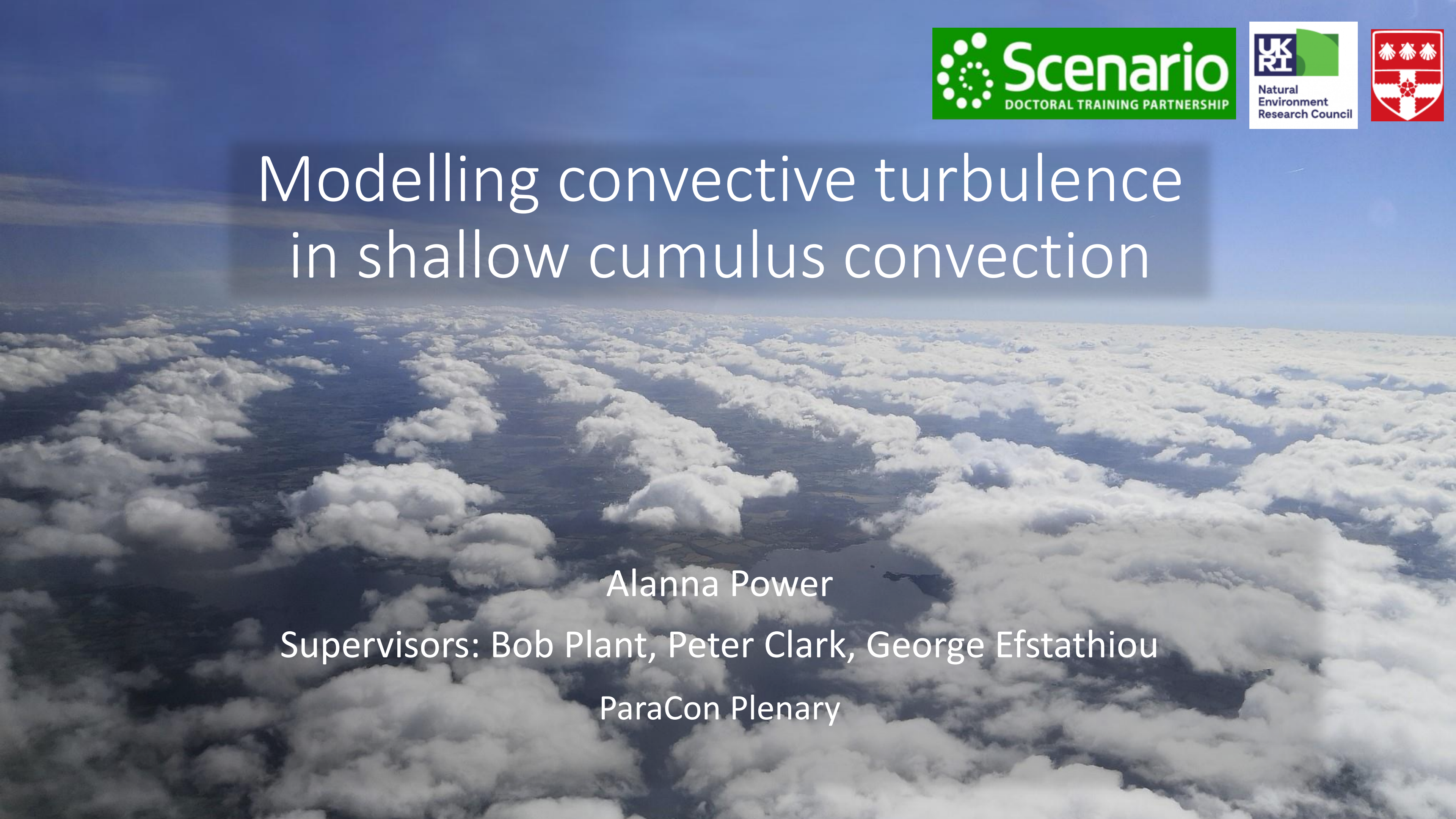

Scenario
DOCTORAL TRAINING PARTNERSHIP
UKRI
Natural Environment Research Council

Modelling convective turbulence in shallow cumulus convection

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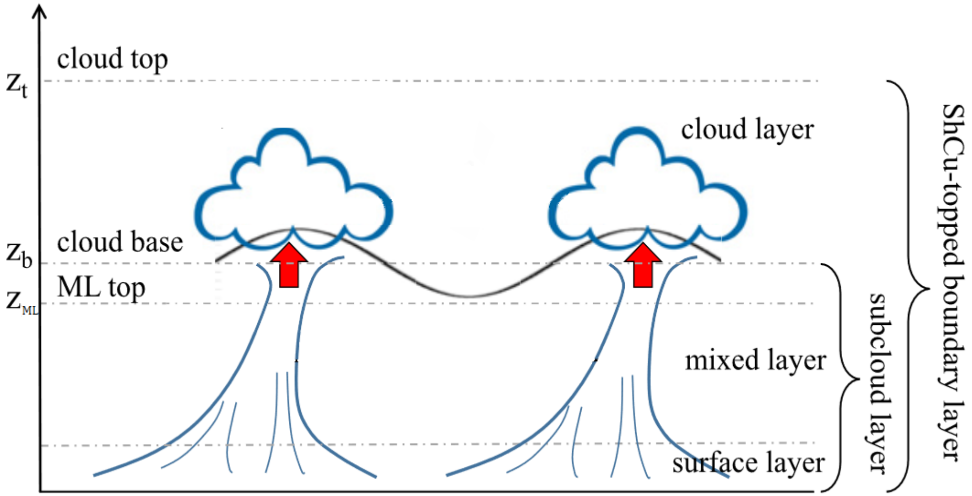
ParaCon Plenary

Modelling convection in the BL

- Shallow cumulus boundary layers are modelled using Large Eddy Simulations (LES).
- At high resolutions, these models can resolve small scale turbulent eddies.
- At coarse resolutions the eddies are not fully resolved on the grid, and therefore the simulation lacks energy: **The grey zone problem.**
- To investigate the grey zone, the mixing lengths that determine energy dissipation are dynamically computed.



