A stochastic framework for modeling the population dynamics of convective clouds

SAMSON HAGOS, ZHE FENG, BOB PLANT, ROBERT HOUZE, HENG XIAO

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Populations of convective clouds cover a spectrum of sizes and lifetimes and are often transitioning.

**Questions:**

I. What are the processes that govern the evolution of the population of convective cells?

II. How can these processes be modelled and represented in global models?
Background

Effort at representing cloud populations and their dynamics

- General energy cycle
  (Arakawa and Schubert 1974)
  \[
  \frac{dA_i}{dt} = -\sum_{j=1}^{N} \gamma_i M_{Bj} + F_i \\
  \frac{dK_i}{dt} = A_i M_{Bi} - \frac{K_i}{\tau_d}
  \]

- Bulk, deviation from quasi-equilibrium
  (Pan and Randall 1998) \[ K = \alpha M_B^2 \]
  (Yano and Plant 2010) \[ K = \beta M_B \]

- Spectral variations about quasi-equilibrium
  (Craig and Plant 2008) Stochastic
  (Wagner and Graf 2010) Population dynamics

- Non-equilibrium cloud population model
  (Plant 2012)
Informed by analysis of radar observations, cloud permitting model simulations and theory, develop a probabilistic model of non-equilibrium dynamics of cloud populations for:

- Testing hypotheses regarding the roles of various physical processes and
- Parameterizing the spectrum of convective clouds (from isolated to MCSs) in a unified framework.
Description of observation and CPM simulation

**C-Pol observation at Darwin**
- 3 winters of C-Pol radar scans are used to identify convective cells.
- The variability in size and number of the convective cells are used in the analysis.

**CPM simulation**
- 2.5 km grid spacing
- Two months long simulation Jan-Feb 2006.
- No cumulus parameterization.
The General Framework

A Master equation representation of population dynamics

\[ \frac{dn_i}{dt} = \sum_{j \neq i} W_{ji} n_j - W_{ij} n_i \]

Transition to size \( a_i \), Transition from size \( a_i \)

Definitions

Cell size \( a_i = i \cdot a_1 \)

Area fraction \( af = \frac{\sum i n_i a_i}{A_{domain}} = \frac{pa_1}{A_{domain}} \)

Cloud base mass flux per cell area \( m_{bi} = w \rho \)

Cloud base mass flux of a cell \( M_{Bi} = m_{bi} a_i \)
Cell size distribution

Assumption

Cloud work function

\[ A = m \bar{A} \]

Discretization

\[ \frac{dA_i}{dt} = -\sum_{j=1}^{N} \gamma_{ij} M_{bj} + F_i \]

\[ \frac{dp}{dt}; \quad \frac{1}{a_i m_{bi}} \left( -\frac{\sum M_{bi}}{\tau} + F \right) \]

The equation gives us \( dp \) but what we are looking for is the new \( \{n_i\} \)

The problem

- For a given area fraction, what is the size distribution of the cells?
- How is the size distribution related to the mass flux?

We need a probability of growth vector for \( i > 0 \)

\[ G_i = f (\text{environment}, n_i, a_i, ...) \]

Such that the probability of new cell formation is

\[ G_0 = 1 - \sum_{i>0} G_i \]
This framework is hereafter referred to as **STO**chastic framework for **M**odelling **P**opulation dynamics of convective cells (STOMP).
(a) A uniform probability model

**Assumption:** \( dp \) convective pixels land on the domain randomly with uniform probability

**Forcing:** Adding \( dp \) pixels one at a time:

\[
G_{i>0} = \frac{n_i a_i}{A_{domain}}
\]

\[
n_{i+1} = n_{i+1} + 1
\]

\[
n_i = n_i - 1
\]

**Damping:** Removing \( dp \) pixels one at a time:

\[
P_n = \frac{n_i a_i}{\sum_{j=1}^{N} n_j a_j}
\]

\[
n_i = n_i - 1
\]

\[
n_{i-1} = n_{i-1} + 1
\]

Mass flux is assumed to be linear function of cell area

\[
M_{Bi} = m_{bi} a_i \quad m_{bi} = 0.78 \text{kg} / m^2 s
\]
(a) A uniform probability model (STOMP-UP)

Size distribution

- Uniform probability results in too many small cells and too few large cells.
- Chance does not explain the existence of large convective cells.
Diurnal cycle of cell count and mean cell size

In the uniform probability model they are **in phase**.

In both the observations and the CPM the mean cell size **lags behind** cell count.
(b) Aggregation probability model (STOMP-AP)

Probability of growth vector

Probability that favors growth

Through detrainment of moisture by existing convection for example (Mack and Craig 2015)

We define an aggregation parameter $\delta$

$$G_{i>0} = \frac{\delta n_i a_i}{A_{\text{domain}}}$$

$$G_0 = \max(1 - \delta a f, 0)$$

Higher probability of growth  
Lower probability of new cell formation
Aggregation with delta=30.0 introduces the observed lag of mean cell size behind the cell count.

It suggests in this case, growth of existing cells is about 30 times more likely than formation of new cells.
Mass flux and convective cell sizes

Why do we care about cell size distribution anyway?

- Larger cells carry more than their share of mass flux.
Closure: Dependence of mass flux on cell area

\[ M_b = \sum_{N} n_i a_i m_b \]

i) Linear:

\[ m_b = 0.78 \text{kg} / \text{m}^2 \text{s} \]

ii) Non-linear

\[ m_b = (0.30 + 0.023 \frac{(a_i - a_i)}{a_i}) \text{kg} / \text{m}^2 \text{s} \quad a_i \leq 500 \text{km}^2 \]

\[ m_b = (0.54 + 0.0027 \frac{(a_i - a_i)}{a_i}) \text{kg} / \text{m}^2 \text{s} \quad a_i > 500 \text{km}^2 \]
Non-linearity introduces a stochastic ‘recharge-discharge’ behavior. The length of the suppressed “recharge” period increases with the adjustment time-scale.
Response to diurnal forcing

- Because of convective damping the mass flux lags behind the forcing.
- The non-linear relationship between cell size and mass flux per cell area further delays the mass flux diurnal cycle.
Aggregation probability model with non-linear mass flux produces the desired frequency of large cells.
Summary

- A framework for stochastic modeling population dynamics of convective clouds is developed.

- A specific model in this framework is defined by the representation of a growth probability vector (G) and decay vector (D) or more generally by a transition rate matrix.

- If convective plumes prefer to form near existing cells and if mass flux is an non-linear function of cell area:
  - **Under steady forcing:** A recharge-discharge response is obtained.
  - **Under diurnally varying forcing:** Peak mass flux is delayed.
Future work

- A more general transition rate matrix $W$ that represents, lifecycle of convection and formation of cold pools and stratiform area will be derived from observations and cloud permitting model simulations.

- This modelling framework will be implemented into a climate model