# Entropy versus APE production: On the buoyancy power input in the oceans energy cycle

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This letter argues that the current controversy about whether $W_{buoyancy}$, the power input due to the surface buoyancy fluxes, is large or small in the oceans stems from two distinct and incompatible views on how $W_{buoyancy}$ relates to the volume-integrated work of expansion/contraction $B$. The current prevailing view is that $W_{buoyancy}$ should be identified with the net value of $B$, which current theories estimate to be small. The alternative view, defended here, is that only the positive part of $B$, i.e., the one converting internal energy into mechanical energy, should enter the definition of $W_{buoyancy}$, since the negative part of $B$ is associated with the non-viscous dissipation of mechanical energy. Two indirect methods suggest that by contrast, the positive part of $B$ is potentially large.

## 1. Introduction

For over a century now, oceanographers have debated about whether $W_{buoyancy}$, the power input due to surface buoyancy fluxes, is negligibly small or large and comparable to the mechanical power input due to the wind and tides? Part of the difficulty in addressing the issue is that while surface buoyancy fluxes give rise to buoyancy forces, they do not themselves exert any force on the fluid, and hence technically produce neither work nor power on their own. This difficulty is compounded by the fact that mechanical energy created by diabatic effects must ultimately result from a conversion from internal energy ($IE$) mediated by compressible effects via the work of expansion/contraction:

$$B = \int_V P \frac{D\upsilon}{Dt} \mathrm{d}m, \qquad (1)$$

as discussed by *Welander* [1991] for instance, where $P$ is the pressure, $\upsilon = 1/\rho$ is the specific volume, $\rho$ is the density, and $\mathrm{d}m = \rho \mathrm{d}V$ is the mass of an elementary fluid parcel. However, because low Mach number fluids such as seawater are generally regarded as nearly incompressible, they are nearly almost tackled by means of the Boussinesq approximation, rarely if ever in the context of the fully compressible Navier-Stokes equations, so that no rigorous exact results currently exist that could give us insights into the actual value of $B$ in the oceans.

*Lorenz* [1955]'s available potential energy (APE) theory has long been the accepted framework to understand how diabatic effects drive motions, by suggesting that they do so by making a certain fraction of the total potential energy (i.e., the sum of gravitational potential and internal energies) available for conversion into kinetic energy, as recently discussed by *Hughes et al.* [2009] and *Tailleux* [2009]. In the APE theory, therefore, the concept of buoyancy power input $W_{buoyancy}$ is naturally identified with the production rate of available potential energy $G(APE)$, which physically represents the amount of potential energy being released per unit time that is in principle convertible into KE. In the oceans, *Oort et al.* [1994] estimated $G(APE) = 1.2 \pm 0.7\,\mathrm{TW}$, and concluded that $W_{buoyancy}$ was comparable to the power input by the mechanical forcing.

Over the past ten years or so, however, APE theory was challenged by *Munk and Wunsch* [1998] and others on account of its apparent conflict with a popular interpretation of *Sandström* [1908]'s "theorem" (see *Kuhlbrodt* [2008] for a translation), according to which $W_{buoyancy}$ can only be significant if heating occurs on average at higher pressure than cooling, whereas in the oceans, heating and cooling are applied at approximately constant pressure. *Sandström* [1908] failed to recognize, however, that molecular diffusion, possibly enhanced by turbulence [1], always induces an additional internal heating and cooling mode such that the net (external+internal) heating always occurs on average at higher pressure that the net cooling regardless of the particular vertical arrangement of the external sources of heating and cooling, as illustrated by *Marchal* [2005]. More useful is *Paparella and Young* [2002]'s anti-turbulence theorem, which for the first time provided a rigorous quantitative constraint linking the net work of expansion contraction $B_{bq}$ with the overall viscous dissipation $D(KE)$ in the idealized context of a Boussinesq ocean with a linear equation of state, suggesting that $D(KE)$ would be several orders of magnitude smaller than observed if the buoyancy forcing acted alone. Subsequently, *Wang and Huang* [2005] suggested that $B_{bq}$ should be regarded as the relevant definition of $W_{buoyancy}$, which they estimated, using typical oceanic values, to be $B_{bq} = O(15\,\mathrm{GW})$.