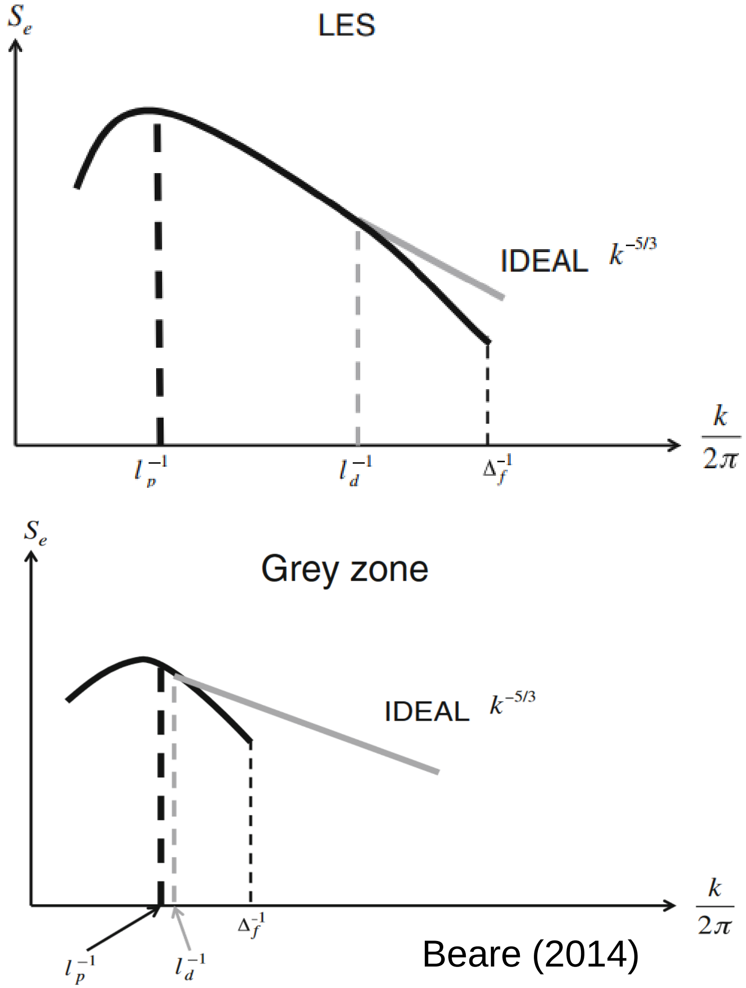
Wang, Y. et al.(2022)

Defining the Grey Zone

- Wyngaard: Grey zone when length scale of the peak is similar to the filter scale: $l_p \approx \Delta$
  - This does not account for dissipation from model dynamics.
- Beare: Grey zone when the dissipation length scale is similar to length scale of the peak: $l_d \approx l_p$.

$$k_{\mathrm{d}}^2 = \frac{\int_{k_0}^{k_1} k^2 S_{\mathrm{e}}(k)\mathrm{d}k}{\int_{k_0}^{k_1} S_{\mathrm{e}}(k)\mathrm{d}k}$$

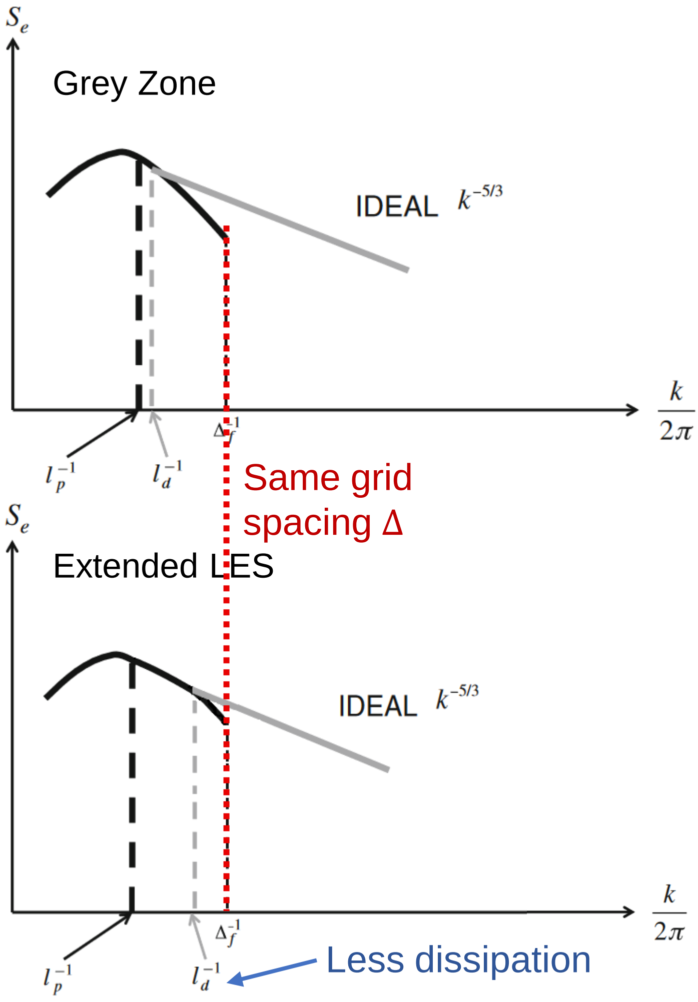
$$l_{\mathrm{d}} = \frac{2\pi}{k_{\mathrm{d}}}$$



Beare (2014)

Delay onset of the Grey Zone

- We know that models in the grey zone are too dissipative.
- Aim: make the model less dissipative by adjusting parameters to more accurately model the flow.
- By retaining more energy, LES methods can be extended to grid spacings that are currently considered to be in grey zone regimes.



Same grid spacing Δ

Less dissipation

Research Questions

- What are the dependencies of the model parameters?
    - Stability dependent, scale dependent, time dependent.

- Do all scalar fluxes behave in the same way?
    - Mixing of moisture vs mixing of heat?
    - Is it reasonable to assume scalar mixing is proportional to that of momentum?

- Is there a relationship between the values of model parameters and their position relative to cumulus clouds?
    - Is it possible to inform the model of any relations, if identified?

