On Sandström's inferences from his tank experiments – a hundred years later

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Abstract

In 1908, J.W. Sandström conducted several tank experiments to illustrate oceanic circulations driven by the wind and by buoyancy fluxes. His main inference from them is: "A circulation can develop from thermal causes only if the level of the heat source lies below the level of the cold source." This inference applies to buoyancy-driven overturning circulations that are steady and closed. The relevance of this inference, which is often quoted as "Sandström's theorem", has been under discussion ever since. It seems that Sandström was not careful enough in observing his experiments. He overlooked diffusively driven circulation patterns. At the same time, many of his pioneering ideas, together with his main inference, still appear qualitatively correct when applied to the observed Atlantic meridional overturning circulation. As a tribute to the centenary of Sandström's publication some common misconceptions about what Sandström (1908) exactly said will be identified here. It is hoped that this clarification is substantiated by a translation of Sandström's original 1908 paper into English.

1. Introduction

Sandström (1908) is often cited in the literature on buoyancy-driven overturning circulations in fluids. Such overturning circulations are found in the atmosphere and in the ocean, and they have been treated in tank experiments in laboratories as well as in numerical models. There is a strong interest to understand how such circulations are maintained against dissipation both from the viewpoints of climate dynamics and fluid dynamics (see the reviews of Kuhlbrodt et al., 2007, and Hughes and Griffiths, 2008, respectively). In the case of the Atlantic meridional overturning circulation (AMOC) it is important to better understand its physics since the large amount of heat it transports (Trenberth and Caron, 2001) influences the climate in several regions, e.g. northwest Europe. This role of the AMOC can be inferred from climate records of the past (Rahmstorf, 2002) as well as from model simulations of the future (Meehl et al., 2007).

It has been debated for a long time whether the AMOC, as a thermally direct circulation, is "pushed" or "pulled" (e.g. Marshall and Schott, 1999; Visbeck, 2007). In the "pushing" mechanism the cooling in high latitudes is seen as the primary driver since it leads to deep water formation and subsequent downwelling. A balancing upwelling is then assumed (Stommel and Arons, 1960). The "pulling" hypothesis asks how exactly the upwelling from the deep ocean is achieved. This question leads to energetic arguments from which two alternative theories of a mechanically driven AMOC emerged: upwelling through the interior of the ocean that is balanced by a mixing-driven downward heat transport, and wind-driven upwelling in the Southern Ocean (reviewed in Kuhlbrodt et al., 2007). Sandström (1908, henceforth cited as S08) pioneered the "pulling" hypothesis in that he inferred from his experiments that "a circulation can develop from thermal causes only if the level of the heat source lies below the level of the cold source." Later on, this inference has often been cited as "Sandström's theorem", and it has also been said that it is not correct. A closer look at the literature reveals that in many instances Sandström is cited only indirectly, via authors that, as it turns out, render Sandström's (1908) results along with their own interpretation. To some extent this is of course due to the fact that Sandström (1908, 1916) published in German, along with some of the later authors (Bjerknes, 1916; Defant, 1929).

The present paper aims at clarifying what S08 actually did and said. To this end we have translated that paper into English (see Appendix). In the following section 2 we discuss S08's experiments, his actual inference, and the treatment of his work in later studies. Our focus is hereby on the relevant points with regard to the AMOC. Section 3 tries to outline why Sandström's early work still today deserves to be appreciated, even if the physical concepts he had were not clear enough to withstand later scrutiny. Some brief conclusions can be found in section 4.

2. Sandström's (1908) statements

2.1 Diffusion and mixing

As a prerequisite to discuss S08's statements, we need to clarify the concepts of conduction (in a fluid), molecular diffusion, salt fingering, turbulent mixing, and what S08 calls "heat convection".

