Boundary Layer Parameterization

Bob Plant

With thanks to: R. Beare, S. Belcher, I. Boutle, O. Coceal, A. Grant, D. McNamara

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The University of Reading

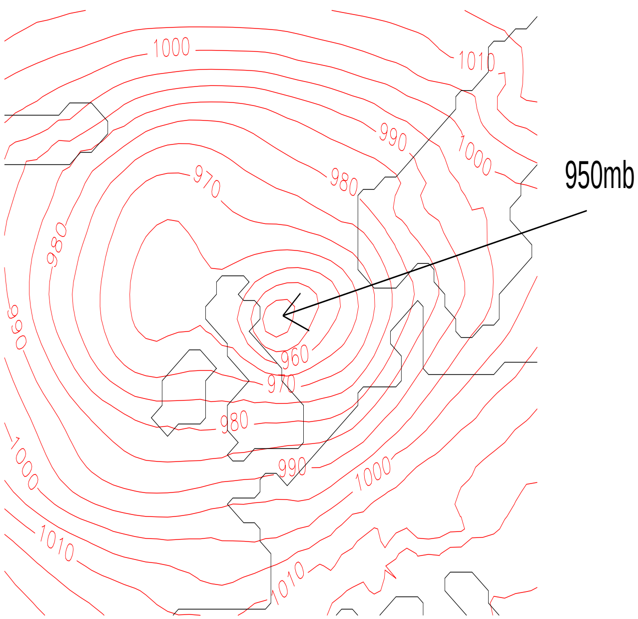
Outline

- Motivation
- Some background on turbulence
- Explicit turbulence simulations
- Organization of the problem
- Surface layer parameterization
- K theory
- The boundary layer grey zone

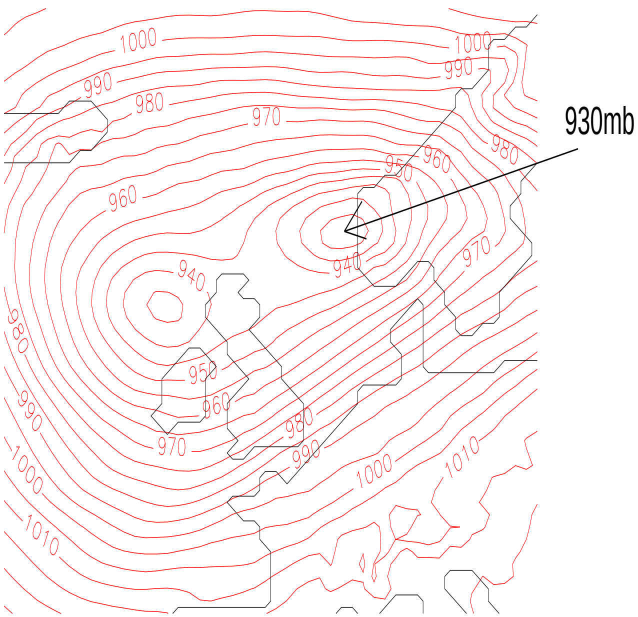
Motivation

Effects of the boundary layer

Control simulation, T+60

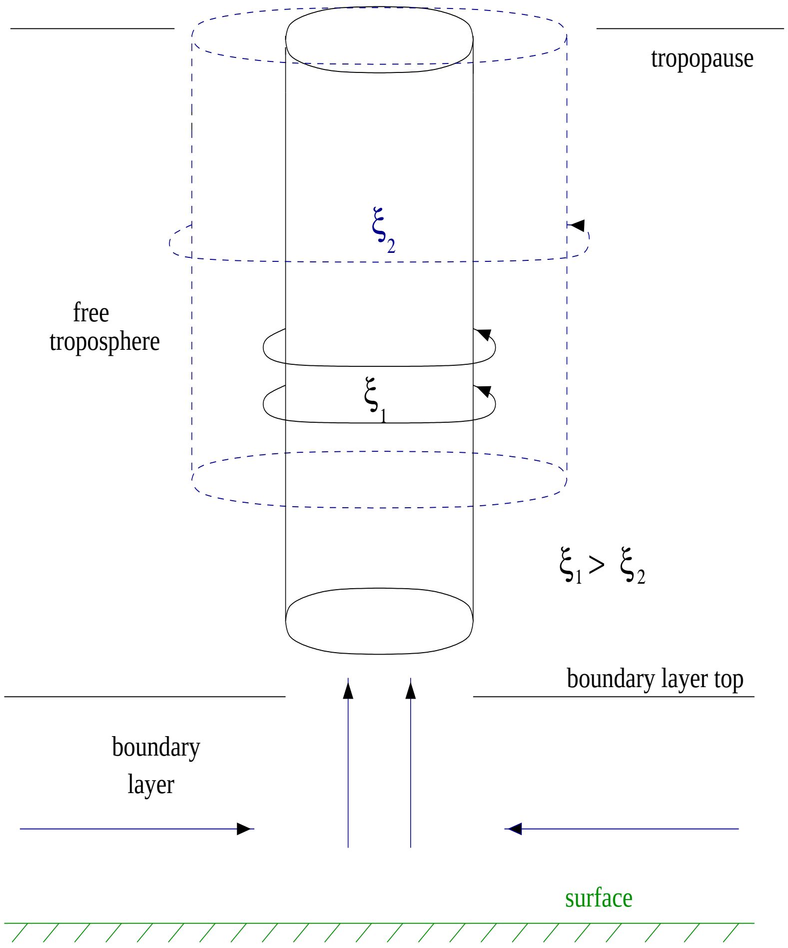


Simulation with no boundary layer turbulence, T+60.



Simulations with (left) and without (right) boundary layer, of storm of 30/10/00

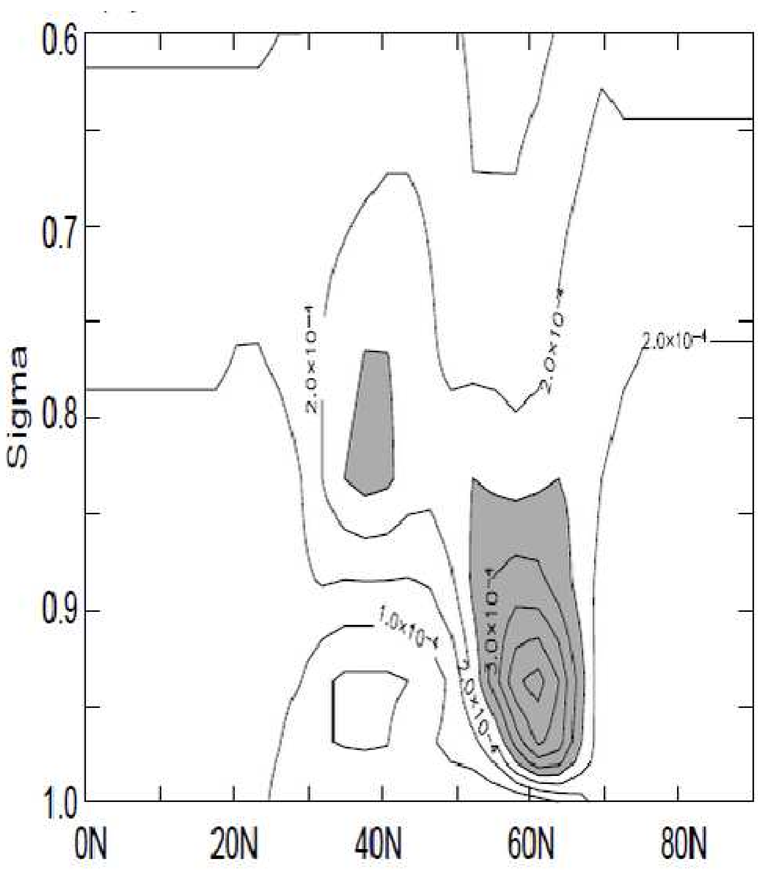
Role of friction: Ekman pumping



- Boundary layer convergence leads to ascent leads to spin-down of a barotropic vortex
- Barotropic vorticity equation,

$$\frac{D\zeta}{Dt} = \zeta\frac{\partial w}{\partial z}, \qquad \zeta = f + \xi$$