LES: Smagorinsky Scheme

- The Smagorinsky scheme assumes **energy production is balanced by dissipation**, i.e. the small scales are in equilibrium.
    a. The eddy viscosity dissipates energy, preventing a build up of energy at the small scales (from the energy cascade).

    b. Eddy viscosity is determined by the mixing length.

    c. The mixing length is set according to the Smagorinsky parameter C_s , and the grid spacing Δ.
- C_s controls the turbulent momentum flux.

$$\overset{\text{stress}}{\tau_{ij}} = 2\nu_t \overset{\text{strain}}{\overline{S}_{ij}}$$

$$\underset{\text{viscosity}}{\nu_t} = l_{\text{mix}}^2 |\overline{S}|$$

$$l_{\text{mix}} = C_s \Delta$$

Standard Smagorinsky Approach

- Current LES models often use the “**Standard Smagorinsky Scheme**”, where C_s is constant.
  - Values between C_s = 0.1 to C_s = 0.23 (used in MONC).
- C_s also controls the scalar fluxes (eg: heat, water) along with the Prandtl number Pr.
  - Pr is the ratio between the rate of diffusion of momentum and the rate of diffusion of a scalar.
- In the Standard model, for all scalars ψ, the Prandtl number is set to the typical value for air.

$$l_{\mathrm{mix}} = C_s \Delta$$

Smagorinsky parameter for a scalar ψ

Smagorinsky parameter for momentum

$$C_\psi^2 = \frac{C_s^2}{Pr_\psi}$$

Prandtl number for a scalar ψ

$$Pr_{air} = 0.7$$

Dynamic Model Equations

- The Smagorinsky parameters are calculated based on the flow at each grid point.
- Defined by Germano (1991) and modified by Lilly (1992).
- High resolution data (denoted by overbars), is filtered to coarser resolutions (denoted by carets).

$$\bar{S}_{ij} = \frac{1}{2}\left(\frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i}\right)$$

$$|\bar{S}| = \sqrt{2\bar{S}_{kl}\bar{S}_{kl}}$$

Momentum:

$$L_{ij} = \widehat{\bar{u}}_i \widehat{\bar{u}}_j - \widehat{\bar{u}_i \bar{u}_j}$$

$$M_{ij} = \widehat{\bar{\Delta}}^2 |\widehat{\bar{S}}| \widehat{\bar{S}}_{ij} - \bar{\Delta}^2 \widehat{|\bar{S}|\bar{S}_{ij}}$$

$$C_s = \sqrt{\frac{1}{2}\left(\frac{\langle L_{ij} M_{ij}\rangle}{\langle M_{kl}^2\rangle}\right)}$$

Scalars:

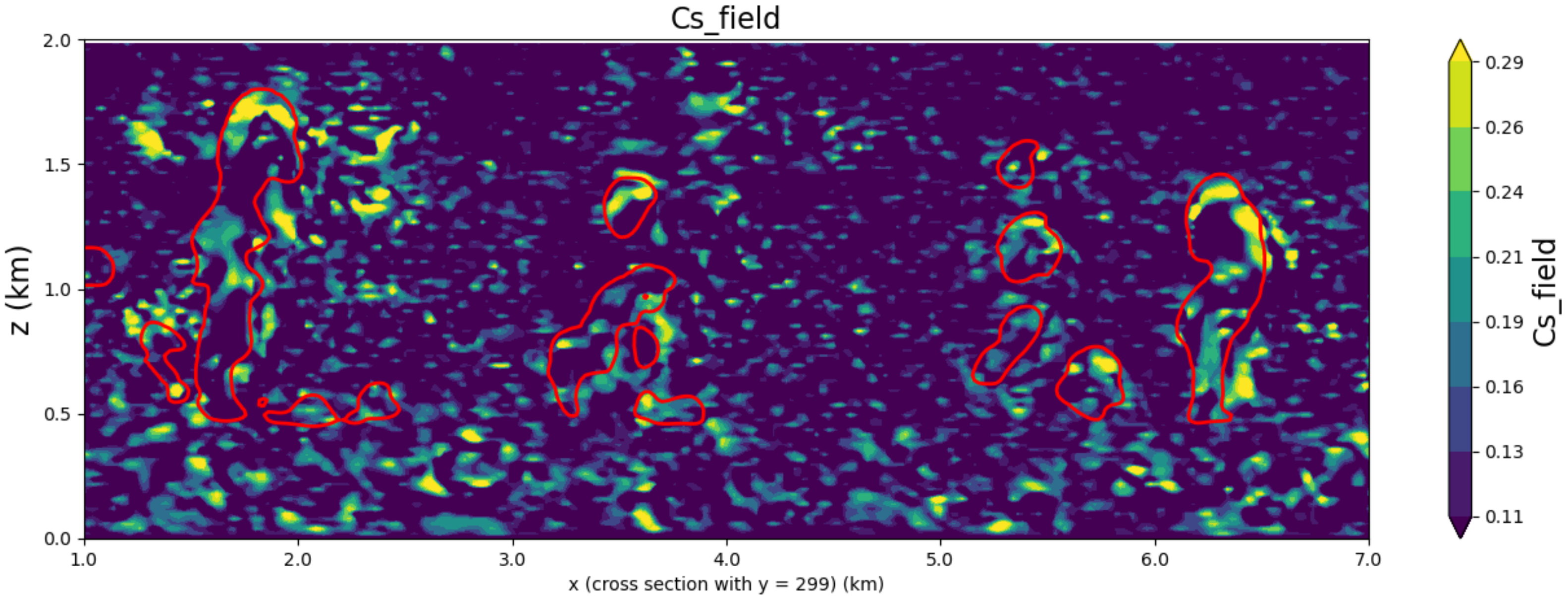
$$H_j = \widehat{\bar{u}}_j \widehat{\bar{\psi}} - \widehat{\bar{u}_j \bar{\psi}}$$

$$R_j = \widehat{\bar{\Delta}}^2 |\widehat{\bar{S}}| \widehat{\frac{\partial \bar{\psi}}{\partial x_j}} - \bar{\Delta}^2 \widehat{|\bar{S}| \frac{\partial \bar{\psi}}{\partial x_j}}$$

