Spectral and Bulk Mass-Flux Convective Parameterizations

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Basic features of mass flux schemes

Single plumes

Construction of equivalent bulk plume

Closure

Conclusions
Outline

1. Basic features of mass flux schemes
   - Some mass flux basics
   - Basic bulk idea

2. Single plumes

3. Construction of equivalent bulk plume

4. Closure

5. Conclusions
The Basic Picture

The Arakawa and Schubert (1974) picture

Scale separation in both space and time between cloud-scale and the large-scale environment ⇒ Convection characterised by ensemble of convective plumes within some area of tolerably uniform forcing

Fig. 1. A unit horizontal area at some level between cloud base and the highest cloud top. The taller clouds are shown penetrating this level and entraining environmental air. A cloud which has lost buoyancy is shown detraining cloud air into the environment.
Mass flux approximation

- Individual plumes described in terms of mass flux, \( M_i = \rho \sigma_i w_i \)
- Effects of the plumes on their environment are very simple under the usual mass flux approximations of \( w \ll w_i \) and \( \sigma_i \ll 1 \).
- For some variable \( \chi \)

\[
\rho \chi' w' = \sum_i M_i (\chi_i - \chi)
\]

where the prime is a local deviation from the horizontal mean.
Mass flux parameterizations

- Many parameterizations have been written based on this picture and the mass flux approximation
- The differences are very interesting
- But here we focus on their similarities
Many parameterizations have been written based on this picture and the mass flux approximation.

The differences are very interesting.

But here we focus on their similarities ⇒

- If I don’t need a decision about the details of the parameterization then I don’t make it.
- If I do need a decision then I do the simplest possible thing, even if it is not realistic (i.e., no mesoscale circulations, no downdrafts and no ice).
- Often this means copying Arakawa and Schubert, AS74.
Spectral Approach

Group the plumes together into types defined by a labelling parameter $\lambda$
- In AS74 this is the fractional entrainment rate, but could be anything
- e.g. the cloud top height $\hat{z}$ is sometimes also used
- Generalization to multiple spectral parameters is trivial
The plumes do not interact directly, only with their environment

⇒ If the plume equations are almost linear in mass flux then a summation over plumes will recover equations with the same form

So the ensemble of plumes can be represented as a single equivalent “bulk” plume
Bulk Approach: The Basic Idea

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⇒ If the plume equations are almost linear in mass flux then a summation over plumes will recover equations with the same form

- So the ensemble of plumes can be represented as a single equivalent “bulk” plume

Aim of this talk is to discuss the words if and almost in the above
More specifically

We will need to use the mass-flux-weighting operation (Yanai et al. 1973)

\[ \chi_B = \frac{\sum_i M_i \chi_i}{\sum_i M_i} \]

\( \chi_B \) is the bulk value of \( \chi \) produced from an average of the \( \chi_i \) for each individual plume
Outline

1. Basic features of mass flux schemes

2. Single plumes
   - Plume equations
   - Detrainment at the plume top

3. Construction of equivalent bulk plume

4. Closure

5. Conclusions
Plumes: what are they?

1. A caricature of an individual cumulus cloud or?
2. A description of a sub-cloud element, each cloud being composed of many such elements
Plume equations

For an entraining/detraining plume

\[
\frac{\partial \rho \sigma_i}{\partial t} = E_i - D_i - \frac{\partial M_i}{\partial z}
\]

\[
\frac{\partial \rho \sigma_i s_i}{\partial t} = E_i s - D_i s_i - \frac{\partial M_i s_i}{\partial z} + L \rho c_i + \rho Q_{R_i}
\]

\[
\frac{\partial \rho \sigma_i q_i}{\partial t} = E_i q - D_i q_i - \frac{\partial M_i q_i}{\partial z} - \rho c_i
\]

\[
\frac{\partial \rho \sigma_i l_i}{\partial t} = -D_i l_i - \frac{\partial M_i l_i}{\partial z} + \rho c_i - R_i
\]

- \(s = c_p T + gz\) is the dry static energy
- \(Q_R\) is the radiative heating rate
- \(R\) is the rate of conversion of liquid water to precipitation
- \(c\) is the rate of condensation.
Using the plume equations

- Average over the plume lifetime to get rid of $\partial/\partial t$
- Various plume models differ in specification of entrainment/detainment and microphysics
- Integrate from cloud base up to terminating level where the in-cloud buoyancy vanishes
Neutral buoyancy level of a plume