In fluids like water or air the molecules always move in a random way. Heat, salt and other tracers are redistributed by this motion. This process is called molecular diffusion. For instance, the molecular heat diffusivity of sea water is 1.39×10^{-7} m^2 s^{-1}. For the purposes of this paper we can assume that, in a fluid, molecular diffusion is identically the same as conduction. Turbulent mixing, by contrast, is a process that is generally associated with the dissipation of kinetic energy. Therefore, for turbulent mixing to be present in a fluid some form of external energy supply is required. In the ocean this is provided by the winds and the tides (about 1 TW each, where 1 TW = 10^{12} W; Munk and Wunsch, 1998). The associated turbulent diffusion coefficient has a typical value of 10^{-5} m^2 s^{-1}. In a few small areas of the ocean very strong surface heat fluxes can sometimes lead to a destabilizing of the water column and ensuing vigorous vertical mixing that is many orders of magnitude stronger than the general turbulent mixing. This process is called convection.

We note that there is an ongoing discussion about the amount of work done by the surface buoyancy fluxes. Partly due to lack of agreement on its definition, the estimates lie in the range from being comparable to the work of the wind, e.g., 1.2 TW ± 0.7 TW (Oort et al., 1994), to being somewhat smaller, e.g., 0.2-0.4 TW (Tailleux, pers. comm.), and eventually to being utterly small or even zero (Wunsch and Ferrari 2004; Wang and Huang, 2005). Hence there is a possibility that the surface buoyancy fluxes might turn out to be a significant source of mechanical energy to the ocean.

It is intriguing that S08 apparently was aware of molecular diffusion, but did not realize its potential to generate an overturning circulation independently of the position of the warm and cold sources. He warns of "diffusive phenomena" (sec. 3) in case of too strong salinity gradients in the tank. He also describes in great detail the mechanism that is today known as salt fingering (sec. 5) and suggests this as a mechanism to carry heat downwards. Salt fingering arises from the fact that, in sea water, the molecular diffusivity of heat is about two orders of magnitude larger than that of salt. It always develops where a warm and saline water mass lies on top of a cooler, fresher and denser one, and where the turbulent mixing is very weak. Sandström himself refers to this process as “heat convection” later in his text (sec. 8), evoking surface evaporation to start it. It is not quite clear to what extent Sandström realized the difference between convection and salt fingering. He seems to have imagined the process he describes in sec. 5 as a large-scale one. Salt fingering as we know it today has however a length scale of centimetres, and it is entirely different from convection as no surface buoyancy fluxes are involved. Its relevance for driving the AMOC is currently being studied (Canuto et al., 2008).

2.2 Conducting the tank experiments

Sandström conducted two series of experiments, in 1908 and in 1916. For the description of the experiments of S08 we refer the reader to the translation in the appendix, specifically sec. 3 (beginning), sec. 6 (beginning), and sec. 7. For the experiments of Sandström (1916, henceforth cited as S16) we will give a brief summary in the following. S16 stresses that he put great care in suppressing temperature fluctuations in the laboratory. The tank was bigger this time (50 cm x 50 cm x 5 cm), but was otherwise constructed in the same way as in S08. The heating and cooling elements in the fluid were “metal cell systems” with water flowing through them, similar to radiators of a central heating system. This seems to be more efficient that the simple tubes used in 1908. The temperature differences applied to the fluid were “small but constant”. This is in contrast to the 40°C temperature difference used in the 1908 paper. The pipes leading to and from the metal cells were carefully isolated. Constant temperature of the cells was ensured by using large barrels as a reservoir for the through-flowing water. The results from the experiments are the same as in S08, with an overturning cell developing between a low heat source and a higher heat sink. This cell is sharply confined to the region between the heat source and sink, with the water being stagnant above and below, exactly like in Fig. 18 of S08.

Coman et al. (2006) re-conducted Sandström’s experiments under conditions very close to the ones from 1908. Remarkably they observed entirely different circulation patterns. Specifically, in their experiments the overturning circulation, consisting of one or more cells, filled the whole depth of the tank, no matter how the heat sink and source were positioned. Coman et al. (2006) could not find stagnant parts of the fluid as Sandström did.