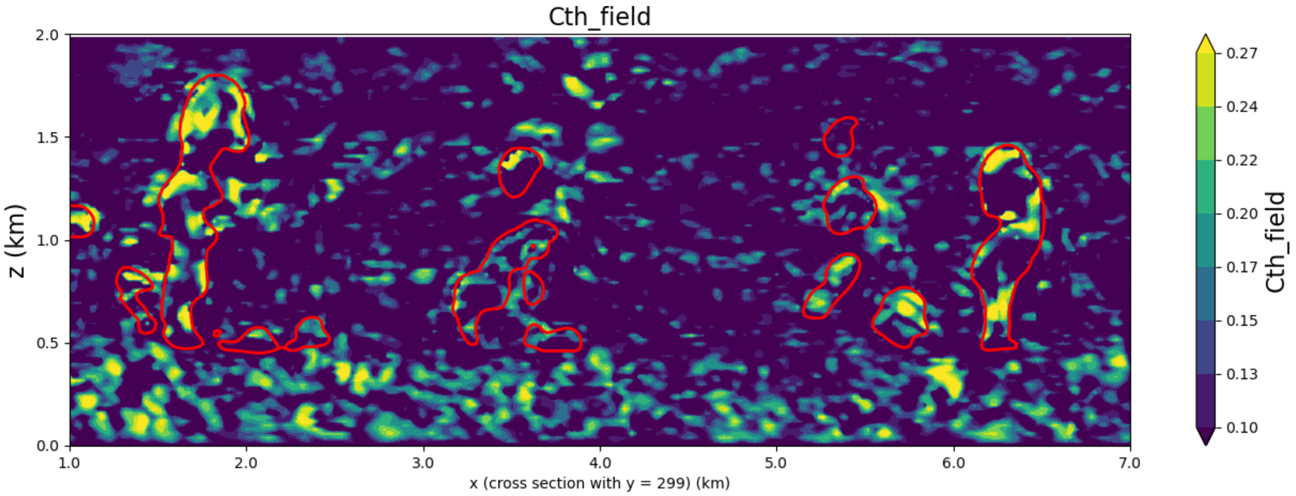
$$C_\psi = \sqrt{\frac{1}{2}\left(\frac{H_j R_j}{R_k^2}\right)}$$

Dynamic Smagorinsky Parameters

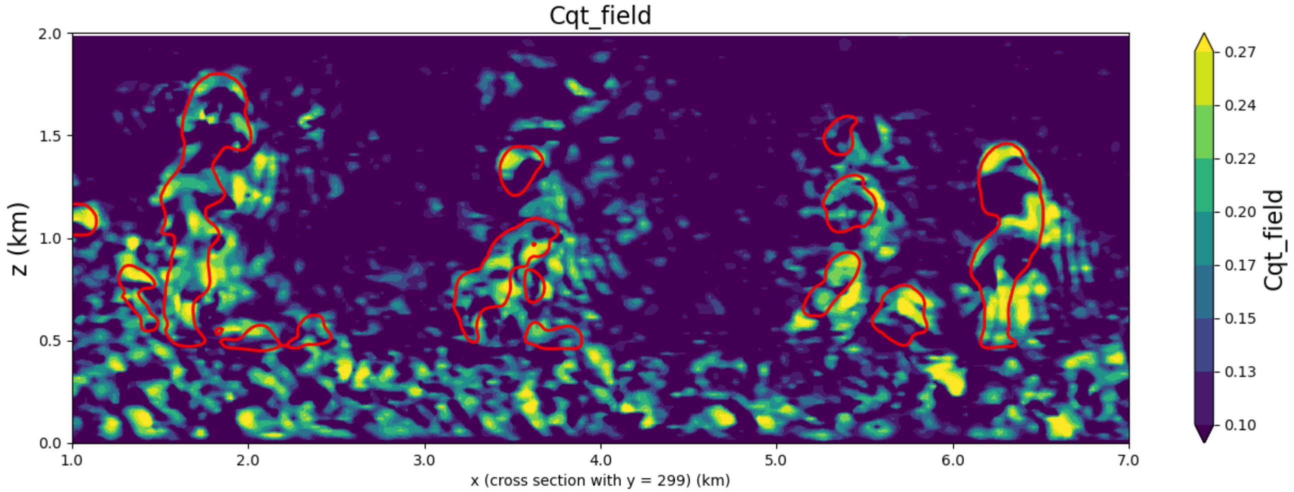
Field of dynamic Smagorinsky parameter values for **Momentum** (BOMEX):



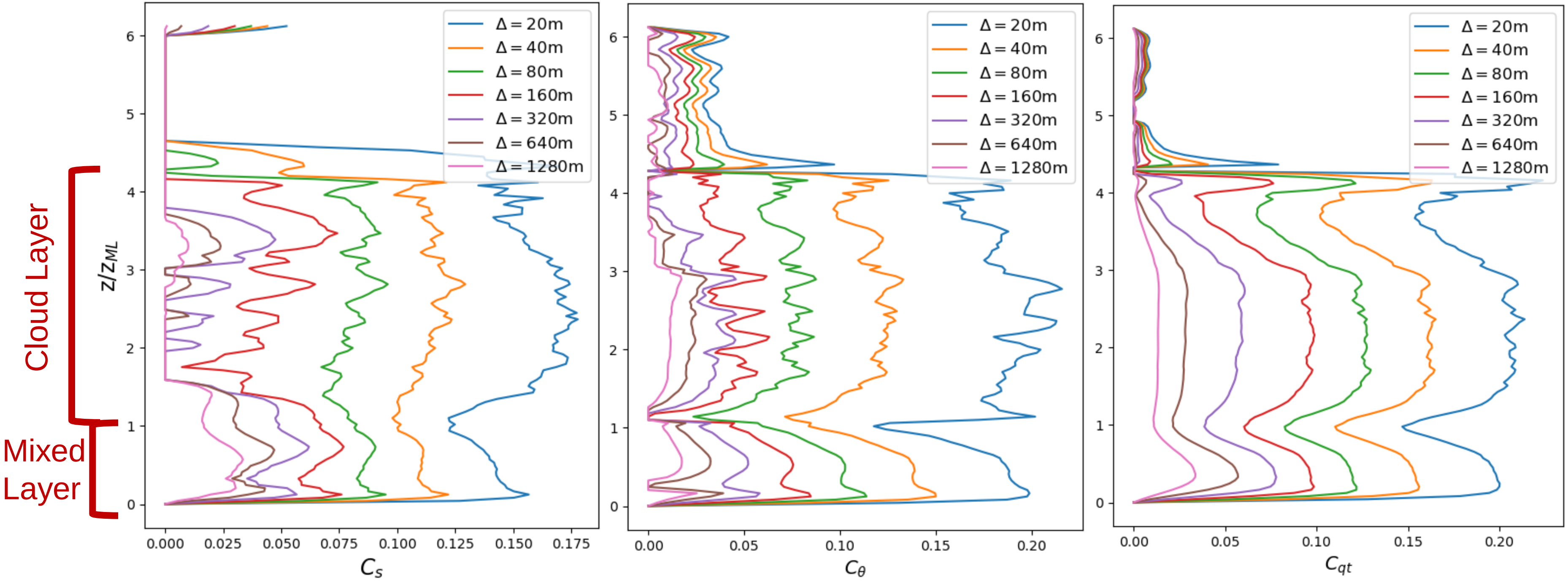
Field of dynamic Smagorinsky parameter values for **heat:**