- Occurs when the in-plume virtual temperature equals that of the environment
- Applying this condition, the values of the detraining variables are

\[ l_i = \hat{l} \]

\[ s_i = \hat{s} = s - \frac{L\epsilon}{1 + \gamma\epsilon\delta} \left( \delta(q^* - q) - \hat{l} \right) \]

\[ q_i = \hat{q}^* = q^* - \frac{\gamma\epsilon}{1 + \gamma\epsilon\delta} \left( \delta(q^* - q) - \hat{l} \right) \]

where

\[ \epsilon = \frac{c_p T}{L} \]

\[ \delta = 0.608 \]

\[ \gamma = \frac{L}{c_p} \left. \frac{\partial q^*}{\partial T} \right|_p \]
Outline

1. Basic features of mass flux schemes
2. Single plumes
3. Construction of equivalent bulk plume
   - Combining the plumes
   - Detrainment ansatz
4. Closure
5. Conclusions
Effect on the environment

Taking a mass-flux weighted average,

\[ \rho \bar{w} \bar{\chi}' = M (\chi_B - \chi) \]

where

\[ M = \sum_i M_i \]

Recall that the aim is for the equations to take the same form as the individual plume equations but now using bulk variables like \( M \) and \( \chi_B \).
Construction of equivalent bulk plume I

Now look at the averaged plume equations

\[ E - D - \frac{\partial M}{\partial z} = 0 \]

\[ Es - \sum_i D_i s_i - \frac{\partial M_{sB}}{\partial z} + L \rho c + \rho Q_R = 0 \]

\[ Eq - \sum_i D_i q_i - \frac{\partial M_{qB}}{\partial z} - \rho c = 0 \]

\[ - \sum_i D_i l_i - \frac{\partial M_{lB}}{\partial z} + \rho c - R = 0 \]

The same bulk variables feature here
Construction of equivalent bulk plume II

\[ E - D - \frac{\partial M}{\partial z} = 0 \]

\[ E_s - \sum_i D_i s_i - \frac{\partial M_{sB}}{\partial z} + L \rho c + \rho Q_R = 0 \]

\[ E_q - \sum_i D_i q_i - \frac{\partial M_{qB}}{\partial z} - \rho c = 0 \]

\[ - \sum_i D_i l_i - \frac{\partial M_{lB}}{\partial z} + \rho c - R = 0 \]

where

\[ E = \sum_i E_i \quad ; \quad D = \sum_i D_i \]

are the total entrainment and detrainment rates from all plumes present at that level.
The trade-off for a bulk scheme

- $E$ and $D$ encapsulate both the entrainment/detrainment process for an individual cloud and the spectral distribution of cloud types.
- Is it better to set $E$ or to set $E_i$ and the distribution?
- Observational data about single clouds gives us information on $E_i$.
- CRM/LES data can be analysed either way, but is more often done in terms of $E$ and $D$. 
Construction of equivalent bulk plume III

\[ E - D - \frac{\partial M}{\partial z} = 0 \]

\[ Es - \sum_i D_i s_i - \frac{\partial M_{sB}}{\partial z} + L\rho c + \rho Q_R = 0 \]

\[ Eq - \sum_i D_i q_i - \frac{\partial M_{qB}}{\partial z} - \rho c = 0 \]

\[ - \sum_i D_i l_i - \frac{\partial M_{lB}}{\partial z} + \rho c - R = 0 \]

where

\[ Q_R(s_B, q_B, l_B, \ldots) = \sum_i Q_{Ri}(s_i, q_i, l_i, \ldots) \]

is something for the cloud-radiation experts to be conscious about.
Construction of equivalent bulk plume IV

\[ E - D - \frac{\partial M}{\partial z} = 0 \]

\[ Es - \sum_i D_i s_i - \frac{\partial M s_B}{\partial z} + L \rho c + \rho Q_R = 0 \]

\[ Eq - \sum_i D_i q_i - \frac{\partial M q_B}{\partial z} - \rho c = 0 \]

\[ - \sum_i D_i l_i - \frac{\partial M l_B}{\partial z} + \rho c - R = 0 \]

where

\[ c(s_B, q_B, l_B, \ldots) = \sum_i c_i(s_i, q_i, l_i, \ldots) \]

\[ R(s_B, q_B, l_B, \ldots) = \sum_i R_i(s_i, q_i, l_i, \ldots) \]

is something for the microphysics experts to be conscious about
Microphysics in AS74