The smallness of $B_{bq}$, in contrast to *Oort et al.* [1994]'s "too-large" value of $G(APE)$, was arguably more in agreement with the prevailing idea at the time that $W_{buoyancy}$ should be small because of Sandstrom's "effect". This undoubtedly significantly contributed to the rapid and widespread acceptance of $B_{bq}$ as the relevant definition of $W_{buoyancy}$, as indicated by the fact that the most recent reviews of ocean energetics by *Wunsch and Ferrari* [2004] and *Kuhlbrodt et al.* [2007] cite *Paparella and Young* [2002] and *Wang and Huang* [2005]'s studies, but elude any discussion of *Oort et al.* [1994]'s views on the oceanic energy cycle. So far, however, it is hard to understand why we should disregard APE theory as the relevant framework to discuss the energetics of buoyancy forcing in the oceans, especially given that APE theory remains the dominant paradigm for understanding the energetics of the global atmospheric circulation. Why APE theory should work for the atmosphere but not for the oceans deserves clarification if we are to make progress. In this letter, our aim is to clarify the conceptual differences between APE theory and *Wang and Huang* [2005]'s approach, in order to help identifying which approach appears to be more suited to quantify $W_{buoyancy}$. To that end, we develop a number of physical arguments rooted in a first principles analysis of the fully compressible Navier-Stokes equations, while also trying to link the issue to the classical thermodynamic theory of heat engines, which has been the main approach to quantify how much power can be created by hot and cold sources in the literature.

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## 2. Theory of the Oceanic Energy Cycle

### 2.1. Forcing Versus Dissipation

In order to understand the nature of the buoyancy forcing in the oceans, it is first necessary to recall some basic ideas about forcing and dissipation. This is done here in the context of the fully compressible Navier-Stokes equations for a binary fluid, which is widely accepted as the relevant theoretical framework to describe the oceanic circulation, viz.,

$$\rho\frac{D\mathbf{v}}{Dt} + 2\mathbf{\Omega}\times(\rho\mathbf{v}) = -\nabla P - \rho\nabla\Phi + \nabla\cdot\mathbf{S} \qquad (2)$$

$$\frac{D\rho}{Dt} + \rho\nabla\cdot\mathbf{v} = 0 \qquad (3)$$

$$\rho\frac{DS}{Dt} = -\nabla\cdot(\rho\mathbf{F}_S) \qquad (4)$$

$$\rho\frac{D\eta}{Dt} = -\nabla\cdot(\rho\mathbf{F}_\eta) + \rho\dot{\eta}_{irr} + \frac{\rho\varepsilon_K}{T}, \qquad (5)$$

where $\mathbf{v} = (u, v, w)$ is the three-dimensional velocity field, $\rho$ is density, $P$ is pressure, $T$ is temperature, $S$ is salinity, $\mathbf{\Omega}$ is Earth's rotation vector, $\Phi$ is the geopotential, $\mathbf{S}$ is the deviatoric stress tensor, $\mathbf{F}_S$ is the molecular diffusive salt flux, $\eta$ is the specific entropy, $\varepsilon_K$ is the viscous rate of kinetic energy dissipation, $\mathbf{F}_\eta$ is the molecular flux of entropy accomplished by all irreversible molecular diffusive processes, $\dot{\eta}_{irr}$ is the local rate of irreversible entropy production due to irreversible molecular diffusive fluxes. These equations are usually closed by an equation of state for the specific internal energy $e = e(\eta, S, \upsilon)$, expressed in terms of specific entropy $\eta$, salinity $S$, and specific volume $\upsilon$, whose partial derivatives define the temperature, pressure, and relative chemical potential as follows: $T = T(\eta, S, \upsilon) = \partial e/\partial\eta$, $P = P(\eta, S, \upsilon) = -\partial e/\partial\upsilon$, and $\mu = \mu(\eta, S, \upsilon) = \partial e/\partial S$.