Why are the results from Coman et al. (2006) so different? There are certainly shortcomings in the way S08 and S16 conducted his experiments. It is obvious that he did not insulate his tanks. Note however that S08 himself implicitly acknowledges this by describing the circulation that ensues when the temperature differences between the different partitions of the fluid and the surrounding air vanish (third-but-last paragraph of sec. 7). It seems that S08 was simply not aware of the relevance of this long-term development of his experiment towards the equilibrium state. Unfortunately a note in S08 and S16 about how long exactly he observed the experiments lacks. Coman et al. (2006) waited for three days before analysing their circulation. They suggest that he failed to observe the equilibrium because the fluid velocities are one order of magnitude smaller then (as compared to the initial transients), qualifying as "minor" in Sandström's view. However, S16 explicitly says that he describes the "stationary state". In addition, S08 and S16 seems to have undergone some effort in observing the spread of the fuchsine dye, using a bright arc lamp. He must have failed in seeing the diffusive spread of the colour, but it remains puzzling that for the case of the heat source below the heat sink S08 (Fig. 18) observes a cell boundary – implying a small velocity – at the position where Coman et al. (2006, Fig. 3a) observe a velocity maximum.

2.3 Sandström's inference

The actual inference that has often been referred to later as "Sandström's theorem" is drawn in S08, sec. 7: "A circulation can develop from thermal causes only if the level of the heat source lies below the level of the cold source." From the context it is clear that closed and steady circulations are meant here. We would like to refer to it as "Sandström's inference" as this seems to be an appropriate translation of how S08 himself called it ("Schluß" in German). Neither S08 nor S16 use the word "theorem" when referring to this statement. The two theorems that are referred to in the title of Sandström (1922) are different statements. They are very similar to the two conclusions drawn in sec. 9 of S08.

How relevant is the paper that Sandström published in 1916? Its title, in translation, is: "Meteorological Studies in the Swedish High Mountains". The largest part of the paper describes an expedition that Sandström – entirely on his own – conducted in the Swedish Mountains in the winter of 1913/ 1914. To explain his observations, he repeats the tank experiments from the 1908 paper (see above) and applies them to atmospheric circulations, specifically to a hypothesized circulation between the Gulf Stream and the European mountain ranges. The progress lies in the analysis of the experiments in terms of a Carnot heat engine. He assumes that the warming and cooling of the air happens under isobaric conditions, and that the air moves adiabatically between the levels of warming and cooling. (See Kuhlbrodt et al., 2007, for a more detailed discussion.)

However, the heat engine concept is not applied to oceanic circulations in S16. In the whole paper the ocean is only indirectly referred to, in that Sandström says that he earlier saw the tank experiments conducted by Prof. O. Petterson, and that Petterson aimed at explaining oceanic circulations with them. This is not further elaborated upon however. Moreover, there is no citation given with this statement, and Sandström does not even mention his own 1908 paper.

As an aside we note that the hypothesized atmospheric circulation in S16 is driven only by pressure gradients. Sandström did not realize that the Coriolis force would balance such large-scale pressure gradients. His view is entirely thermodynamic. In addition, Sandström does not use the term "pressure gradient", but rather invokes "Bjerknes forces" (S08).

In the following paragraphs we will briefly review how Sandström's experiments and his inference have been discussed by later authors. The paper by Bjerknes (1916) discusses heat engines in form of fluid loops. In terms of the application to the atmosphere and the ocean it adds nothing new to S08 and S16. We note in passing that Bjerknes (1916) speaks of Sandström's "principle".

Substantial progress in understanding buoyancy-driven overturning was achieved by Jeffreys (1925). He explicitly used the hydrostatic equation to describe Sandström's experiment (as opposed to S08's "Bjerknes forces"). In addition, Jeffreys (1925) was fully aware of the role of molecular diffusion. He concluded that molecular diffusion should always lead to a heat transport and an ensuing flow in the fluid, irrespective of the position of the heat source and the heat sink. Given this heat transport throughout the fluid, Jeffreys (1925) inferred that one cannot maintain S16's assumption that expansion and contraction happen under isobaric conditions. In order to obtain a net gain from the work of expansion and contraction, the only condition is that "the path from the cold region to the hot one must lie below the return path". Jeffreys (1925) also noted that turbulent mixing would be much more important than molecular diffusion, at least in the atmosphere; he was not aware, however, of turbulence in the ocean below the mixed layer.

According to Jeffreys (1925), in the absence of turbulent mixing an equilibrium would be established between molecular diffusion and convection. It seems that all later experiments, be they numerical or in an actual tank (see the recent review by Hughes and Griffiths, 2008), confirm Jeffreys' (1925) theoretical results in that some kind of overturning always develops. However, the exact shape may depend sensitively on parameters like the fluid's viscosity (Rossby, 1998), the tank dimensions and the applied temperature difference. It is even possible that the overturning does not penetrate the whole depth of the tank, or that no steady state is reached (Wang and Huang, 2005).