Field of dynamic Smagorinsky parameter values for **total moisture:**

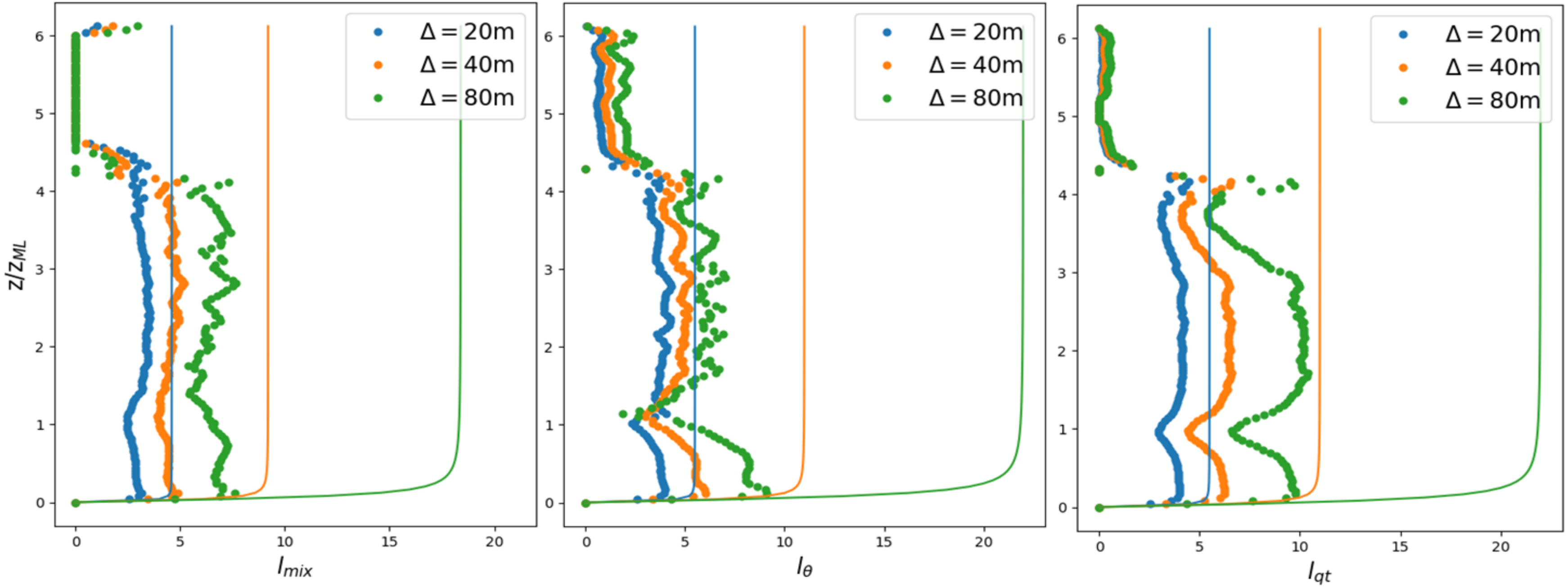


Horizontal averaging: stability dependencies

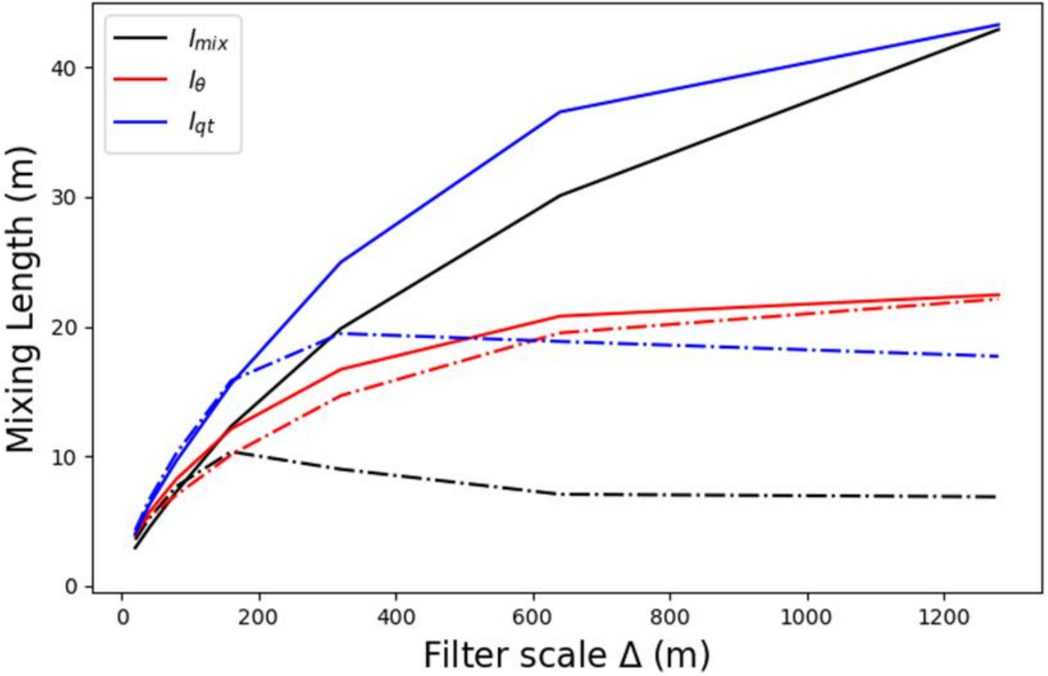


Dynamic vs Standard Mixing Lengths

Profiles of the mixing length for 3 different filter scales Δ. Dynamic values are shown as points, standard values as a solid line.



