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On the seemingly incompatible parcel and globally integrated views of the energetics of triggered atmospheric deep convection over land

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SUMMARY

The energetics of the diurnal cycle of atmospheric deep convection over land remain difficult to understand and simulate accurately with current cumulus parametrizations. Furthermore, a proper formulation has remained elusive owing to seeming incompatibilities between, on the one hand, the parcel view of energetics which relies on such concepts as Convective Available Potential Energy (CAPE) and Convective Inhibition (CIN), and, on the other hand, the globally integrated view, which relies on such concepts as Moist Available Energy (MAE), reference states, and energy conversion terms. While the MAE is intuitively the global counterpart of the parcel-defined CAPE, there seems to be no global analogue to the parcel-defined concept of energy barrier attached to CIN. To gain insights into this issue, a new cost function PE is introduced to quantify the amount of positive or negative energy required for a given sounding to undergo an arbitrary adiabatic rearrangement of mass, and which encompasses both the parcel-defined and global energy concepts as particular cases. The function PE offers a complementary view of the stability and energy characteristics of atmospheric soundings, whose local minima are naturally associated with the reference states of the system.

It is established that:

(a) MAE is essentially equivalent to CAPE multiplied by a mass conversion factor $M_{\rm b}$ which scales as the amount of convectively unstable boundary-layer air. Using the available summer 1997 IOP data from the ARM–SGP site, $M_{\rm b}$ is found to correlate with CAPE, which suggests the existence of a functional relationship between CAPE and MAE; if further confirmed, this result would considerably simplify the computation of MAE.

(b) A global counterpart to the parcel-defined concept of energy barrier can only be defined if the system considered admits several reference states, and not one as is classically assumed. In that case, energy barriers naturally arise as the amount of energy required to switch from one reference state to another. In the context of triggered deep convection, there must be at least two reference states: a shallow one, which is the actual state (or a slightly modified one if there is boundary layer CAPE), and a deep one associated with the release of MAE/CAPE; the energy barrier separating these two reference states naturally defines a generalized CIN.

In the limited context of the above-mentioned IOP ARM data, it is further shown that:

(c) Spatially averaged conditions exhibiting potential instability to deep convection may be associated with individual soundings having widely different stability characteristics, including absolute stability, potential instability, and absolute instability; this suggests that triggered deep convection may not necessarily be the result of a parcel's vertical kinetic energy exceeding its negative buoyancy, but rather from the destruction of convective inhibition (i.e. production of absolute instability) in a local region.

(d) A few local soundings exhibit multiple reference states, corresponding roughly to multiple levels of neutral buoyancy. These may allow for convective clouds with cloud-top heights significantly lower than those classically predicted by the undiluted ascent of a boundary-layer air parcel up to its highest level of neutral buoyancy, even in the absence of complex entrainment scenarios.

KEYWORDS: Convective available potential energy Convective inhibition energy Energy cycle Multiple reference states

1. INTRODUCTION

Deep atmospheric convection over land often shows a marked diurnal cycle that is coupled to the diurnal cycle of the boundary-layer heating. Three main stages characterize its temporal evolution:

Stage I: Development of a (dry) convectively unstable boundary layer (BL) as solar heating sets in.

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Stage II: Unstable moist BL parcels reach their condensation level, and shallow cumulus form.

Stage III: Unstable moist BL parcels reach their Level of Free Convection (LFC) and deep convection develops, accompanied by heavy rainfall, usually in the late afternoon. Current Atmospheric General Circulation Models (AGCMs) show marked difficulties in correctly reproducing stage II, with the result that stage III and hence rainfall are initiated too early. (See Guichard *et al.* (2004) for an introduction to the problem.) Correcting this problem by tuning the parameters of existent cumulus parametrizations has proved difficult so far, hinting at intrinsic problems in the current design of the latter.

To gain insights into the transient dynamics of triggered deep convection, energetics considerations have been found to be useful, e.g. Guichard *et al.* (2004); Chaboureau *et al.* (2004). Particularly popular is the 'parcel view' of energetics, which is centred on three main quantities, namely: Convective Available Potential Energy (CAPE), Convective INhibition energy (CIN) and subgrid-scale triggering energy (STE, Mapes (2000)). In the following, we shall preferentially use the term Available Lifting Energy (ALE) instead of STE, to avoid the problem of making explicit reference to the scales and subgrid-scales of the problem, which is beyond the scope of this paper. As is well known, CIN and CAPE represent the negative and positive work done by buoyancy forces acting on a BL air parcel lifted adiabatically from its initial position up to its level of free convection (LFC), and from its LFC up to its level of neutral buoyancy (LNB) respectively (with more complicated definitions required, of course, in presence of several LNBs). In this view, the presence of CIN prevents the spontaneous release of the stored CAPE, so that deep convection can only be triggered when ALE becomes sufficient to overcome CIN. Such quantities define a natural framework for the design of a triggering function in which convection is triggered when ALE > CIN. Such a triggering function was used in the early stages of EUROCS, and was found to work better than the buoyancy threshold criterion used in many standard convection schemes. In the parcel view, the concept of environment plays a crucial role to define the buoyancy of the lifted air parcels.

The other main approach to energetics is that of the so-called energy cycles which rely on the use of energy balance equations of the form:

$$\frac{\mathrm{d}E_i}{\mathrm{d}t} = \sum_j C(E_i, E_j) - D(E_i) + G(E_i), \tag{1}$$

as is commonly done in the atmospheric literature (e.g. see Peixoto and Oort (1992)), building upon the pioneering work on the global energy cycle of the atmospheric circulation by Lorenz (1955). In this approach, E_i denotes the vertically integrated and domain-averaged value of a given energy reservoir, $C(E_i, E_j) = -C(E_j, E_i)$ the conversion term of the energy form E_j into the energy form E_i, while $G(E_i)$ and $D(E_i)$ represent the generation and dissipation of E_i respectively. Although recent work has attempted to extend the ideas of Lorenz (1955) to deal with atmospheric dry and moist convection (e.g. Marquet 1993; Haimberger and Hantel 2000; Hantel and Haimberger 2000), it is not clear to us that they have reached a mature stage yet because of potential problems with the current existing definitions of reference states discussed in this paper. In practice, two energy reservoirs at least are needed, namely one for the vertically integrated kinetic energy (KE), and one for the available nonkinetic energy (NKE). In the present case, the available potential energy (i.e. the part of the total potential energy available for conversion into kinetic energy) was termed Moist Available Energy (MAE) by Lorenz (1978, 1979), who defined it as the vertical integral of the enthalpy difference between the actual state and that of a reference state obtained by an adiabatic rearrangement of mass. The MAE is a generalization of the concept of available potential energy (APE) previously introduced by Lorenz (1955) for a dry atmosphere. The concept of 'reference state' is central to all current definitions of utilizable energy, whether formulated in terms of APE (Lorenz 1955, 1978), exergy (Marquet 1991, 1993), or pseudo-energy (Shepherd 1993). Physically, the reference state is meant to represent that part of a stratified fluid whose potential energy cannot be converted into kinetic energy, i.e. its 'dead' part, so that what is available is the difference between the total potential energy of the state considered (i.e. the 'actual' state) minus that of the reference state. Although there are still many ambiguities in the literature as to how one should define and interpret a reference state, we shall adopt the standard view of defining it as a particular adiabatic rearrangement of mass of the actual state minimizing the total nonkinetic energy of the system in some sense. In the present context, Randall and Wang (1992; RW92 hereafter) suggested that the reference state thus computed is the state toward which the atmosphere tends to adjust under the action of deep convection (i.e. the convectively adjusted state), and suggested that it can be used as a theoretical justification for the parametrization of Betts and Miller (Betts 1986; Betts and Miller 1986). They later proposed a parametrization based on this idea (Wang and Randall 1996).

The global energy approach—which accounts for all the forces acting on the system—is a priori more satisfactory than the less general and more ambiguous parcel approach. On the other hand, it is widely accepted that the work done by the environment, which is neglected in the parcel approach, is small in comparison to that done by the buoyancy forces acting on the air parcels feeding the convective updraughts (Bjerknes 1938; Bretherton 1987). For this reason, one should expect the two approaches to be somehow connected, even if the nature of this connection has not been much discussed so far, and is hence currently not well understood. In this paper, our purpose is to clarify this issue by seeking to understand how each particular concept of a given approach translates into the other approach. Specifically, the two main questions addressed in this paper are:

(a) What is the link between the parcel-defined CAPE and the MAE of Lorenz (1978,1979)?

(b) What is the global counterpart of the parcel-defined concept of convective inhibition or equivalently of an energy barrier to deep convection?