Defant (1929) speaks of Sandström's "principle". He acknowledges Jeffreys' (1925) objection that an overturning must always develop under differential heating, but is convinced of Bjerknes' (1916) result that the efficiency of a heat engine is enhanced if the heat source is lower than the heat sink. Defant (1961) goes as far as saying that Sandström's "deductions" (as they are called there) are correct.

The term "Sandström theorem" seems to appear the first time in Defant (1961). Here the description in terms of a Carnot cycle is reiterated, but in the form of S16 with isobaric expansion and contraction, not taking Jeffreys' (1925) generalization into account. In addition, Sandström's inference is expressed in terms of the circulation theorem. Assuming that the circulation is steady, and assuming closed streamlines in the fluid, one can show that the work of expansion and contraction against pressure is balanced by the frictional losses. Equivalent statements are found later e.g. in Dutton (1986) and Colin de Verdière (1993). The circulation theorem is however not applicable to buoyancy-driven overturning circulations in general, as (i) the streamlines are not necessarily closed (Vallis, 2006), (ii) the flow is not steady in all cases

(Wang and Huang, 2005), and (iii) in shear flows the work of friction might be locally positive.

The valid statement from the current understanding is that in any fluid to which a heat source and a heat sink are applied some kind of overturning always develops, irrespective of the relative positions of the heat source and the heat sink. The only condition is that “the path from the cold region to the hot one must lie below the return path”, as was found by Jeffreys (1925). This statement can be proven by analysing the energetics of the equations of motion in the Boussinesq approximation (Paparella and Young, 2002; Vallis, 2006). This proof is more general than the one relying on the circulation theorem since a steady flow is the only assumption needed. An alternative to this proof is based on the streamfunction of the fluid (Nycander et al., 2007). Paparella and Young (2002) studied the case of vanishing molecular diffusion using a very strict definition of turbulence. In the light of the above discussion it is however not entirely clear what their results imply for the fluids of interest here in which molecular diffusion is always present.

3. The relevance of Sandström (1908) today

In analysing why Sandström’s (1908) study is still of relevance today, it is useful to distinguish between the overturning circulation in the tank and in that in the ocean. Obviously S08 and S16 failed to observe, in his tank, the overturning circulation that is driven by molecular diffusion. But would surface buoyancy fluxes and molecular diffusion alone be sufficient to drive the observed AMOC? While this does not seem to be the case as far as we know, this question is a matter of ongoing research (see sec. 2.1). S08 says in sec. 5 that “radiation and conduction from layer to layer are too minor to carry downwards such amounts of heat as are necessary here”, i.e. to drive the “Gulf Stream”. Of course, the circulation pattern that is today called “Gulf Stream” is wind-driven and not a deep circulation. However, S08’s use of the term indicates that in using the term “Gulf Stream” he meant the deep overturning circulation that we today call the AMOC (see Fig. 13; note he that is nevertheless well aware of the wind-driven gyres in the subtropics [end of sec. 4]). Then one might say that he intuitively grasped that, presumably, molecular diffusion of heat is indeed too weak to drive the observed overturning.

In other words, whilst Sandström’s (1908) inference does not hold in the strict sense, as discussed in sec. 2, it still seems true that an externally driven heat transport from the surface to depth is necessary to maintain an overturning circulation of the strength observed in the Atlantic. This vertical heat transport provides the “deep heat source” S08 identifies in his inferences as necessary for a buoyancy-driven overturning. Certainly a heat source is not the same as a heat transport, but it is clear from S08 that, for the application to the oceans, he thought of heat transports rather than sources.

It is remarkable that S08 identified a rate-limiting mechanism for the AMOC. S08 concludes that the water volume driven by his “Gulf Stream” depends on the amount of heat that the deep layer gains in the Tropics and that the upper layer loses in the Arctic (sec. 5). S08 was not aware of turbulent mixing, the main driver of this diapycnal heat transport.The hypothesis of the diapycnal heat transport as a rate-limiting driver of the AMOC however is still very relevant. The balance of vertical upward advection and turbulent downward mixing of heat (parameterized as diffusion) is the core assumption in a thread of papers that attempt to constrain the strength of the AMOC by the amount of turbulent kinetic energy available for vertical mixing (e.g. Munk and Wunsch, 1998).