Effects on low-level stability



- Mid-level feature associated with dry intrusion
- Baroclinic frictional effects increase low-level stability over the low centre
- Reduces the strength of coupling between tropopause-level PV feature and surface temperature wave

Some background on turbulence

TKE

$$\text{KE} = \frac{1}{2}\underline{u}^2 = \frac{1}{2}\left(\overline{u}^2 + \overline{v}^2 + \overline{w}^2\right) + \frac{1}{2}\left(u'^2 + v'^2 + w'^2\right) + \left(uu' + vv' + ww'\right)$$

inc. KE of mean flow, KE of turbulence and cross terms

- Reynolds average gives KE=mean flow KE + TKE

$$e = \frac{1}{2}\left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2}\right)$$

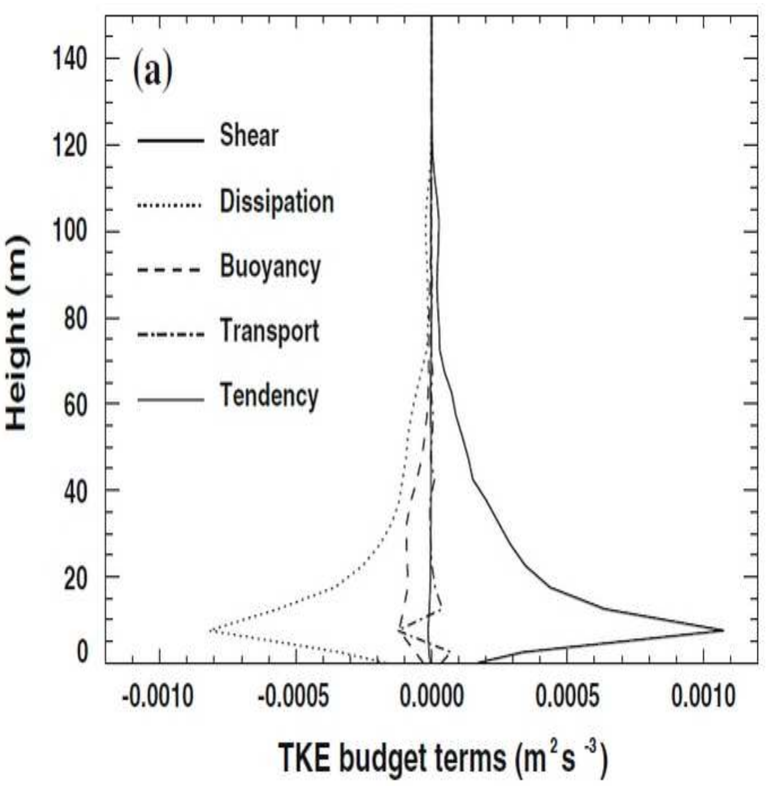
TKE Evolution

$$\frac{D\overline{e}}{Dt} = -\overline{u'w'}\frac{\partial\overline{u}}{\partial z} + \frac{g}{\theta_0}\overline{w'\theta'} - \frac{1}{\rho}\frac{\partial}{\partial z}\overline{w'p'} - \frac{\partial}{\partial z}\overline{w'e} - \varepsilon$$

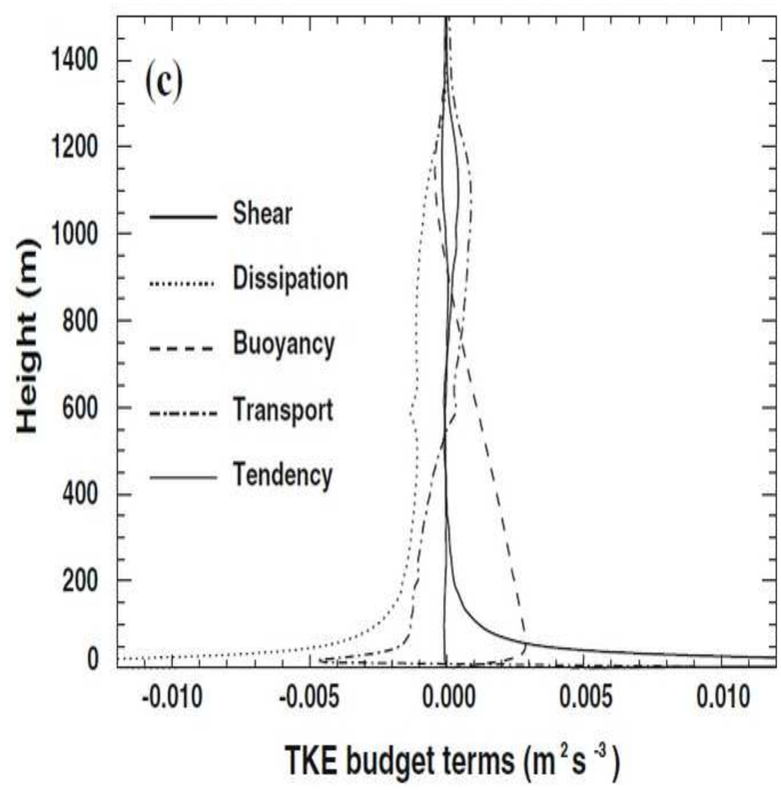
Storage + Advection = Shear + Buoyancy + Pressure correlations + Dissipation

- A crucial distinction is between the convective BL (buoyancy generates TKE) and the stable BL (buoyancy destroys TKE)

Typical TKE Budgets

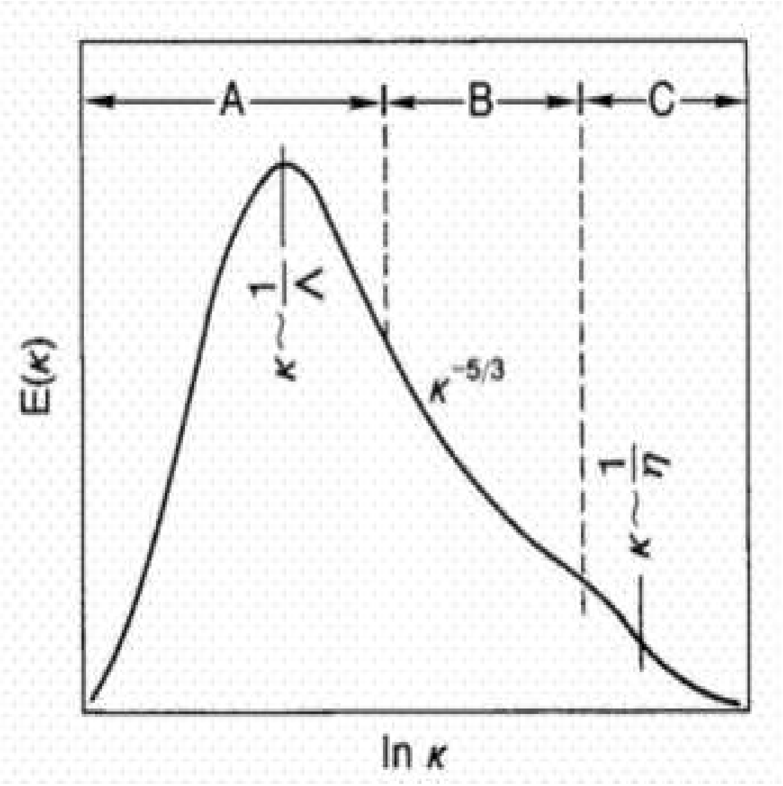


Nocturnal, stable boundary layer



Morning, well-mixed boundary layer

Split of eddy sizes



Fourier transform of TKE energy into different $k = 2\pi/\lambda$

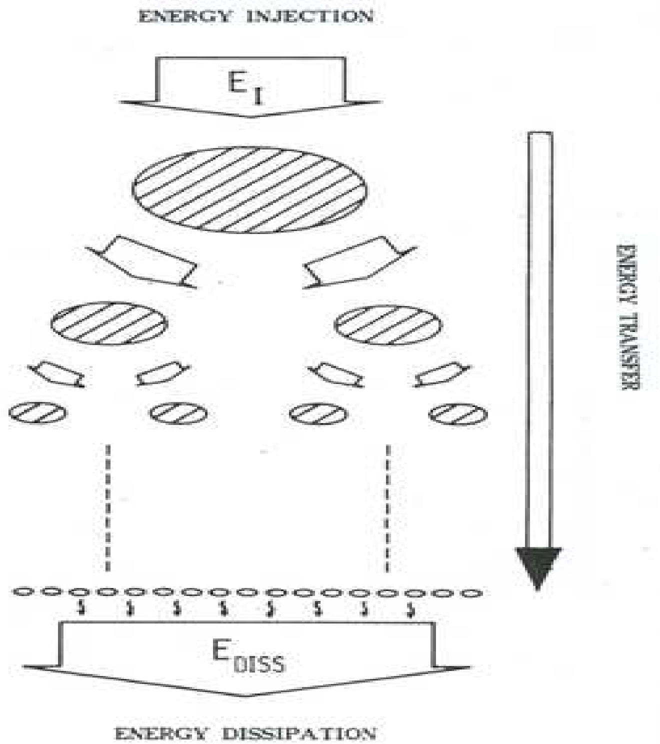
A Energy-containing range.