Concerning (a), RW92 seem to be the first to implicitly recognize the existence of a link between CAPE and MAE, by proposing to define a Generalized (i.e. parcel-independent) CAPE (GCAPE) as the MAE divided by the total mass of the local atmospheric column. This terminology is perhaps questionable, however, because CAPE and GCAPE are in general very different numbers, which makes the physical meaning of GCAPE hard to grasp. To add to the confusion, RW92 showed their GCAPE to be strongly correlated to the cloud work function of Arakawa and Shubert (1974), which is interesting but also intriguing because the cloud work function is by construction related to a conversion term $C(E_i, E_j)$ of (1), and not to an energy reservoir E_i, as one expects MAE and GCAPE to be. In fact, since GCAPE and MAE differ only by a more or less constant proportionality factor, the study of RW92 does not tell us much about the links between MAE and CAPE. This link was clarified by Emanuel (1994) which established the approximate formula:

$$\mathrm{MAE} \approx M_{\mathrm{b}} \times \mathrm{CAPE}_{\mathrm{b}}, \tag{2}$$

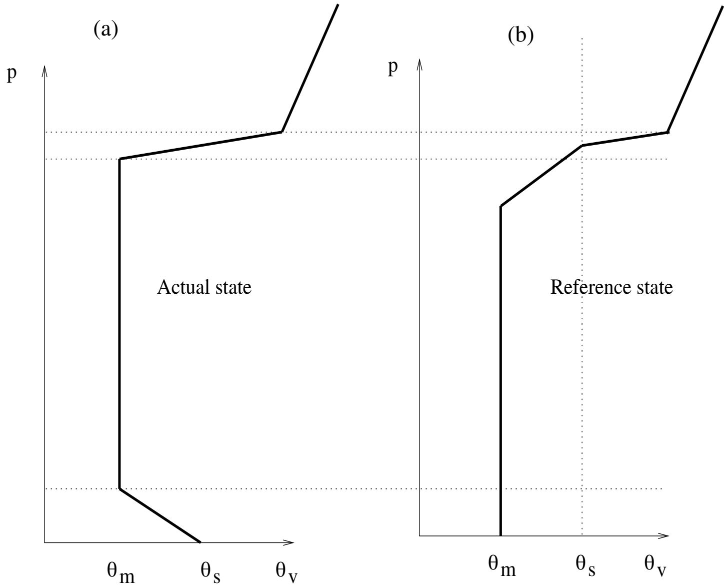

Figure 1. Schematic depiction of (a) a convectively unstable profile of an atmospheric boundary layer topped by an inversion and (b) the associated natural reference state. The abscissa is virtual potential temperature and the ordinate is pressure. Note that the reference state is not the same as the convectively adjusted state as defined in Emanuel (1994).

under the assumption that the conditionally unstable BL air parcels have all the same CAPE $=$ CAPE$_\mathrm{b}$. Here, M_b is a proportionality mass factor that scales as the mass per unit area of potentially unstable BL air (i.e. to sufficiently large amplitude lifting), with a weak dependence on the environment. This result therefore establishes clearly that the global and parcel approaches to the energetics of deep convection are strongly linked. According to (2), all one needs to go from one approach to the other is to be able to compute the mass conversion factor M_b. Since these issues are crucial to understand the links between MAE and CAPE, they are discussed in detail in sections 2 and 3.

Question (b) is a less trivial problem because no previous study seems to have needed such a concept to discuss global energy cycles. In this paper, we argue that this is because all published studies of global energetics have systematically used a single reference state to define what is the available or utilizable part of the potential energy, whereas we believe that at least two different reference states are required in the context of triggered deep convection. This issue is actually an old one and is related to the classical problem that most reference states in use are often not optimal in the sense that there can still be a large fraction of the APE that is not convertible into kinetic energy (e.g. see Codoban and Shepherd 2003). In the present context, it is intuitively clear that during the above-mentioned stage I, the atmosphere does not adjust to a deep convectively adjusted state even though the BL air parcels possess CAPE. In other words, the kinetic energy that is created during stage I is not tapped from the CAPE reservoir, but rather from the ‘BL CAPE’ associated with absolutely unstable near-surface BL air, an idealized typical situation being depicted in Fig. 1(a). It is easily seen that the parcels can be adiabatically rearranged to yield the virtual potential temperature, θ_v, profile of Fig. 1(b), which is statically stable, and a local minimum in potential energy. Clearly, one has thus explicitly constructed a shallow reference state which is not that discussed by RW92.

Reference states are by construction stable in some sense, which can be expressed in terms of the local convexity of some cost function, PE (Kucharsky 1997). Physically, PE must quantify the change in potential energy achieved by an arbitrary adiabatic rearrangement of mass of the actual state. If one admits the possibility of multiple reference states, then energy barriers naturally arise as the amount of energy required to go from one reference state to another. To compute reference states in practice, one needs an explicit expression for PE, which requires identifying a suitable set of coordinates $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \dots)$ to characterize an arbitrary particular adiabatic rearrangement. By construction, $\boldsymbol{\lambda} = 0$ must refer to the actual state, while $\mathrm{PE}(0) = 0$. In such a description, the reference states are defined by the local maxima of PE, whereas MAE refers by definition to the global maximum of PE achieved for $\boldsymbol{\lambda} = \boldsymbol{\lambda}_{\mathrm{max}}$, say. In the global view, the analogue of the convective inhibition translates into the existence of an intermediate state $\boldsymbol{\lambda}_{\mathrm{CIN}} = (\alpha_1\lambda_1, \alpha_2\lambda_2, \dots) = \boldsymbol{\alpha} \cdot \boldsymbol{\lambda}_{\mathrm{max}}$, such that $\mathrm{PE}(\boldsymbol{\lambda}_{\mathrm{CIN}}) < 0$, with all the α_i such that $0 \leqslant \alpha_i \leqslant 1$. Obviously, $\mathrm{PE}(\boldsymbol{\lambda}_{\mathrm{CIN}})$ is a natural candidate for generalizing the concept of CIN. From a practical viewpoint, a description of the paths is only possible if it is possible to identify a subspace of finite dimensions allowing us to describe the maximization process in terms of a finite number of parameters. It turns out that this is possible for computing the MAE, as discussed in section 3.

Section 4 provides additional illustrations and applications of the above ideas. Specifically, it explores the horizontal variability of the stability characteristics of particular individual soundings as compared to that estimated from the ‘mean’ sounding. This issue is important to understand better the limitations of the assumptions of a barotropic atmosphere in the current theory of MAE. It also provides observational evidence for multiple deep reference states which could possibly account for various cloud-top height regimes from purely adiabatic processes without the need to invoke entrainment. Section 5 discusses the present results.

2. CAPE, GCAPE, AND THE APE OF A BAROTROPIC MOIST ATMOSPHERE

(*a*) *Review of the theoretical links between APE and CAPE*

The APE of a conditionally unstable moist atmosphere is currently well-defined only for a barotropic atmosphere. This is in contrast with the case of a dry atmosphere, for which the concept of APE is meaningful only in the baroclinic case (Lorenz 1955), as it vanishes identically otherwise. If the atmosphere is moist, however, a purely barotropic atmosphere may have a non-zero APE, as shown by Lorenz (1978, 1979). This is because the dry APE is associated with the horizontal departure of the isopycnals from their equilibrium resting position, whereas the moist APE is associated with the existence of BL parcels possessing CAPE, which refers only to the vertical displacements of the air parcels. In this respect, it may be said that the APE of a general moist atmosphere possesses both a horizontal and vertical component (RW92). In this paper, we shall be only concerned with the latter, assuming implicitly that the two components of the APE can be studied separately owing to the physical processes responsible for their release, i.e. deep convection and baroclinic instability respectively, having well-separated time-scales in general (RW92). How to compute and define the APE of a general moist atmosphere to deal consistently with both its horizontal and vertical components is still an open question beyond the scope of this paper. According to Emanuel (1994), the APE of a barotropic moist atmosphere can be expressed as the total enthalpy of the actual state considered minus the total enthalpy of a reference state obtained by an adiabatic rearrangement of mass, viz.

$$\mathrm{APE} = \frac{1}{g}\int_0^{p_0} (k' - k'_{\mathrm{ref}})\,\mathrm{d}p, \tag{3}$$

where k' is the enthalpy per unit total mass

$$k' = \frac{(C_{\mathrm{pd}} + r_{\mathrm{tot}}C_{\ell})T + Lr}{1 + r_{\mathrm{tot}}},$$

with C_{pd} the specific heat capacity for dry air at constant pressure, C_{ℓ} the specific capacity for liquid water, L the latent heat for water vapour, r the mixing ratio for water vapour, r_{tot} the total mixing ratio (liquid + vapour) for water, T the temperature, and p_0 the surface pressure. In the following, we shall denote by MAE the APE of a moist barotropic atmosphere, to avoid confusion with the APE of a dry baroclinic atmosphere, as in Lorenz (1978, 1979).

