# Spectral and Bulk Mass-Flux Convective Parameterizations

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1 Basic features of mass flux schemes

2 Single plumes

3 Construction of equivalent bulk plume

4 Closure

5 Conclusions

# Outline


1. Basic features of mass flux schemes
   - Some mass flux basics
   - Basic bulk idea
2. Single plumes
3. Construction of equivalent bulk plume
4. Closure
5. Conclusions

# The Basic Picture

The Arakawa and Schubert (1974) picture

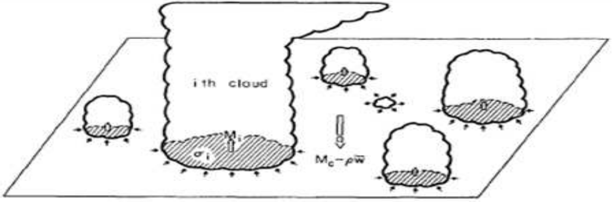


FIG. 1. A unit horizontal area at some level between cloud base and the highest cloud top. The taller clouds are shown penetrating this level and entraining environmental air. A cloud which has lost buoyancy is shown detraining cloud air into the environment.

Scale separation in both space and time between cloud-scale and the large-scale environment ⇒ Convection characterised by ensemble of convective plumes within some area of tolerably uniform forcing

# Mass flux approximation

- Individual plumes described in terms of mass flux, $M_i = \rho\sigma_i w_i$
- Effects of the plumes on their environment are very simple under the usual mass flux approximations of $w \ll w_i$ and $\sigma_i \ll 1$.
- For some variable $\chi$

$$\rho\overline{\chi' w'} = \sum_i M_i(\chi_i - \chi)$$

where the prime is a local deviation from the horizontal mean

# Mass flux parameterizations

- Many parameterizations have been written based on this picture and the mass flux approximation
- The differences are very interesting
- But here we focus on their similarities

# Mass flux parameterizations

- Many parameterizations have been written based on this picture and the mass flux approximation
- The differences are very interesting
- But here we focus on their similarities ⇒
  - If I don't need a decision about the details of the parameterization then I don't make it
  - If I do need a decision then I do the simplest possible thing, even if it is not realistic
    (i.e., no mesoscale circulations, no downdrafts and no ice)
  - Often this means copying Arakawa and Schubert, AS74

# Spectral Approach

Group the plumes together into types defined by a labelling parameter $\lambda$

- In AS74 this is the fractional entrainment rate, but could be anything
- e.g. the cloud top height $\widehat{z}$ is sometimes also used
- Generalization to multiple spectral parameters is trivial

# Bulk Approach: The Basic Idea

- The plumes do not interact directly, only with their environment

$\Rightarrow$ If the plume equations are almost linear in mass flux then a summation over plumes will recover equations with the same form

- So the ensemble of plumes can be represented as a single equivalent "bulk" plume

# Bulk Approach: The Basic Idea

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Aim of this talk is to discuss the words if and almost in the above

# More specifically

We will need to use the mass-flux-weighting operation (Yanai *et al.* 1973)

$$\chi_B = \frac{\sum_i M_i \chi_i}{\sum_i M_i}$$

$\chi_B$ is the bulk value of $\chi$ produced from an average of the $\chi_i$ for each individual plume

# Outline

1 Basic features of mass flux schemes

2 Single plumes
- Plume equations
- Detrainment at the plume top

3 Construction of equivalent bulk plume

4 Closure

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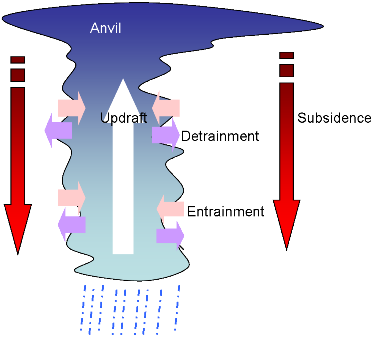
# Plumes: what are they?

