



The University of Reading



**NERC Centres for
Atmospheric Science**
NATURAL ENVIRONMENT RESEARCH COUNCIL

Adaptive Mesh Modelling of the Global Atmosphere

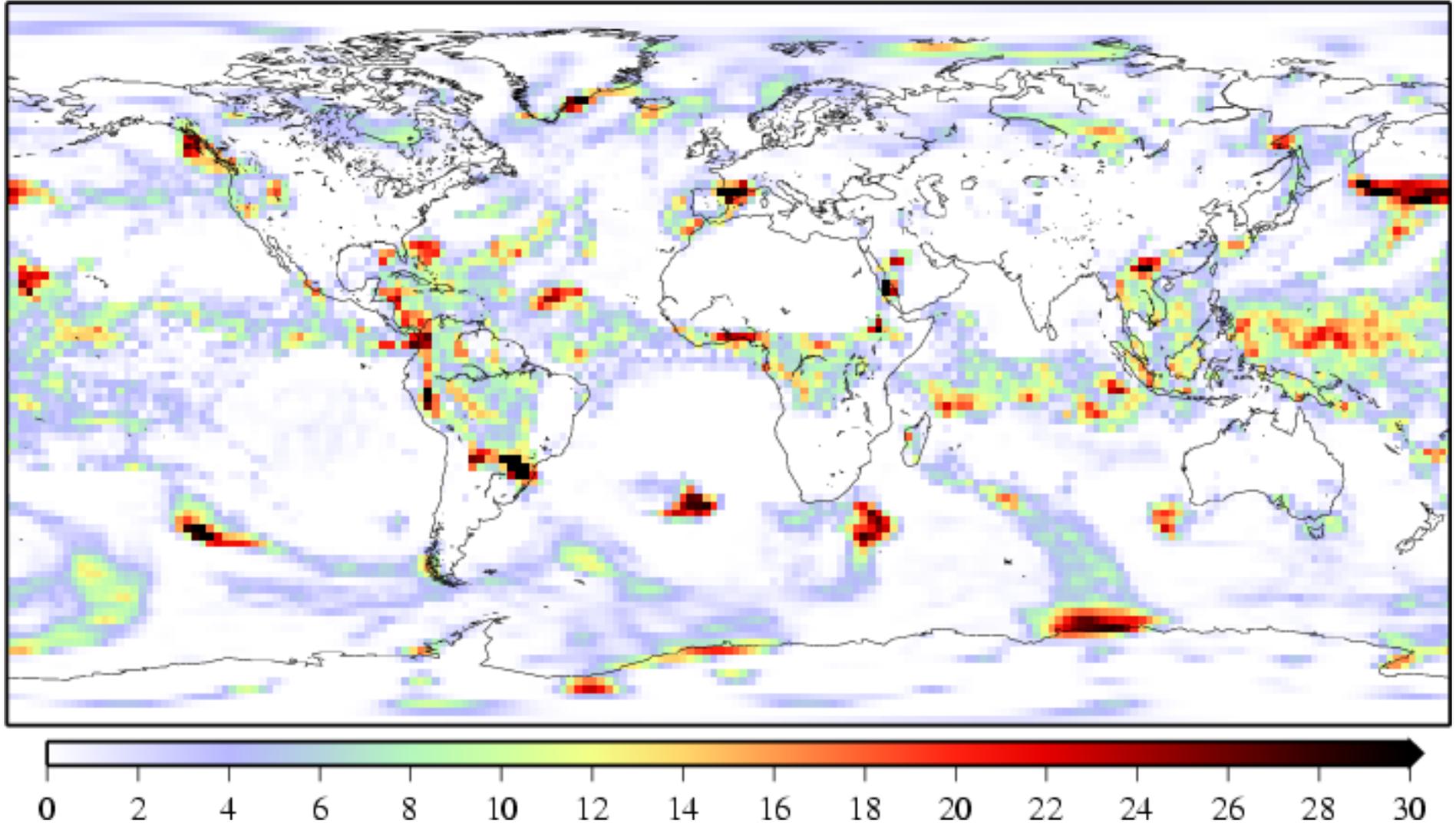
Hilary Weller

NCAS - Climate

November 2008

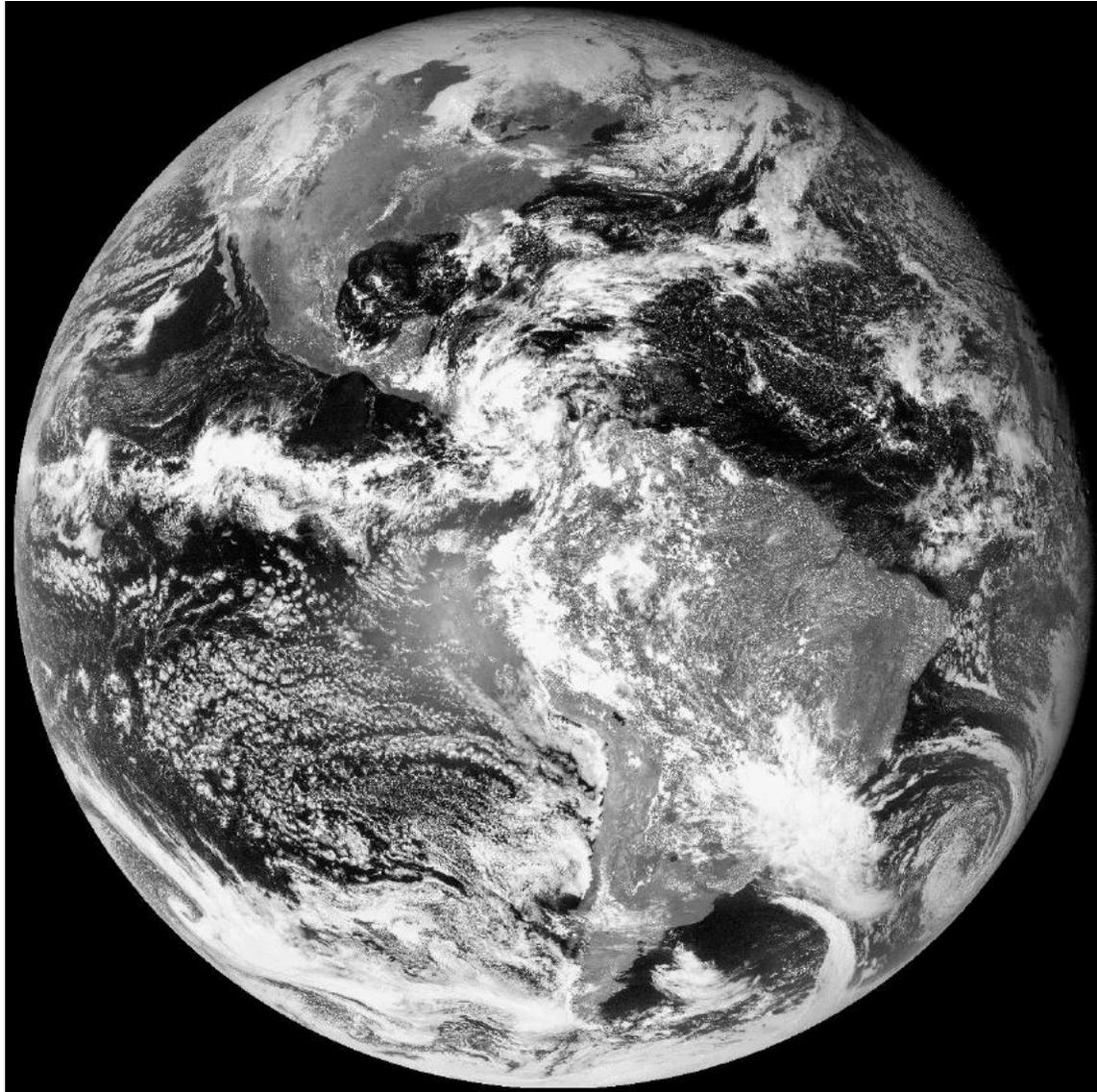
Motivation: More Resolution where we really need it

Precipitation rate on 2 November 2008



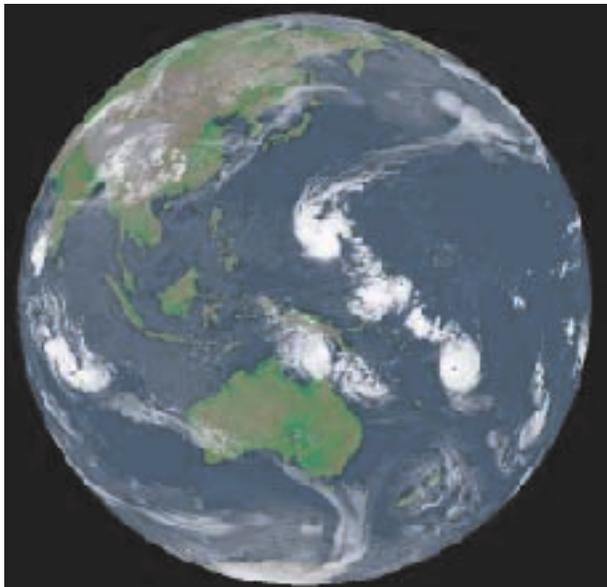
But fine scale detail is ubiquitous

NEODAAS GOES East 075.0W 2 Nov 2008 18:00hours Channel: 1 (Visible)

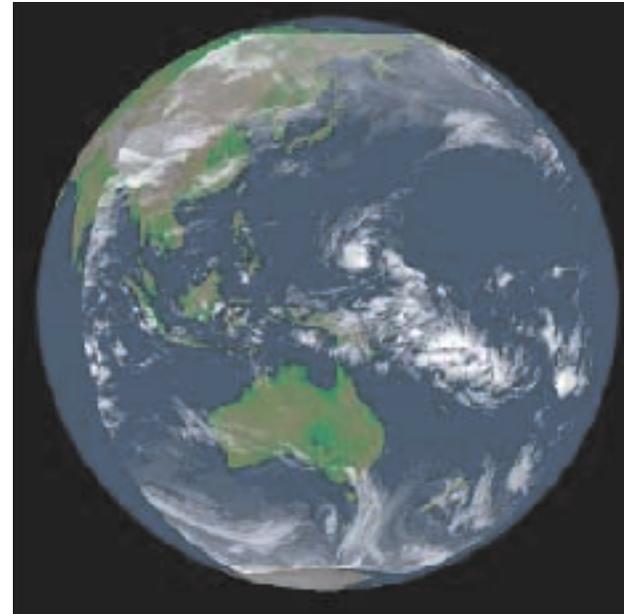


NICAM uses a hexagonal icosahedral mesh at fixed resolutions down to 3.5km on the Earth Simulator and captures features of tropical cloud clusters:
(see work of Masaki Satoh et al, JAMSTEC)