The main issue to compute MAE is in finding the particular adiabatic rearrangement of mass that maximizes (3). This is addressed in more detail in the next section. Physically, one intuitively expects MAE to be linked with the standard parcel-defined CAPE. To show that this is indeed the case, Emanuel (1994) assumes that the reference state maximizing (3) is achieved by lifting a mass $\Delta p_{\mathrm{b}}/g$ of BL air—assuming its parcels to have uniform $\mathrm{CAPE} = \mathrm{CAPE}_{\mathrm{b}}$—up to their LNB, with a compensating subsidence of environmental air. If so, the change in potential energy achieved by an adiabatic rearrangement of mass bringing an amount of BL air of thickness Δp_{b} near their LNB, with a compensating subsidence, is the following function of Δp_{b}:

$$\mathrm{PE}(\Delta p_{\mathrm{b}}) \approx \frac{\Delta p_{\mathrm{b}}}{g}\left(\mathrm{CAPE}_{\mathrm{b}} - \frac{1}{2}\Delta p_{\mathrm{b}}\overline{p^{\kappa-1}}\;p_0^{-\kappa}\;R_{\mathrm{d}}\Delta\theta_{\mathrm{v}}\right), \tag{4}$$

where $\kappa = R_{\mathrm{d}}/C_{\mathrm{pd}}$, R_{d} is the gas constant for dry air, $\Delta\theta_{\mathrm{v}}$ is the total change of θ_{v} across the system, and the overbar represents a θ_{v}-weighted average over the system, see Emanuel (1994), pp. 179–185. Equation (4) is a quadratic polynomial in Δp_{b}; it therefore reaches its extreme value for:

$$\Delta p_{\mathrm{b,max}} = \frac{\mathrm{CAPE}_{\mathrm{b}}}{\overline{p^{\kappa-1}}\;p_0^{-\kappa}\;R_{\mathrm{d}}\Delta\theta_{\mathrm{v}}}, \tag{5}$$

in which case the maximum achieved is given by

$$\mathrm{MAE} = \mathrm{PE}(\Delta p_{\mathrm{b,max}}) \approx \frac{\Delta p_{\mathrm{b,max}}\,\mathrm{CAPE}_{\mathrm{b}}}{2g}. \tag{6}$$

Equation (6) is an important result that states that the MAE of a moist barotropic atmosphere, which is an integral quantity involving all parcels and expressed in J m^{-2}, is different from zero only if CAPE, which is a parcel-dependent quantity expressed in J kg^{-1}, is also different from zero. As a result, it makes sense to define a parcel-independent index of moist convective instability based on the concept of MAE. This is what RW92 proposed to do, by defining a generalized CAPE as the MAE divided by the total mass of the air column. From (6), an approximate expression for GCAPE is as follows:

$$\mathrm{GCAPE}_{\mathrm{RW92}} = \frac{g\,\mathrm{MAE}}{p_0} \approx \frac{\Delta p_{\mathrm{b,max}}}{2p_0}\mathrm{CAPE}_{\mathrm{b}}. \tag{7}$$

According to (7), GCAPE is only a fraction of $\mathrm{CAPE}_{\mathrm{b}}$ since $\Delta p_{\mathrm{b,max}}/p_0 \ll 1$. In RW92, the link between GCAPE and CAPE was not clearly established, although the authors

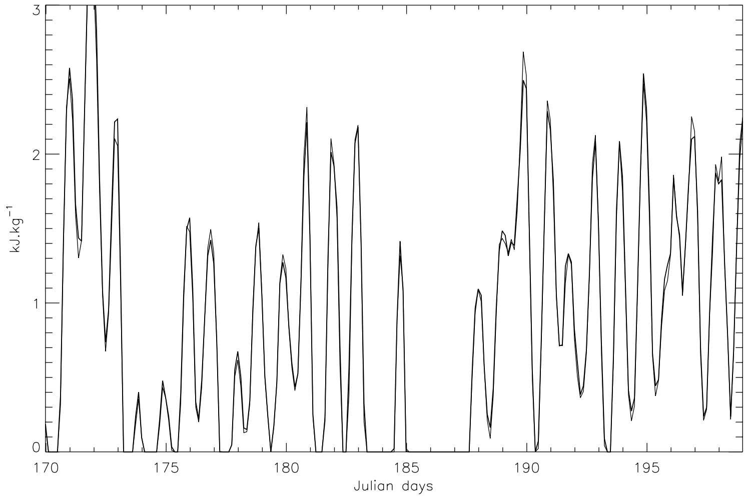

Figure 2. Comparison of parcel-defined CAPE (thin line) with parcel-independent mean CAPE (thick line) for the Southern Great Plains ARM data, for the period June–July 1997.

showed their GCAPE to be correlated with the cloud work function of Arakawa and Shubert (1974). However, knowledge of (6) suggests the following alternative parcel-independent definition of CAPE:

$$\overline{\mathrm{CAPE}} = \frac{2g\,\mathrm{MAE}}{\Delta p_{\mathrm{b,max}}}, \tag{8}$$

since from the above results one should have approximately

$$\overline{\mathrm{CAPE}} \approx 2\,\mathrm{CAPE_b} - \Delta p_{\mathrm{b,max}}\overline{p^{\kappa-1}}\;p_0^{-\kappa}\;R_\mathrm{d}\Delta\theta_\mathrm{v} = \mathrm{CAPE_b}, \tag{9}$$

making $\overline{\mathrm{CAPE}}$ closer by construction to standard measures of CAPE than the GCAPE of RW92. The formula (9) may become ambiguous when the CAPE of BL air parcels vary sensitively over height, as may happen sometimes, e.g. see de la Torre *et al.* (2004) for a particular example. For the present purposes, we tested the accuracy of (9) by comparing the CAPE of the lowermost parcel with the mean CAPE defined by (8), for the data from the Atmospheric Radiation Measurement (ARM) Intensive Observation Period in summer 1997, from 18 June to 17 July at a site in Southern Great Plains (SGP), Oklahoma. The data used are those obtained by the constrained variational analysis described in Zhang and Lin (1997) and Zhang *et al.* (2001), which describe the average conditions at the location. In this case, Fig. 2 shows that the agreement between $\overline{\mathrm{CAPE}}$ and CAPE is particularly good, probably because of the quite low resolution of the vertical soundings of the constrained analysis. Based on this example, we shall make no distinction in the following between the parcel-dependent and parcel-independent definitions of CAPE.

(*b*) *The conversion factor between MAE and CAPE*

In the absence of significant differences between the parcel-dependent and parcel-independent measures of CAPE, the link between MAE and CAPE can be simply expressed as follows (from (8)):

$$\mathrm{MAE} = M_b\,\mathrm{CAPE}, \tag{10}$$

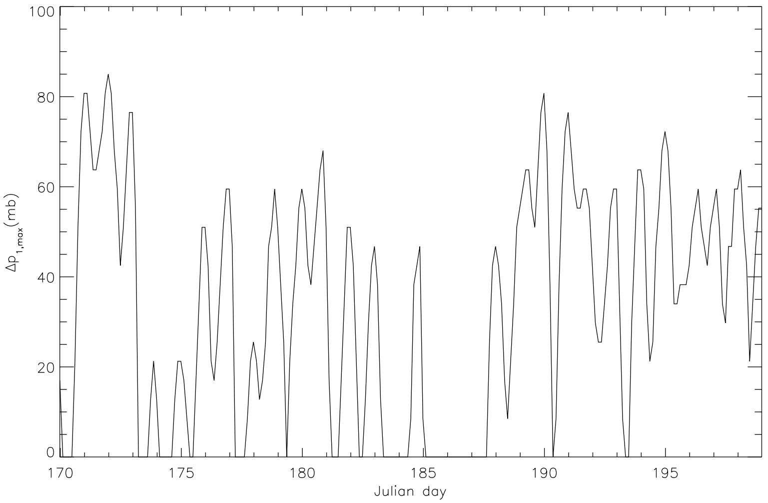

Figure 3. Time evolution of the boundary-layer thickness realizing the global minimum in the Moist Available Energy.

where $M_b = \Delta p_{b,\max}/(2g)$ is the conversion factor between the two quantities. Although such a result is implicitly contained in Emanuel (1994), it does not appear to have been clearly realized before. This result is interesting in the context of shifting the present theoretical focus on CAPE to the more general MAE. Indeed, the former has been extensively studied theoretically (e.g. Williams and Renno (1993); Emanuel and Bister (1996); Renno and Ingersoll (1996); Parker (2002)), whereas MAE is still poorly understood. Yet, it is MAE not CAPE that needs to be understood to understand the global energetics of deep convection. Since CAPE is well-known and well-studied, (10) shows that we simply need to study the mass factor M_b to understand MAE. To that end, we computed the temporal evolution of $\Delta p_{b,\max}$ for the ARM–SGP data described above, as depicted in Fig. 3. In this example, $\Delta p_{b,\max}$ is found to vary within the interval (20 mb, 85 mb), which is consistent with the value of 50 mb estimated by Emanuel (1994) for typical atmospheric conditions.

