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Radar scattering by realistic ice aggregates

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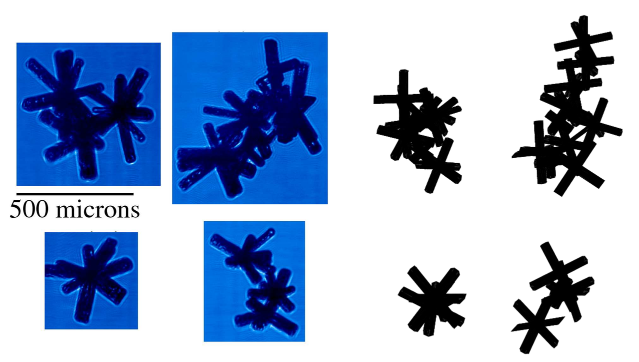
1 Introduction

Ice clouds have an important influence on the climate. Uncertainties about the microphysics of ice clouds make quantitative estimates of their radiative properties difficult, and these estimates are key to accurate climate modelling given the extensive global coverage of such clouds.

Cloud radar offers a powerful tool to probe the microphysics of such clouds, but accurate models of millimetre wave scattering by natural ice particles is needed to interpret the observations.



CPI simulation

In-situ aircraft studies indicate that aggregates (see CPI, left) are the dominant habit for particles larger than a few hundred microns, and it is these large particles which contribute most to the radar reflectivity.

A recent model of ice aggregation[a] has allowed realistic 'synthetic' aggregates to be simulated (shown alongside CPI above) and here we calculate the scattering from these in the expectation that the results should be a good approximation to natural ice aggregates.

[a]Westbrook *et al* (2004). *Geophys. Res. Lett.* **31** L15104

2 Rayleigh Scattering

Particles much smaller than the wavelength scatter coherently, and the reflectivity of an individual ice aggregate is:

$$Z_{\text{rayleigh}} = \frac{36}{0.93\pi^2} \times \gamma^2 v^2$$

where v is the volume of ice in the aggregate, and γ depends on the shape and dielectric constant (for ice $\epsilon \simeq 3.2$).
For a solid ice sphere $\gamma = |\frac{\epsilon-1}{\epsilon+2}| = 0.423$ (ie $Z = 0.19 \times \text{Diameter}^6$).

We used the discrete dipole approximation (DDA) to calculate Z for small, randomly oriented, aggregates of columns, and inferred γ from the results.

γ is insensitive to the size of the aggregate or its composing crystals, but does depend on the aspect ratio of the columns:

· For aggregates of squat columns (aspect ratio=1) there was a 9% (0.4dB) increase in z over an equivalent volume sphere ($\gamma = 0.440$).
· For aggregates of thin columns (aspect ratio=4) the reflectivity was 16% (0.6dB) larger, with $\gamma = 0.455$.

Unresolved issue: do aggregates have a preferential orientation? (max. area $\perp$ to flow? distribution of canting angles?) $\Rightarrow$ will increase Z.

3 Non-Rayleigh I: Rayleigh-Gans approximation

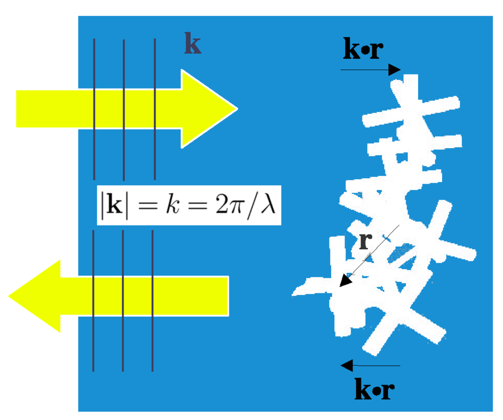
When the size of the aggregate approaches the wavelength λ, the reflectivity is reduced relative to the Rayleigh formula.

Well known exact Mie theory for homogeneous spheres. But ice aggregates are non-spherical and inhomogeneous!

We use the Rayleigh-Gans theory as a first approximation for our non-spherical aggregates. The contributions from each of the ice crystals in the aggregate interfere, reducing the reflectivity relative to the Rayleigh formula by a factor $0 < f < 1$:

$$Z = Z_{\text{rayleigh}} \times f$$

We assume that although the aggregate is close to the wavelength, the composing crystals are much smaller and act as Rayleigh scatterers:



Consider the waves at a crystal, position $\mathbf{r}$ relative to the centre of mass:

Phase difference in applied wave = $\mathbf{k} \cdot \mathbf{r}$

Phase difference in reflected wave = $\mathbf{k} \cdot \mathbf{r}$

Total phase difference is $2\mathbf{k} \cdot \mathbf{r}$.