NICAM at 3.5km

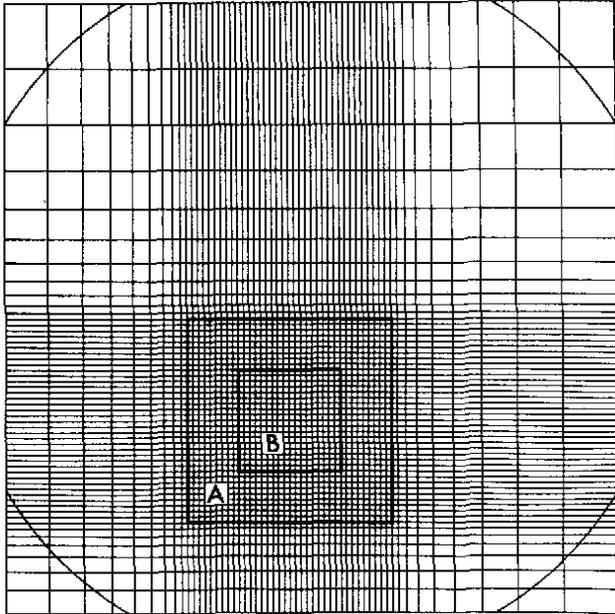


GMS/GOES-9 at Apr. 6, 2004, 00UTC

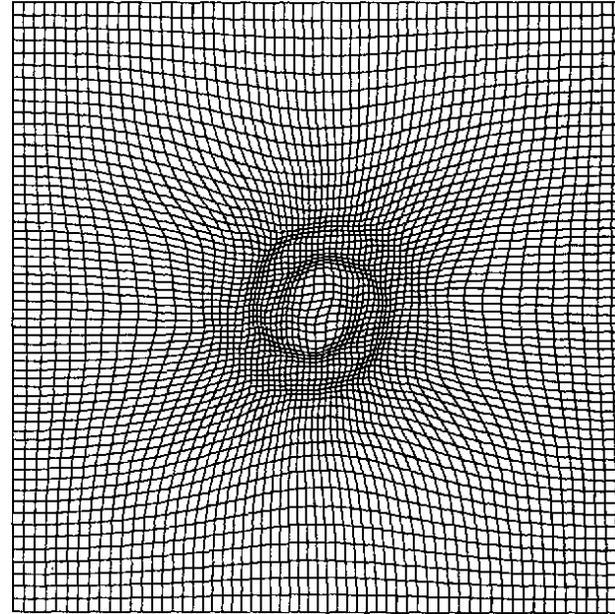


- But these “Grand Challenge” experiments are too expensive for operations
- And 3.5km is still not really fine enough to *resolve* convection

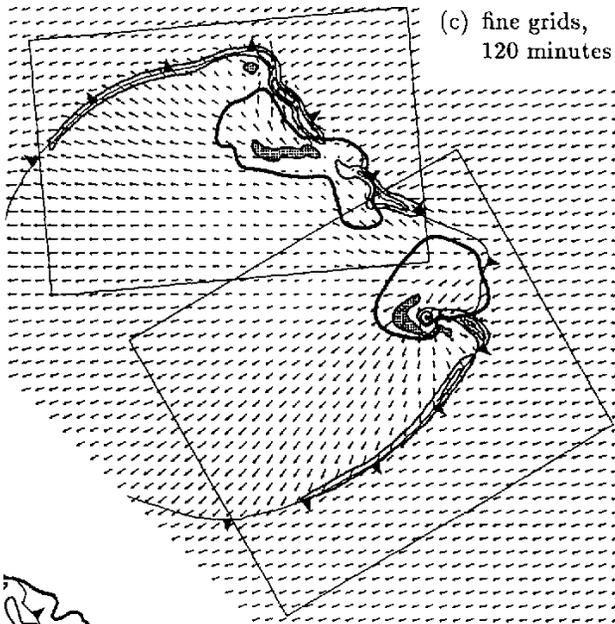
Adaptive Meshes have a long history in Atmospheric Science



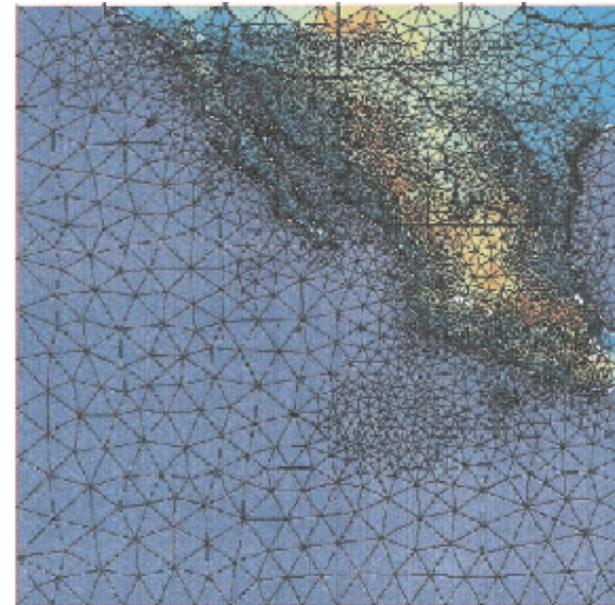
Staniforth and Mitchell, 1978. 2D fixed grid finite elements:



Dietachmayer and Droegemeier, 1992. Continuous Dynamic Grid Adaption



Skamarock and Klemp, 1993 Explicitly coupled rotated blocks



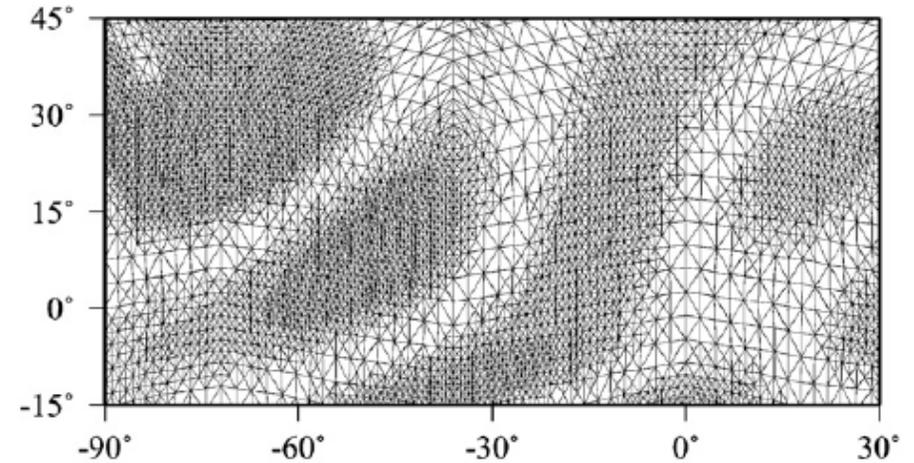
Bacon et al, 2000. Finite volume triangular prisms

Some More Recent Work in the Field:

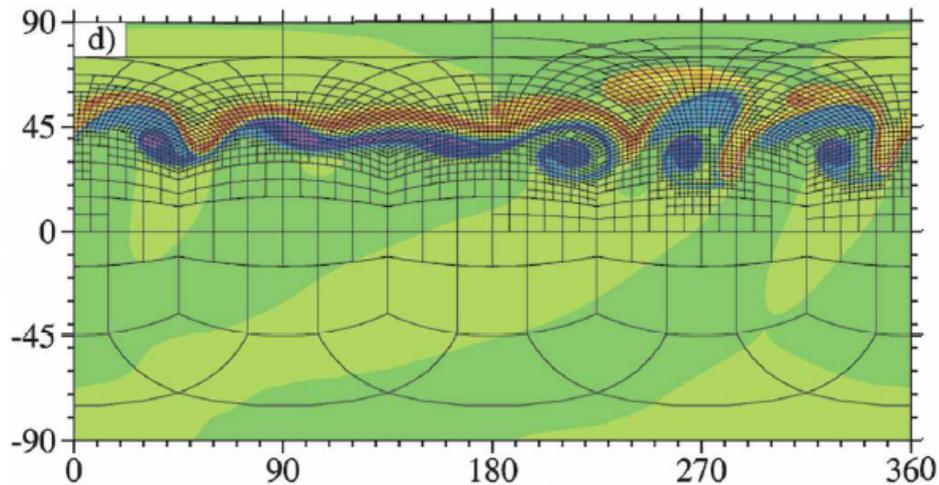
- Smith and Dritschel, 2006, CASL



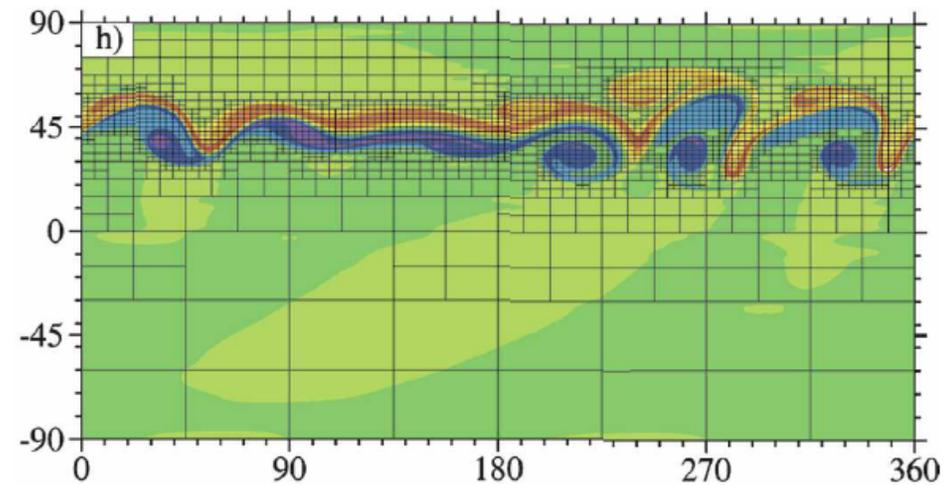
- Lauter et al 2007: Barotropic model



- St-Cyr et al 2008: A Comparison of Two Shallow-Water Models ...
Spectral Element on Cubed Sphere



- Finite Volume on lat-lon



Challenges

- Adaptive Meshing of:
 - Deep convection
 - Tropical cyclones
 - Orography
 - Fronts
- Unstructured or block-structured mesh adaptation
- Efficiency and accuracy
- Adaptation criteria and adaptation frequency
- Mesh to mesh mapping
- Ever more parallelisable algorithms
- Physical parametrisations for adaptive meshes
- Data assimilation

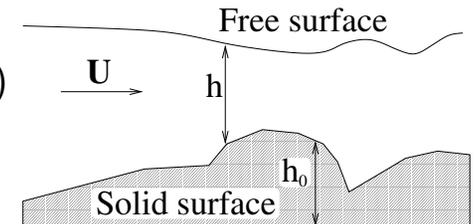
AtmosFOAM: Shallow water solver for any mesh (see Weller and Weller, 2008)

- Written using open source package OpenFOAM www.opencfd.co.uk

- Shallow water equations (SWEs):

$$\frac{\partial h\mathbf{U}}{\partial t} + \nabla \cdot (h\mathbf{U} \otimes \mathbf{U}) = -\boldsymbol{\Omega} \times h\mathbf{U} - gh\nabla(h + h_0)$$

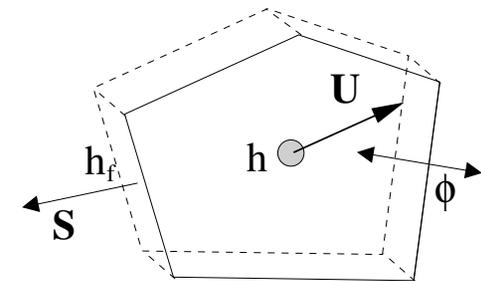
$$\frac{\partial h}{\partial t} + \nabla \cdot (h\mathbf{U}) = 0$$



- Co-located finite volume on polyhedra: Volumetric mean gradients and divergences converted to sums of mass fluxes over faces using Gauss' theorem:

$$\int_V \nabla \cdot h\mathbf{U} dV = \sum_{\text{faces}} \phi$$

$$\int_V \nabla h dV = \sum_{\text{faces}} h_f \mathbf{S}$$

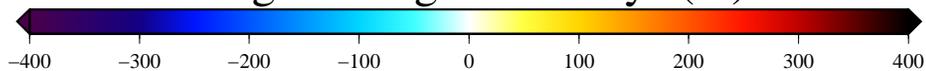


- ϕ as well as h and $h\mathbf{U}$ evolved from old time step using SWEs to avoid computational mode
- Matrices constructed to solve each component of momentum and continuity equations implicitly using linear differencing \implies very sparse diagonally dominant matrices – easy to solve in parallel
- Explicit corrections – bi-quadratic or bi-cubic differencing
 - non-linear advection
 - Coriolis
- No diffusion or filtering