Interestingly, a close comparison between Figs. 2 and 3 suggests that $\Delta p_{b,\max}$ and CAPE are correlated. To test this hypothesis, we plotted in Fig. 4 the value of $\Delta p_{b,\max}$ as a function of $\overline{\mathrm{CAPE}}$. Clearly, this suggests the existence of a functional relationship between the two quantities, although the underlying physical reason of why this should be so is admittedly still unclear to us. For lack of theoretical grounds to suggest admissible functional forms between M_b and $\overline{\mathrm{CAPE}}$, the cloud of points was fitted visually by the curve $\Delta p_{b,\max} = \mu\, \overline{\mathrm{CAPE}}^{1/2}$, with $\mu = 130\,\mathrm{kg\,m^{-2}s^{-1}}$ having the physical dimensions of a mass flux, and the square root being chosen for convenience. This result is potentially important, because the only known way so far of computing M_b (and hence MAE) is by minimizing (3), which is computationally expensive. If further confirmed, the existence of a functional relationship between M_b and $\overline{\mathrm{CAPE}}$ would imply the same for MAE and $\overline{\mathrm{CAPE}}$, which, if of universal validity, would relax the need for the minimization of (3), making MAE no more computationally expensive than CAPE, and significantly advancing our theoretical knowledge of the global energetics of deep convection.

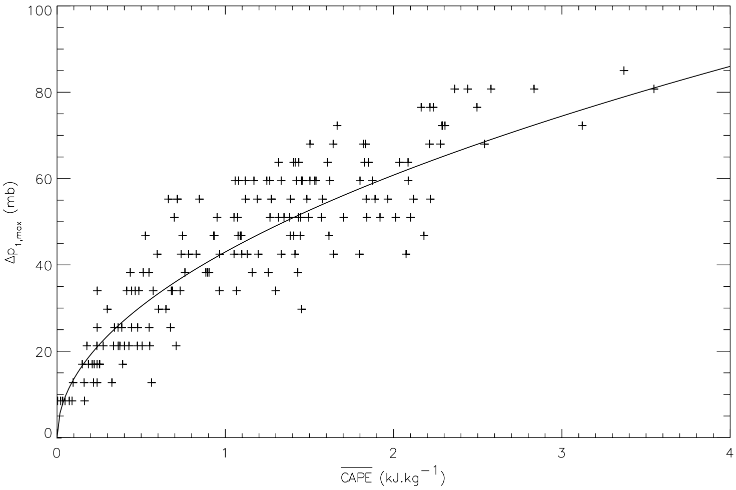

Figure 4. Scatter plot of $\Delta p_{\mathrm{b,max}}$ versus $\overline{\text{CAPE}}$, suggesting a correlation between the two quantities.

3. HOW TO GENERALIZE THE PARCEL-DEFINED CONCEPT OF ENERGY BARRIER TO DEEP CONVECTION WITHIN THE FRAMEWORK OF GLOBAL ENERGETICS?

(*a*) *Description and parametrization of an adiabatic rearrangement of mass*

The previous paragraph sought to clarify the links between parcel-dependent and parcel-independent measures of moist convective instability, but said little about how one could similarly define a parcel-independent measure of convective inhibition. Intuitively, this is because the current theory of MAE (and for that matter of all current theories of utilizable energies) fails to describe the path(s) followed to reach the sought-for reference state. Indeed, the knowledge of such path(s), as well as a way to define a 'cost' for each path, seems to be required by a parcel-independent definition of CIN. To that end, one first needs to find explicit expressions for the particular one-to-one map $p_t = f(p)$ describing a particular adiabatic rearrangement. The natural AGCM-oriented way, followed by RW92, is to discretize the sounding considered into N equal elements of mass $\Delta p/g = p_0/(g\,N)$, in which case the computation of the MAE becomes equivalent to maximizing the following discrete sum:

$$\mathrm{PE}_{\mathrm{discrete}} = \sum_{i=1}^{N} (k'_{i,i} - k'_{i,\sigma(i)}) \frac{\Delta p}{g}$$

over all possible permutations σ of N elements, where $k'_{i,j}$ denotes the enthalpy of a parcel initially at the pressure level $p_i = p_0 - (i - 0.5)\Delta p$ brought adiabatically to the pressure level $p_j = p_0 - (j - 0.5)\Delta p$. This type of problem is known in combinatorial mathematics as a so-called Asymmetric Travelling Salesman Problem (ATSP), where $k'_{i,j}$ is here the analog to the cost of travelling from city i to city j, the asymmetry resulting from that $k'_{i,j} \neq k'_{j,i}$ in general. The brute force approach to solving ATSP problems requires exploring the $N!$-dimensional space of all permutations of N elements, which makes it computationally prohibitive as the number is extremely large even for small N. At present, only heuristics are available to solve ATSP problems in reasonable time, although it is in general difficult to ascertain that they have found the global extremum. Fortunately, our experience with potentially unstable soundings gives us important a priori information about the most likely relevant permutations, which favours permutations bringing BL air parcels near the tropopause with an accompanying compensating subsidence. The mass-flux approach of RW92, which makes use of a particular permutation called 'penetrator', is thus easily understood in this context.

In the context of defining reference states from a convexity argument, as in Kucharsky (1997), the discrete nature of RW92's approach is inconvenient. For this reason, we propose an alternative continuous description of the one-to-one maps $p_t = f(\Delta p_1, \ldots, \Delta p_M; p)$ dependent on M parameters Δp_i, $i = 1, \ldots, M$, such that whatever M, $\sum_{i=1}^{M} \Delta p_i \leqslant p_0$. For instance, for $M = 2$, we shall have:

$$p_t = f(p; \Delta p_1, \Delta p_2) = \begin{cases} p - \Delta p_2 & \text{if } p_0 \geqslant p \geqslant p_0 - \Delta p_1 \\ p + \Delta p_1 & \text{if } p_0 - \Delta p_1 \geqslant p \geqslant p_0 - \Delta p_1 - \Delta p_2 \\ p & \text{if } p \leqslant p_0 - \Delta p_1 - \Delta p_2, \end{cases} \tag{11}$$

which is illustrated in Fig. 5. This transformation obviously is mass conserving, and can be regarded as a continuous equivalent of the 'penetrator' of RW92. Likewise, the classes of 3-parameter transformations would be defined as illustrated in Fig. 6, and so on. With the present parametrization of adiabatic rearrangements, PE becomes a function of M parameters only, i.e. $\mathrm{PE} = \mathrm{PE}(\Delta p_1, \ldots, \Delta p_M)$. Following Kucharsky (1997), we can therefore define any reference state $(\Delta p_{1,\mathrm{ref}}, \ldots, \Delta p_{M,\mathrm{ref}})$ to be such that there exists some positive radius R such that locally

$$\mathrm{PE}(\Delta p_1, \ldots, \Delta p_M) - \mathrm{PE}(\Delta p_{1,\mathrm{ref}}, \ldots, \Delta p_{M,\mathrm{ref}}) < 0 \tag{12}$$

for all values of Δp_i such that $\sqrt{(\Delta p_{1,\mathrm{ref}} - \Delta p_1)^2 + \cdots + (\Delta p_{M,\mathrm{ref}} - \Delta p_M)^2} < R$. This is the same convexity argument as in Kucharski (1997), where global convexity is replaced by local convexity to allow for the possibility of multiple reference states. Any local maximum of PE is therefore a reference state by (12). Clearly, the class of all M-parameter transformations thus defined includes as a particular subclass all $(M-1)$-parameter transformations. The whole issue is therefore about finding the minimum M that does the job. In this paper, we find that $M = 2$ is sufficient for the present purposes, leaving for a subsequent study the question of whether the present results would significantly differ for $M > 2$.