B Inertial subrange, $k^{-5/3}$

C Viscous dissipation at Kolmogorov microscale

$$\eta = \left(\frac{\nu^3}{\varepsilon}\right)^{1/4} \sim 1\text{mm}$$

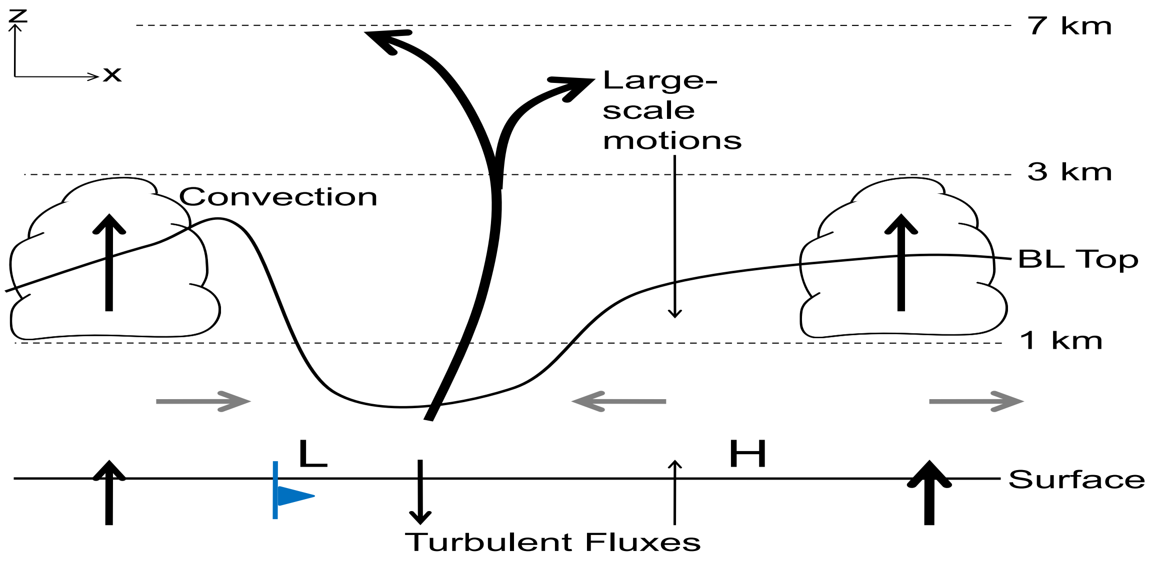
Split of eddy sizes



The turbulent energy cascade

Moisture transports

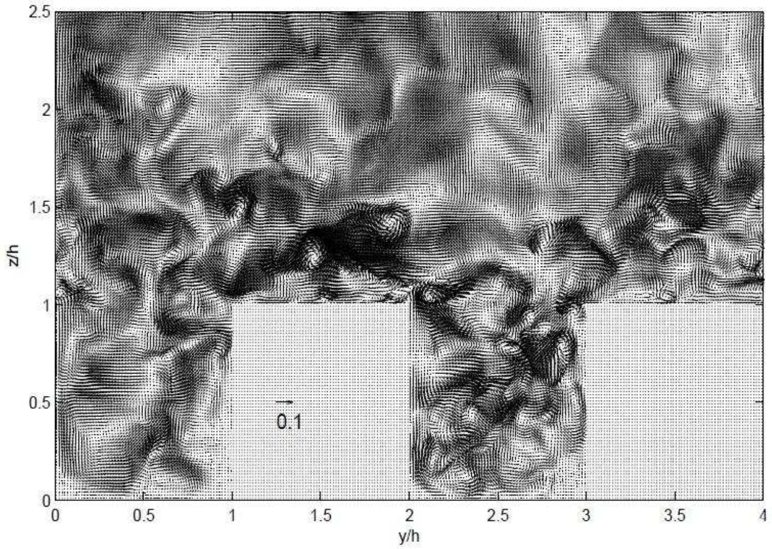
Schematic based on boundary-layer moisture budget analysis:



Divergence from high and convergence towards low within the boundary layer necessary to supply WCB with moisture

Explicit turbulence simulations

Direct Numerical Simulation



Snapshot of DNS at Re$=500$. yz slice through an array of cubes with large-scale flow out of page

- Viscous dissipation important at ~ 1mm, and BL height ~ 500m
- Fully resolving turbulence with DNS of Navier-Stokes needs $\sim 10^{18}$ grid points
- Inflate viscous scale and assume Reynolds number indpendence

Large Eddy Simulation

- Simulate only as far as the inertial subrange: capture the large eddies

$$E = \int_0^{\infty} E(k)dk \approx \int_0^{k_c=\pi/\Delta x} E(k)dk$$

- Dissipation rate is

$$\varepsilon = 2\nu \int_0^{\infty} k^2 E(k)dk \not\approx 2\nu \int_0^{k_c} k^2 E(k)dk \approx 0$$

- Without viscous scales there is no sink of TKE

Energy loss from LES

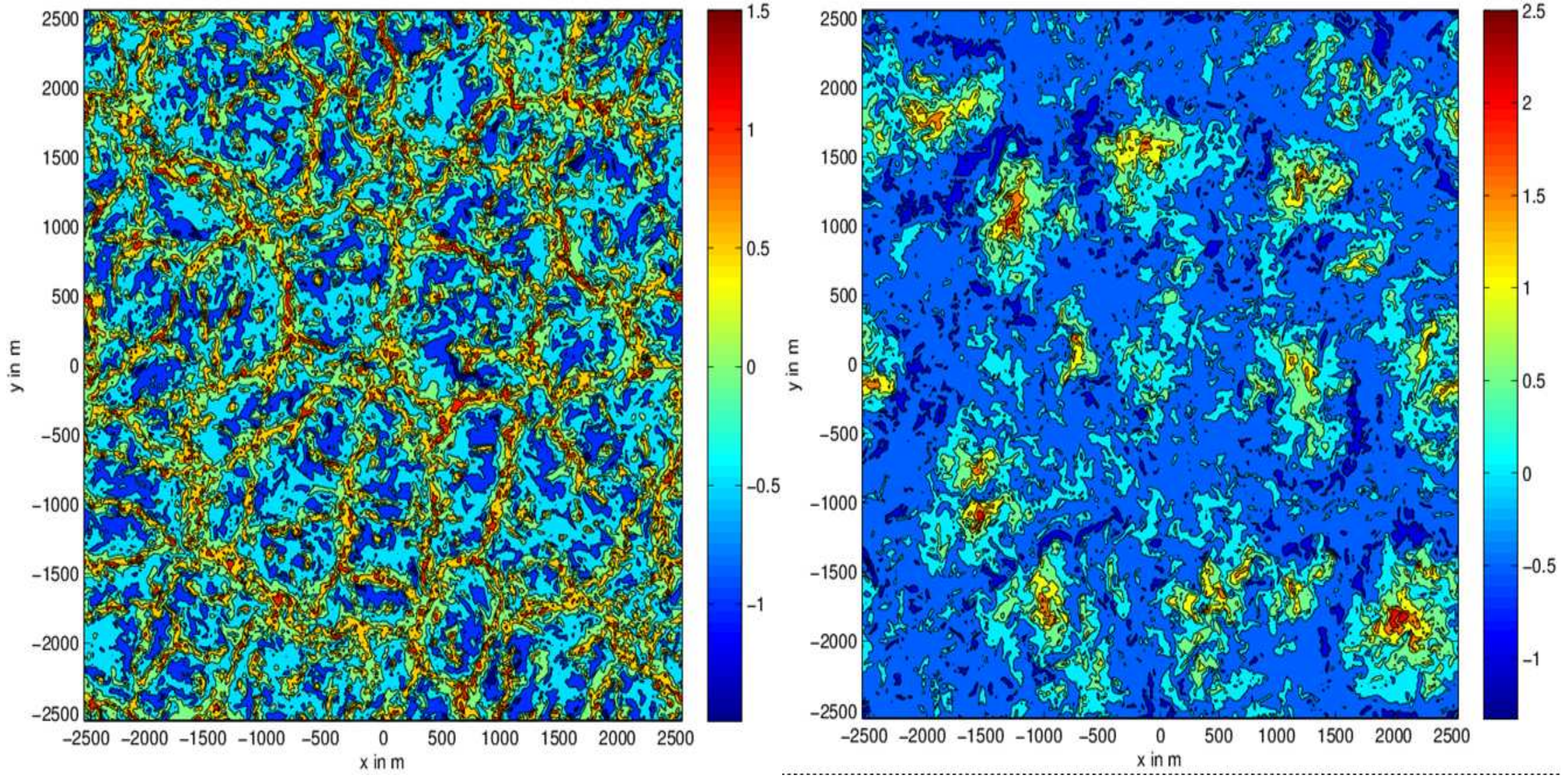
- Need a parameterization P of the energy drain