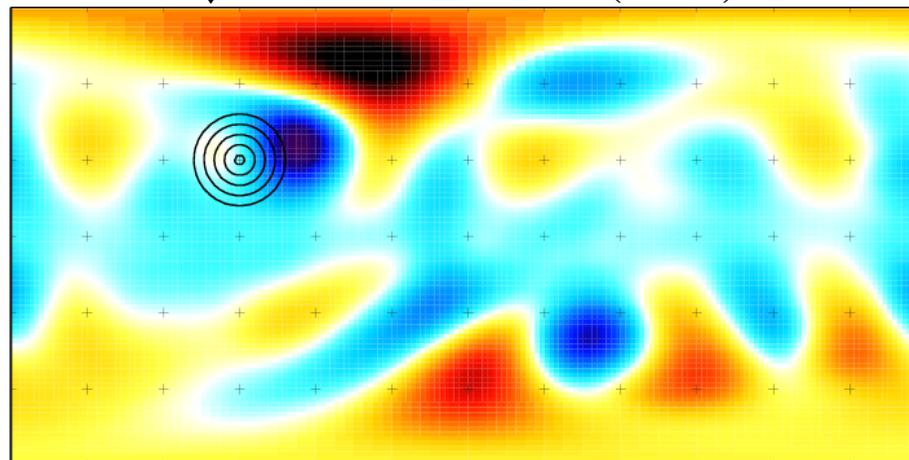
Shallow Water Flow over a Mountain

(Test case of Williamson et al, 1992)

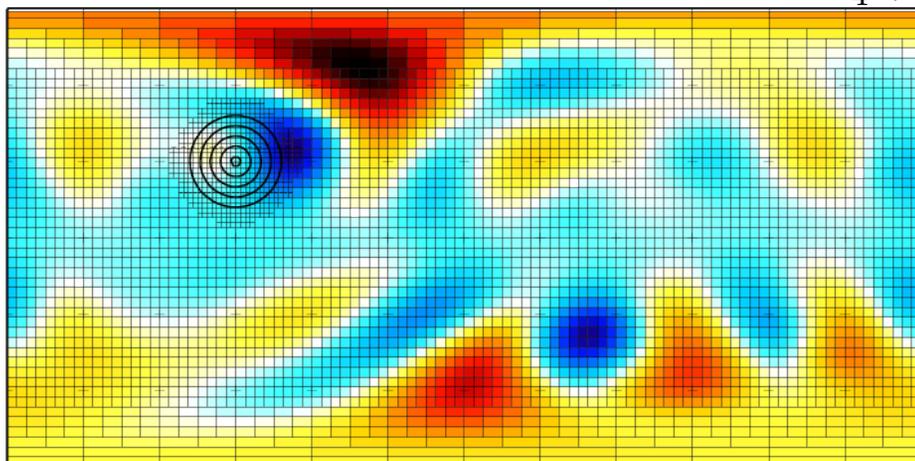
Height change in 15 days (m)



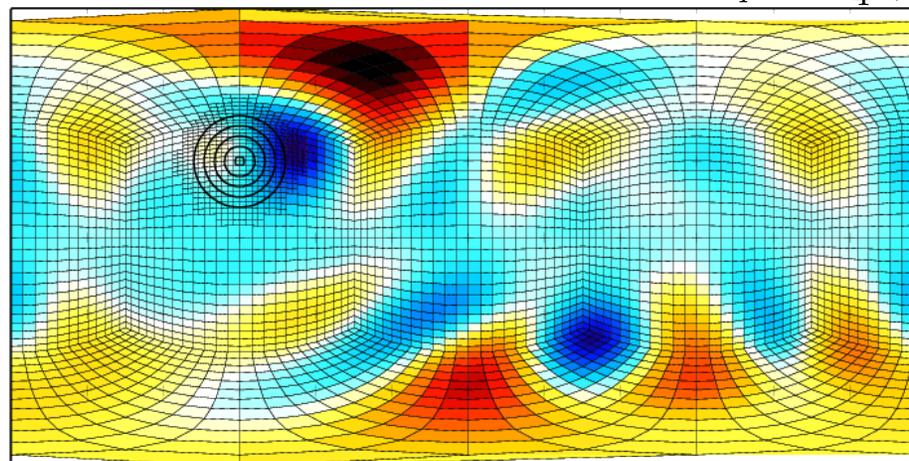
↓ Reference solution (T213)



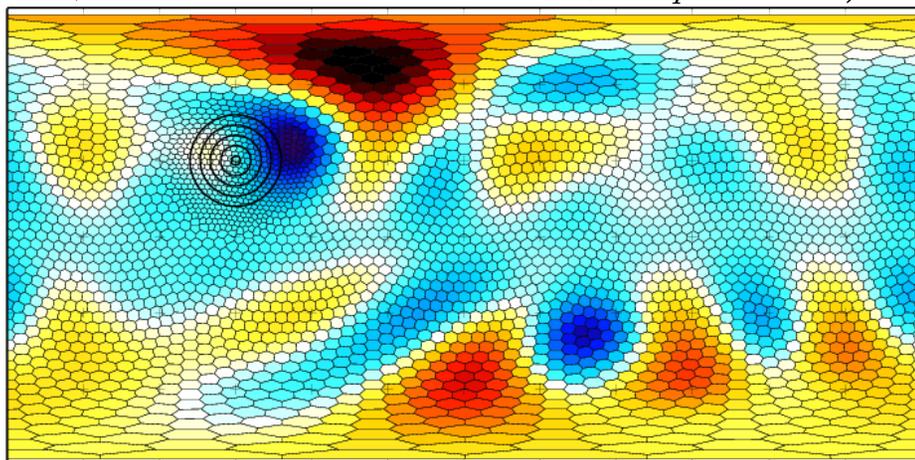
↓ Reduced lat-lon with refinement ($\delta x = 3\frac{3}{4}^\circ$)



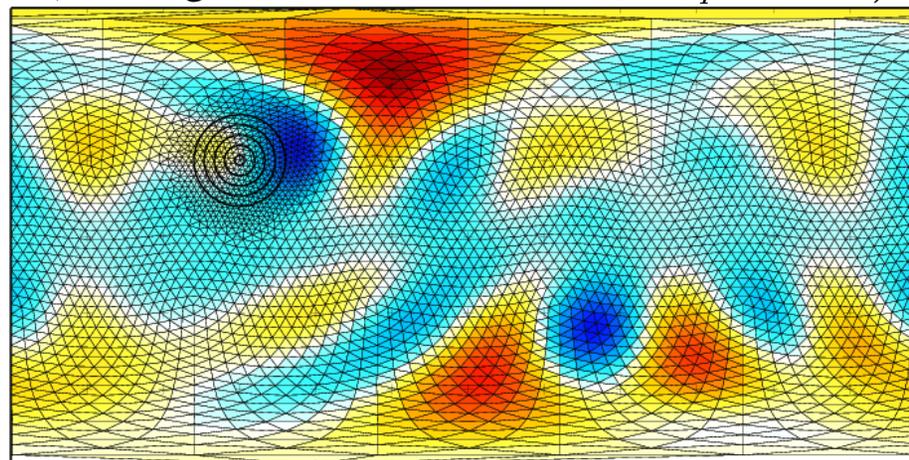
↓ Cubed sphere with refinement ($\delta x_{eq} = 3\frac{3}{4}^\circ$)



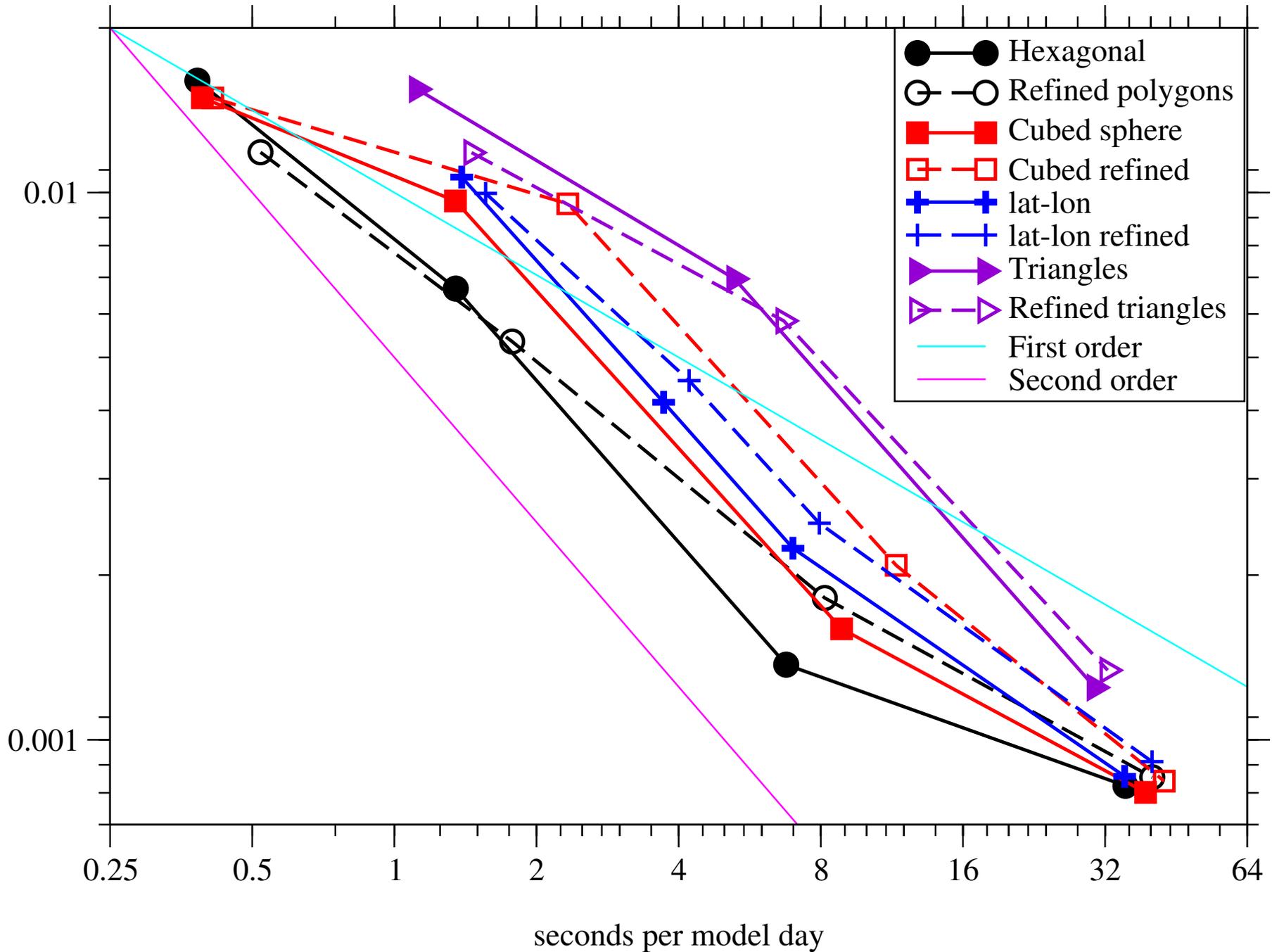
↓ Voronoi with refinement ($\delta x_{eq} = 4.3^\circ$)



↓ Triangular with refinement ($\delta x_{eq} = 4.3^\circ$)