There is one more point that deserves to be mentioned. S08 hypothesizes that there should be an abyssal overturning cell because the abyssal ocean is populated by animals which in turn need oxygen (sec. 8). Thus, he continues, these abyssal layers must be ventilated somehow. This thought shows Sandström's good oceanographic intuition. Later authors like Bjerknes (1916) and Defant (1929, 1961) were convinced that the abyssal ocean is stagnant. Defant (1929) confines the overturning circulation to the upper 200 m. By contrast, S08 concludes from observations that the return flow of the AMOC lies below 500 m and that therefore the downward heat transport must reach depths of up to 1000 m, which is much closer to the values observed today.

From the assumption of such an abyssal overturning cell S08 moves on to think about its possible driving mechanisms. These include viscous friction, which transports momentum downwards, and geothermal heating. S08 acknowledges that the amount of heat provided by the latter is tiny, but he suspects that it plays a role nevertheless, mainly because he cannot think of any other mechanism that could set the abyssal water in motion. Mullarney et al. (2006) have recently pointed out that the geothermal heat fluxes, small as they are, might have a significant destabilizing influence on the stratification.

4. Conclusion

Our discussion of Sandström's (1908) experiments and his inference as well as the ensuing literature has shown that on the practical side he was not careful enough in setting up and carrying out his experiments, and that on the theoretical side he was not fully aware of the presence and the role of molecular diffusion. His inference is therefore not valid in general. However, he did have a good intuitive grasp of the buoyancy-driven overturning circulation in the ocean. He identified the pivotal role of vertical heat transport, suggested that the strength of this heat transport limits the overturning rate, and he hypothesized an additional abyssal overturning cell. These ideas sparked many further studies on the subject of buoyancy-driven overturning flows in tanks and in geophysical fluids, both experimentally and numerically.

From our viewpoint it seems unhelpful to speak of Sandström's "theorem", as this term implies that it can be proved, which S08 and S16 did not do. In addition, Sandström did not use this term – apparently it was Defant (1961) who dubbed Sandström's inference a "theorem". Rather, it seems appropriate to speak of Sandström's "inference", translating the word he used himself. This inference still holds true today insofar as, very likely, an externally driven vertical heat transport is necessary to maintain the observed buoyancy-driven overturning in the ocean.

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Appendix

Dynamic Experiments with Sea Water.

By J. W. Sandström, University of Kristiania[1].

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Translation by Till Kuhlbrodt, Department of Meteorology, NCAS-Climate, University of Reading, UK, 2008.

1.

The motive for these experiments was the following observation I made at the Bornö station in the Gullmarfjord on the west coast of Sweden. When the wind swept over the fjord, the water at the surface flowed in the direction of the wind. Yet, as soon as the wind ceased, it flowed back in the opposite direction.

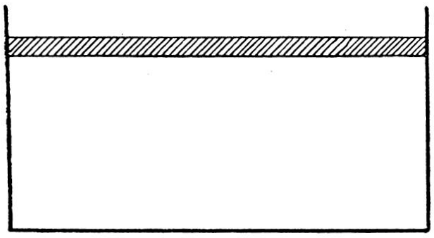
It seemed to me worth the effort to study this remarkable phenomenon further. To this end I tried to produce water that had the same properties as the fjord water. I achieved this in the following way:

I took samples of water from various depths of the fjord and poured these carefully into a glass tank, such that the various samples had the same sequence in the tank as in the fjord. Now, as I blew over the water in the tank, here as well the water at the surface flowed in the direction of the wind, and as soon as I stopped blowing, it flowed back in the opposite direction.

In order to investigate the reason for this phenomenon, I conducted a second experiment with new samples of water in the same way as before, with the sole difference that this time I coloured the uppermost layer with ink before I poured it into the tank. This black water settled as a horizontal layer of uniform thickness on top of the underlying clear water, as shown in Fig. 1. When I then blew over the water, the black water flowed at the surface in the direction of the wind and formed a wedge-shaped layer at the side of the tank the wind was blowing against – i.e. against which I blew – (see Fig. 2); but as soon as I stopped blowing, the black water flowed back to its former position, such that it formed anew a horizontal layer of uniform thickness, as in Fig. 1.