Since the central issue about $W_{buoyancy}$ revolves on understanding how internal energy created by the surface buoyancy forcing is converted into mechanical energy, we form two separate evolution equations for the mechanical energy $\mathbf{v}^2/2 + \Phi$ and specific internal energy as follows:

$$\rho\frac{D}{Dt}\left(\frac{\mathbf{v}^2}{2} + \Phi\right) + \nabla\cdot(P\mathbf{v}) = \rho P\frac{D\upsilon}{Dt} + \mathbf{v}\cdot\nabla\cdot\mathbf{S}, \qquad (6)$$

$$\rho\frac{De}{Dt} = -\nabla\cdot(\rho\mathbf{F}_q) - \rho P\frac{D\upsilon}{Dt} + \rho\varepsilon_K, \qquad (7)$$

where the diffusive "heat" flux $\mathbf{F}_q$ is shown by *Tailleux* [2010] to be related to the diffusive fluxes of entropy and salt, as well as to the irreversible entropy production, by the following relations:

$$\mathbf{F}_\eta = \frac{\mathbf{F}_q - \mu\mathbf{F}_S}{T}, \qquad \dot{\eta}_{irr} = \mathbf{F}_q\cdot\nabla\frac{1}{T} - \mathbf{F}_S\cdot\nabla\frac{\mu}{T}, \qquad (8)$$

the precise forms of $\mathbf{F}_q$, $\mathbf{F}_S$ and $\mathbf{F}_\eta$ being left unspecified as unimportant for our argument. Upon integration over the whole ocean volume, the mechanical energy equation becomes:

$$\frac{d}{dt}\int_V \rho\left(\frac{\mathbf{v}^2}{2} + \Phi\right) dV$$
$$= -P_a\frac{dV_{ol}}{dt} + \int_V P\frac{D\upsilon}{Dt}\,\mathrm{d}m + \int_V \mathbf{v}\cdot\nabla\cdot\mathbf{S}\,\mathrm{d}V \qquad (9)$$

This result shows that mechanical energy changes as the result of: 1) the work of the atmospheric pressure $P_a$ against volume changes ($V_{ol}$ is the ocean volume); 2) conversion with/from internal energy via the work of expansion/contraction, 3) the work done against the stress tensor. As is well known, integrating the latter over the whole ocean volume allows one to rewrite the overall work against the stress tensor as the difference between the work done by the wind stress on the ocean surface velocity minus the total viscous dissipation as follows:

$$\int_V \mathbf{v}\cdot\nabla\mathbf{S}\,\mathrm{d}V = \underbrace{\int_S \mathbf{v}_S\cdot\tau\,\mathrm{d}S}_{G(KE)} - \underbrace{\int_V \rho\varepsilon_K\,\mathrm{d}V}_{D(KE)} \qquad (10)$$

where $G(KE)$ is in general positive, whereas $D(KE)$ is always positive definite. Physically, the work of expansion/contraction is related to the heat engine behavior of the oceans, but note here that the cases $PD\upsilon/Dt > 0$ and $PD\upsilon/Dt < 0$ are physically fundamentally different, given that only in the first case is internal energy converted into mechanical energy and hence acting as a heat engine, since the opposite conversion implies a non-viscous dissipation of mechanical energy. To formalize this distinction, we define $\dot{w} = PD\upsilon/Dt$ to simplify notations, and write:

$$B = \underbrace{\int_V \frac{\dot{w} + |\dot{w}|}{2}\,\mathrm{d}m}_{B^+} - \underbrace{\int_V \frac{|\dot{w}| - \dot{w}}{2}\mathrm{d}m}_{B^-}, \qquad (11)$$

where $|.|$ denotes the absolute value of a quantity, so that $B^+$ and $B^-$ are both positive by construction. Assuming now a (statistically) steady-state, Eqs. (10) and (11) suggest to write the mechanical energy balance under the form:

$$\underbrace{\overline{G(KE) + B^+}}_{forcing} = \underbrace{\overline{B^- + D(KE)}}_{dissipation}, \qquad (12)$$

where the overbar denotes a long term temporal average, assuming for simplicity that only the wind contributes to the mechanical forcing [2]. (The work done by the atmospheric pressure vanishes on average if atmospheric pressure is constant). The corresponding energy diagram is illustrated in Fig. 1. Physically, such a decomposition suggests to regard $B^+$ as part of the forcing, since it acts as a net source of mechanical energy, and $B^-$ as a form of non-viscous dissipation, since it acts as a net sink of mechanical energy. The central question here is whether such a decomposition implies that $B^+$ should be identified with $W_{buoyancy}$? The answer is not straightforward, however, because it is not immediately apparent that $B^+$ should necessarily be linked to