Scale Dependency of Mixing Lengths



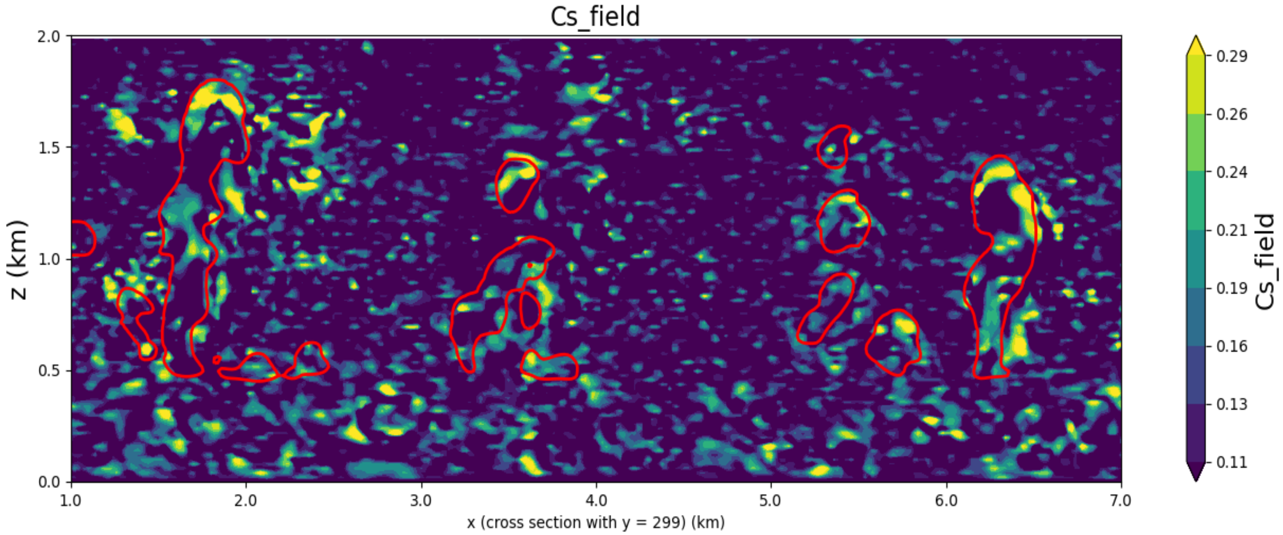
Dynamic mixing length vs filter scale Δ.

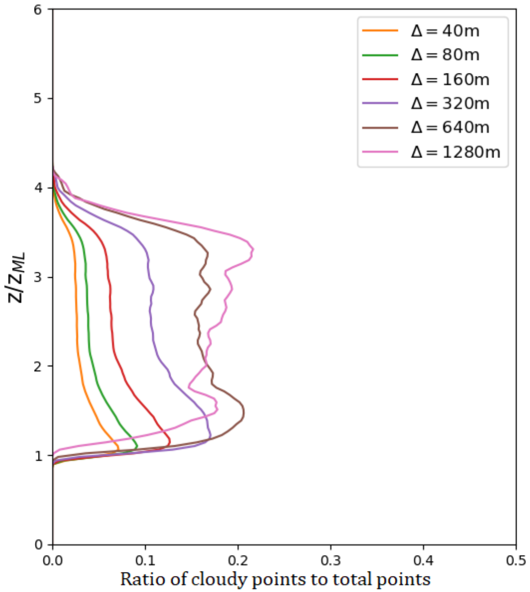
Solid line: average mixing length value mid mixed layer ($0 \leq z/z_{ML} \leq 1$).

Dashed line: average mixing length value mid cloud layer ($1 \leq z/z_{ML} \leq 4$).

In-cloud vs Environment

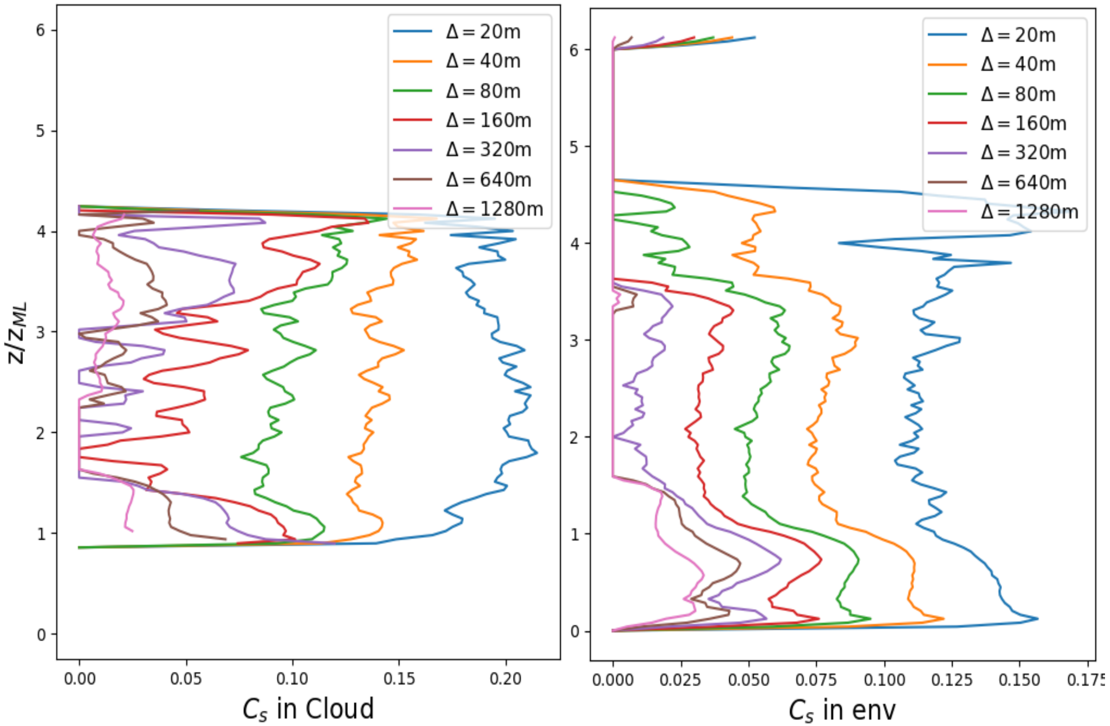
In-cloud Smagorinsky parameter values substantially higher than non-cloudy environment in the cloud layer