I A caricature of an individual cumulus cloud
**or?**

II A description of a sub-cloud element, each cloud being composed of many such elements



# Plume equations

For an entraining/detraining plume

$$\frac{\partial \rho \sigma_i}{\partial t} = E_i - D_i - \frac{\partial M_i}{\partial z}$$

$$\frac{\partial \rho \sigma_i s_i}{\partial t} = E_i s - D_i s_i - \frac{\partial M_i s_i}{\partial z} + L \rho c_i + \rho Q_{\mathrm{R}i}$$

$$\frac{\partial \rho \sigma_i q_i}{\partial t} = E_i q - D_i q_i - \frac{\partial M_i q_i}{\partial z} - \rho c_i$$

$$\frac{\partial \rho \sigma_i l_i}{\partial t} = -D_i l_i - \frac{\partial M_i l_i}{\partial z} + \rho c_i - R_i$$

- $s = c_p T + gz$ is the dry static energy
- $Q_{\mathrm{R}}$ is the radiative heating rate
- $R$ is the rate of conversion of liquid water to precipitation
- $c$ is the rate of condensation.

# Using the plume equations

- Average over the plume lifetime to get rid of $\partial/\partial t$
- Various plume models differ in specification of entrainment/detrainment and microphysics
- Integrate from cloud base up to terminating level where the in-cloud buoyancy vanishes

# Neutral buoyancy level of a plume

- Occurs when the in-plume virtual temperature equals that of the environment
- Applying this condition, the values of the detraining variables are

$$l_i = \widehat{l}$$

$$s_i = \widehat{s} = s - \frac{L\epsilon}{1 + \gamma\epsilon\delta}\left(\delta(q^* - q) - \widehat{l}\right)$$

$$q_i = \widehat{q}^* = q^* - \frac{\gamma\epsilon}{1 + \gamma\epsilon\delta}\left(\delta(q^* - q) - \widehat{l}\right)$$

where

$$\epsilon = \frac{c_p T}{L} \quad ; \quad \delta{=}0.608 \quad ; \quad \gamma = \frac{L}{c_p} \left.\frac{\partial q^*}{\partial T}\right|_p$$

# Outline


1 Basic features of mass flux schemes

2 Single plumes

3 Construction of equivalent bulk plume
- Combining the plumes
- Detrainment ansatz

4 Closure

5 Conclusions

# Effect on the environment

Taking a mass-flux weighted average,

$$\rho\overline{\chi' w'} = M\left(\chi_B - \chi\right)$$

where

$$M = \sum_i M_i$$

Recall that the aim is for the equations to take the same form as the individual plume equations but now using bulk variables like $M$ and $\chi_B$

# Construction of equivalent bulk plume I

Now look at the averaged plume equations

$$E - D - \frac{\partial M}{\partial z} = 0$$

$$Es - \sum_i D_i s_i - \frac{\partial M s_B}{\partial z} + L\rho c + \rho Q_R = 0$$

$$Eq - \sum_i D_i q_i - \frac{\partial M q_B}{\partial z} - \rho c = 0$$

$$-\sum_i D_i l_i - \frac{\partial M l_B}{\partial z} + \rho c - R = 0$$

The same bulk variables feature here

# Construction of equivalent bulk plume II

$$E - D - \frac{\partial M}{\partial z} = 0$$

$$E s - \sum_i D_i s_i - \frac{\partial M s_B}{\partial z} + L\rho c + \rho Q_R = 0$$

$$E q - \sum_i D_i q_i - \frac{\partial M q_B}{\partial z} - \rho c = 0$$

$$- \sum_i D_i l_i - \frac{\partial M l_B}{\partial z} + \rho c - R = 0$$

where

$$E = \sum_i E_i \quad ; \quad D = \sum_i D_i$$

are the total entrainment and detrainment rates from all plumes present at that level

# The trade-off for a bulk scheme

- $E$ and $D$ encapsulate both the entrainment/detrainment process for an individual cloud and the spectral distribution of cloud types
- Is it better to set $E$ or to set $E_i$ and the distribution?
- Observational data about single clouds gives us information on $E_i$
- CRM/LES data can be analysed either way, but is more often done in terms of $E$ and $D$