$$\varepsilon \approx 2 \int_0^{k_c} P(k) k^2 E(k) dk$$

- Ideally P acts close to k_c only so well-resolved eddies are not affected
- Popular (and very simple) choice is a Smagorinsky scheme, which is effectively a diffusion with coefficient $\propto \Delta x$

LES snapshots of w

CBL simulated at $\Delta x = \Delta y = 10$m, $\Delta z = 4$m



$z = 80$m (left) and 800m (right)

Organization of the problem

NWP and GCMs: Closure Problem

- On an NWP grid, no attempt to simulate turbulent eddies
- Parameterize full turbulent spectrum.

$$\frac{\partial \overline{u}}{\partial t} + \overline{u}\frac{\partial \overline{u}}{\partial x} + \overline{v}\frac{\partial \overline{u}}{\partial y} + \overline{w}\frac{\partial \overline{u}}{\partial z} - f\overline{v} + \frac{1}{\rho}\frac{\partial \overline{p}}{\partial x} = -\frac{\partial \overline{u'u'}}{\partial x} - \frac{\partial \overline{v'u'}}{\partial y} - \frac{\partial \overline{w'u'}}{\partial z}$$

- Effects of turbulence are described by fluxes like $\overline{u'w'}$
- Evolution equation for $\overline{u'w'}$ includes $\overline{u'w'w'}$ etc

Mellor-Yamada hierachy

Level 4	Carry (simplified) prognostic equations for all 2nd order moments with parameterization of 3rd order terms
Level 3	Carry (simplified) prognostic equations for $\overline{\theta'^2}$ and TKE with parameterization of 3rd order terms
Level 2.5	Carry (simplified) prognostic equation for TKE with parameterization of 3rd order terms
Level 2	Carry diagnostic equations for all 2nd order moments
Level 1	Carry simplified diagnostic equations 2nd order moments, K theory

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Level 3

$$\frac{\partial \overline{e}}{\partial t} = -\overline{u'w'}\frac{\partial \overline{u}}{\partial z} + \frac{g}{\theta_0}\overline{w'\theta'} - \frac{1}{\rho}\frac{\partial}{\partial z}\overline{w'p'} - \frac{\partial}{\partial z}\overline{w'e} - \varepsilon$$

$$\frac{\partial \overline{\theta'^2}}{\partial t} = -2\overline{w'\theta'}\frac{\partial \overline{\theta}}{\partial z} - \frac{\partial}{\partial z}\overline{w'\theta'^2} - \varepsilon_\theta$$

- NWP/GCMs usually have diagnostic treatment but some use TKE approach
- Drop terms in red to get to the level 2.5, TKE approach
- Not well justified theoretically though: no fundamental reason to prefer kinetic to potential energy

Parameterization of terms

- After Kolmogorov,

$$\varepsilon = \frac{e}{\tau} \propto \frac{e^{3/2}}{l} \qquad \varepsilon_\theta \propto \frac{\overline{\theta'^2} e^{1/2}}{l}$$

- Terms involving pressure treated using return-to-isotropy ideas (Rotta)
- Many possibilities for 3rd order terms, from simple downgradient forms: eg,

$$\overline{w'u'^2} \propto -\frac{\partial \overline{u'^2}}{\partial z}$$

to much more “sophisticated” (ie, complicated!) methods

Surface layer parameterization

Surface Layer Similarity

Similarity theory requires us to:

1. write down all of physically relevant quantities that we believe may control the strength and character of the turbulence
2. put these together into dimensionless combinations
3. any non-dimensional turbulent quantity must be a function of the dimensionless variables: we just need to measure that function

Monin-Obukhov theory: Step 1

Postulate that surface layer turbulence can be described by

- height z
- friction velocity (essentially the surface drag)

$$u_* = \sqrt{-\overline{u'w'}_0}$$

- a temperature scale that we can get from the surface heat flux

$$T_* = \frac{\overline{w'\theta'}_0}{u_*}$$

which is more conveniently expressed as a turbulent production of buoyancy $(g/\theta_0)T_*$

Monin-Obukhov theory: Step 2

Construct dimensionless combinations from these three. Here z/L where L is the Obukhov length

$$L = \frac{-u_*^2 \theta_0}{kgT_*}$$

- L measures relative strength of shear and buoyancy
- buoyancy becomes as important as shear at height $z \sim |L|$
- $+$ve in stable conditions
- k is von Karman's constant, $= 0.4$

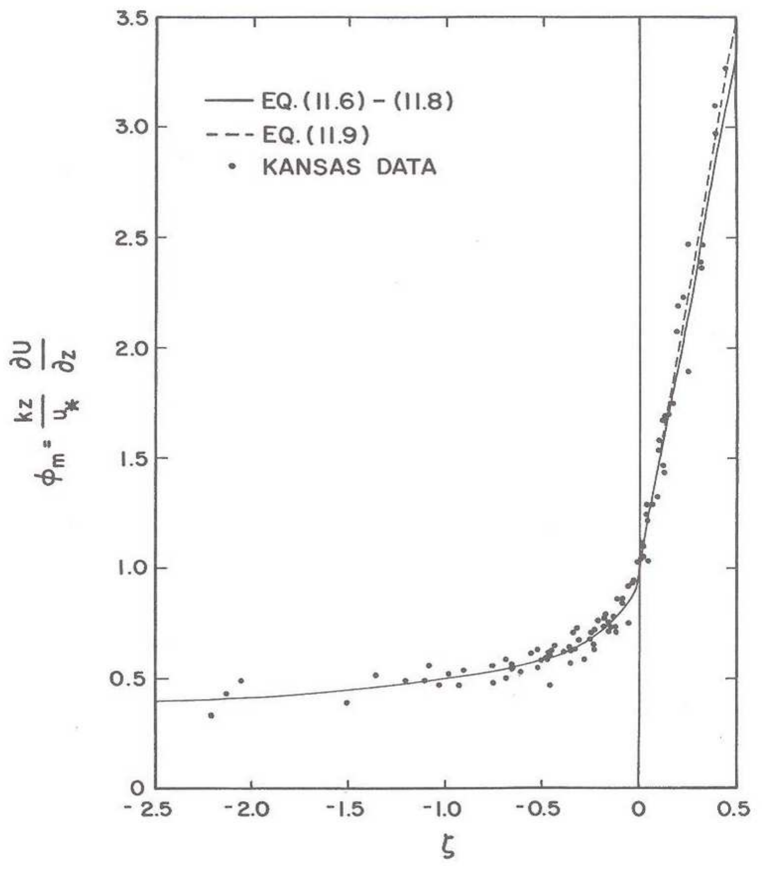
Monin-Obukhov theory: Step 2

- Express the variables we need in terms of dimensionless quantities

$$\frac{\partial \overline{u}}{\partial z} = \frac{u_*}{kz}\phi_m \quad ; \quad \frac{\partial \overline{\theta}}{\partial z} = \frac{T_*}{kz}\phi_h$$

- Now measure the dimensionless functions (obs or LES)
- Which must be universal if the scaling is correct

Monin-Obukhov Theory: Step 3



re 11.2 Dimensionless wind shear as a function of the M–O stability par
ısas data from Izumi (1971).]