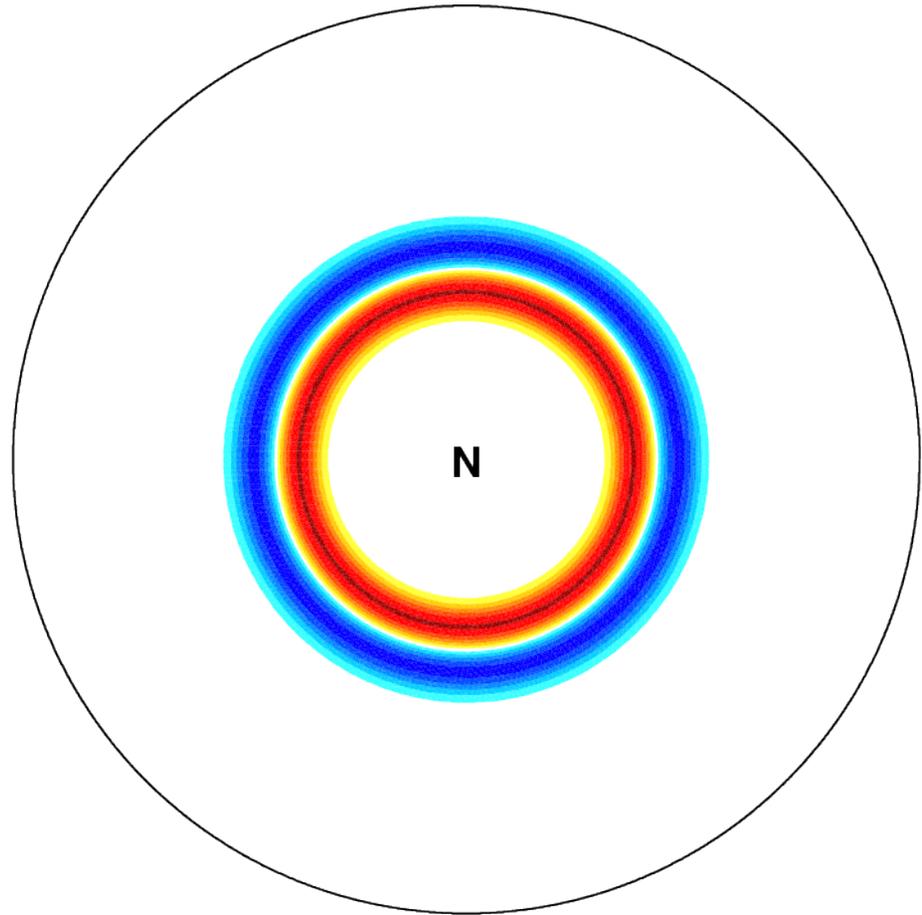
Root mean square errors after 15 days. Relative to T213 Spectral reference solution.
Models with different resolutions



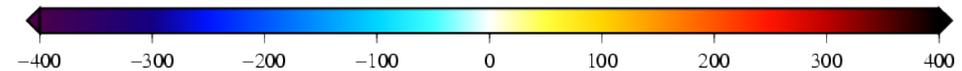
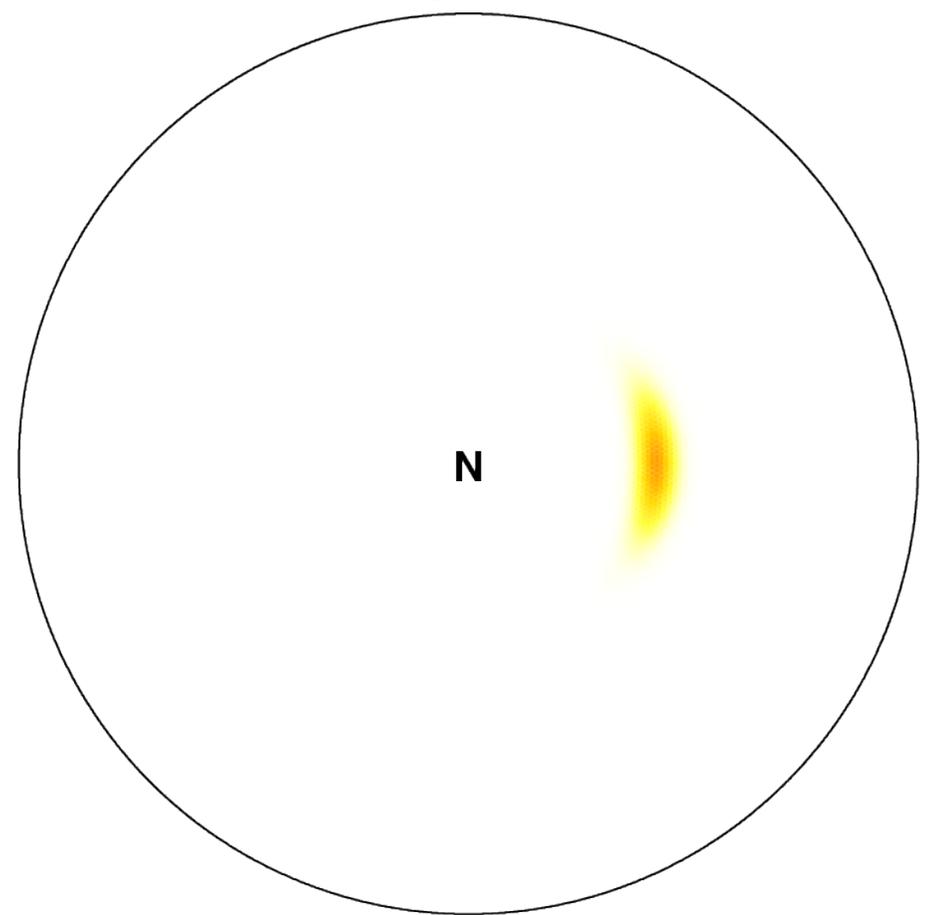
Barotropically Unstable Jet Test Case (Test case of Galewsky et al 2004)

Initially geostrophically balanced mid-latitude jet with very small unbalanced height perturbation

Initial vorticity (s^{-1})



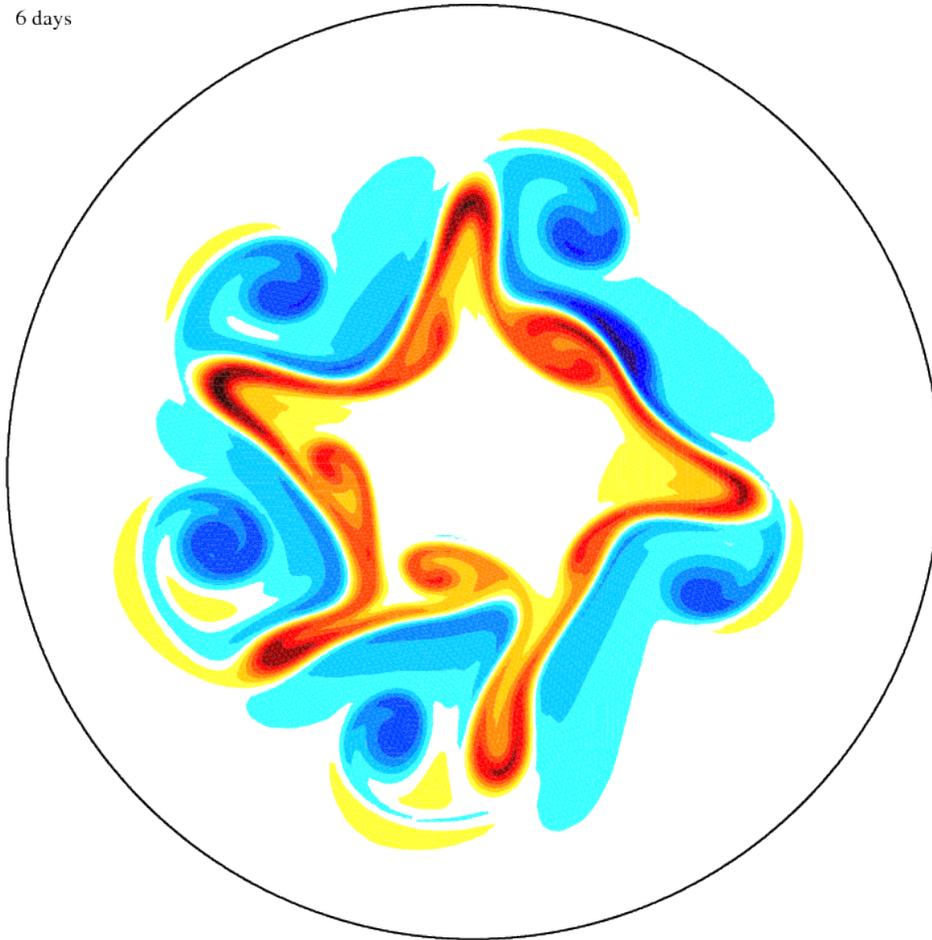
Initial height perturbation (m)



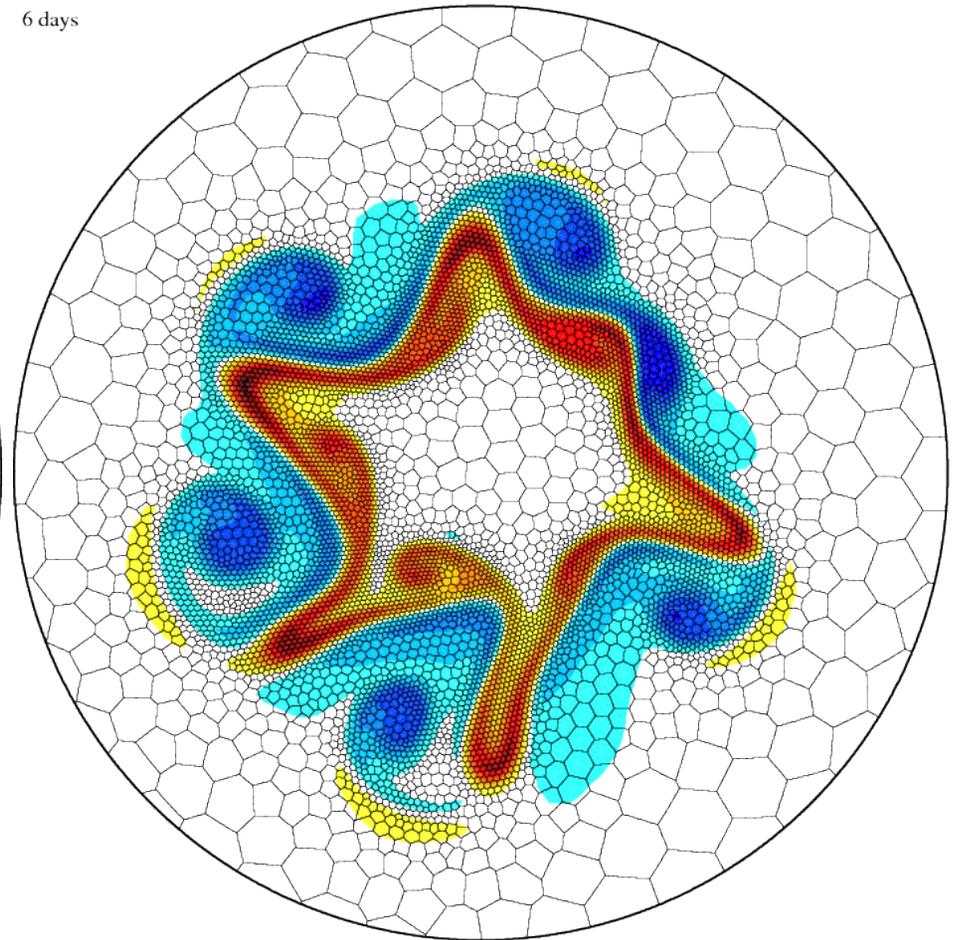
Adaptive Meshing of a Barotropically Unstable Jet

- Fixed hexagonal icosahedral mesh
- 40,962 cells, $\delta x \approx 100km$
- Relative vorticity (s^{-1})
- Adapt based on vorticity gradient
- Mesh adapts every 3 hours (18 time steps)
- 8,162 cells after 6 days