# Construction of equivalent bulk plume III

$$E - D - \frac{\partial M}{\partial z} = 0$$

$$Es - \sum_i D_i s_i - \frac{\partial M s_B}{\partial z} + L\rho c + \rho Q_R = 0$$

$$Eq - \sum_i D_i q_i - \frac{\partial M q_B}{\partial z} - \rho c = 0$$

$$- \sum_i D_i l_i - \frac{\partial M l_B}{\partial z} + \rho c - R = 0$$

where

$$Q_R(s_B, q_B, l_B, \ldots) = \sum_i Q_{Ri}(s_i, q_i, l_i, \ldots)$$

is something for the cloud-radiation experts to be conscious about

# Construction of equivalent bulk plume IV

$$E - D - \frac{\partial M}{\partial z} = 0$$

$$Es - \sum_i D_i s_i - \frac{\partial M s_B}{\partial z} + L\rho c + \rho Q_R = 0$$

$$Eq - \sum_i D_i q_i - \frac{\partial M q_B}{\partial z} - \rho c = 0$$

$$-\sum_i D_i l_i - \frac{\partial M l_B}{\partial z} + \rho c - R = 0$$

where

$$c(s_B, q_B, l_B, \ldots) = \sum_i c_i(s_i, q_i, l_i, \ldots)$$

$$R(s_B, q_B, l_B, \ldots) = \sum_i R_i(s_i, q_i, l_i, \ldots)$$

is something for the microphysics experts to be conscious about

# Microphysics in AS74

In Arakawa and Schubert 1974, the rain rate is

$$R_i = C_0 M_i l_i$$

where $C_0$ is a constant. Hence,

$$R = C_0 M l_B$$

- If $C_0$ were to depend on the plume type then we couldn't write $R$ as a function of the bulk quantities but would need to know how $l_B$ is partitioned across the spectrum
  $\Rightarrow$ A bulk scheme is committed to crude microphysics
- But microphysics in any mass-flux parameterization has issues anyway

# Construction of equivalent bulk plume V

$$E - D - \frac{\partial M}{\partial z} = 0$$

$$Es - \sum_i D_i s_i - \frac{\partial M s_B}{\partial z} + L\rho c + \rho Q_R = 0$$

$$Eq - \sum_i D_i q_i - \frac{\partial M q_B}{\partial z} - \rho c = 0$$

$$-\sum_i D_i l_i - \frac{\partial M l_B}{\partial z} + \rho c - R = 0$$

How can we handle these terms?

(a) Below the plume tops?

(b) At the plume tops?

# (a) Below the plume tops

One option is to consider all the constitutent plumes to be *entraining-only* (except for the detrainment at cloud top)

- If $D_i = 0$ then $\sum D_i \chi_i = 0$ and the problem goes away!
- This is exactly what Arakawa and Schubert did

# (a) Below the plume tops

Alternatively, if we want to retain entraining/detraining plumes then we have

$$\sum_i D_i \chi_i \equiv D_\chi \chi_B$$

$$D_\chi = M \frac{\sum_i D_i \chi_i}{\sum_i M_i \chi_i}$$

- The detrainment rate is $\neq \sum D_i$
  i.e., it is different from the $D$ that we see in the vertical mass flux profile equation
- and it is different for each in-plume variable

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- and it is different for each in-plume variable

⇒ A bulk parameterization can only be equivalent to a spectral parameterization of entraining plumes

# (b) At the plume tops

- As well as the possibility of detrainment during ascent, there are the contributions to $\sum_i D_i \chi_i$ from plumes the that have reach neutral buoyancy at the current level
- For such plumes, we can simply use our earlier formulae for $s_i$ etc. coming from the neutral buoyancy condition.

$$Es - D\widehat{s} - \frac{\partial M s_B}{\partial z} = 0$$

$$Eq - D\widehat{q}^* - \frac{\partial M q_B}{\partial z} = 0$$

$$-D\widehat{l} - \frac{\partial M l_B}{\partial z} = 0$$

so now these equations use the same $D$ as in the mass flux profile equation

- But what about $\widehat{s}$, $\widehat{q}$, $\widehat{l}$?

