a plume model

Buoyancy and entrainment are two major balanced terms.

Effect of pressure perturbation is incorporated into the reduced buoyancy term

\[
\frac{1}{2} \frac{\partial w_C^2}{\partial z} = a B_C - b \varepsilon v_C^2, \]

It is the pressure drag, not the entrainment, that balances most the buoyancy acceleration. (DeRoode et al. 2012; Sherwood et al. 2013; Romps and Charn 2015; Romps and Oktem 2015)

Parameterization of pressure gradient force

Thermodynamic pressure perturbation accounts for most of the pressure drag on the central axis of updrafts

\[
\frac{1}{2} \frac{\partial (w^2)}{\partial z} = \left(1 + \frac{a^2 R_0^2}{H_1^2} \right)^{-1} \frac{B - b \varepsilon v^2}{\varepsilon} \quad z \leq z_M \quad \text{and} \quad (5)
\]

\[
\frac{1}{2} \frac{\partial (w^2)}{\partial z} = \left[1 + \gamma_1 + \frac{(z - z_M)}{H_2} (\gamma_2 - \gamma_1)\right]^{-1} B - \left[1 + \gamma_1 + \frac{(z - z_M)}{H_2} (\gamma_2 - \gamma_1)\right]^{-1} \frac{B}{\varepsilon} dz - b \varepsilon v^2 \quad z > z_M. \quad (6)
\]

\[
\nabla^2 p \approx \nabla^2 p_D + \nabla^2 p_B; \quad \nabla^2 p_D = -\nabla \cdot (\rho \mathbf{u} \cdot \nabla \mathbf{u}) ; \quad \nabla^2 p_B = \frac{\partial (\rho B)}{\partial z},
\]

\[
\nabla^2 p = \nabla^2 p_D + \nabla^2 p_B \approx \frac{\partial (\rho B)}{\partial z}.
\]
Motivation

1. How can the pressure drag of the cloud ensemble be related to that of a single cloud or successive rising thermals within the cloud?

2. What about the pressure gradient force off the central axis?

3. Does pressure gradient force always act like a drag?

“Core-cloak” conceptual model
(Gu et al. 2020 JAS)
Methodology

• Large eddy simulation
  
• BOMEX [Met Office-NERC Cloud (MONC) model]
    15 km × 15 km × 3 km @ 25 m resolution (both horizontal and vertical)
    Most configurations follow the inter-comparison study of BOMEX (Siebesma et al. 2003)
    Smagorinsky-Lilly turbulence scheme
    6 hour simulation, last hour simulation (equilibrium state, 1 min output frequency) is taken for analysis

• 3D Cloud tracking
  
• An extension of Muetzelfeldt’s 2D cloud tracking algorithm
  
• For complicated life cycles, only the cloud object that has the closest cloud depth with previous time is kept.
  
• A total of 4448 cloud objects have been tracked

Muetzelfeldt 2020
Constructing the vertical momentum budget for cloud ensemble from single cloud

Single cloud

\[
\frac{\partial (\rho a_i)}{\partial t} + \frac{\partial (\rho a_i w_i)}{\partial z} = E_i - D_i
\]

\[
\frac{\partial (\rho a_i w_i)}{\partial t} + \frac{\partial (\rho a_i w_i^2)}{\partial z} = E_i w_0 - D_i w_i + \rho a_i B_i - a_i \left( \frac{\partial p}{\partial z} \right)_i + \rho a_i S_i
\]

Cloud ensemble

\[
\frac{\partial a}{\partial t} + \frac{\partial \sum_i a_i w_i}{\partial z} = \frac{1}{\rho} \sum_i E_i - \frac{1}{\rho} \sum_i D_i
\]

\[
\frac{\partial (\sum_i a_i w_i)}{\partial t} + \frac{\partial (\sum_i a_i w_i^2)}{\partial z} + \frac{\partial \sum_i a_i w_i^2}{\partial z} = \frac{1}{\rho} \sum_i E_i w_0 - \frac{1}{\rho} \sum_i D_i w_i + \sum_i a_i B_i
\]

