A nomogram for the atmosphere

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When our departmental library cleared out some of their old and largely unused books —not always a reflection on their quality: there were two copies of Lamb’s “Hydrodynamics”— I came across a book by H. J. Allcock and J. Reginald Jones called “The nomogram,” a title intriguing enough to warrant a closer look.

A nomogram (or nomograph) is a graphical computation chart. Like a slide rule, it works by reading off numbers on graduated scales, but unlike a slide rule, a nomogram is designed to perform only one calculation or a few related calculations efficiently. Like the slide rule, nomograms have been largely displaced by electronic calculators, but unlike the slide rule, some nomograms are still in use. A web search will reveal several examples. A tephigram is a type of nomogram: it is more than just a graph; it also represents relationships between several variables and can be used for graphical computations such as the determination of atmospheric stability, water vapour mixing ratios, or convective available potential energy.

In meteorology we often need to convert between pressure, density, temperature, and potential temperature. Following Allcock and Jones, I constructed a nomogram to do these computations. We further need a device to lay an “index line” over the nomogram: a ruler or, more practically, an overhead transparancy with a single straight line drawn on it. The nomogram itself can be seen in Figure 1.

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The nomogram contains four graduated scales: for potential temperature, density, pressure, and temperature. We can choose any two of these and read off the other two using the ruler: put the ruler against the values of the two chosen variables and read off the other two missing values. But there is more: the temperature scale also shows a saturated vapour pressure scale, and the pressure scale also shows a 1976 US standard atmosphere height scale, so these two quantities can be thrown into the mix as well. For good measure I included a Celsius-Fahrenheit converter, which consists of a separate linear scale with two graduations, the simplest form of nomogram possible. All calculations used are detailed in Box 1.

As an example, consider an air parcel at 500hPa and -10°C. A ruler crossing these scales at the given pressure and temperature values crosses the density scale between 0.65 and 0.675 kg m\(^{-3}\) and the potential temperature scale just above 320K. The corresponding saturated vapour pressure is between 2.75 and 3 hPa and the corresponding standard height slightly above 5.5km.

Using this nomogram reveals the great advantage of graphical computation methods: qualitative and quantitative relationships between variables become immediately clear. Not just single point values can be read off, their ranges and gradients also become obvious. We can see where small changes in one variable can lead to big changes in another variable or vice versa. Answers to questions such as "how much would the temperature decrease if an air parcel is lifted adiabatically to such-and-such a pressure level" can be read off easily as well. The nomogram places a single computation in a continuum of similar computations.

Figure 2 is a schematic to get an idea how the nomogram works. The red index line crosses the three parallel equidistant lines \(A\), \(B\), and \(C\). It is now easy to see that for any index line the locations of the crossing points will satisfy \(p + r = 2q\), so this way we can perform additions. The coefficients can be changed by changing the relative distances. If we now put logarithmic scales on each parallel line it is clear that we can also do multiplications (including fixed powers, depending on the scalings and relative distances of the parallel lines.) The atmospheric nomogram is essentially a scaled and sheared version of this basic design with the added complexity of two simultaneous multiplications where all values fall within specified ranges.

I have found my nomogram quite satisfying to use and I have been show-
ing it to students who were intrigued to figure out how, and why, it works: many of today’s students are unaware of logarithmic tables and slide rules. Readers are invited to download an electronic copy of the nomogram (a pdf file) from the web: search for "nomogram ambaum" in a web search engine.

Reference:


Box 1 — Computations presented in the nomogram.

The nomogram computes the ideal gas law for dry air:

\[ p = \rho RT \]

with \( p \) the pressure, \( \rho \) the density, \( R \) the specific gas constant for dry air (287 J kg\(^{-1}\) K\(^{-1}\)) and \( T \) the absolute temperature. The potential temperature \( \theta \) for dry air is given by:

\[ \theta = T \left( \frac{p_0}{p} \right)^{R/c_p} \]

with \( p_0 \) a reference pressure of 1000hPa and \( c_p \) the specific gas constant for dry air at constant pressure (1004 J kg\(^{-1}\) K\(^{-1}\)). The saturated vapour pressure \( e_s \) for water is related to the temperature by Teten’s empirical formula:

\[ e_s = 6.1129 \exp \left( \frac{17.2694 T(C)}{T(C) + 237.3} \right) \]

where \( T(C) \) is the temperature in Celsius.
**Figure 1:** The nomogram as described in the article. To use it, a ruler is required to lay a straight index line over the nomogram. For any straight index line the values of potential temperature $\theta$, density $\rho$, pressure $p$ and temperature $T$ will be consistent with each other.
Figure 2: Schematic of a basic nomogram. Lines A, B, and C are parallel and equidistant. For any randomly drawn straight index line (in red) we will have \( p + r = 2q \).