# (b) At the plume tops

Recall:

$$l_i = \widehat{l}$$

$$s_i = \widehat{s} = s - \frac{L\epsilon}{1+\gamma\epsilon\delta}\left(\delta(q^* - q) - \widehat{l}\right)$$

$$q_i = \widehat{q}^* = q^* - \frac{\gamma\epsilon}{1+\gamma\epsilon\delta}\left(\delta(q^* - q) - \widehat{l}\right)$$

- Everything on the RHS is known in the bulk system, apart from $\widehat{l}$
- $\widehat{l}(z)$ can only be calculated by integrating the plume equations for a plume that detrains at $\widehat{z}_i = z$

# Key bulk assumption

At the heart of bulk models is an ansatz that the liquid water detrained *from each individual plume* is given by the *bulk* value

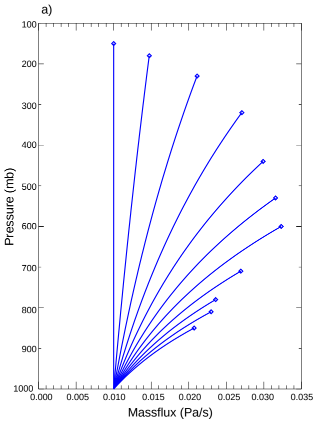
$$l_i = l_B$$

Yanai *et al* (1973): "a gross assumption but needed to close the set of equations"

# Example

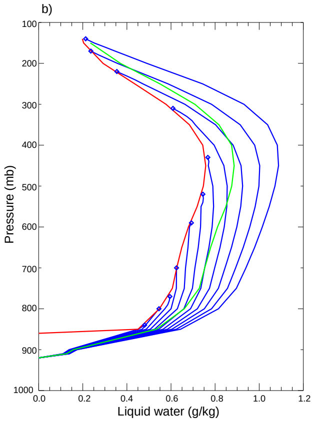
Calculate entraining plumes for Jordan's sounding

- each with an arbitrary mass flux at cloud base
- a range of entrainment rates



# Example

- Blue: in-plume liquid water, $l_i$
- Red: detrained liquid water, $\widehat{l}$
- Green: bulk liquid water, $l_B$
- Liquid water is detrained throughout profile
- and is over-estimated (the detraining plumes have lower $l_i$)



# Neutral buoyancy level

Recall again:

$$l_i = \widehat{l}$$

$$s_i = \widehat{s} = s - \frac{L\epsilon}{1 + \gamma\epsilon\delta}\left(\delta(q^* - q) - \widehat{l}\right)$$

$$q_i = \widehat{q}^* = q^* - \frac{\gamma\epsilon}{1 + \gamma\epsilon\delta}\left(\delta(q^* - q) - \widehat{l}\right)$$

- Given that $\widehat{l}$ is not known, Yanai *et al.* (1973) neglected the virtual contributions
- Nordeng (1994) suggests some practical sensitivities to this

# Other transports

- Contributions to $\sum_i D_i \chi_i$ from detrainment at plume top can be simplified for $s$, $q$ and $l$ from the neutral-buoyancy condition (with $l$ ansatz)
- But no simplification occurs for other transports (e.g., tracer concentrations, momentum)
- Needs further ansatze, $\chi_i = \chi_B$
- Or decompose bulk plume into spectrum of plumes (Lawrence and Rasch 2005)

# Outline


1 Basic features of mass flux schemes

2 Single plumes

3 Construction of equivalent bulk plume

4 Closure
- Normalization transformations
- Closure Equivalence
- The quasi-equilibrium closure of Arakawa and Schubert
- Equivalent closures for a bulk system?
- Stochastic aspects of convection

5 Conclusions

# Normalization transform: Definition

- If $\mathcal{M}(z_{\rm base}, \lambda)$ denotes cloud-base mass flux for plume type $\lambda$ then we can define a *normalization transform* $\mathcal{T}$ as a positive rescaling

$$\mathcal{M}(z_{\rm base}, \lambda) \rightarrow \mathcal{M}(z_{\rm base}, \lambda)\mathcal{T}(\lambda)$$