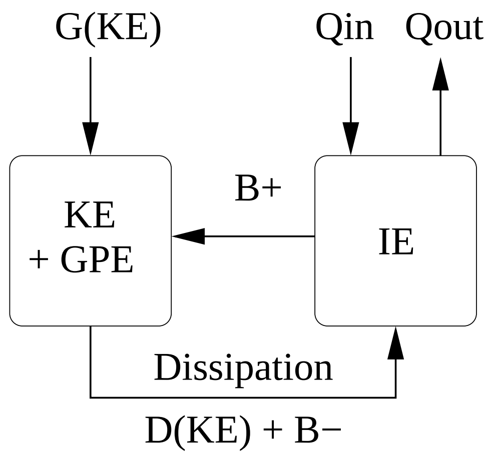


**Figure 1.** Simple schematics of the energetics of a wind and buoyancy forced ocean. The positive part of $B$ acts as a forcing, whereas the negative part contributes to the non-viscous dissipation of the fluid.

the surface buoyancy forcing. Clarifying this link is what we try to do next.

### 2.2. Linking $B$ to the thermodynamic forcing and irreversible molecular processes

In trying to understand how to link $B^+$ with the surface buoyancy forcing, it is useful first to recall how in classical thermodynamics, the first and second laws of thermodynamics are generally combined to yield an explicit expression for the work produced by a heat engine functioning between two heat sources in presence of irreversibilities (see *Ambaum* [2010] or *Lucarini* [2009] for particular examples). Thus, for a closed cycle, the energy and entropy balances are:

$$W = Q_{in} - Q_{out} \tag{13}$$

$$\frac{Q_{in}}{T_{in}} - \frac{Q_{out}}{T_{out}} + \Delta\Sigma_{irr} = 0, \tag{14}$$

which can be manipulated to yield:

$$W = \frac{T_{in} - T_{out}}{T_{in}} Q_{in} - T_{out}\Delta\Sigma_{irr}. \tag{15}$$

This famous result, known as the Gouy-Stoudola theorem (see *Ambaum* [2010]), states that the work produced by a heat engine functioning between a hot and cold source can be written down as the difference between the optimal Carnot work $(1-T_{out}/T_{in})Q_{in}$ minus the "lost work" due to the irreversible entropy production $\Delta\Sigma_{irr}$ arising from irreversible processes taking place in the system. This result is interesting, for it naturally casts the thermodynamic work in the production/destruction form given by Eq. (11). The following seeks to derive a Guouy-Stoudola theorem for the oceanic case. First, the total energy budget yields:

$$Q_{in} - Q_{out} + G(KE) = 0, \tag{16}$$

where $Q_{in}$ and $Q_{out}$ represent respectively the positive and negative contribution to the $IE$ input due to the molecular diffusive flux $\mathbf{F}_q$ through the ocean surface. Physically, Eq. (16) states that the sum of thermal and mechanical energy must balance each other in a steady-state. As the mechanical energy input is positive, it follows that there must be a net $IE$ loss (cooling) to compensate for the net $IE$ gain due to the Joule heating resulting from the ultimate viscous dissipation of the wind and buoyancy power input. Next, the volume-integrated internal energy budget yields:

$$0 = \underbrace{Q_{in} - Q_{out} + D(KE)}_{\delta Q} - \underbrace{B}_{\delta W}. \tag{17}$$

This is of the classical form $de = \delta Q - \delta W$, where the heat transfer is associated with the air-sea interaction terms $Q_{in}/Q_{out}$ and Joule heating term $D(KE)$, whereas the work transfer is due to the work of expansion/contraction $B$. Finally, the global entropy budget yields:

$$0 = \frac{Q_{in}}{T_{in}} - \frac{Q_{out}}{T_{out}} + \dot{\Sigma}_\mu + \frac{D(KE)}{T_\varepsilon} + \dot{\Sigma}_{irr}, \tag{18}$$

where $T_{in}$ and $T_{out}$ represent averaged surface temperatures weighted by the regions of net heat gain and loss respectively, $T_\varepsilon$ a dissipation temperature (see *Lucarini* [2009] for definition), $\dot{\Sigma}_{irr}$ the net irreversible entropy production by all molecular diffusive processes, and $\dot{\Sigma}_\mu$ the net entropy change linked to evaporation/precipitation freshwater fluxes. Now, combining Eqs. (17) and (18) yields:

$$B = W_{carnot} + \left(1 - \frac{T_{out}}{T_\varepsilon}\right) D(KE) - T_{out}(\dot{\Sigma}_{irr} + \dot{\Sigma}_\mu) \tag{19}$$