(*b*) *A parcel-independent measure of convective inhibition*

In order to understand how to define a parcel-independent measure of convective inhibition, let us first examine a particular example of the function $\mathrm{PE} = \mathrm{PE}(\Delta p_1, \Delta p_2)$ (normalized by the total mass of the atmosphere p_0/g to facilitate comparison with RW92) computed for the particular GATE* sounding used in RW92 and depicted in Fig. 7(a). This example displays features typical of most potentially unstable soundings analyzed so far, namely the existence of two main regions, one negative and one positive, the latter exhibiting a global maximum for the particular values $\Delta p_{1,\max}$ and $\Delta p_{2,\max}$ which define the reference state associated with MAE, i.e. $\mathrm{MAE} = \mathrm{PE}(\Delta p_{1,\max}, \Delta p_{2,\max})$. With the present notations, our previous parcel-independent definition (8) of CAPE becomes:

$$\frac{1}{2}\overline{\mathrm{CAPE}} = \max_{\Delta p_2} \left\{ \frac{g\,\mathrm{PE}(\Delta p_{1,\max}, \Delta p_2)}{\Delta p_{1,\max}} \right\} = \frac{g\,\mathrm{MAE}}{\Delta p_{1,\max}}. \tag{13}$$

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* Global Atmospheric Research Program Atlantic Tropical Experiment.

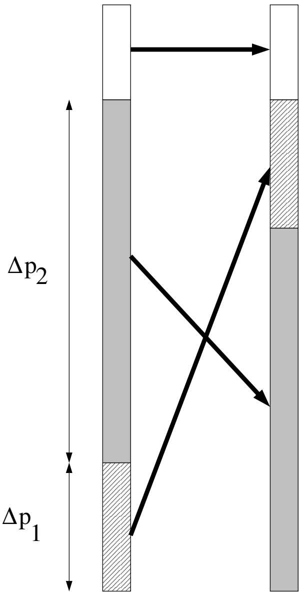

Figure 5. Schematic depiction of the two-parameter transformation.

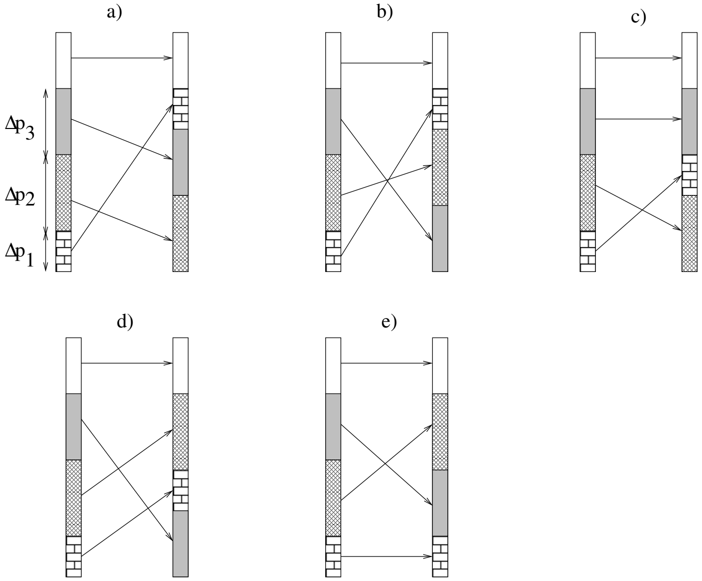

Figure 6. Possible vertical rearrangements associated with a three-parameter transformation. Note that only the transformations (b) and (e) are not reducible to the two-parameter transformation described in the text.

Physically, all adiabatic rearrangements belonging to the region PE > 0 (PE < 0) have less (greater) total NKE than the actual state, and thus are able to release energy (require external energy supply). Since the actual state lies in the negative region and is a zero of PE, it is a reference state by the convexity definition (12). Obviously, our sought-for parcel-independent definition of convective inhibition must be linked to the negative

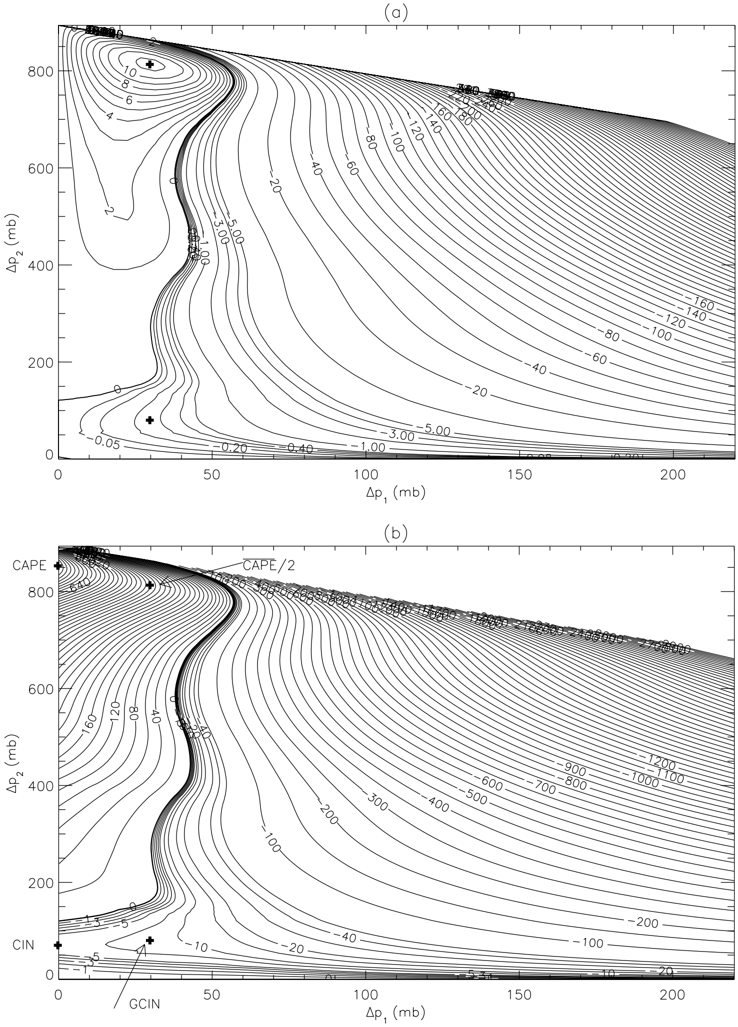

Figure 7. The function PE normalized by (a) the mass of the atmospheric column, i.e. $g\,\mathrm{PE}/p_0$ (J kg^{-1}) and by (b) the function $g\,\mathrm{PE}/\Delta p_1$ (also J kg^{-1}) as a function of the two parameters Δp_1 and Δp_2. (a) displays a global maximum of about 11.2 J kg^{-1}, which is about the same as computed by RW92, and defines the coordinates $(\Delta p_1, \Delta p_2)$ where $\overline{\mathrm{CAPE}}/2$ is located in (b). Also indicated in (b) are the locations of GCIN and of the classical CAPE and CIN defined for the lowermost air parcel.

values of PE. A possible and natural way to define a generalized CIN, which is consistent with (13), is as follows:

$$\mathrm{GCIN} = \min_{0 \leqslant \Delta p_2 \leqslant \Delta p_{2,\max}} \left\{ \frac{g\,\mathrm{PE}(\Delta p_{1,\max}, \Delta p_2)}{\Delta p_{1,\max}} \right\}. \tag{14}$$

The locations where $\mathrm{PE}(\Delta p_{1,\max}, \Delta p_2)$ reaches its two extreme values are visualized in Fig. 7(a) as the crosses. Another quantity of interest is the normalized function $g\,\mathrm{PE}(\Delta p_1, \Delta p_2)/\Delta p_1$, which has the same dimension as CAPE and CIN, and which is depicted in Fig. 7(b). This function allows us to recover the classical definitions of

CAPE and CIN for the lowermost parcel of the sounding as follows:

$$\mathrm{CAPE} = \max_{0 \leqslant \Delta p_2 < p_0} \left\{ \lim_{\Delta p_1 \to 0} \frac{g\,\mathrm{PE}(\Delta p_1, \Delta p_2)}{\Delta p_1} \right\}, \tag{15}$$

$$\mathrm{CIN} = \min_{0 \leqslant \Delta p_2 < p_0} \left\{ \lim_{\Delta p_1 \to 0} \frac{g\,\mathrm{PE}(\Delta p_1, \Delta p_2)}{\Delta p_1} \right\}. \tag{16}$$

A visual comparison of the parcel-dependent and parcel-independent definitions of CAPE and CIN is provided in Fig. 7(b); the relevant values are simply those of the contours on which the crosses sit. In general, we always have $|\mathrm{GCIN}| > |\mathrm{CIN}|$, with the discrepancy between the two quantities increasing with increasing $\Delta p_{1,\max}$, and vanishing in the limit $\Delta p_{1,\max} = 0$. Physically, this occurs because GCIN takes into account by construction the work done by the compensating subsiding environment which is always negative. For this reason, the standard CIN is expected to underestimate the actual convective inhibition.

