Pressure drag for shallow cumulus clouds -- from thermals to the cloud ensemble

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Improvement and calibration of clouds in models
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Parameterization of vertical velocity in a plume model

Buoyancy and entrainment are two major balanced terms

Effect of pressure perturbation is incorporated into the reduced buoyancy term

\[
\frac{1}{2} \frac{\partial w_c^2}{\partial z} = aB_c - bw_c^2,
\]

It is the pressure drag, not the entrainment, that balances most the buoyancy acceleration. (DeRoode et al. 2012; Sherwood et al. 2013; Romps and Charn 2015; Romps and Oktem 2015)

Parameterization of pressure gradient force

Thermodynamic pressure perturbation accounts for most of the pressure drag on the central axis of updrafts

\[
\frac{1}{2} \frac{\partial (w^2)}{\partial z} = \left(1 + \frac{\alpha^2 R^3}{H_1^2}\right)^{-1} B - bw^2 \quad z \leq z_M \quad \text{and}
\]

\[
\frac{1}{2} \frac{\partial (w^2)}{\partial z} = \left[1 + \frac{\gamma_1}{H_2}(\gamma_2 - \gamma_1)\right]^{-1} B - \left[1 + \frac{\gamma_1}{H_2}(\gamma_2 - \gamma_1)\right] \int_{z_M}^z B \ dz - bw^2 \quad z > z_M.
\]

\[
\nabla^2 p \approx \nabla^2 p_D + \nabla^2 p_B; \quad \nabla^2 p_D = -\nabla \cdot (\rho \mathbf{u} \cdot \nabla \mathbf{u}); \quad \nabla^2 p_B = \frac{\partial (\rho B)}{\partial z},
\]

\[
\nabla^2 p = \nabla^2 p_D + \nabla^2 p_B \approx \frac{\partial (\rho B)}{\partial z}.
\]
Motivation

1. How the pressure drag of the cloud ensemble can be related to that of a single cloud or successive rising thermals within the cloud?

2. What about the pressure gradient force off the central axis?

3. Does it always serve as a drag?

(He et al. 2021, under review)

(Gu et al. 2020)
Methodology

- Large eddy simulation
- BOMEX (MONC model)
  15 km × 15 km × 3 km @ 25 m resolution (both horizontal and vertical)
  Most configurations follow the inter-comparison study of BOMEX (Siebesma et al. 2003)
  3D Smagorinsky turbulence scheme
  6 hour simulation, last hour simulation (equilibrium state, 1 min output frequency) is taken for analysis

- 3D Cloud tracking
  - An extension of Muetzelfeldt’s 2D cloud tracking algorithm
  - For complicated life cycles, only the cloud object that has the closest cloud depth with previous time is kept.
  - A total of 4448 cloud objects have been tracked

Muetzelfeldt 2020
Constructing the vertical momentum budget for cloud ensemble from single cloud

**Single cloud**

\[
\frac{\partial (\rho a_i)}{\partial t} + \frac{\partial (\rho a_i \overline{w_i})}{\partial z} = E_i - D_i
\]

\[
\frac{\partial (\rho a_i \overline{w_i})}{\partial t} + \frac{\partial (\rho a_i \overline{w_i^2})}{\partial z} + \frac{\partial a_i \overline{w_i^2}}{\partial z} + \frac{\partial a_i \overline{w_i^2}}{\partial t} - a_i \frac{\partial \overline{p}}{\partial z} = E_i \overline{w_0} - D_i \overline{w_i} + \rho a_i \overline{B_i} - a_i \frac{\partial \overline{p}}{\partial z} + \rho a_i \overline{S_i}
\]

\[
\frac{\partial \overline{w_i}}{\partial t} = -\frac{\partial (\frac{1}{2} a_i \overline{w_i^2})}{\partial z} - \frac{\partial a_i \overline{w_i^2}}{\partial z} + \epsilon_i \overline{w_i (w_0 - w_i)} + B_i - \frac{1}{\rho} \frac{\partial \overline{p}}{\partial z} + \overline{S_i}
\]