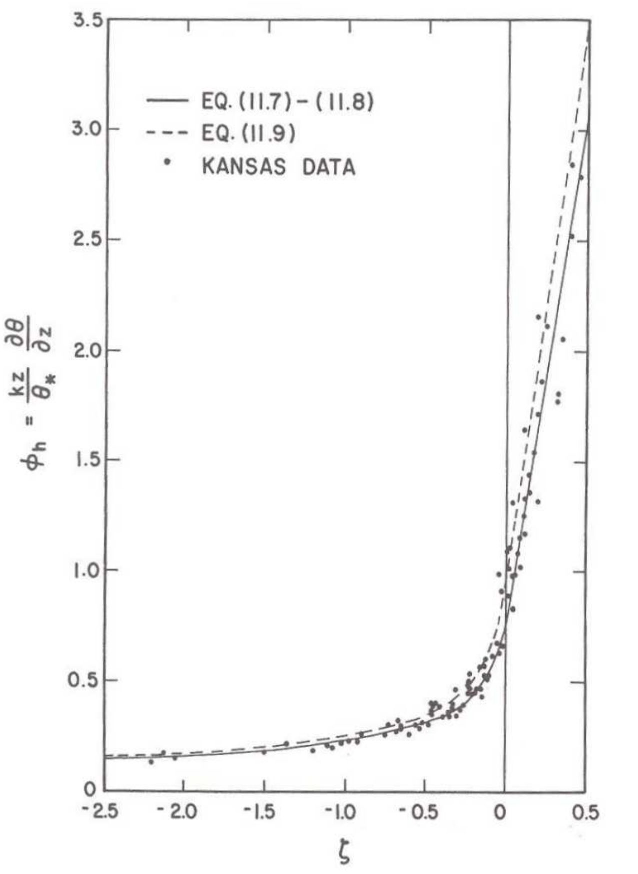


Figure 11.3 Dimensionless potential temperature gradient as a function of the M–O stability parameters. [Kansas data from Izumi (1971).]

Functions ϕ_m (left) and ϕ_h (right)

Computing surface-layer fluxes

- Lowest model level typically at $\sim$ 10m
- Use Monin-Obukhov similarity theory

$$\frac{\partial \overline{u}}{\partial z} = \frac{u_*}{kz}\phi_m(z/L)$$

- Integrate with height to get $\overline{u}$ for lowest level

$$\overline{u} = \frac{u_*}{k}\left[\ln\frac{z}{z_0} - \psi_m\right]$$

$$\psi_m = \int_{z_0/L}^{z/L}(1-\phi_m)(z/L)^{-1}d(z/L)$$

- Iterative calculations needed: $\overline{u}_{10} \longrightarrow u_* \longrightarrow L \longrightarrow \overline{u}_{10} \longrightarrow \ldots$

Connection to diagnostic equations

- We can rewrite MOST in the form

$$\overline{w'u'} = -K_m \frac{\partial \overline{u}}{\partial z}$$

$$K_m = (kz)^2 \phi_m^{-2} \frac{\partial \overline{u}}{\partial z}$$

- So that in the main $\overline{u}$ equation we have a diffusion structure

$$\left.\frac{\partial \overline{u}}{\partial t}\right|_{\text{turbulence}} = -\frac{\partial}{\partial z}\overline{w'u'} = \frac{\partial}{\partial z} K_m \frac{\partial \overline{u}}{\partial z}$$

The outer boundary layer

Mixed Layer Similarity: Step 1

- Surface heat flux $\overline{w'\theta'}_0$ drives convective eddies of the scale of h
- Scaling velocities for vertical velocity and temperature

$$w_* = \left[\frac{g}{\theta_0}\overline{w'\theta'}_0 h\right]^{1/3}$$

$$\theta_* = \frac{\overline{w'\theta'}_0}{w_*}$$

- Dimensionless quantity z/h

Mixed Layer Similarity: Step 2

$$\overline{w'^2} = w_*^2 f_1(z/h)$$

$$\overline{u'^2} = w_*^2 f_2(z/h)$$

$$\overline{\theta'^2} = \theta_*^2 f_3(z/h)$$

$$\overline{w'\theta'} = w_* \theta_* f_4(z/h)$$

$$\varepsilon = \frac{w_*^3}{h} f_5(z/h)$$

etc

Mixed layer similarity: Step 3

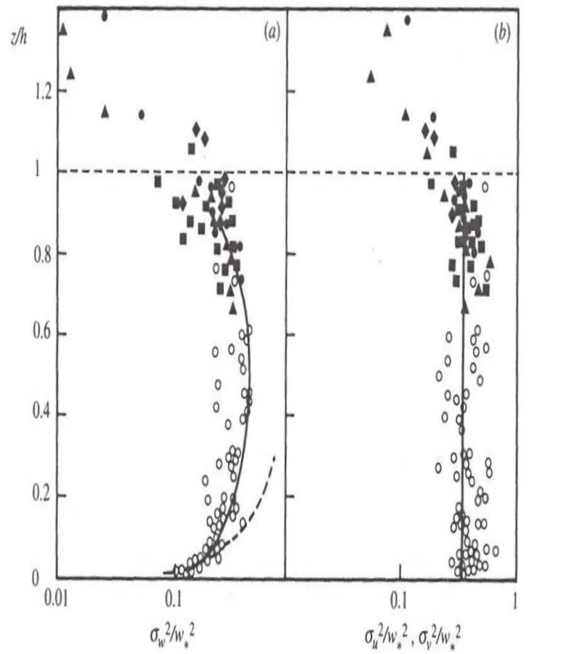


Fig. 3.21 (*a*) Normalized vertical velocity variance in unstable conditions as a function of normalized height throughout the CBL. The pecked curve represents Eq. 3.101 with $a_1 = 1.8$, whilst the solid curve represents Eq. 3.109 according to Sorbjan (1989). The behaviour represented by this expression is similar to that found by Lenschow *et al.* (1980) for the CBL over the sea during cold air outbreaks. (*b*) As in (*a*), but for the horizontal wind components. The vertical straight line represents $\sigma_u^2/w_*^2 = \sigma_v^2/w_*^2 = 0.36$. Atmospheric data from Caughey and Palmer (1979).

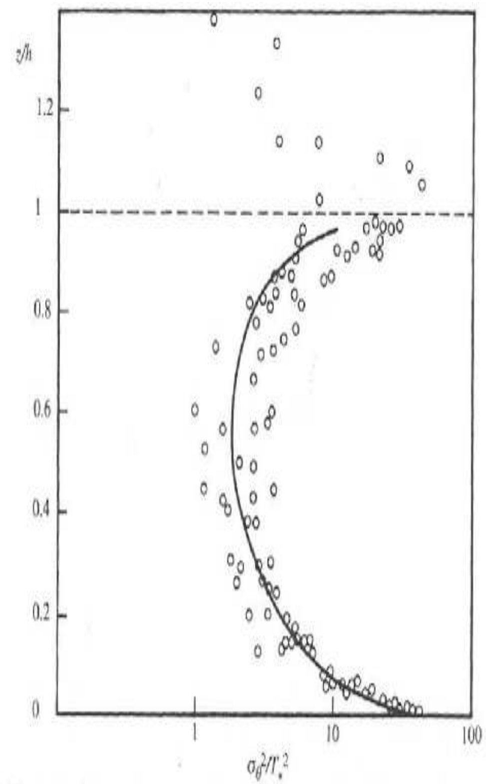


Fig. 3.22 Normalized temperature variance profiles in the CBL, showing atmospheric observations from Caughey and Palmer (1979). The solid line is the prediction given by Eq. 3.110, based on Sorbjan (1989). The expression uses the value −0.2 for the flux ratio β (see Section 6.1.6), based on the work of Moeng and Wyngaard (1984).