6 days



6 days



Predict mesh density requirement using coarse mesh solution

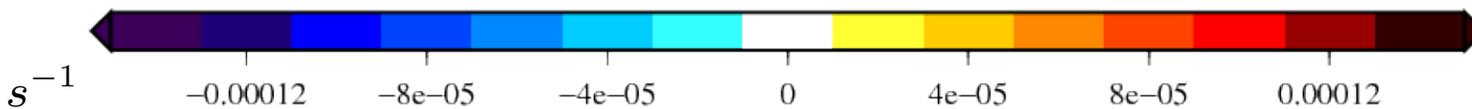
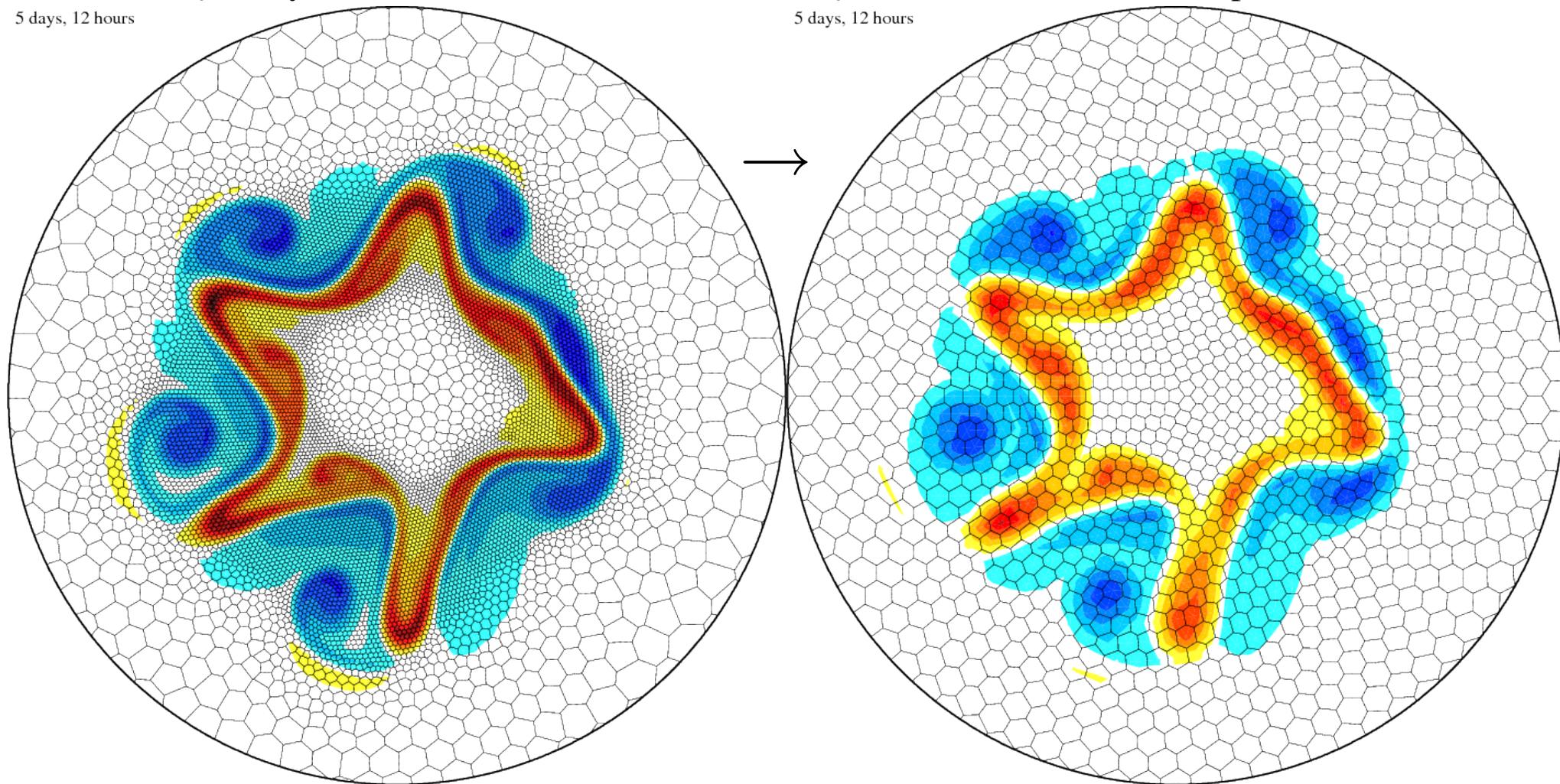
- Starting from adapted mesh solution (at time 5.5 days) ✓
- Map velocity and height onto coarse mesh and calculate vorticity on coarse mesh ↓ (2,562 cells)

ξ at day 5.5 on refined mesh

ξ on coarse mesh after interpolation of h and U

5 days, 12 hours

5 days, 12 hours

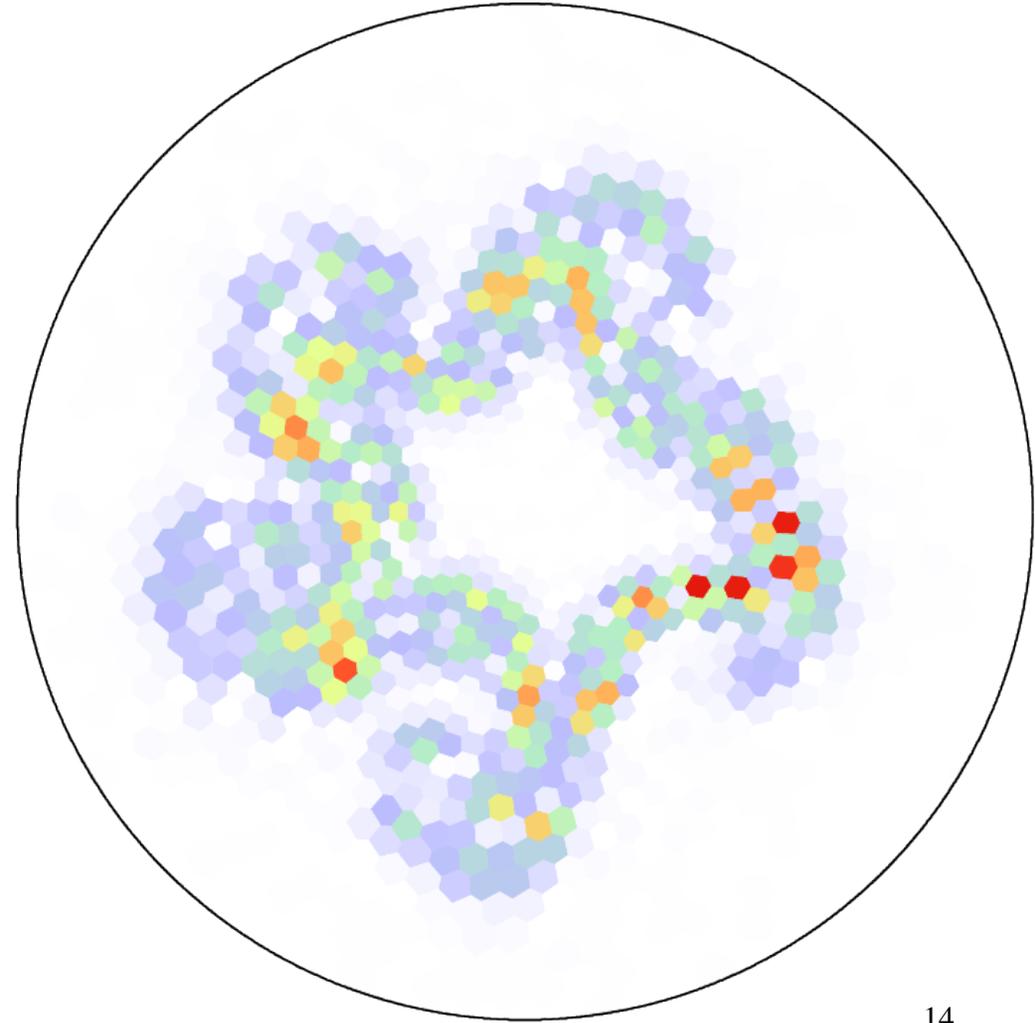
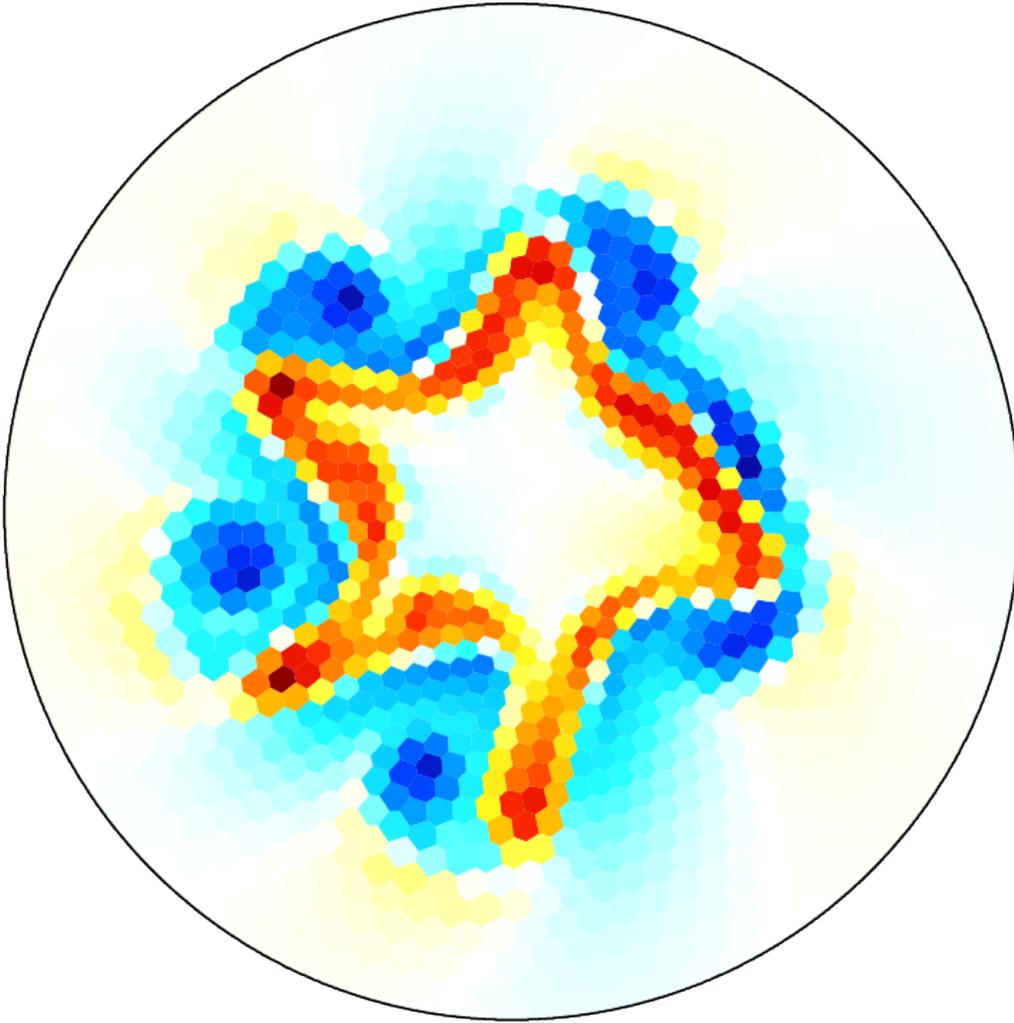


Predict mesh density requirement using coarse mesh solution

- Solve SWEs on coarse mesh with initial conditions of h and U interpolated from adapted mesh
- Advect the unresolved vorticity as a tracer within this solution
- Run for 12 hours (8 time steps), $\delta t = 90$ mins, $\delta x \approx 480km$