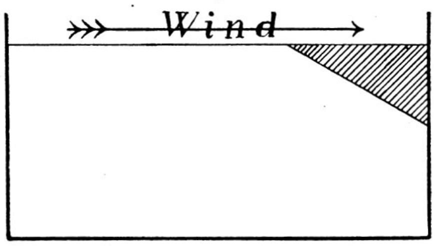
[1] Kristiania is the former name of Oslo, Norway.

Fig. 1.



Geschichtetes Fjordwasser. Oberflächenschicht mit Tusche gefärbt.

Fig. 2.



Das mit Tusche gefärbte Oberflächenwasser der Fig. 1 ist vom Wind zu einer keilförmig. Schicht zusammengeschoben.

Fig.1. Stratified fjord water. Surface layer coloured with ink.

Fig. 2. The ink-coloured surface water of Fig. 1 has been pushed into a wedge-shaped layer by the wind.

After this experiment the reason for the phenomenon was perfectly clear and finds its explanation in the circumstance that the water layer at the surface – the *surface water*[2], as I would like to call it for brevity – is less dense[3] than the underlying water and thus cannot be forced to intrude into this *lower water*.

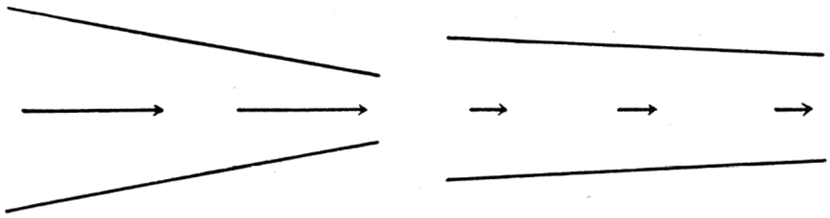
Bjerknes' circulation theory[4] gives the simplest explanation of this phenomenon. This theory says that, if a fluid is composed of parts of various densities, it strives to form horizontal layers of uniform thickness whose density increases with depth. We shall call the forces that rearrange the fluid in this way Bjerknes forces. These forces seek to shape every layer in the fluid horizontally and of the same thickness throughout. If a layer is initially not equally thick throughout (see Fig. 3 and 4), the Bjerknes forces attempt to move fluid from the thicker to the thinner parts of the fluid in order to render the fluid equally thick throughout. If the layer thickness changes markedly, as in Fig. 3, the Bjerknes forces are large; if the layer thicknesses are only marginally varying, as in Fig.4, the Bjerknes forces are small.

[2] The s p a c e d words in the original text have been replaced by *italics*.

[3] The original text uses "specific weight" throughout, which is defined as the product of density and the gravity acceleration. For clarity we will use "density" instead, and likewise "less dense" instead of "specifically lighter" etc. This leaves the physical statements of the original text intact.

[4] Sandström gives no citation. However, the forces he addresses in the following are simply pressure gradients.

Fig. 3. Fig. 4.



Dicke der Schicht sehr verschieden; die Bjerknesschen Kräfte sind deshalb groß. | Dicke der Schicht nur wenig verschieden; die Bjerknesschen Kräfte sind deshalb klein.

Größe und Richtung der Bjerknesschen Kräfte in der keilförmigen Oberwasserschicht.

Fig. 3. Layer thickness very different; the Bjerknes forces are therefore large.	*Fig. 4. Layer thickness marginally different; the Bjerknes forces are therefore small.*
Strength and direction of the Bjerknes forces in the wedge-shaped surface water layer.	

If we now apply this theory to the experiment described above, we have a layer that is not equally thick throughout, namely the wedge-shaped layer of the surface water in Fig. 2. In this layer the Bjerknes forces are directed from the thicker part to the thinner part, i.e. against the wind. These forces are balanced by the wind as long as it is blowing, but when the wind ceases they start to act and drive the surface water back in the direction whence the wind was blowing.