- A *global transform* is one where $\mathcal{T}$ is the same for all $\lambda$
- Evolution of the plume-ensemble between any two times can be represented as a normalization transformation
$\Rightarrow$ different transformation properties under $\mathcal{T}$ define characteristic timescales of evolution of the ensemble

# Transform properties

1. *Normalization-invariant* variables are unaffected by any $\mathcal{T}$. Evolve only in response to changes in the large-scale, with timescale $\tau_{\rm LS}$
2. *Globally-invariant* variables are unaffected by a global $\mathcal{T}$. Evolve in response to changes in spectral plume distribution but not the overall strength of convection. Timescale $\tau_{\rm spec}$
3. *Normalization-rescaled* variables $V$ depend extensively on one plume type only $V \rightarrow V\mathcal{T}(\lambda)$, with timescale $\tau_\lambda$
4. *Globally-rescaled* variables depend extensively on a global $\mathcal{T}$. Evolve in response to overall strength of convection and are sensitive to the spectrum, with timescale $\tau_{\rm adj}$

# Closure Equivalence

- Normalization transforms are very relevant for closure because starting from some guess about $\mathcal{M}(z_{\rm base}, \lambda)$ closure means finding a special normalization transform
- For equivalence of bulk and spectral methods, require that the closure transform for a bulk model respects the same physical constraints that were specified to formulate the closure transform of the spectral model

# The AS74 closure

Evolution of the kinetic energy $\mathcal{K}$ for each plume type

$$\frac{\partial \mathcal{K}(\lambda)}{\partial t} = A(\lambda)\mathcal{M}(z_{\mathrm{base}}, \lambda) - \mathcal{D}(\lambda)$$

where $\mathcal{D}$ is the dissipation and $A$ is the cloud work function,

$$A(\lambda) \equiv \int_{z_{\mathrm{base}}}^{\widehat{z}(\lambda)} \frac{g}{T} \frac{\mathcal{M}(z, \lambda)}{\mathcal{M}(z_{\mathrm{base}}, \lambda)} \left(T_{\mathrm{vp}}(\lambda) - T_v\right) dz$$

Take a time derivative produces

$$\frac{dA}{dt} = \left.\frac{dA}{dt}\right|_{\mathrm{LS}} + \left.\frac{dA}{dt}\right|_{\mathrm{C}} \equiv \dot{A}_{\mathrm{LS}} + \dot{A}_{\mathrm{C}}$$

where LS and C are large-scale and cloud contributions.

# In terms of normalization transforms...

- $A(\lambda)$ is normalization invariant
- $dA/dt$ has contributions
  - which are normalization invariant ($\dot{A}_{\rm LS}$)
  - which are globally rescaled ($\dot{A}_{\rm C}$)
- Physical constraint is that $\tau_{\rm LS} \gg \tau_{\rm adj}$, which defines the AS74 quasi-equilibrium closure, $dA/dt \approx 0$

# Sum over plumes

Applying bulk averaging

$$\frac{\partial K}{\partial t} = A_B M(z_{\mathrm{base}}) - \mathrm{DIS}$$

where

$$K = \int \mathcal{K} d\lambda \quad ; \quad \mathrm{DIS} = \int \mathcal{D} d\lambda$$

$$A_B \equiv \frac{\int \mathcal{M}(z_{\mathrm{base}}, \lambda) A(\lambda) d\lambda}{M(z_{\mathrm{base}})} = \int_{z_{\mathrm{base}}}^{\widehat{z}(0)} \frac{g}{c_p T} \frac{M}{M(z_{\mathrm{base}})} \left( T_{vB} - T_v \right) dz$$

is the bulk equivalent of $A(\lambda)$.
Everything is defined in terms of bulk variables

# Closure based on $A_B$?

$$\frac{dA_B}{dt} = \int \frac{\mathcal{M}(z_{\rm base}, \lambda)}{M(z_{\rm base})} \frac{dA(\lambda)}{dt} d\lambda + \cdots$$

$$= \frac{\int \mathcal{M}(z_{\rm base}, \lambda)(\dot{A}_{\rm LS}(\lambda) + \dot{A}_{\rm C}(\lambda))d\lambda}{M(z_{\rm base})} + \cdots$$