ξ solved on coarse mesh

$|\xi_f - \xi_c|$ interpolated onto coarse mesh and advected

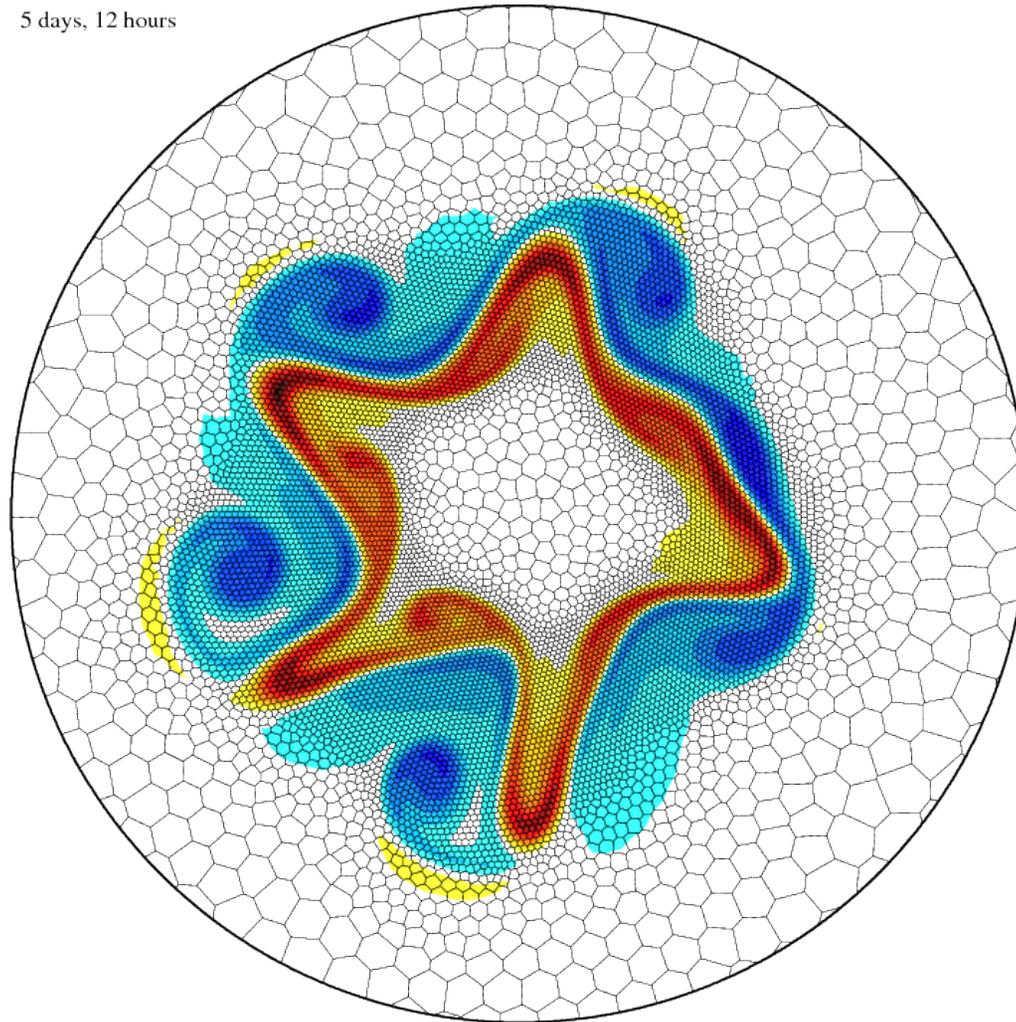


Predict mesh density requirement using coarse mesh solution

- These are combined to form a new “required resolution” field
- A new mesh is generated to satisfy the new “required resolution” field
- h and U are interpolated from the old to new adapted meshes
- AtmosFOAM run for a further 12 hours (72 time steps with 10 min time step)

5 days, 12 hours

ξ at 5.5 days on old mesh

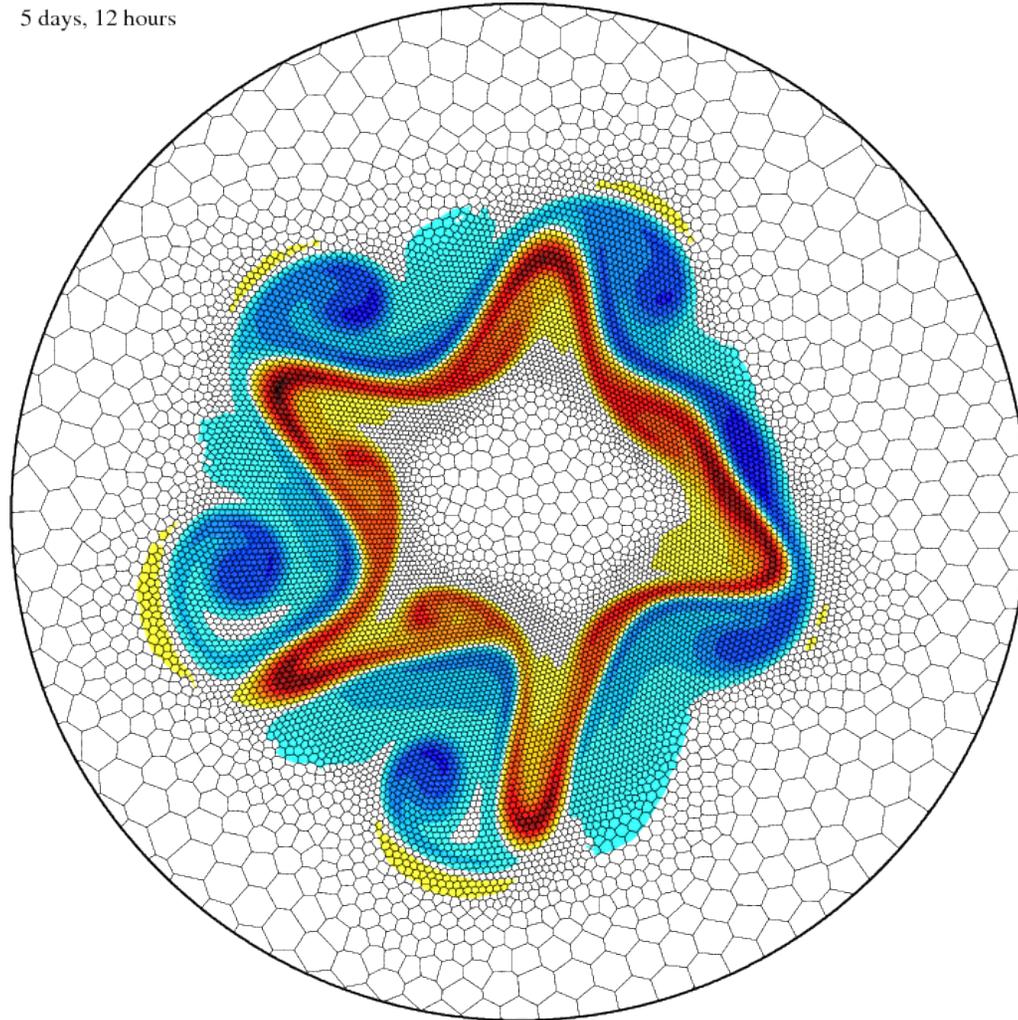


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5 days, 12 hours

ξ at 5.5 days on new mesh

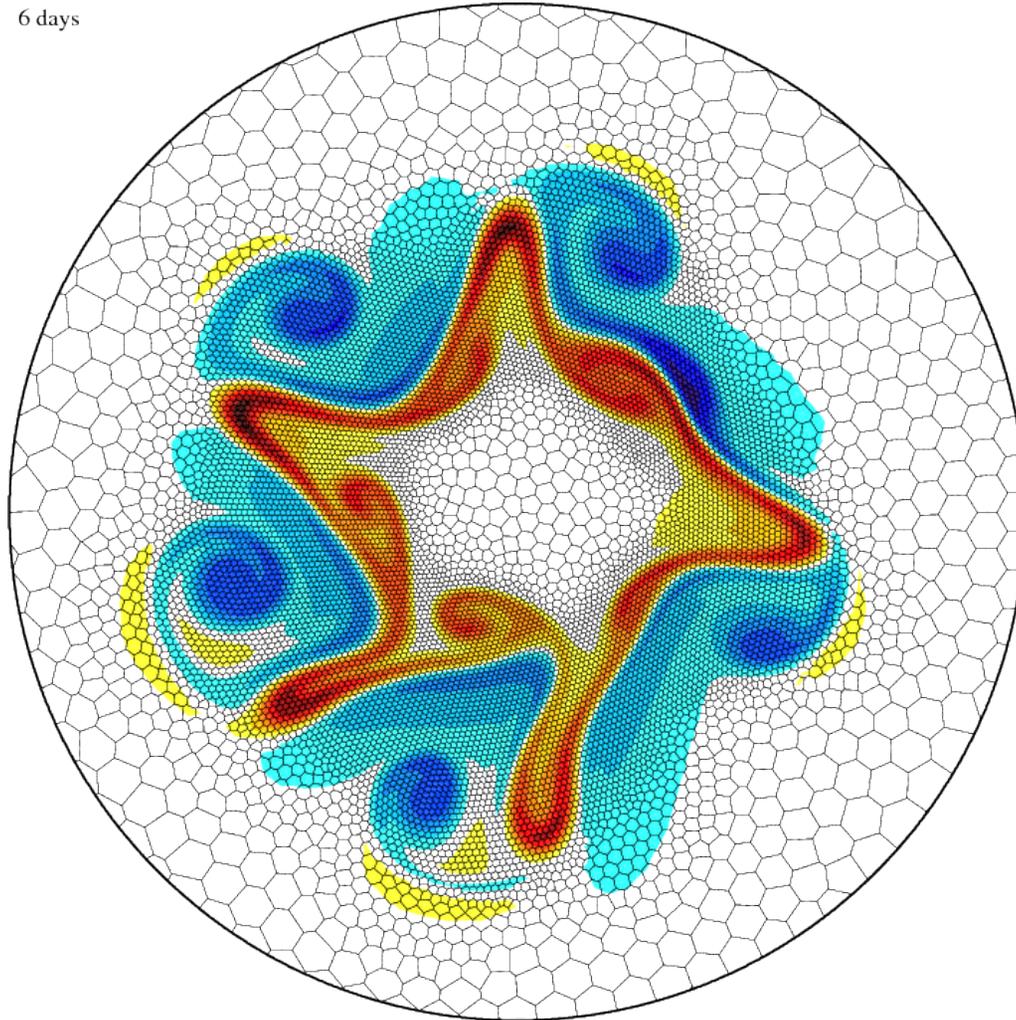


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6 days

ξ at 6 days on old mesh



Summary

- Local refinement of orography

Can be cost effective even without the effect of small scale diabatic processes
But gradual refinement might reduce truncation errors

- Unstructured or block-structured mesh adaptation?