4. HORIZONTAL VARIABILITY OF MOIST APE AND MULTIPLE REFERENCE STATES

(*a*) *Horizontal variability of APE using mean versus local soundings*

So far, the main underlying assumption used throughout this paper about triggered convection is that the latter occurs when subgrid-scale triggering energy becomes large enough to overcome convective inhibition. In this view, convective inhibition is generally assumed to pertain to the large-scale or mesoscale mean environmental conditions, and not to that of a particular individual sounding. At least, that is what needs to be assumed in cumulus parametrizations for large-scale AGCMs for consistency. In reality, however, one may plausibly conceive that deep convection originates at places where the locally defined convective inhibition vanishes, i.e. at the places where the sounding is absolutely unstable, not potentially unstable. To gain insights into this issue, it is of interest to examine how the function $\mathrm{PE}(\Delta p_1, \Delta p_2)$ estimated for the mean conditions is representative of that estimated for each particular individual sounding. This is useful, for instance, to assess the potential importance of subgrid-scale variability. As an example, we depict in Figs. 8, 9 and 10 the normalized function $g\,\mathrm{PE}(\Delta p_1, \Delta p_2)/p_0$, both from the mean soundings and from the five particular soundings available for 2330 UTC on 14 July 1997. The sounding locations are given in Table 1.

The function PE for the averaged conditions (Fig. 10(b)) is seen to share the same qualitative features as the GATE sounding of Fig. 7, namely two broad regions, one positive and one negative, with a well-defined deep reference state, and the actual state being also a reference state. The corresponding sounding (Fig. 10(d)) shows BL air to be conditionally unstable, and to possess both well-defined CIN and CAPE. However, examination of each particular sounding at the five sites reveals stability characteristics that vary greatly from one place to another. Specifically, the B1 and B4 sites (Figs. 8(a) and (b)) appear to be absolutely stable in the sense that their PE is mostly strictly negative. In both cases, this can be accounted for by the corresponding soundings (Figs. 8(c) and (d)) displaying quite large CIN and negligible CAPE. The B5 and C1 sites (Figs. 9(a) and (b)), on the other hand, display features very similar to that of the mean sounding, except for the evidence of a small positive area associated with a shallow reference state. The corresponding soundings (Figs. 9(c) and (d)) show respectively small CIN and large CAPE at the B5 site, which explains the large positive area of PE, while the large CIN seen at the C1 site explains why the same positive area is so small despite quite significant CAPE. Most interesting, however, is the case of the

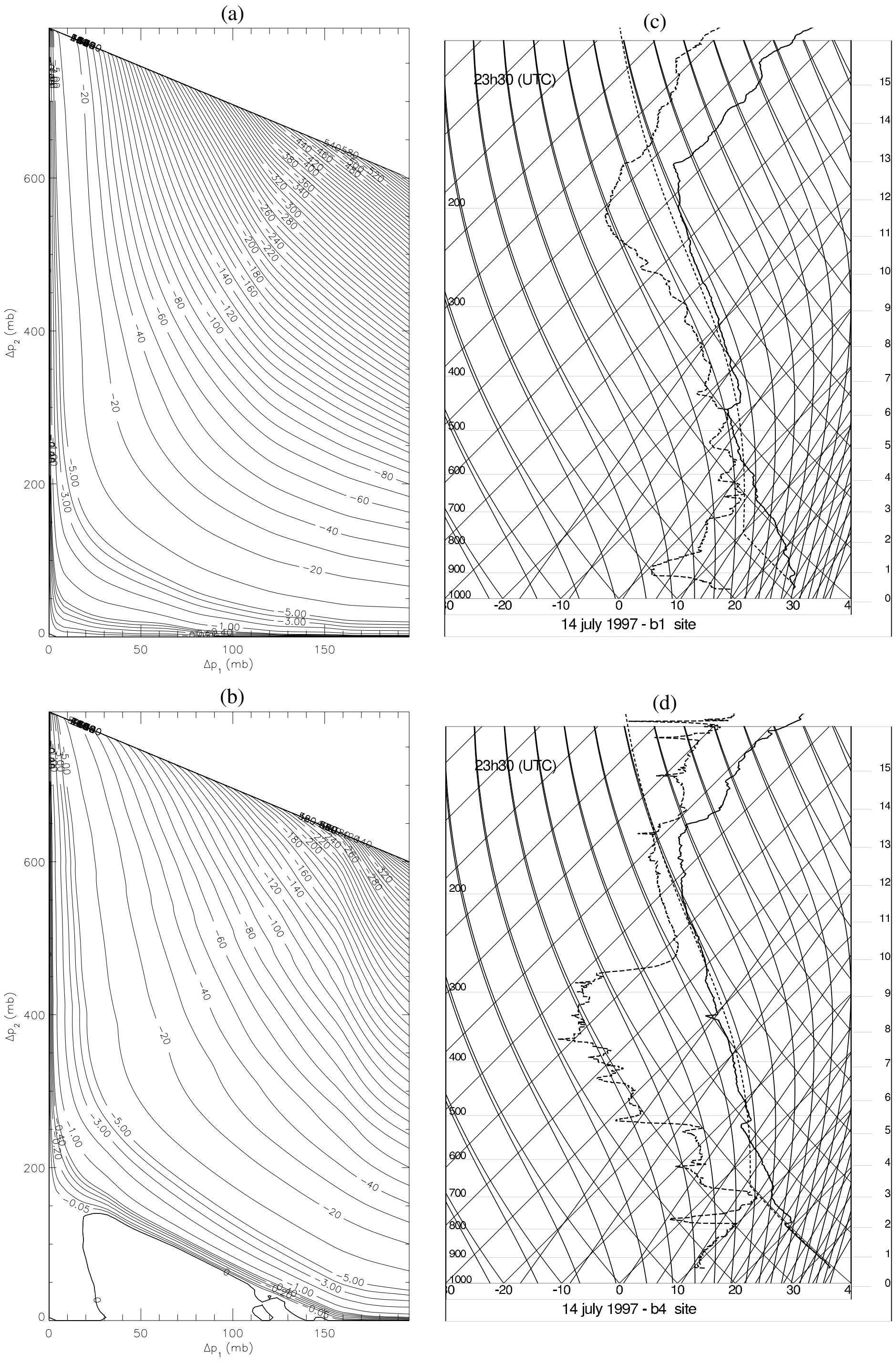

Figure 8. The normalized function $g\,\mathrm{PE}(\Delta p_1, \Delta p_2)/p_0$ (J kg^{-1}) at (a) the B1 and (b) the B4 sites at 2330 UTC on 14 July 1997. The corresponding soundings are at (c) and (d), respectively, and display temperature (solid line), dew-point temperature (dashed line), and the density temperature of an adiabatically lifted boundary-layer air parcel (short-dashed line).