**Cloud ensemble**

\[
\frac{\partial a}{\partial t} + \frac{\partial \sum_i a_i \overline{w_i}}{\partial z} = \frac{1}{\rho} \sum_i E_i - \frac{1}{\rho} \sum_i D_i
\]

\[
\frac{\partial (\sum_i a_i \overline{w_i})}{\partial t} + \frac{\partial (\sum_i a_i \overline{w_i^2})}{\partial z} + \frac{\partial \sum_i a_i \overline{w_i^2}}{\partial z} = \frac{1}{\rho} \sum_i E_i \overline{w_0} - \frac{1}{\rho} \sum_i D_i \overline{w_i} + \sum_i a_i \overline{B_i}
\]

\[
- \frac{1}{\rho} \sum_i a_i \frac{\partial \overline{p}}{\partial z} + \sum_i a_i \overline{S_i}
\]

\[
\frac{\partial \overline{w_c}}{\partial t} = -\frac{\partial (\frac{1}{2} a \overline{w_c^2})}{a \partial z} - \frac{1}{a} \frac{\partial a}{\partial z} \sum_i a_i (w_i - \overline{w_c})^2 - \frac{1}{a} \frac{\partial \sum_i a_i \overline{w_i^2}}{\partial z} + \frac{1}{\alpha a \rho} \sum_i E_i (w_0 - \overline{w_c})
\]

\[
- \frac{1}{\alpha a \rho} \sum_i D_i (w_i - \overline{w_c}) + \frac{1}{a} \sum_i a_i \overline{B_i} - \frac{1}{\alpha a \rho} \sum_i a_i \frac{\partial \overline{p}}{\partial z} + \sum_i a_i \overline{S_i}
\]
Despite the vertical extent, our budget results capture reasonable vertical distribution of each terms and also their values, being consistent with DeRoode et al. (2012).

Main points:
1. Thermodynamic pressure gradient force changes consistently with buoyancy source but in an opposite phase;
2. Dynamical pressure gradient force dominates the pressure gradient force, both in terms of magnitude and vertical variations;
Budget for tracked clouds with different life time

(a) < 5 min (2829)

(b) 5-15 min (1307)

(c) 15-25 min (275)

(d) > 25 min (37)

Legend:
- PGF
- BUOY
- PBGF
- PDGF
Budget for single cloud

Cloud 1 (16 min)  
Cloud 92 (32 min)  
Cloud 222 (46 min)

(a)  
(b)  
(c)

-1.5 -1.0 -0.5 0.0 0.5 1.0 1.5 1e-2  
-1.5 -1.0 -0.5 0.0 0.5 1.0 1.5 1e-2  
-1.5 -1.0 -0.5 0.0 0.5 1.0 1.5 1e-2

w tendency (ms^{-2})

PGF  
BUOY  
PBGF  
PDGF

2000  
1750  
1500  
1250  
1000  
750  
500  
250  
0

4Z_b  
3Z_b  
2Z_b  
Z_b

0  
0  
0
Physical interpretation

Multiple couplets of pressure perturbation in the vertical results in the frequent oscillation of pressure gradient force. Increased magnitude of pressure perturbation leads to the increased negative tendency of pressure gradient force.
The couplets of pressure perturbations off the central axis is closely related with successive thermals within the cloud. Downdrafts on the downshear side further complicated the distribution of pressure perturbation.

How can we explain the distribution of pressure perturbation using the circulations of thermals?
Decomposition of dynamical pressure perturbation

The dynamical pressure perturbation can be further decomposed into two components: deformation and rotation

\[ \frac{1}{\rho} \nabla^2 p_D = -e_{ij}^2 + \frac{1}{2} |\boldsymbol{\omega}|^2, \quad (5) \]

where \( e_{ij} \) is the deformation tensor and \( \boldsymbol{\omega} \) is the vorticity, such that

\[ e_{ij}^2 = \frac{1}{4} \sum_{ij} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)^2, \quad (6) \]

\[ \boldsymbol{\omega} = \nabla \times \mathbf{u}, \quad (7) \]

where \( u_1 = u, \ u_2 = v, \ u_3 = w, \ x_1 = x, \ x_2 = y, \) and \( x_3 = z. \)
\[
\frac{d\omega}{dt} = (\omega + fk) \cdot \nabla u + \nabla \times Bk,
\]

\[
\frac{d\omega}{dz} = \nabla \times Bk,
\]

tilting/stretching

baroclinic generation

baroclinic generation
Summary

Continuous baroclinic generation of horizontal vorticity associated with rising thermals leads to amplified local minimum of dynamical pressure perturbation.

Within individual cloud, the dynamical pressure drag dominates the total pressure drag off the central axis of updraft. Successive rising thermals within the cloud results in the frequent oscillations. The downdrafts outside the cloud further complicated the picture of oscillation.

The pressure drag of cloud ensemble shares similar features with dominant contribution from dynamical pressure drag and increased magnitude with height, but much smoother behaviour.

For parameterization, the thermodynamic pressure drag can be incorporated into a reduced buoyancy source, but the dynamic pressure drag cannot.