where $W_{carnot} = (1 - T_{out}/T_{in})Q_{in}$ is reminiscent of the power generated by an ideal Carnot heat engine. Assuming that $T_\varepsilon > T_{out}$ and that $\dot{\Sigma}_\mu > 0$, Eq. (19) suggests a decomposition $B = B_I^+ - B_I^-$ with:

$$B_I^+ = W_{carnot} + \left(1 - \frac{T_{out}}{T_\varepsilon}\right) D(KE), \tag{20}$$

$$B_I^- = T_{out}(\dot{\Sigma}_{irr} + \dot{\Sigma}_\mu). \tag{21}$$

This result, which can be regarded as a Gouy-Stoudola theorem for the oceans, is important, for it shows that $B$ can be in principle decomposed as the difference between a term related to the external buoyancy forcing (plus a small term associated with the recycling of a fraction of the Joule heating due to viscous dissipation into work) minus a term depending on irreversible molecular diffusive processes. It is important to realize here, however, that this decomposition by no means imply that $B_I^+$ and $B_I^-$ are necessarily suitable estimates for $B^+$ and $B^-$, because it is not possible to ascertain that $B_I^+$ and $B_I^-$ actually measure physical work actually taking place in the system. In fact, as discussed next, numerical estimates based on typical observed values suggest that $B_I^+$ and $B_I^-$ most likely represent considerable overestimates of $B^+$ and $B^-$.

### 2.3. Comparison with APE theory

Despite the fact that the above decomposition given by Eqs. (20-21) is the one that seems to most naturally follow from the direct application of the first and second laws of thermodynamics, physical considerations suggest that it fails to be really satisfactory for reasons clarified below. In fact, as we now show, APE theory provides a physically much more interesting decomposition. Indeed, as shown by *Tailleux* [2009], it can be shown from first principles that the mechanical energy balance can also be written under the following form:

$$G(APE) + G(KE) = D(APE) + D(KE) \tag{22}$$

where $G(APE)$ is the production rate of APE, while $D(APE)$ is the dissipation rate of $APE$ by molecular diffusive processes, which naturally suggests the following decomposition:

$$B_{II}^+ = G(APE), \qquad B_{II}^- = D(APE). \tag{23}$$

*Oort et al.* [1994] estimated $G(APE) = 1.2\pm0.7\,\mathrm{TW}$. Given that the mechanical power input by the wind and tides is of the order of $O(2\,\mathrm{TW})$, e.g., *Munk and Wunsch* [1998], *Oort et al.* [1994] concluded that the buoyancy power input was comparable to that due to the mechanical forcing.

It is useful, for comparison, to estimate the Carnot work $W_{carnot}$, which can be done from the ocean entropy budget. Recently, *Pascale et al.* [2010] estimated that the rate of entropy production by the surface buoyancy fluxes was $O(1\,\mathrm{mW.m^{-2}.K^{-1}})$. Taking $S_{ocean} = 3.10^{14}\,\mathrm{m}^2$ as an estimate of the ocean surface area, and $T_{in} = 20^\circ\mathrm{C} = 293\,\mathrm{K}$, one obtains:

$$W_{carnot} \approx 10^{-3} \times 3.10^{14} \times 293 \approx 90\,\mathrm{TW}, \tag{24}$$

which is nearly two orders of magnitude larger than $G(APE)$'s estimate. The fact that classical thermodynamics and APE theory both provide a decomposition of $B$ as the difference between a positive component controlled by the surface buoyancy forcing minus a term controlled by molecular diffusive processes strongly suggests that this is also true of $B^+$ and $B^-$. However, because both approaches are indirect, the precise link between $W_{carnot}$ and $G(APE)$ and $B^+$ can only be tentative at present, so that additional physical considerations are requiblack to clarify this link. To that end, it is fundamental to realize that $B^+$ is related to the work done by buoyancy forces created at the surface by surface buoyancy fluxes. To compute this work, as for instance in the case of surface parcels made denser by cooling, it is well known that the full knowledge of the stratification along the path followed by the descending plume is requiblack. On this account, it is very implausible that $W_{carnot}$ can be a good estimate of $B^+$, since its determination only requires the knowledge of the surface stratification and buoyancy fluxes. By contrast, the computation of $G(APE)$ is well known to require the full knowledge of the ocean stratification. In fact, $G(APE)$ is also related to the work done by buoyancy forces, but along paths involving the reference stratification rather than the actual stratification. Since the reference stratification only differs from the reference stratification by relatively small adiabatic modifications to the density, it seems plausible that $B^+$ should be comparable with $G(APE)$, but again, we need to stress that this cannot be ascertained with full certainty yet owing to the indirect character of APE theory in decomposing $B$.