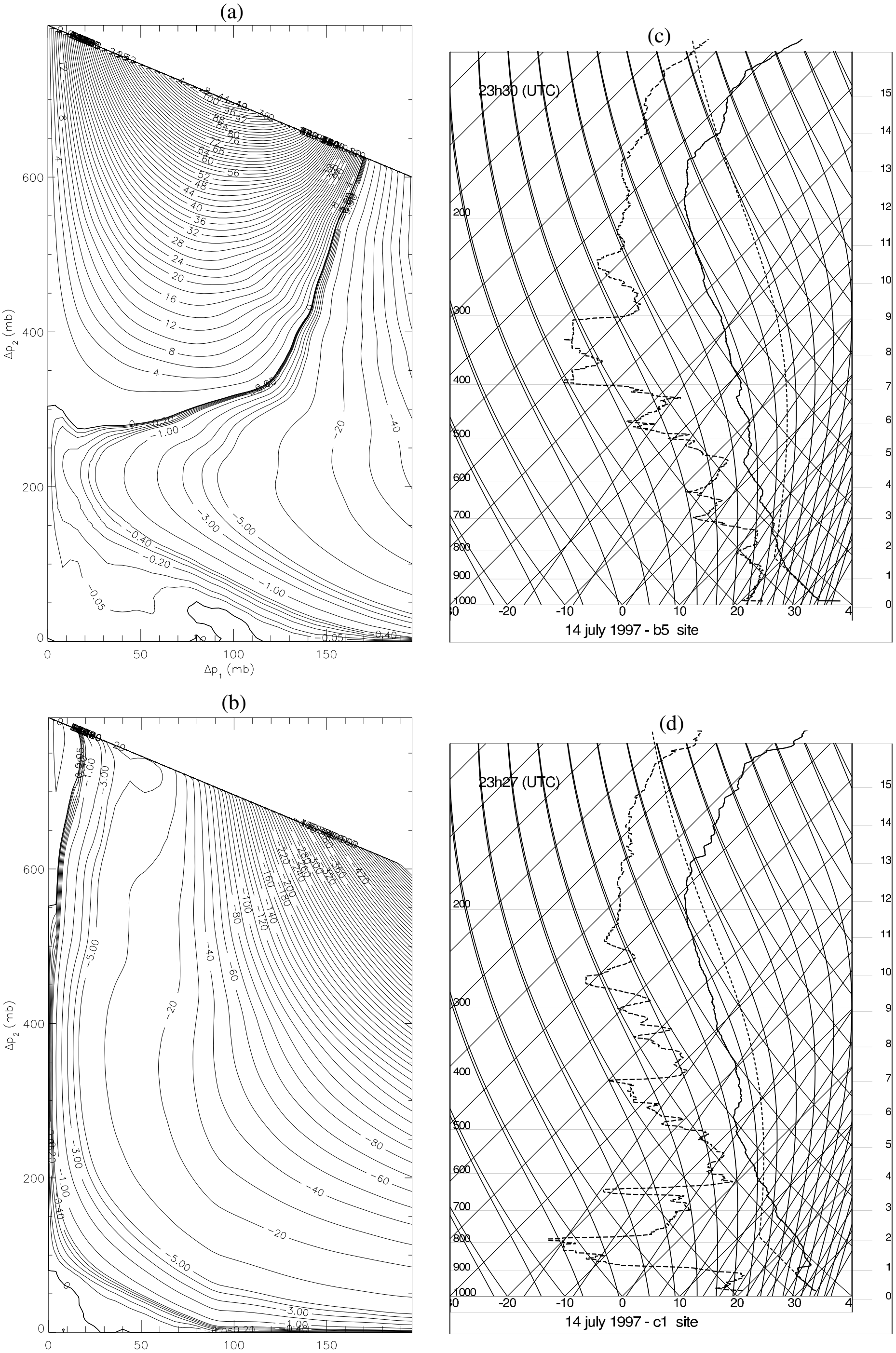

Figure 9. As Fig. 8, but for (a, c) the B5 and (b, d) the C1 sites.

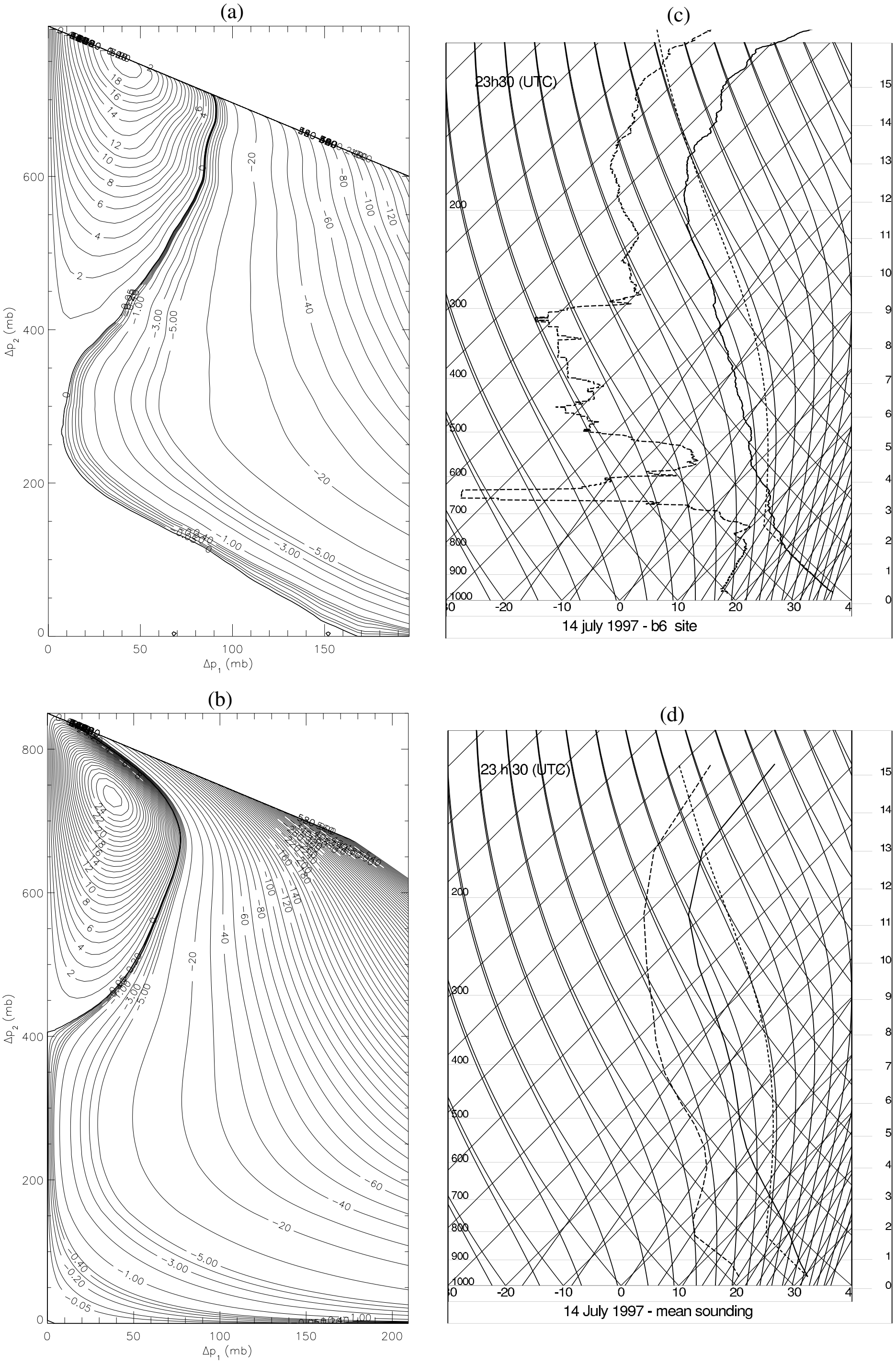

Figure 10. As Fig. 8, but for (a, c) the B6 site and (b, d) the mean conditions.

TABLE 1. DETAILS OF THE FIVE SOUNDING SITES

Site	Latitude (°N)	Longitude (°W)	Altitude (m)
B1	38.30	97.30	447
B4	36.07	99.20	622
B5	35.68	95.85	217
B6	34.97	97.42	344
C1	36.61	97.49	315

B6 site (Fig. 10(a)), where the positive area is seen to connect the shallow and deep reference states, suggesting that the atmosphere is locally absolutely unstable in some global sense. The corresponding sounding (Fig. 10(c)) is seen to possess a very deep well-mixed layer that extends up to 800 mb, which is furthermore topped by a thin layer of very dry air around 650 mb.

The study of this particular example reveals that the stability characteristics inferred from averaged conditions, which in that case suggest potential instability, may translate into widely different local stability characteristics including mostly absolute stability, potential instability, and absolute instability. Here, the concepts of stability and instability are to be understood in a global sense, in relation to the amount of energy that needs to be provided or extracted from the atmosphere in an adiabatic rearrangement of mass of the air column. The existence of absolutely unstable soundings is found to be intriguing, because it suggests that deep convection might be able to develop spontaneously at such places, without the need for triggering processes to overcome convective inhibition, as is often believed. However, additional data, or case-studies using cloud-resolving models for instance, are required to provide conclusive evidence.