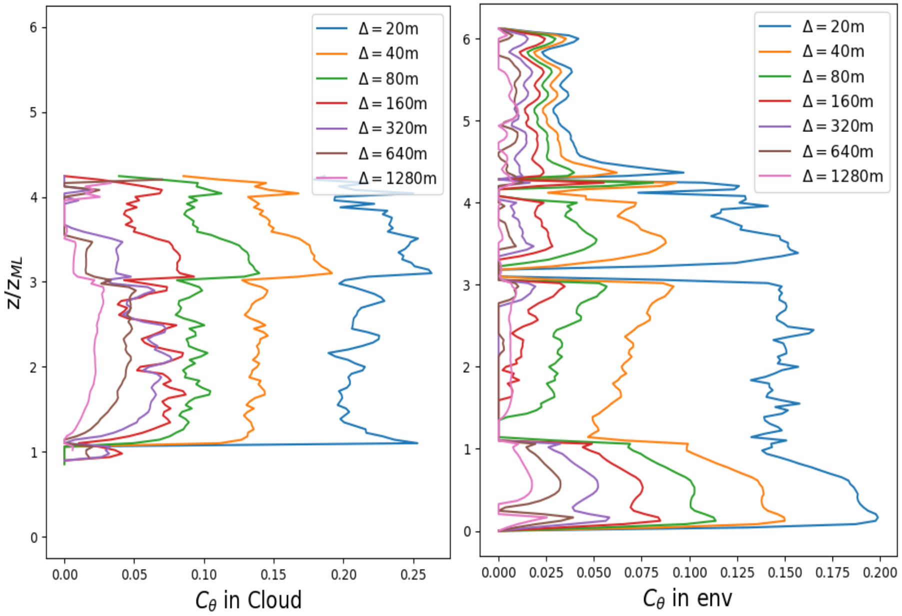
Horizontal averaging: In-cloud vs Environment

- Higher Smagorinsky parameter for momentum: C_s values in-cloud than in the ML and in the non-cloudy environment of the CL.
- Consistent decreases in value at the surface, mixed layer capping inversion/cloud base ($z/z_{ML} \approx 1$), and at the cloud top ($z/z_{ML} \approx 4.5$).
- Maximum values are in the middle of the mixed layer (ML) and middle of the cloud layer (CL).