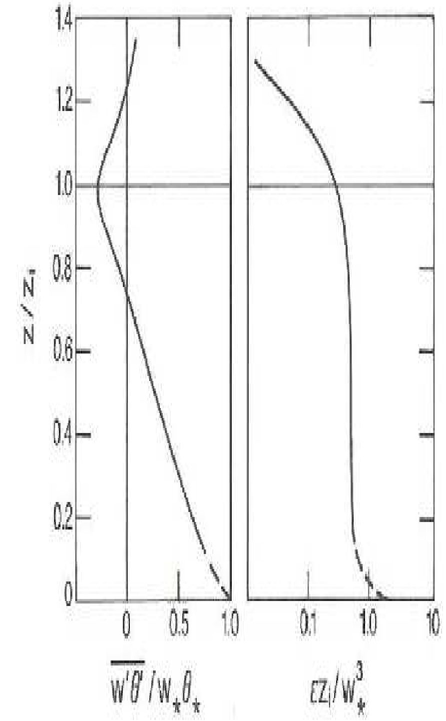


Fig. 1.10. Vertical profiles of nondimensional mixed layer parameters (after Caughey and Palmer, 1979). Dashed portions of curves imply extrapolations through the surface layer.

Left to right: $\overline{w'^2}$, $\overline{u'^2}$, $\overline{\theta'^2}$, $\overline{w'\theta'}$, ε

K theory

- For the turbulent fluxes within the boundary layer, the K-theory formula is

$$\overline{w'u'} = -K_m \frac{\partial \overline{u}}{\partial z}$$

$$K_m = \lambda_m^2 f_m(\text{Ri}) \frac{\partial \overline{u}}{\partial z} \qquad \text{compare} \quad K_m = (kz)^2 \phi_m^{-2} \frac{\partial \overline{u}}{\partial z}$$

- ie, generalizes MOST
- Typical eddy size $\sim kz$ or λ_m

$$\frac{1}{\lambda_m} = \frac{1}{kz} + \frac{1}{l} \quad \text{where} \quad l \propto h$$

Stability dependence

- z/L and Richardson number Ri play similar role in measuring relative importance of buoyancy and shear

$$Ri = \frac{-\frac{g}{\theta_0}\frac{\partial\overline{\theta}}{\partial z}}{\left(\frac{\partial\overline{u}}{\partial z}\right)^2}$$

- The flux Richardson number is sometimes used instead

$$Rf = \frac{-(g/\theta_0)\overline{w'\theta'}}{\overline{w'\theta'}\frac{\partial\overline{u}}{\partial z}}$$ ratio of terms in TKE equation

Properties of K

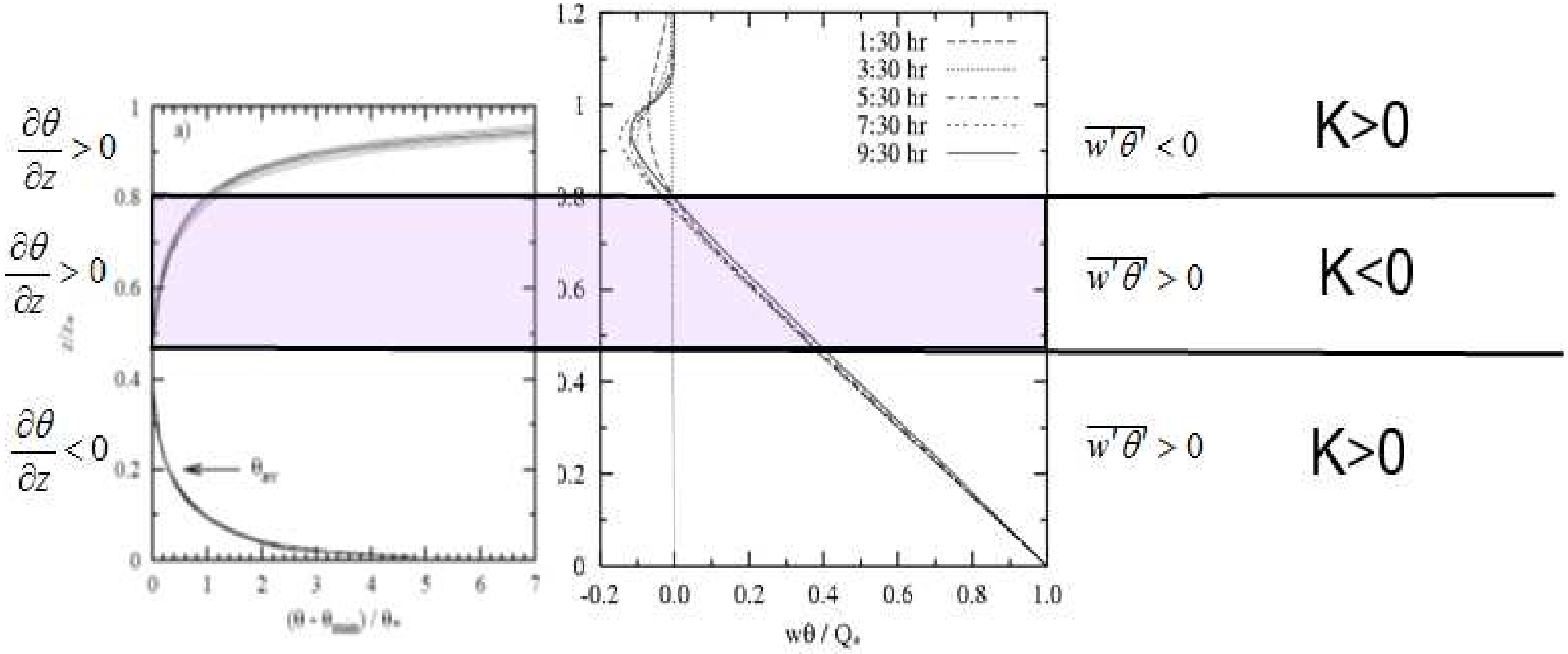
- K theory should match to MOST for small z
- and to the relevant similarity theory in the outer layer, being based on the appropriate scaling parameters
- eg, for the well-mixed CBL a suitable choice is

$$K_m = w_* h g_1(z/h) = k w_* z \left(1 - \frac{z}{h}\right)^2$$

- eg, Holstlag and Bolville 1993; as used at ECMWF

A problem in the CBL

$$\overline{w'\theta'} = -K_h \frac{\partial\overline{\theta}}{\partial z} \quad \Longrightarrow \quad \text{can diagnose } K_h \text{ from LES/obs data}$$



A solution for the CBL

Simplest possibility is to introduce a non-local contribution

$$\overline{w'\theta'} = -K_h \frac{\partial \overline{\theta}}{\partial z} + K_h \gamma$$

where γ is simply a number

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Parameterizations Based on Scalings

Possible difficulties are:

- Developing good scalings for each boundary layer regime
- Good decison making needed for which regime to apply
- Handling transitions between regimes