(*b*) *Empirical evidence for multiple 'deep' reference states*

We conclude this empirical examination of stability characteristics of atmospheric soundings based on the function PE by presenting a case displaying multiple reference states. Indeed, most of the soundings examined so far and possessing CAPE exhibited in general only two distinct references states, namely one deep reference state defining MAE, and a shallow reference state differing or not from the actual state depending on the sign of PE for infinitesimal values of Δp_1 and Δp_2. In some isolated cases, however, such as the one depicted in Fig. 11, the function PE(Δp_1, Δp_2) is found to possess at least two extrema associated with two deep reference states, in addition to a shallow reference state associated with near-surface unstable air. Qualitatively, this case shares the feature of the B6 site of Fig. 10 to be absolutely unstable for arbitrary small Δp_1, a feature only found in general for particular individual soundings, never for the 'averaged condition'. The analysis of the corresponding sounding (Fig. 11(b)) reveals that the BL air parcels possess intermediate LNBs around 450 mb and 385 mb respectively, as well as a deep LNB around 255 mb; in this case the buoyancy is computed for a purely adiabatic displacement with water loading. However, only the deeper of these LNBs remains under a pseudo-adiabatic displacement, which indicates that the intermediate LNBs are only marginally well-defined. Curiously, the intermediate reference state of the function PE does not appear to be correlated to these intermediate LNBs, but rather to a buoyancy minimum that exists near 555 mb. Therefore, although multiple reference states appear to be obviously correlated to multiple LNBs or buoyancy minima, the correlation is not completely straightforward.

The existence of multiple deep reference states raises the issue of towards which one, if any, the atmosphere will tend to adjust. Physically, it is conceivable that the atmosphere would initially prefer to evolve toward the 'shallowest' deep reference state,

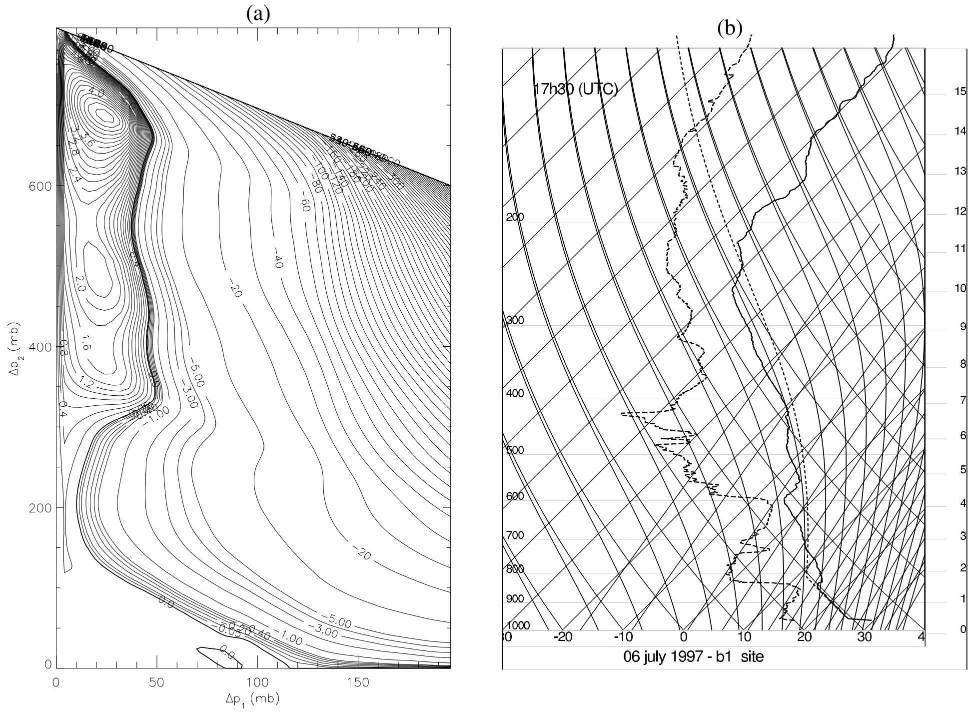

Figure 11. Particular example of (a) the function PE(Δp_1, Δp_2) at the B1 site at 1730 UTC on 6 July 1997 exhibiting multiple local extrema and (b) the corresponding sounding. Displayed quantities on the sounding are same as in Figs. 8, 9, and 10.

which is nearer to the actual state and hence easier to reach than the deepest reference state. Since by definition a reference state is stable, external energy is required to evolve toward a different reference state, so that the atmosphere could possibly reside in such a state for a while. If so, this could have consequences for the observed cloud-top heights, which would not immediately reach the highest level predicted by the undiluted adiabatic ascent of BL air parcels up to their LNB, as is often done. What determines the cloud-top heights in reality was recently addressed by Redelsperger *et al.* (2002), who suggested that entrainment could be the main reason why the latter method sometimes grossly overestimates actual cloud-top heights. However, in the presence of multiple reference states, as proposed here, it would be still possible for the cloud-top heights to be mainly determined by adiabatic processes. To test the present results, it would be of interest to try to correlate the observed cloud-top heights with extrema of the function PE(Δp_1, Δp_2). This requires a separate study which will be reported elsewhere.

5. DISCUSSION

In this study, we examined some fundamental and basic issues pertaining to the understanding and description of the global energetics of continental deep convection as part of the effort undertaken within the EUROCS project to improve the representation of the diurnal cycle of deep convection over land in atmospheric GCMs. Energetics considerations play indeed a central role in our current view of triggered convection, which relies mostly on the idea that it occurs when subgrid-scale triggering energy becomes sufficient to overcome convective inhibition. To put this idea on solid theoretical grounds, it is essential to develop a good understanding of the global energetics of the diurnal cycle of deep convection over land. However, progress has been hindered because of seeming incompatibilities between the two main extant views of energetics, namely the parcel view and the global view. Indeed, the parcel view is centred on the concepts of CAPE and CIN, whereas the global view is centred on the concepts of APE, reference state, and energy conversion terms. In this paper, we sought to understand how each particular concept defined in a particular approach translates into the other approach.

To that end, we first invoked previous work by Emanuel (1994) to clarify the links between the parcel-defined CAPE and the globally-defined MAE. Building upon this work, we introduced a parcel-independent definition of CAPE, which was found to be close to the parcel-dependent definition for low-resolution atmospheric soundings. In that case, there is a simple conversion factor M_b between MAE and CAPE, which physically scales at leading order as the mass per unit area of conditionally unstable BL air. Interestingly, we found empirical evidence that M_b strongly correlates to CAPE, suggesting the existence of a functional relationship between CAPE and MAE. If further confirmed, this result would be very important, as it would permit a computationally inexpensive determination of MAE, which is not presently the case. This also shows that there is additional information content in MAE as compared to CAPE, which is encompassed in the factor M_b. This suggests that future theoretical studies should aim at using MAE rather than CAPE to better understand how the knowledge of M_b could possibly improve our knowledge and understanding of triggered deep convection.

In a second step, we argued that the lack of an analogue to the concept of energy barrier in the current approaches to utilizable energy, whether framed in terms of APE, exergy, or pseudo-energy, probably stems from the failure to recognize the need for multiple reference states to define the latter optimally, as implicitly suggested recently by Codoban and Shepherd (2003), but already hinted at by Lorenz (1955). As a result, the reference states in use are rarely attractors of the system, nor necessarily attainable 'dead states'. The idea of multiple reference states, however, enables energy barriers to arise naturally as the amount of energy required to pass from one reference state to the other, with some reference states hopefully becoming attainable. In the case of triggered deep convection, we find that two reference states at least are required to describe its transient dynamics, namely a shallow one, which is either the actual state or a closely related one, and a deep reference one which is associated with the MAE of Lorenz (1978,1979). Physically, the need for at least two reference states stems from the fact that neither CAPE nor MAE are readily available for conversion into kinetic energy before deep convection is actually triggered, i.e. during stage III.

The concept of multiple reference states might prove useful to account, at least partially, for observed cloud-top height distributions in some cases, by invoking only purely adiabatic processes. In contrast, Redelsperger *et al.* (2002) argue that entrainment might explain why sometimes cloud-top height appears to be overestimated by the computation of the adiabatic undiluted ascent of BL air parcels. However, our explanation in terms of multiple reference states requires further study and confirmation, owing to the small number of soundings found to exhibit this property. Finally, we also find that the stability characteristics estimated from the mean sounding often translate into very different stability characteristics for local soundings, which suggests that our classical view that deep convection over land is triggered when ALE becomes able to overcome CIN might be an artifact of assessing stability from mean soundings, as it seems possible for local soundings to be absolutely unstable.

In conclusion, while we believe that the present study offers significant new insights into the energetics of triggered deep convection, even if admittedly only with regard to its concepts and often only speculatively, it seems fair to say that many more issues need to be resolved to organize the present new ideas into a really coherent picture. For instance, it is still unclear how one should modify the energy balance equations (1) to deal with multiple reference states and energy barriers, as well as to account for non-conservative processes such as entrainment. To answer these questions, further work is required to understand better the physical nature of the reference states, and to confirm their hypothesized attracting properties. These remain challenging issues. We hope that the present study will help to stimulate research aimed at resolving them.

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