\[
- \frac{1}{\rho} \sum_i \frac{\partial p}{\partial z} + \sum_i a_i S_i
\]

\[
\frac{\partial w_c}{\partial t} = - \frac{\partial (\frac{1}{2} a w_c^2)}{\partial z} - \frac{1}{\rho} \sum_i \frac{\partial}{\partial z} \sum_i a_i (w_i - \bar{w})^2 - \frac{1}{\rho} \sum_i \frac{\partial}{\partial z} \sum_i a_i w_i^2
\]

\[
- \frac{1}{\rho} \sum_i D_i (w_i - \bar{w}) + \frac{1}{\rho} \sum_i a_i B_i - \frac{1}{\rho} \sum_i \frac{\partial p}{\partial z} + \sum_i a_i S_i
\]
Despite the vertical extent, our budget results capture reasonable vertical distribution of each terms and also their values, being consistent with DeRoode et al. (2012).

Main points:
1. Thermodynamic pressure gradient force changes consistently with buoyancy source but in an opposite phase;
2. Dynamical pressure gradient force dominates the pressure gradient force, both in terms of magnitude and vertical variations;

DeRoode et al. 2012
Budget for tracked clouds with different life time

(a) < 5 min (2829)
(b) 5-15 min (1307)
(c) 15-25 min (275)
(d) > 25 min (37)

Legend:
- PGF
- BUOY
- PBGF
- PDGF
Budget for single cloud

Cloud 1 (16 min)

Cloud 92 (32 min)

Cloud 222 (46 min)

$w$ tendency ($ms^{-2}$)

$P_{GF}$

$BUOY$

$PBGF$

$PDGF$
Physical interpretation

Multiple couplets of pressure perturbation in the vertical results in the frequent oscillation of pressure gradient force.

Increased magnitude of pressure perturbation leads to the increased negative tendency of pressure gradient force.
The couplets of pressure perturbations off the central axis is closely related with successive thermals within the cloud. Downdrafts on the downshear side further complicated the distribution of pressure perturbation.

How can we explain the distribution of pressure perturbation using the circulations of thermals?
Decomposition of dynamical pressure perturbation

The dynamical pressure perturbation can be further decomposed into two components: deformation and rotation

\[
\frac{1}{\rho} \nabla^2 p_D = -e^2_{ij} + \frac{1}{2} |\omega|^2, \tag{5}
\]

where \( e_{ij} \) is the deformation tensor and \( \omega \) is the vorticity, such that

\[
e^2_{ij} = \frac{1}{4} \sum_{lj} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)^2, \tag{6}
\]

\[
\omega = \nabla \times u, \tag{7}
\]

where \( u_1 = u, \ u_2 = v, \ u_3 = w, \ x_1 = x, \ x_2 = y, \) and \( x_3 = z. \)
Physical interpretation

\[ \frac{d\omega}{dt} = (\omega + fk) \cdot \nabla u + \nabla \times Bk, \]

\[ w \frac{d\omega}{dz} = \nabla \times Bk, \]

\[
\frac{1}{2} |\omega|^2 \text{ (s}^{-2})
\]

\( (e) \)

\( (f) \)

PD (Pa)
Physical interpretation

600.0 m (7434 slices)  
1000.0 m (2696 slices)  
1800.0 m (534 slices)
Summary

Continuous baroclinic generation of horizontal vorticity associated with rising thermals leads to amplified local minimum of dynamical pressure perturbation.

Within individual cloud, the dynamical pressure drag dominates the total pressure drag off the central axis of updraft. Successive rising thermals within the cloud results in the frequent oscillations. The downdrafts outside the cloud further complicated the picture of oscillation.

The pressure drag of cloud ensemble shares similar features with dominant contribution from dynamical pressure drag and increased magnitude with height, but much smoother behaviour.

For parameterization, the thermodynamic pressure drag can be incorporated into a reduced buoyancy source, but the dynamic pressure drag cannot.

Pressure gradient force does not always act like a drag, sometimes it accelerates the vertical motion. This might be important for cloud life cycle and may need to be considered in the parameterization.