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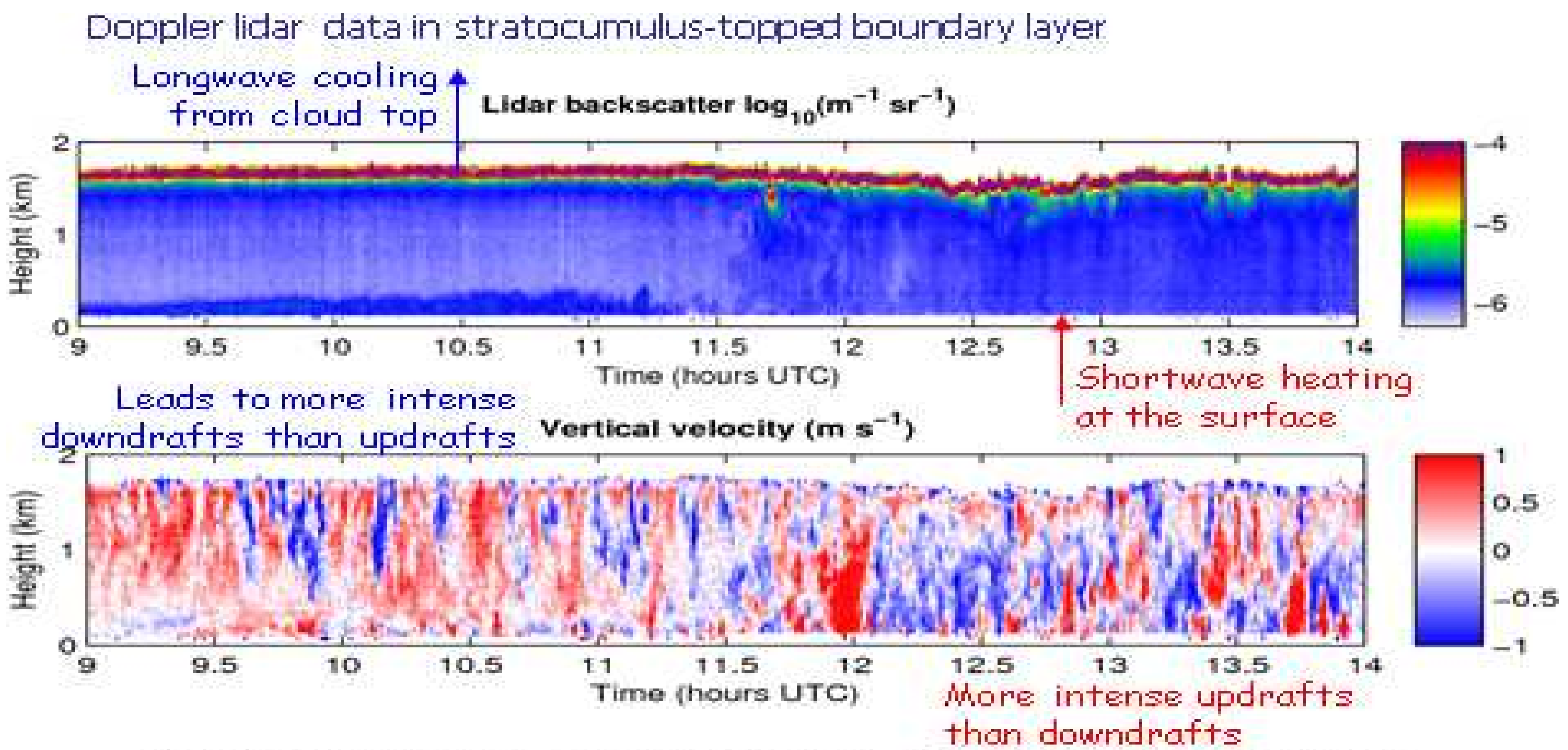
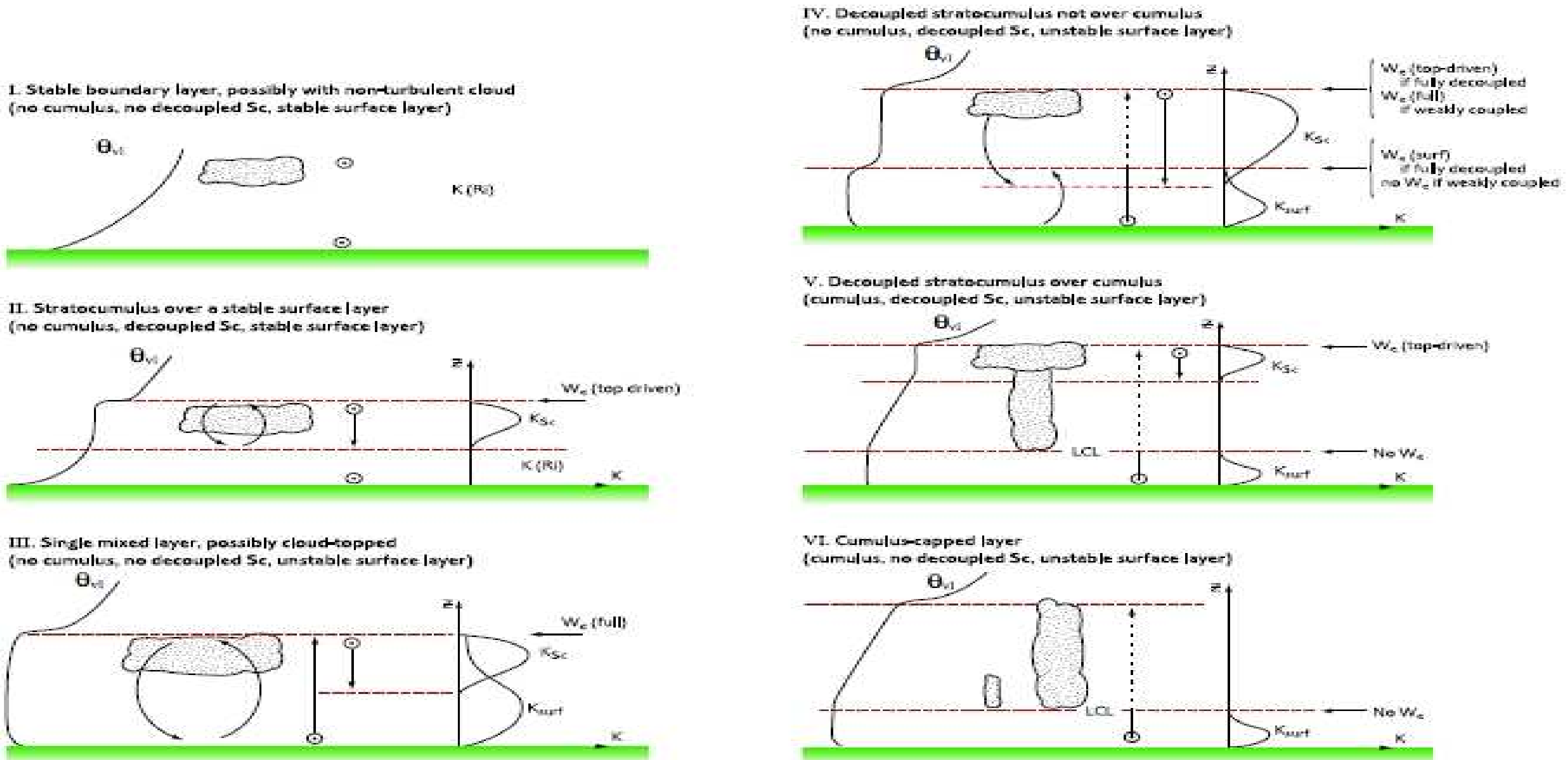
Effects of radiation on buoyancy



- K profile specified across the unstable layer by a *nonlocal* scheme

- LW cooling at cloud top; SW heating within cloud produces source of buoyant motions