In Arakawa and Schubert 1974, the rain rate is

\[ R_i = C_0 M_i l_i \]

where \( C_0 \) is a constant. Hence,

\[ R = C_0 M l_B \]

- If \( C_0 \) were to depend on the plume type then we couldn’t write \( R \) as a function of the bulk quantities but would need to know how \( l_B \) is partitioned across the spectrum
  \[ \Rightarrow \text{A bulk scheme is committed to crude microphysics} \]
- But microphysics in any mass-flux parameterization has issues anyway
Construction of equivalent bulk plume V

\[ E - D - \frac{\partial M}{\partial z} = 0 \]

\[ Es - \sum_i D_i s_i - \frac{\partial M_{sB}}{\partial z} + L \rho c + \rho Q_R = 0 \]

\[ Eq - \sum_i D_i q_i - \frac{\partial M_{qB}}{\partial z} - \rho c = 0 \]

\[ -\sum_i D_i l_i - \frac{\partial M_{lB}}{\partial z} + \rho c - R = 0 \]

How can we handle these terms?

(a) Below the plume tops?

(b) At the plume tops?
(a) Below the plume tops

One option is to consider all the constituent plumes to be *entraining-only* (except for the detrainment at cloud top)

- If $D_i = 0$ then $\sum D_i \chi_i = 0$ and the problem goes away!
- This is exactly what Arakawa and Schubert did
(a) Below the plume tops

Alternatively, if we want to retain entraining/detraining plumes then we have

$$\sum_i D_i \chi_i \equiv D_{\chi} \chi_B$$

$$D_{\chi} = M \frac{\sum_i D_i \chi_i}{\sum_i M_i \chi_i}$$

- The detrainment rate is $\neq \sum D_i$
  i.e., it is different from the $D$ that we see in the vertical mass flux profile equation
- and it is different for each in-plume variable
(a) Below the plume tops

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- The detrainment rate is \( \neq \sum D_i \)
  - i.e., it is different from the \( D \) that we see in the vertical mass flux profile equation
- and it is different for each in-plume variable

\[ \Rightarrow \text{ A bulk parameterization can only be equivalent to a spectral parameterization of entraining plumes} \]
(b) At the plume tops

- As well as the possibility of detrainment during ascent, there are the contributions to $\sum_i D_i \chi_i$ from plumes that have reached neutral buoyancy at the current level.

- For such plumes, we can simply use our earlier formulae for $s_i$ etc. coming from the neutral buoyancy condition.

\[
Es - D\hat{s} - \frac{\partial M_{SB}}{\partial z} = 0
\]

\[
Eq - D\hat{q}^* - \frac{\partial M_{qB}}{\partial z} = 0
\]

\[
- D\hat{l} - \frac{\partial M_{lB}}{\partial z} = 0
\]

so now these equations use the same $D$ as in the mass flux profile equation.

- But what about $\hat{s}$, $\hat{q}$, $\hat{l}$?
(b) At the plume tops

Recall:

\[ l_i = \hat{l} \]

\[ s_i = \hat{s} = s - \frac{L\epsilon}{1 + \gamma\epsilon\delta} \left( \delta(q^* - q) - \hat{l} \right) \]

\[ q_i = \hat{q}^* = q^* - \frac{\gamma\epsilon}{1 + \gamma\epsilon\delta} \left( \delta(q^* - q) - \hat{l} \right) \]

- Everything on the RHS is known in the bulk system, apart from \( \hat{l} \)
- \( \hat{l}(z) \) can only be calculated by integrating the plume equations for a plume that detrains at \( \hat{z}_i = z \)
Key bulk assumption

At the heart of bulk models is an ansatz that the liquid water detrained from each individual plume is given by the bulk value

\[ l_i = l_B \]

Yanai et al (1973): “a gross assumption but needed to close the set of equations”
Calculate entraining plumes for Jordan’s sounding

- each with an arbitrary mass flux at cloud base
- a range of entrainment rates
Example

- **Blue**: in-plume liquid water, \( l_i \)
- **Red**: detrained liquid water, \( \hat{l} \)
- **Green**: bulk liquid water, \( l_B \)

Liquid water is detrained throughout profile and is over-estimated (the detraining plumes have lower \( l_i \))
Neutral buoyancy level

Recall again:

\[ l_i = \hat{l} \]

\[ s_i = \hat{s} = s - \frac{L \epsilon}{1 + \gamma \epsilon \delta} \left( \delta (q^* - q) - \hat{l} \right) \]

\[ q_i = \hat{q}^* = q^* - \frac{\gamma \epsilon}{1 + \gamma \epsilon \delta} \left( \delta (q^* - q) - \hat{l} \right) \]