Meshes of polygons have accuracy advantages

- No directional bias (eg. no alignment of flow with grid)
- Gradual refinement possible
- No pole or cube corner problems

- Adaptation criteria and adaptation frequency

Predicting mesh density for infrequent adaptation allows a more flexible compromise between efficiency, balance and conservation under adaptation

Future Work

- More work on long range prediction of mesh density requirements

- eg. where will mesh density be required to resolve convection over the next few hours

- Conservative mapping

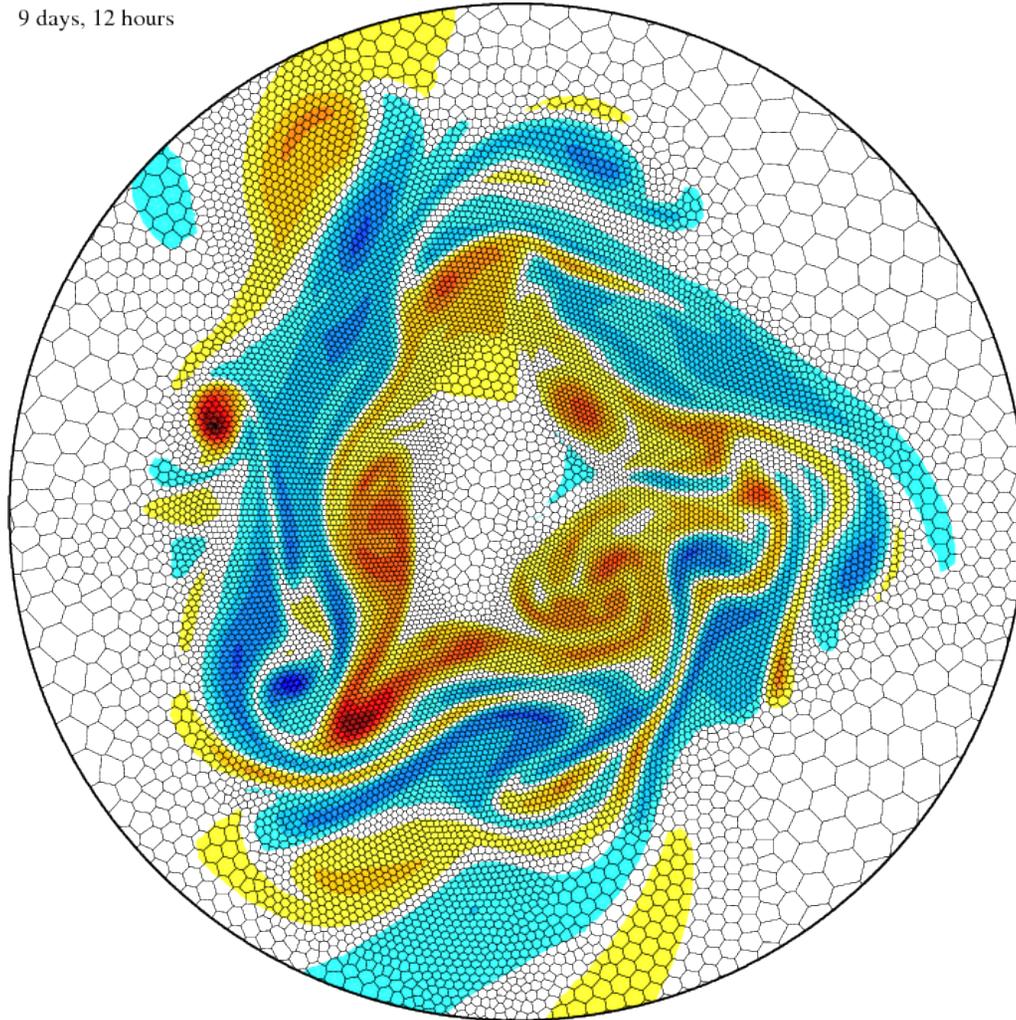
- Gradual anisotropic refinement of polygons

Predict mesh density requirement using coarse mesh solution

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- AtmosFOAM run for a further 12 hours (72 time steps with 10 min time step)

9 days, 12 hours

ξ at 9.5 days on old mesh

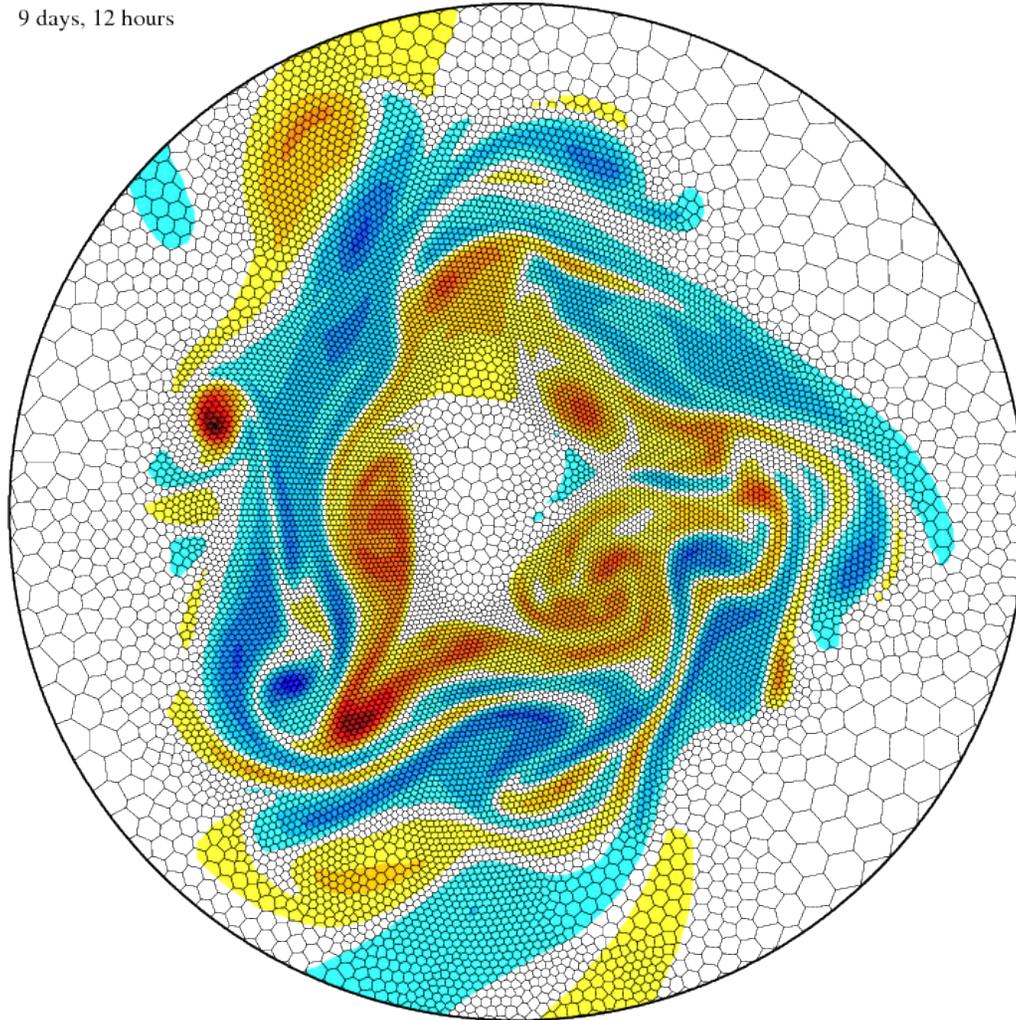


Predict mesh density requirement using coarse mesh solution

- These are combined to form a new “required resolution” field
- A new mesh is generated to satisfy the new “required resolution” field
- h and U are interpolated from the old to new adapted meshes
- AtmosFOAM run for a further 12 hours (72 time steps with 10 min time step)

9 days, 12 hours

ξ at 9.5 days on old mesh

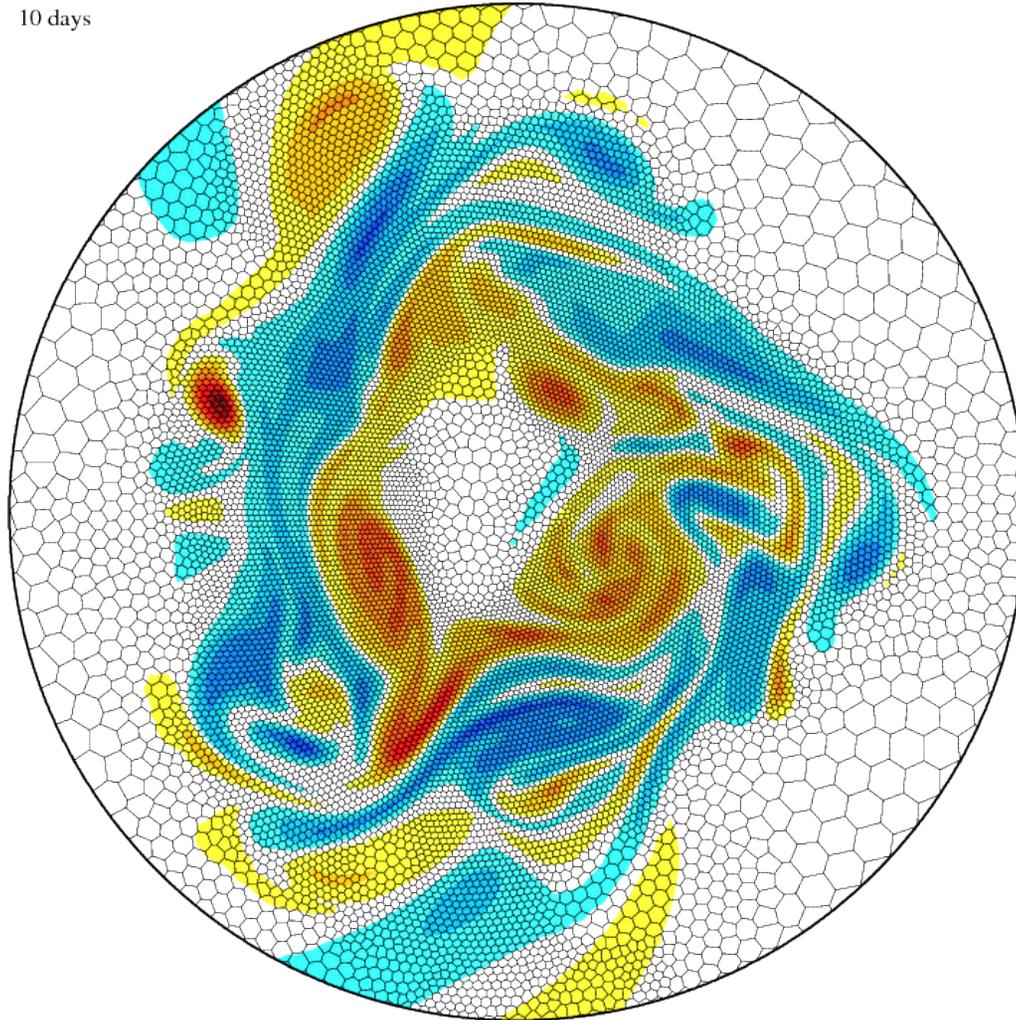


Predict mesh density requirement using coarse mesh solution

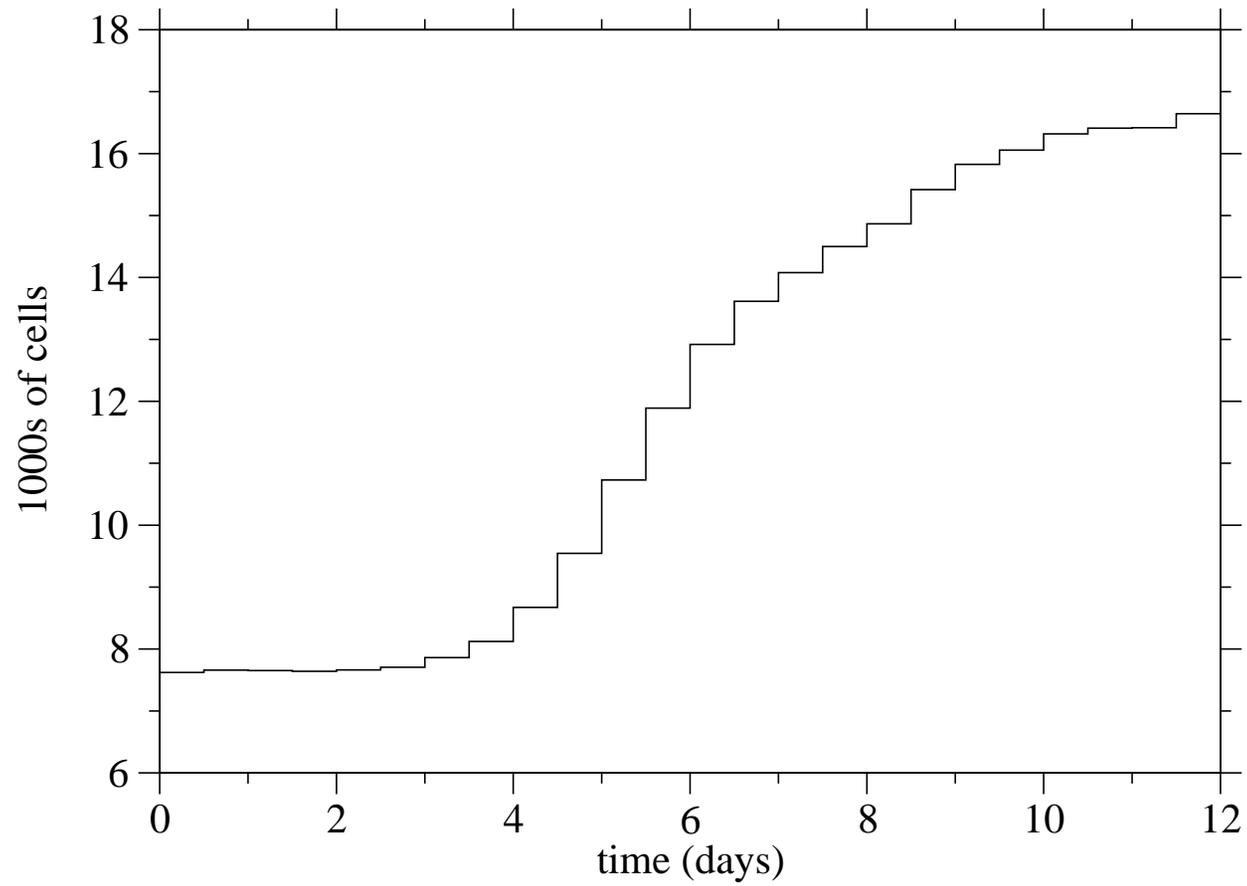
- These are combined to form a new “required resolution” field
- A new mesh is generated to satisfy the new “required resolution” field
- h and U are interpolated from the old to new adapted meshes
- AtmosFOAM run for a further 12 hours (72 time steps with 10 min time step)