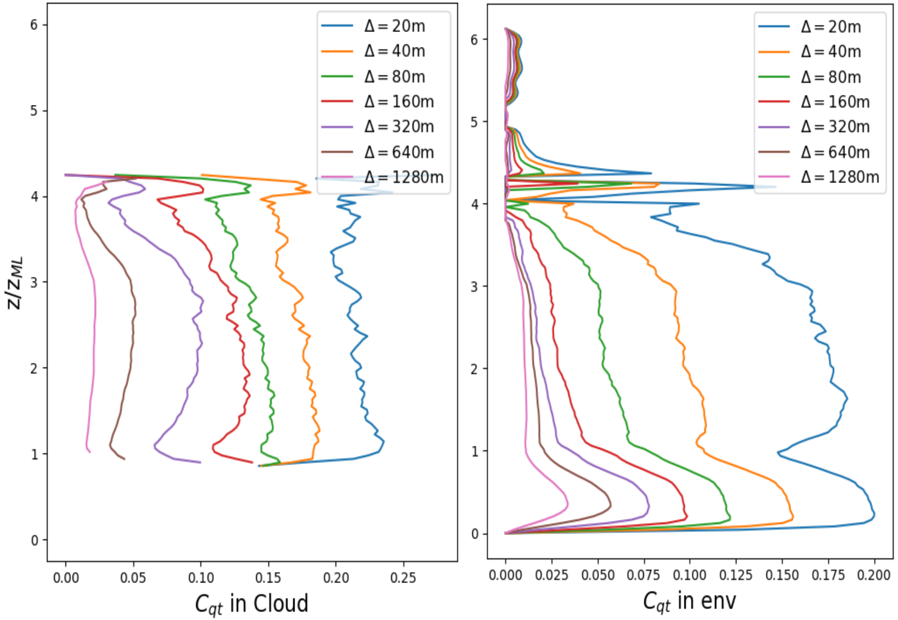


Horizontal averaging: In-cloud vs Environment

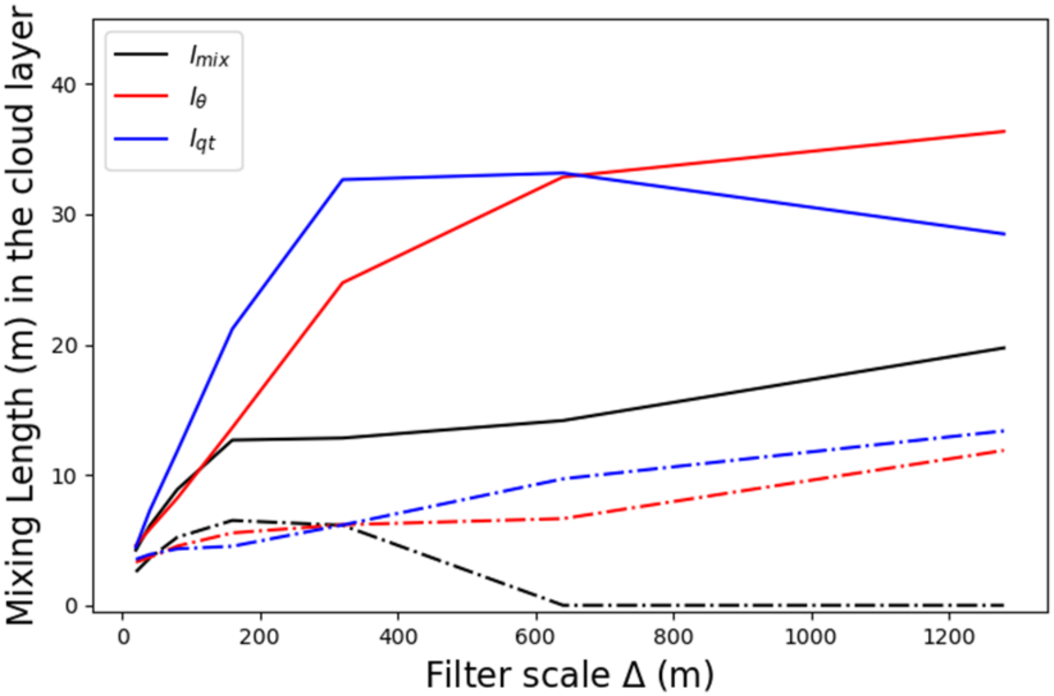
- Smagorinsky parameter for heat (C_θ).



- Smagorinsky parameter for total moisture (C_{qt}).



Mixing lengths in the Cloud Layer



Dynamic mixing length vs filter scale Δ.

Solid line: average mixing length value for cloudy points mid cloud layer.

Dashed line: average mixing length value for non-cloudy points mid cloud layer.

Conclusions

Standard LES models dissipate too much energy at coarse resolutions. Dynamic analysis suggests some reasons for this:

1. The standard values of mixing lengths may be too large.
2. Fixed value mixing lengths cannot account for the effects of stability, particularly at boundaries and temperature inversions, where turbulent eddies have much smaller scales. Using stability functions in the standard model is recommended.
3. The assumption that the mixing of heat/moisture is proportional to that of momentum, and that it is not scalar dependent, does not hold.

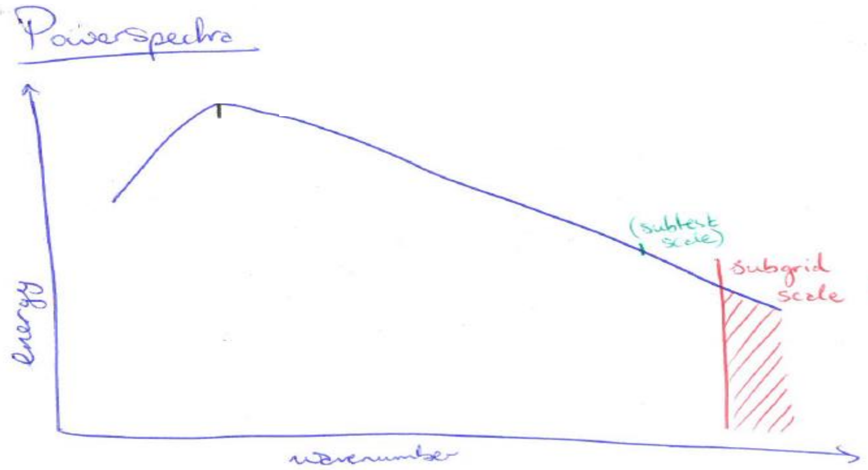
These findings suggest that the turbulent structures which mix momentum, heat, and moisture all differ in scale. Turbulence is also affected by proximity to boundaries, inversions, and clouds. Accounting for this in LES models may allow for more energy to be resolved at coarser resolutions.

References

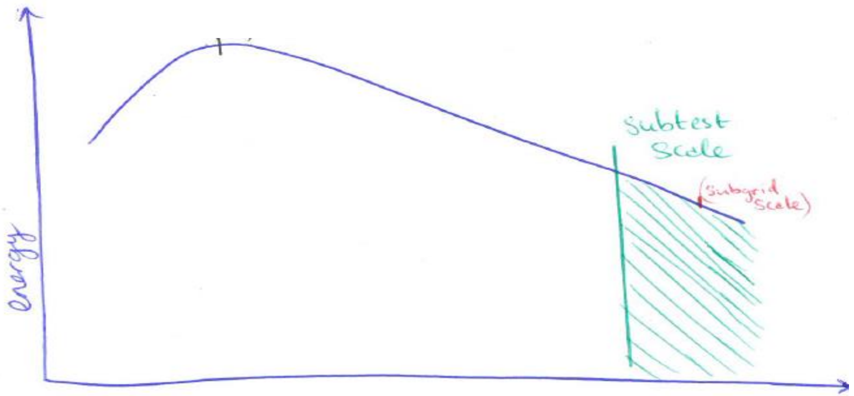
- Beare, R. J., 2014: A Length Scale Defining Partially-Resolved Boundary-Layer Turbulence Simulations. Boundary-Layer Meteorology,151 (1), 39–55, doi:10.1007/s10546-013-9881-3.
- Wang, Y., Cheng, X., Fei, J. & Zhou, B., 2022: Modeling the Shallow Cumulus-topped Boundary Layer at Gray Zone Resolutions. Journal of the Atmospheric Sciences doi:10.1175/JAS-D-21-0339.1.
- Wyngaard, J. C., 2004: Toward Numerical Modeling in the “Terra Incognita”. Journal of the Atmospheric Sciences,61, 11.

Dynamic Smagorinsky

- High resolution data set.
    - Turbulence is resolved down to the **subgrid scale (SGS).**



- These filtered data set (original high res data has been filtered to coarser resolutions).
    - Turbulence is now represented down to **subfilter test scale (STS)**.



Dynamic Smagorinsky

- In the ISR (turbulence is isotropic): turbulence properties scale with wavenumber.
  - Can compute C_s and C_ψ as a function of grid spacing Δ.

Difference in stress between SGS & STS

$$C_s^2 = \frac{1}{2}\left(\frac{L_{ij}M_{ij}}{M_{kl}^2}\right)$$

Difference in strain rate between SGS & STS