- Given that \( \hat{l} \) is not known, Yanai et al. (1973) neglected the virtual contributions.
- Nordeng (1994) suggests some practical sensitivities to this.
Other transports

- Contributions to $\sum_i D_i \chi_i$ from detrainment at plume top can be simplified for $s$, $q$ and $l$ from the neutral-buoyancy condition (with $l$ ansatz)
- But no simplification occurs for other transports (e.g., tracer concentrations, momentum)
- Needs further ansatze, $\chi_i = \chi_B$
- Or decompose bulk plume into spectrum of plumes (Lawrence and Rasch 2005)
Outline

1. Basic features of mass flux schemes
2. Single plumes
3. Construction of equivalent bulk plume
4. Closure
   - Normalization transformations
   - Closure Equivalence
   - The quasi-equilibrium closure of Arakawa and Schubert
   - Equivalent closures for a bulk system?
   - Stochastic aspects of convection
5. Conclusions
Normalization transform: Definition

- If \( M(z_{\text{base}}, \lambda) \) denotes cloud-base mass flux for plume type \( \lambda \) then we can define a *normalization transform* \( T \) as a positive rescaling

\[
M(z_{\text{base}}, \lambda) \rightarrow M(z_{\text{base}}, \lambda)T(\lambda)
\]

- A *global transform* is one where \( T \) is the same for all \( \lambda \)
- Evolution of the plume-ensemble between any two times can be represented as a normalization transformation

\[ \Rightarrow \text{different transformation properties under } T \text{ define characteristic timescales of evolution of the ensemble} \]
Transform properties

1. **Normalization-invariant** variables are unaffected by any $T$. Evolve only in response to changes in the large-scale, with timescale $\tau_{LS}$

2. **Globally-invariant** variables are unaffected by a global $T$. Evolve in response to changes in spectral plume distribution but not the overall strength of convection. Timescale $\tau_{\text{spec}}$

3. **Normalization-rescaled** variables $V$ depend extensively on one plume type only $V \rightarrow V T(\lambda)$, with timescale $\tau_{\lambda}$

4. **Globally-rescaled** variables depend extensively on a global $T$. Evolve in response to overall strength of convection and are sensitive to the spectrum, with timescale $\tau_{\text{adj}}$
Normalization transforms are very relevant for closure because starting from some guess about $M(z_{\text{base}}, \lambda)$ closure means finding a special normalization transform.

For equivalence of bulk and spectral methods, require that the closure transform for a bulk model respects the same physical constraints that were specified to formulate the closure transform of the spectral model.
The AS74 closure

Evolution of the kinetic energy $K$ for each plume type

$$\frac{\partial K(\lambda)}{\partial t} = A(\lambda)M(z_{\text{base}}, \lambda) - D(\lambda)$$

where $D$ is the dissipation and $A$ is the cloud work function,

$$A(\lambda) \equiv \hat{z}(\lambda) \int_{z_{\text{base}}}^{\hat{z}(\lambda)} \frac{\mathcal{M}(z, \lambda)}{\mathcal{T}} \mathcal{M}(z_{\text{base}}, \lambda) \left( T_{vp}(\lambda) - T_v \right) dz$$

Take a time derivative produces

$$\frac{dA}{dt} = \left. \frac{dA}{dt} \right|_{\text{LS}} + \left. \frac{dA}{dt} \right|_{\text{C}} \equiv \dot{A}_{\text{LS}} + \dot{A}_{\text{C}}$$

where LS and C are large-scale and cloud contributions.
In terms of normalization transforms...

- $A(\lambda)$ is normalization invariant
- $dA/dt$ has contributions
  - which are normalization invariant ($\dot{A}_{LS}$)
  - which are globally rescaled ($\dot{A}_C$)
- Physical constraint is that $\tau_{LS} \gg \tau_{adj}$, which defines the AS74 quasi-equilibrium closure, $dA/dt \approx 0$
Sum over plumes

Applying bulk averaging

\[ \frac{\partial K}{\partial t} = A_B M(z_{\text{base}}) - \text{DIS} \]

where

\[ K = \int K d\lambda \quad ; \quad \text{DIS} = \int D d\lambda \]

\[ A_B \equiv \int \frac{M(z_{\text{base}}, \lambda) A(\lambda) d\lambda}{M(z_{\text{base}})} = \int \frac{\hat{z}(0)}{c_p T} \frac{M}{M(z_{\text{base}})} (T_{vB} - T_v) dz \]

is the bulk equivalent of \( A(\lambda) \).