10 days

ξ at 10 days on old mesh



Number of cells used during the run



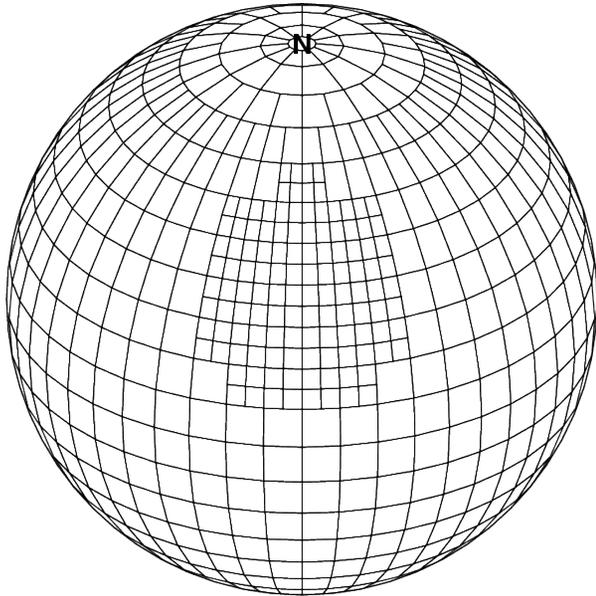
Mechanical Energy Conservation Errors

Williamson et al (1992) test case 5: flow over a mid-latitude mountain after 15 days.

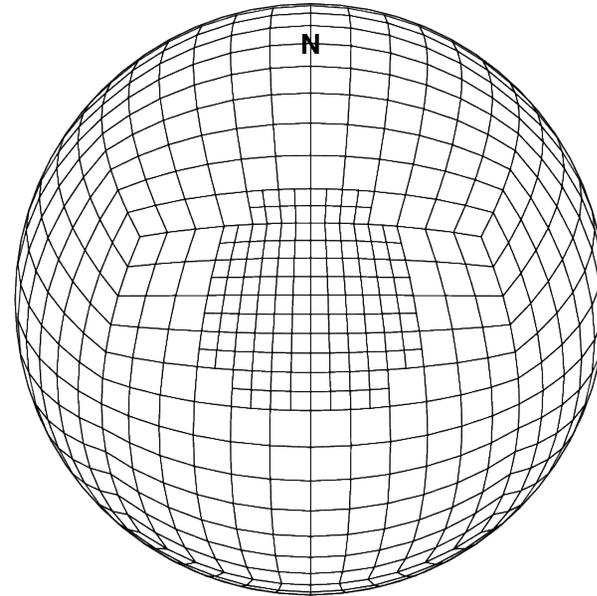
	Grid and resolution	Space order	Normalised Energy
AtmosFOAM	208km lat-lon	2	-5.1×10^{-6}
Spectral (Jakob et al., 1993)		T63	4.6×10^{-6}
AtmosFOAM	250km cubed sphere	2	1.0×10^{-6}
Spectral element (Thomas and Loft, 2002)		8	$\sim 10^{-6}$
AtmosFOAM	250km hexagons	2	-1.7×10^{-5}
PV (Thuburn, 1996)		1-3	1×10^{-5}
AtmosFOAM	120km triangles	2	-9.4×10^{-5}
ICON (Bonaventura and Ringler, 2005)		1-2	-10^{-4}
AtmosFOAM	156km cubed sphere	2	2.8×10^{-7}
Multi-moment finite volume (Chen and Xiao, 2008)		4	-4×10^{-6}
Discontinuous Galerkin (Läuter et al., 2008)	180km	$k = 6$	5.5×10^{-8}

- Which mesh structure is most accurate

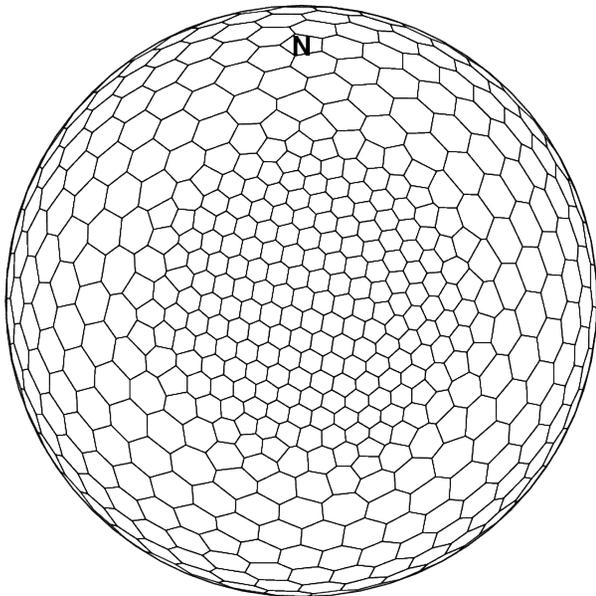
How best to achieve local refinement on the sphere



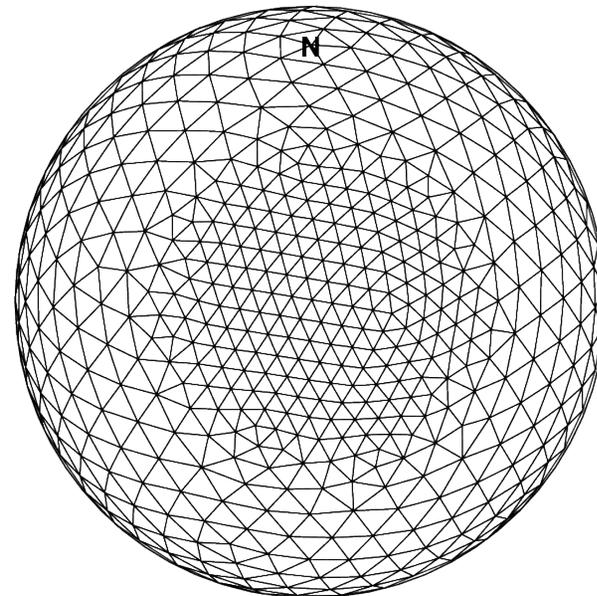
- Reduced lat-lon
- 2:1 refinement



- Equal angle cubed sphere
- 2:1 refinement

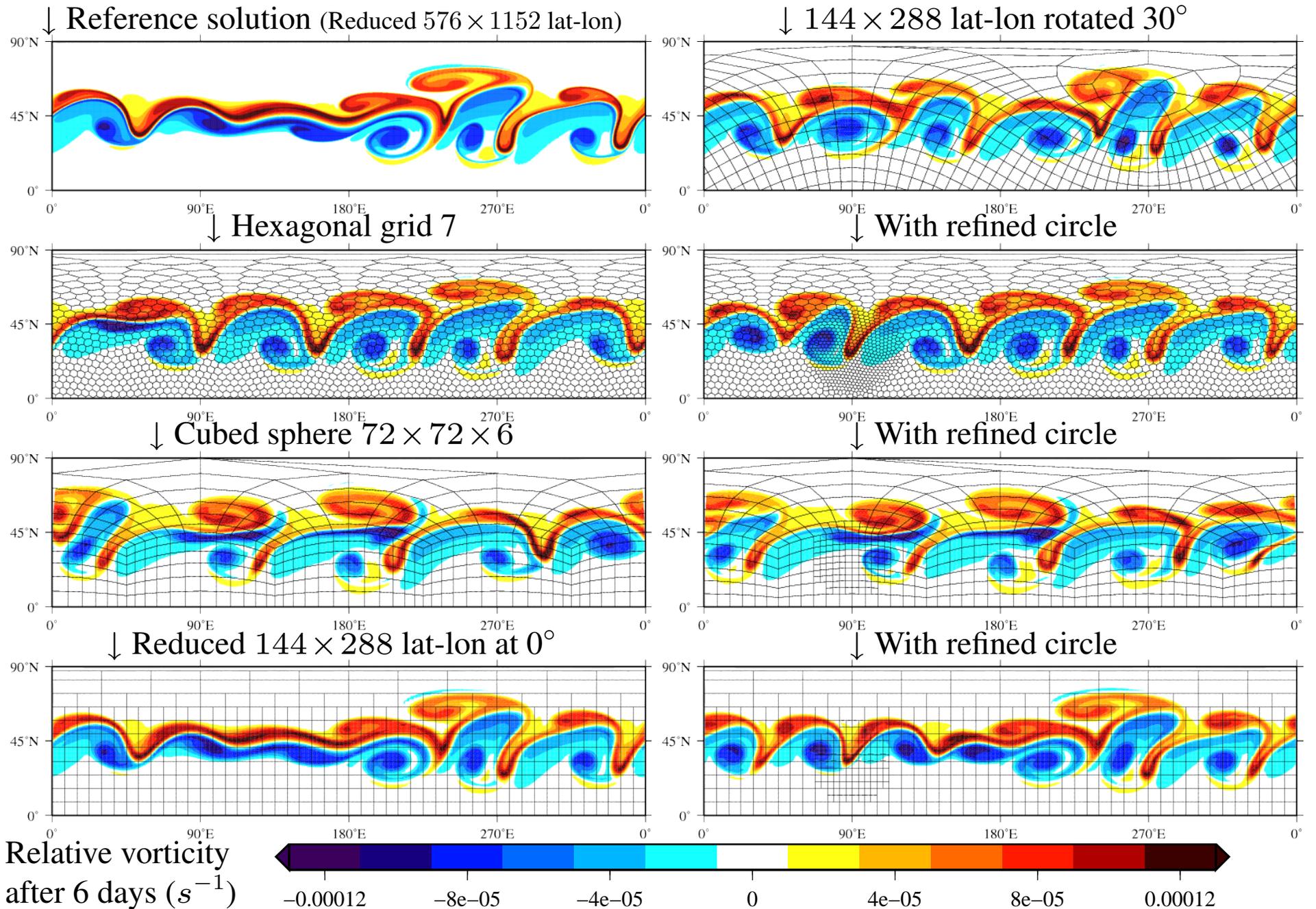


- Hexagonal icosahedral
- Voronoi refinement
- Triangulation of varying springs



- Triangular icosahedral
- Delaunay refinement
- Triangulation of varying springs

Barotropically Unstable Jet (Test case of Galewsky et al 2004)



Meshes plotted of squares are $\times 6$ coarser and of polygons $\times 4$ coarser for clarity