UM scheme

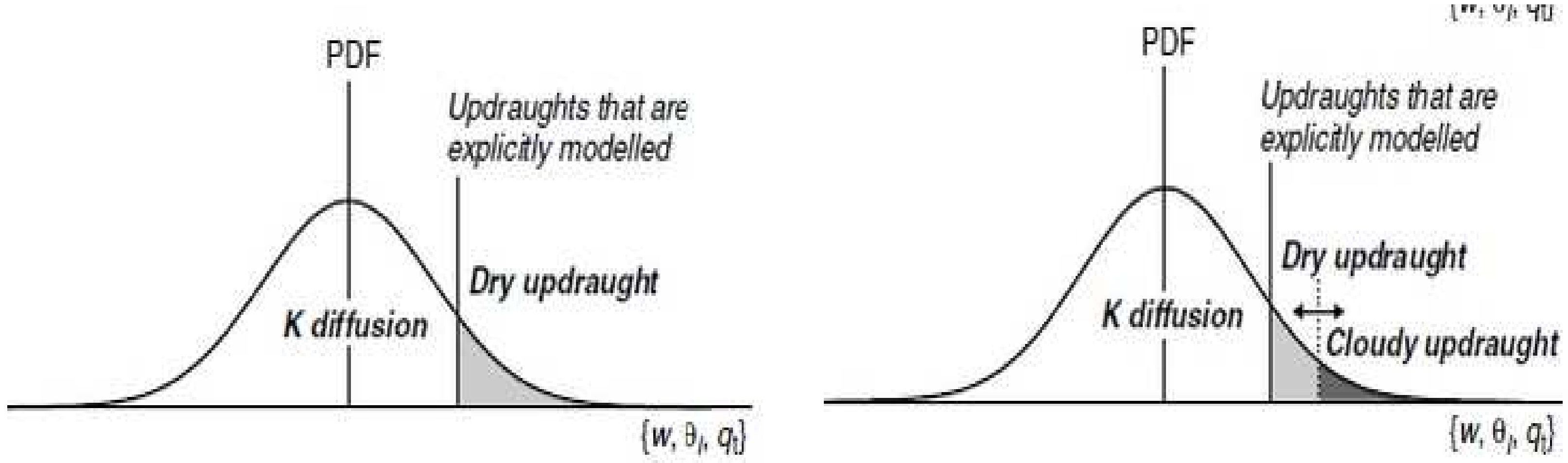


- Decision making about type is important
- Sc treated as BL cloud; shallow Cu separately

EDMF

Eddy-diffusivity mass-flux treatment,

$\overline{w'\phi'} = -K d\phi/dz + \sum_i M_i(\phi_i - \overline{\phi})$



- Mass flux component for largest, most energetic eddies of size $\sim h$ which produce the non-local transport
- Can be used as a treatment for Sc and (especially) shallow Cu
- Can be high sensitivity to bulk entrainment rate

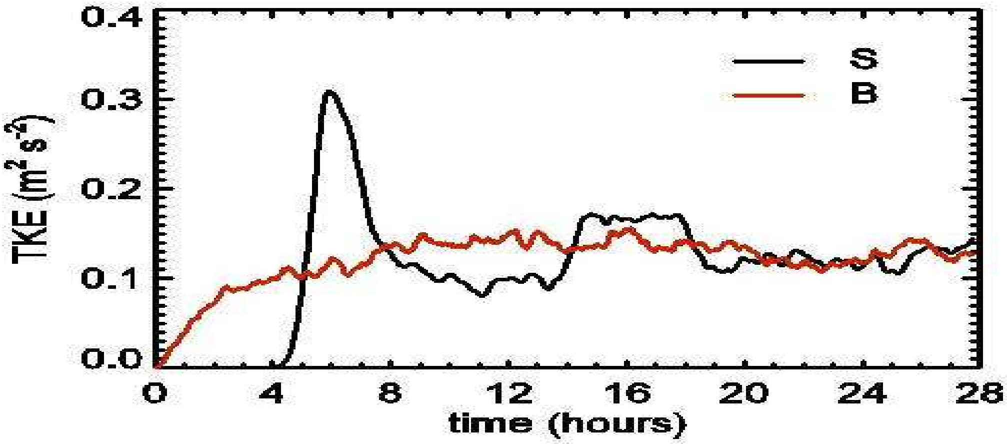
The boundary layer grey zone

The grey zone

- NWP approaches to turbulence valid when all turbulence is parameterized
- LES approaches to turbulence valid
- Grey zone is difficult middle range when model grid is comparable to the size of energy-containing eddies
- Should we try to ignore or such eddies and use an NWP approach? Double counting?
- Or allow them and use an LES approach? Risks under counting?

A Perspective from LES

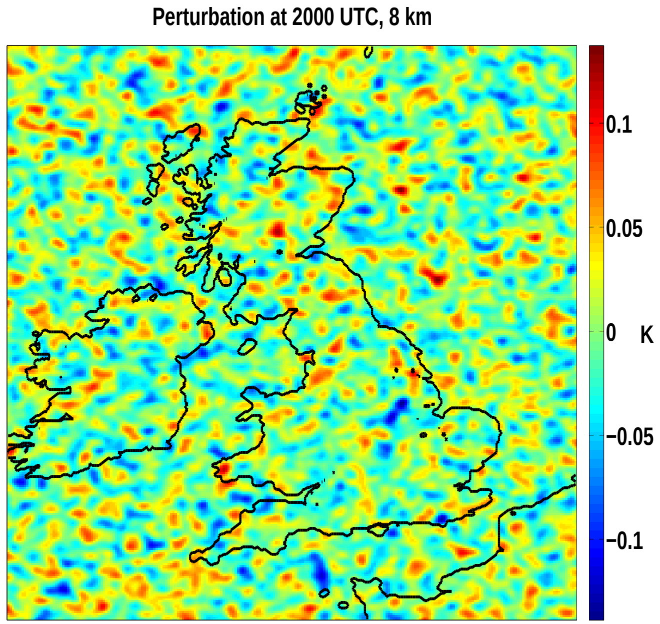
- Stochastic backscatter useful very near surface where $\Delta \ll l$ breaks down
- eg, improves profiles of dimensionless wind shear near surface
- Other LES models proposed for grey zone: dynamic model, tensorial model...

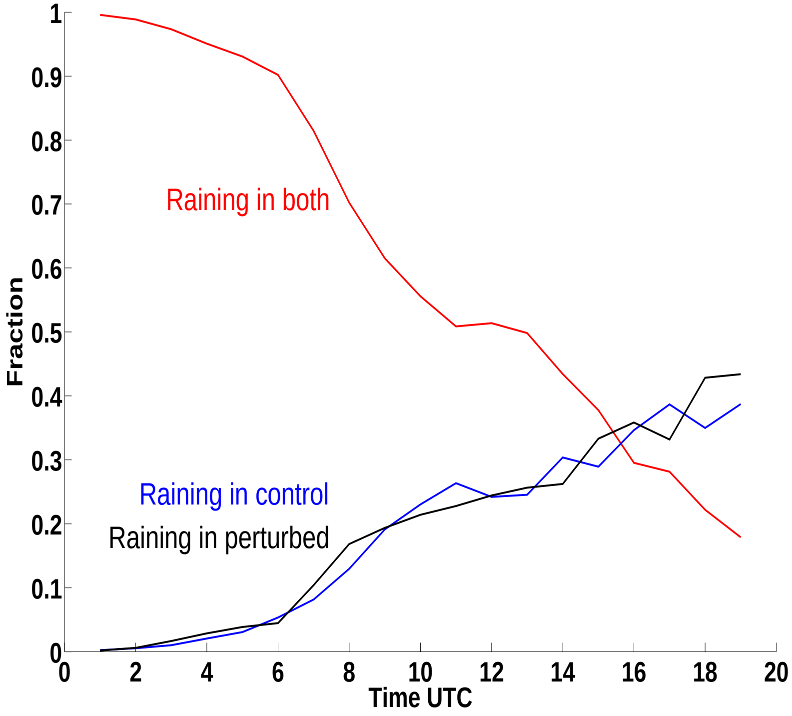


Dry, neutral boundary layer, Weinbrecht 2006

Perspective from NWP

- Small boundary layer fluctuations (~ 0.1K) important for convective initiation
- Can easily shift the locations of precipitating cells e.g. Leoncini et al (2010)





Conclusions

- Most NWP models have relatively simple turbulence modelling based on K-theory
- Simple methods are able to make use of similarity arguments
- These often work very well in the appropriate regimes, although transitions between regimes can be awkward and somewhat ad hoc
- ...because the performance is not very much worse than using very much more complex and more expensive turbulence modelling approaches
- But as resolutions $\Delta x \longrightarrow h$ new approaches may be needed; ensemble-based modelling breaks down