## 3. Summary and discussion

In this paper, we argue that the current controversy about the relative importance of the buoyancy forcing in the oceans stems mostly from two incompatible views on how the latter relates to the work of expansion/contraction. The current prevailing view, mostly popularized over the past decade, has been that $W_{buoyancy}$ should be identified with the *net* value of $B$, which current theories, mostly based on the Boussinesq approximation of $B$, suggest is several orders of magnitude smaller than the mechanical energy input due to the wind and tides, e.g., *Wang and Huang* [2005]. This view, to a large extent, is very similar to that adopted in thermodynamic engineering textbooks, which tends to regard thermodynamic systems as black boxes characterized only by their bulk properties. The alternative view, defended here, is that only the positive component of $B$ should enter the definition of $W_{buoyancy}$, given that the negative component of $B$ acts as a non-viscous dissipation mechanism for mechanical energy. The basis for this view is that a heat engine is by definition a device converting internal energy into mechanical energy; as a result, understanding how buoyancy drives motion requires focusing on the positive part of $B$, since the opposite conversion is of no interest for that purpose. The current view, by contrast, has tended to interpret the occurrence of a negative $B$ as the "opposite" of a heat engine, by itself a puzzling thermodynamic concept, or as proof of the dominance of mechanical forcing, without apparently fully realizing that a negative $B$ simply correspond to a non-viscous dissipation mechanism for mechanical energy. Two independent approaches, respectively based on APE theory and on classical thermodynamics, support this interpretation by providing in both cases a decomposition of $B$ in terms of a positive contribution controlled by the large scale features of the ocean stratification and buoyancy forcing, and negative contribution controlled by molecular diffusive processes acting on the very small diffusive scales.

The proposition that $W_{buoyancy}$ should be identified with the positive part of $B$ appears to be new, because even though APE theory can be regarded as offering a practical way of decomposing $B$ as the sum of a forcing and dissipative terms, the motivation and physical basis underlying the construction of APE theory in *Lorenz* [1955] appears to be very different. Indeed, *Lorenz* [1955]'s paper deals mostly with how to construct APE, and how to estimate its rate of production by diabatic effects, but fails to mention the need or existence for any non-viscous dissipation mechanisms. If one accepts *Wang and Huang* [2005]'s result that $B$ is small, and that $B^+$ is potentially large, then $B^-$ must also be potentially large. The possibility that non-viscous dissipation mechanisms could play a significant role in the oceans is very intriguing, and should be exploblack further, owing to its potential implications for the design of numerical ocean models for instance.

To make progress, further work is needed to further constrain the net value of $B$ in the most general settings, as well as for understanding how best to split $B$ as the sum of a forcing and dissipative terms. Recently, significant new insights into the net value of $B$ for horizontal convection was achieved by *Nycander* [2010] and *McIntyre* [2010], which represent important generalizations of *Paparella and Young* [2002]'s work for a realistic nonlinear equation of state. Regarding the second issue, an important question is whether it is possible to demonstrate the superior dynamical significance of $B^+$ over $B$ to characterize the buoyancy power input. That this is indeed the case is suggested by the recent work of *Gregory and Tailleux* [2010], in which the authors found a strong correlation between the decadal variations of the numerical ocean model equivalent of $B^+$ and the corresponding variations in the strength of the Atlantic meridional overturning circulation. The next step would be to examine in more details the precise link between $G(APE)$ and $B^+$ in the context of numerical ocean models, since the present theoretical considerations developed here could not fully address this link.

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## Notes

1. Turbulence increases the area of isothermal surfaces, thereby enhancing the net diathermal molecular diffusive heat flux, as discussed by *Winters and d'Asaro* [1996]
2. Tidal forcing introduces an additional forcing term in the gravitational potential energy equation, e.g., *Wunsch and Ferrari* [2004], which can be simply added to the left-hand side of Eq. (12). We avoid discussing tidal forcing here, however, because we remain uncertain about how to generalize APE theory for a time-dependent geopotential $\Phi$.

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