Everything is defined in terms of bulk variables.
Closure based on $A_B$?

\[
\frac{dA_B}{dt} = \int \frac{M(z_{base}, \lambda)}{M(z_{base})} \frac{dA(\lambda)}{dt} d\lambda + \cdots
\]

\[
= \int \frac{M(z_{base}, \lambda)(\dot{A}_{LS}(\lambda) + \dot{A}_C(\lambda))}{M(z_{base})} d\lambda + \cdots
\]

- $\dot{A}_C$ produces a globally-rescaled contribution to $dA_B/dt$, with timescale $\tau_{adj}$
- $\dot{A}_{LS}(\lambda)$ is normalization invariant but produces a globally-invariant contribution to $dA_B/dt$, with timescale $\tau_{spec}$

- An equilibrium closure applied requires a different timescale separation, $\tau_{spec} \gg \tau_{adj}$
- This is a different physical constraint from AS74 quasi-equilibrium
CAPE closure?

\[
\text{CAPE} = A(0) = \int_{z_{\text{base}}}^{\hat{z}(0)} \frac{g}{c_p T} (T_{vp}(0) - T_v) \, dz
\]

- CAPE is a special case of \( A(\lambda) \) so its quasi-equilibrium is based on \( \tau_{LS} \gg \tau_{\text{adj}} \)
- Note that the “cloud” terms in \( d\text{CAPE}/dt \) include detrainment contributions like

\[
\dot{A}_C(0) = \frac{gL}{c_p} \int_{z_{\text{LCL}}}^{\hat{z}(0)} dz \frac{1}{\rho T} D(z) [1 - (1 + \delta)\epsilon] \hat{l} + \cdots
\]

- This requires \( \hat{l}(z) \)
CAPE closure

- We can close a spectral or a bulk system using CAPE
- For a spectral system this is not sufficient (need spectral distribution)
- For a bulk system, we have to invoke the Yanai et al. (1973) ansatz again
- i.e., this has an impact on closure too. Practical impact is probably small?

(Same issue for dilute CAPE, $T_{vp}(0) \rightarrow T_{vB}$ in the definition)
Stochastic Effects

We assumed that there are enough plumes to be treated statistically, such as might be found within “a region of space-time large enough to contain an ensemble of cumulus clouds but small enough to cover only a fraction of a large-scale disturbance” (AS74)

But:

- Convective instability is released in discrete events
- The number of clouds in a GCM grid-box is not large enough to produce a steady response to a steady forcing
Convective variability

- Convection on the grid-scale is unpredictable, but randomly sampled from a pdf dictated by the large scale.
- To describe the variability arising from fluctuations about equilibrium, we must consider the partitioning of the total mass flux $M$ into individual clouds, $M_i$. 
Outline

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General remarks

- A bulk model of plumes does not follow immediately from averaging over bulk plumes, but requires some extra assumptions.
- Entrainment is a big issue (as always!)
  - Spectral model has simple $E_i$, $D_i$ that become $E_\chi(z)$ and $D_\chi(z)$ that are complicated functions of the environment. This makes a spectral model natural?
  - Bulk model deals with $E(z)$ and $D(z)$ for which there is arguably better information? No need to consider explicitly the spectrum of plumes.

For some people, may be the comparison stops here!? 
But worth being aware that...

In bulk systems...

- Cloud-radiation interactions have to be estimated using bulk variables
- Microphysics has to be calculated using bulk variables
  - This implies very simple, linearized microphysics
  - But microphysics is problematic for mass flux methods anyway, owing to non-separation of $\sigma_i$ and $w_i$
Also...

- A bulk plume is an entraining/detraining plume that is equivalent to an ensemble of entraining plumes.
- A bulk system needs a “gross assumption” that $\hat{l} = l_B$
  - Not often recognized by later authors, but relevant when detrained condensate is used as a source term for prognostic representations of stratiform cloud (for example).
  - Detrained condensate from a bulk scheme is an overestimate that is not intended to be reliable.
- Natural bulk analogue for AS74 quasi-equilibrium requires a different timescale separation.
- A closure based on CAPE is fine (in terms of equivalence) though note that $\hat{l} = l_B$ crops up again.
Conclusions

- A spectral parameterization of multiple plumes types needs many more computations of course
- It should not be given up lightly, but it does have its limitations