A theory of growth by differential sedimentation, with application to snowflake formation

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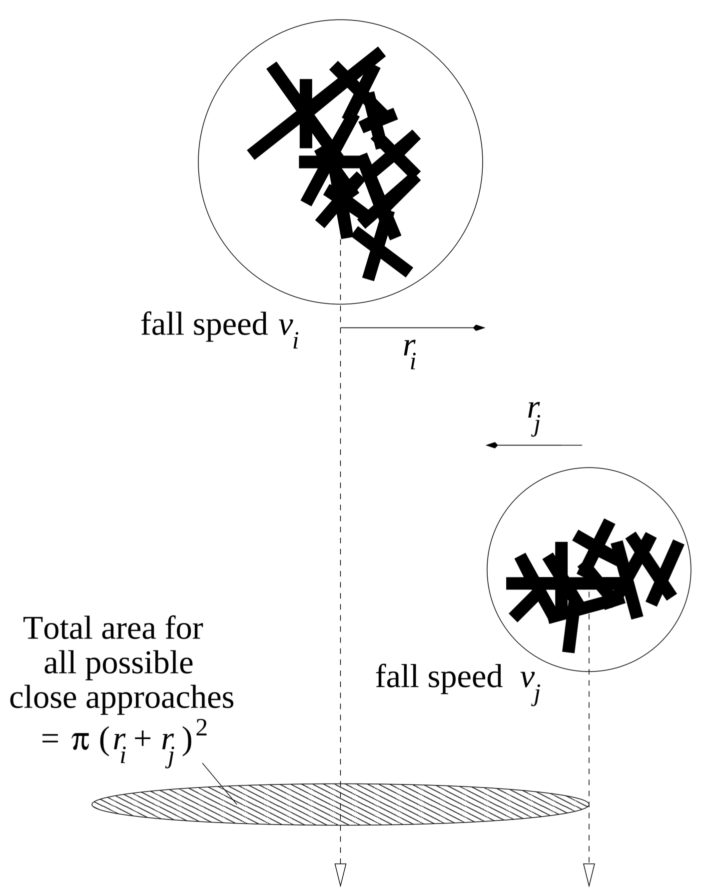
1 Introduction

Aggregation through differential sedimentation of particles in a fluid plays an important role in several physical systems, such as ice crystal aggregation in clouds (to produce snowflakes), or coagulation of clay in estuaries.

A simple model of this process has been constructed, and the geometry of the aggregates is found to feed back on the dynamics in such a way as to stabilise the fractal dimension and growth exponents at readily predictable values.

The model appears to capture the essential physics of snowflake aggregation in Cirrus cloud, our theoretical results providing good agreement with experimental cloud data. Cirrus clouds cover 20% of the earth's surface on average, and play a key role in its radiation balance. Modelling snowflakes in Cirrus is therefore important for understanding the effect these clouds have on the weather, as well as for interpreting radar and satellite data.

2 Aggregation Model



A rate of 'close approach' between two sedimenting particles is calculated:

$$K_{ij} = \pi (r_i + r_j)^2 |v_i - v_j|$$

In our simulations we check for collision along a possible close approach trajectory. In our theory we take K_{ij} to represent the true collision rate.

We assume that the aggregates re-orient themselves at random between collisions, and that collisions result in rigid, irreversible sticking on contact.
The monomer particles were simple rods or crosses: however the results presented here are universal, and insensitive to the monomer geometry.

3 Fall speeds

We assume that the drag on an aggregate is proportional to the drag on a sphere of the same weight and size. This is:

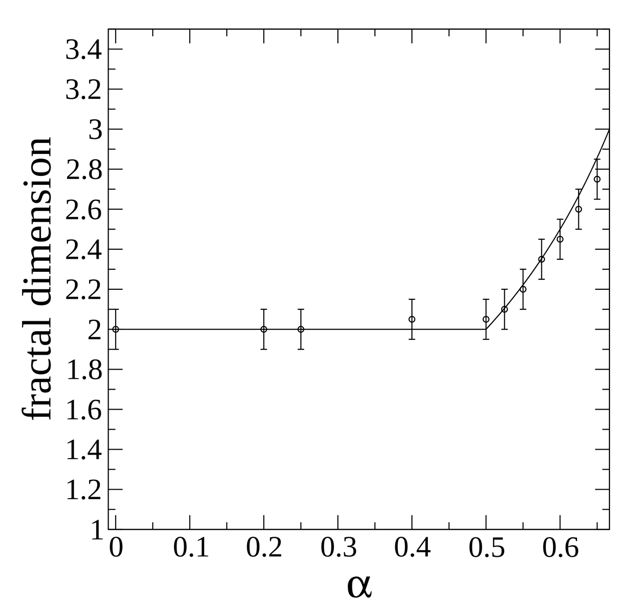
$$v \sim \frac{\nu_k}{r} \left(\frac{mg}{\rho \nu_k^2} \right)^{\alpha}$$

in the viscous ($\alpha = 1$) and inertial ($\alpha = \frac{1}{2}$) limits, where m is the mass of the aggregate, ρ is the density of the fluid, and ν_k is the kinematic viscosity.