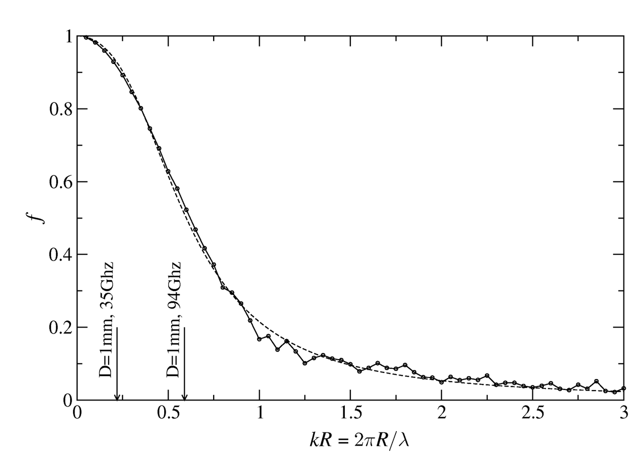
So the reduction in Z relative to the Rayleigh formula is:

$$f = \left[\frac{1}{v}\sum e^{i2\mathbf{k}\cdot\mathbf{r}} dv\right]^2$$

summing over each crystal (volume dv) in the aggregate.
$\Rightarrow$ **We can calculate the reflectivity Z very easily.**

· f is independent of the shape or size of the composing ice crystals (the comparatively long wavelength of the radar can't resolve these details).

· Averaged over fluctuations in detailed geometry depends only on aggregate radius R relative to λ:



Arrows show values of kR for typical cirrus aggregates (1mm maximum dimension) at 35 and 94-GHz frequencies.

Given the volume v and size R of an aggregate $\Rightarrow$ know Z.

The only problem is that it neglects the influence the scattered wave from one crystal has on the incident wave on another, so it's **only accurate for small, low density aggregates** (ie. aggregates of small, thin crystals).

$\Rightarrow$ **TO DO BETTER = DISCRETE DIPOLE CALCULATIONS**

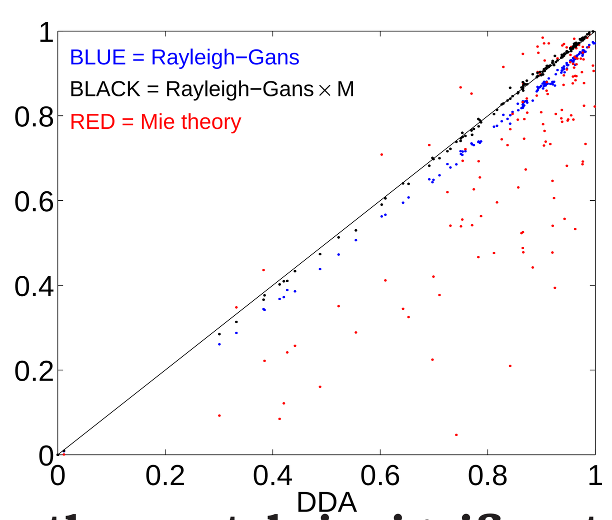
4 Non-Rayleigh II: DDA calculations

The discrete dipole approximation includes the interaction between the scattered electric fields from the different crystals (ie. 'multiple scattering' within the aggregate).

In principle DDA is as accurate as you like $\rightarrow$ just need to make the dipole spacing small enough. Here we use 200 dipoles for each crystal.

We have calculated the ratio of Z_{real} (=DDA) to Z_{rayleigh} for aggregates of realistic size/density at 94-Ghz (=x axis & solid line):

The pristine ice crystals composing the aggregates were columns, length 160 and 200μm, aspect ratio=1.



Blue points are Rayleigh-Gans (one point = one aggregate), underestimates DDA reflectivity (solid line) by up to 20%

So **multiple scattering between the crystals is significant**.

Red is Mie theory with Maxwell-Garnett mixing rule. Much more scatter in the data because variability in the aggregate structure is not captured by the 'homogenised' sphere. Overall trend is that **Mie theory underestimates the reflectivity by as much as 50%**

We have used our DDA calculations to parameterise a multiple scattering correction $\mathcal{M}$ to the RG formula so that:

$$Z_{\text{real}} = Z_{\text{rayleigh}} \times f \times \mathcal{M}$$

Mean-field theory of multiple scattering[a] leads us to expect:

$$\mathcal{M} = \left[\frac{1}{1 - \gamma F_v S_0(kR)^2}\right]^2$$

where $F_v \equiv v/\frac{4}{3}\pi R^3$ and we estimate from our data that $S_0 \simeq 1.2$.

Rayleigh-Gans f corrected by this factor shown by black points $\Rightarrow$ matches DDA almost perfectly.
Have found it works well for a variety of (realistic) aggregate sizes, densities, and crystal aspect ratios.

We are now in a position to accurately estimate the reflectivity from an ice aggregate $Z = Z_{\text{rayleigh}} \times f \times \mathcal{M}$, given knowledge of its volume, size and the geometry of its composing crystals.

Future work: Integrate Z over particle size distribution to give total reflectivity per unit volume cloud. The challenge is to use the scattering physics above to interpret dual-wavelength data from real ice clouds.

[a]Berry and Percival (1986) *Optica Acta* **33** 577