- $\dot{A}_{\rm C}$ produces a globally-rescaled contribution to $dA_B/dt$, with timescale $\tau_{\rm adj}$
- $\dot{A}_{\rm LS}(\lambda)$ is normalization invariant but produces a globally-invariant contribution to $dA_B/dt$, with timescale $\tau_{\rm spec}$

- An equilibrium closure applied requires a different timescale separation, $\tau_{\rm spec} \gg \tau_{\rm adj}$
- This is a different physical constraint from AS74 quasi-equilibrium

# CAPE closure?

$$\mathrm{CAPE} = A(0) = \int_{z_{\mathrm{base}}}^{\widehat{z}(0)} \frac{g}{c_p T} \left( T_{\mathrm{vp}}(0) - T_v \right) dz$$

- CAPE is a special case of $A(\lambda)$ so its quasi-equilibrium is based on $\tau_{\mathrm{LS}} \gg \tau_{\mathrm{adj}}$
- Note that the "cloud" terms in $d\mathrm{CAPE}/dt$ include detrainment contributions like

$$\dot{A}_{\mathrm{C}}(0) = \frac{gL}{c_p} \int_{z_{\mathrm{LCL}}}^{\widehat{z}(0)} dz \frac{1}{\rho T} D(z) \left[ 1 - (1+\delta)\epsilon \right] \widehat{l} + \cdots$$

- This requires $\widehat{l}(z)$

# CAPE closure

- We can close a spectral or a bulk system using CAPE
- For a spectral system this is not sufficient (need spectral distribution)
- For a bulk system, we have to invoke the Yanai *et al* (1973) ansatz again
- i.e., this has an impact on closure too. Practical impact is probably small?

(Same issue for dilute CAPE, $T_{\rm vp}(0) \rightarrow T_{\rm vB}$ in the definition)

# Stochastic Effects

We assumed that there are enough plumes to be treated statistically, such as might be found within "a region of space-time large enough to contain an ensemble of cumulus clouds but small enough to cover only a fraction of a large-scale disturbance" (AS74)

But:

- Convective instability is released in discrete events
- The number of clouds in a GCM grid-box is not large enough to produce a steady response to a steady forcing

# Convective variability

- Convection on the grid-scale is unpredictable, but randomly sampled from a pdf dictated by the large scale
- To describe the variability arising from fluctuations about equilibrium, we must consider the partitioning of the total mass flux $M$ into individual clouds, $M_i$

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# General remarks

- A bulk model of plumes does not follow immediately from averaging over bulk plumes, but requires some extra assumptions
- Entrainment is a big issue (as always!)
  - Spectral model has simple $E_i$, $D_i$ that become $E_\chi(z)$ and $D_\chi(z)$ that are complicated functions of the environment. This makes a spectral model natural?
  - Bulk model deals with $E(z)$ and $D(z)$ for which there is arguably better information? No need to consider explicitly the spectrum of plumes.

For some people, may be the comparison stops here!?

# But worth being aware that...

In bulk systems...

- Cloud-radiation interactions have to be estimated using bulk variables
- Microphysics has to be calculated using bulk variables
  - This implies very simple, linearized microphysics
  - But microphysics is problematic for mass flux methods anyway, owing to non-separation of $\sigma_i$ and $w_i$

# Also...

- A bulk plume is an entraining/detraining plume that is equivalent to an ensemble of entraining plumes
- A bulk system needs a "gross assumption" that $\widehat{l}{=}l_B$
  - Not often recognized by later authors, but relevant when detrained condensate is used as a source term for prognostic respresentations of stratiform cloud (for example)
  - Detrained condensate from a bulk scheme is an overestimate that is not intended to be reliable
- Natural bulk analogue for AS74 quasi-equilibrium requires a different timescale separation
- A closure based on CAPE is fine (in terms of equivalence) though note that $\widehat{l} = l_B$ crops up again

# Conclusions

- A spectral parameterization of multiple plumes types needs many more computations of course
- It should not be given up lightly, but it does have its limitations