We consider a general α to demonstrate the feedback between the geometry of the aggregates and the growth exponents.

4 Fractal dimension

The simulated aggregates were found to have a fractal geometry, characterised by the mass-radius scaling $m \sim r^{d_f}$ with $d_f < 3$.



The fractal dimension d_f varies with the hydrodynamic parameter α, as left (points are simulation data). The collision rate for big + little scales as:

$$K(m_i \ll m_j) \sim (m_i)^{\mu} (m_j)^{\nu}$$

where $\mu = \min(0, \alpha - d_f^{-1})$ and $\nu = \max(\alpha + d_f^{-1}, 2d_f^{-1})$.

Van Dongen (1987) has shown that if $\nu > 1$, big + little collisions completely dominate the aggregation kinetics. In this case small aggregates 'fill up' the large ones, making them compact ($d_f \to 3$), lowering ν. If $\nu < 1$, the distribution is relatively monodisperse, yielding open structures with low d_f, pulling ν back up.
The system self-organises then to settle at $\nu = 1$. The fractal dimension of the aggregates is therefore:

$$d_f = \max\{2, (1 - \alpha)^{-1}\},$$

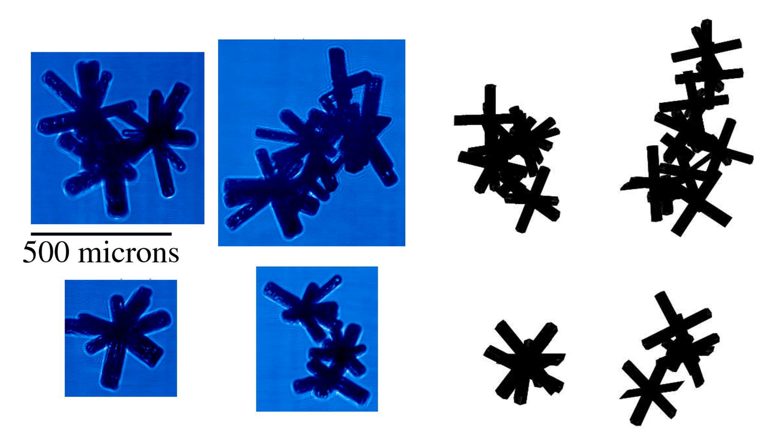
shown as the solid line on the figure.

Heymsfield *et al* (2002) have analysed experimental aircraft data, and found $d_f = 2.04$. This compares to our simulations ($d_f = 2.05 \pm 0.1$) and theory ($d_f = 2$) for inertial flow (the regime appropriate to snowflakes).

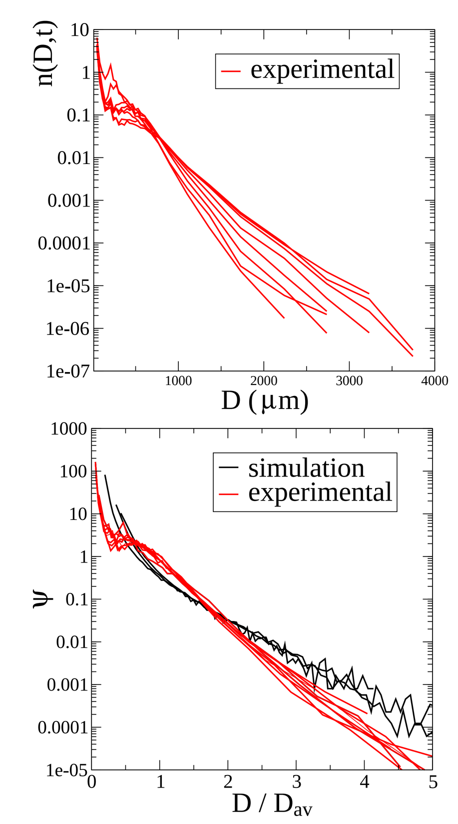
5 Experimental cloud images

Cloud particle imagers mounted on aircraft have captured ice aggregates in Cirrus, as shown below.

Also shown for comparison are some of our simulated aggregates. From these images, Korolev & Isaac (2003) measured the aspect ratio, and found an asymptotic value of $\simeq 0.6 - 0.7$. In simulations this ratio is 0.65 ± 0.05.



6 Size distribution



The size distribution scales dynamically [see Westbrook *et al* (2004)]:

$$n(D, t) = D_{av}^{3} \psi(D/D_{av})$$

where D is the maximum dimension, $D_{av}(t)$ is an average span, and ψ is a universal function. We use this formula to rescale experimental size spectra (top figure) collapsing them onto the curve ψ (lower figure). Overlaid is simulation data: the correspondance between experiment and simulation is good, though not perfect: in particular the kink around $D/D_{av} \simeq 1$ remains unexplained.

7 References

A.J. Heymsfield *et al*, J. Atmos. Sci. 59, 3 (2002)
A. Korolev and G. Isaac, J. Atmos. Sci. 60, 1795 (2003)
P.G. Van Dongen, J. Phys. A 20 1889 (1987)
and for further information on this poster, see: C.D. Westbrook *et al*, submitted to Phys. Rev. E (2004): preprint on http://arxiv.org
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