Simple reasoning shows that Bjerknes forces in the surface water always manifest themselves such as to become just as large as the force with which the wind is affecting the water surface, and directed opposite to this force. Thus, if the Bjerknes forces are smaller than the action of the wind, the wedge-shaped surface layer (see Fig. 2) cannot resist the wind and is compressed to a greater degree by it. Hence the Bjerknes forces in the layer grow. If on the other hand these forces are larger than the action of the wind, the latter is not able to compress the surface water as much, so the surface water flows back in the direction opposite to the wind and thus the layer gets thinner. Hence the Bjerknes forces decrease. Thus the Bjerknes forces grow if they are smaller than the force with which the wind affects the water surface, while they decrease if they are larger than that force. From this it follows that they always attempt to have the same magnitude as that force.

It emerges from this reasoning that one can compute the force with which the wind affects the water surface from the properties of the sea water. That is, since this force is the same size as the Bjerknes forces and directed against them, and since furthermore the direction and strength of these forces can be computed from the density distribution of the water, one also obtains from that density distribution the direction and strength of the force with which the wind affects the water surface.

The reasoning has been founded here on static considerations only and so is not always strictly valid in moving water. However, later I will discuss the dynamical aspect of the matter too.

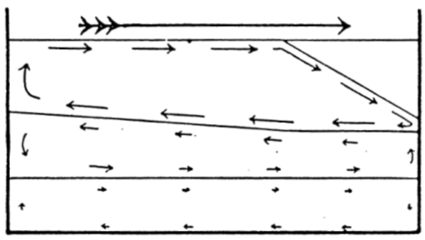
2.

In order to be able to observe the motion of the fjord water in the glass tank small amounts of fuchsine[5] solution were injected into different parts of the water using a capillary tube. It turned out then that the wind induced as many completely separate currents as there were layers in the water. Each current was confined to a single layer and could not penetrate the boundary of that layer at all.

I blew over the water so strongly that the surface layer formed a wedge and that the next layer was exposed for the most part (see Fig. 5). The surface of this layer then started to flow in the direction of the wind, and the layer continued to flow in the same direction so that a closed circulation developed, as Fig. 5 shows. Owing to friction between this layer and the underlying one, the latter then started to move as well, but in the opposite direction. This layer again set in motion its underlying layer and so on.

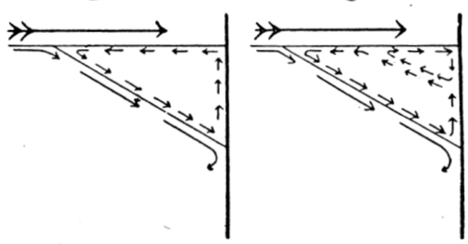
The wedge-shaped surface layer is affected by the wind as well as by the underlying layer (see Fig. 6). The wind tries to excite a circulation in its own direction, and the underlying layer one against the wind. It turns out however that the lower circulation exerts the stronger influence; for the layer circulates against the wind (see Fig. 6). Yet it sometimes happens that the wind is able to produce a small circulation too, and then two circulations develop in this one layer (see Fig. 7).

Fig. 5.



Vom Winde erzeugte Zirkulationen in geschichtetem Wasser.

Fig. 6. Fig. 7.



Vom Wind erzeugte Zirkulationen in der keilförmigen Oberwasserschicht.

Fig. 5. Wind-driven circulations in stratified water.

Figs. 6 and 7. Wind-driven circulations in the wedge-shaped surface layer.

From Figs. 6 and 7 comes the remarkable fact that, if the interface of two layers intersects the water surface, the surface water converges from both sides towards the intersecting line of these surfaces.

[5] Fuchsine, or rosaniline hydrochloride, is a magenta dye.

It is not difficult to produce water artificially that has the same properties as fjord water. One only has to pour table-salt solutions of various strengths carefully on top of one another such that less dense water always lies on top of denser water. In doing so it is advisable not to choose excessively concentrated solutions because otherwise diffusive phenomena could have a perturbing effect. Therefore I worked with solutions of about 20‰, 10‰, and 0‰ salinity.

As a tank one uses two evenly cut glass sheets of 1 m length, 25 cm height and about 6 mm thickness each. Between these two glass sheets one puts three wooden strips of 1½ cm x 2½ cm width such that a tank of 2½ cm inner width and with an open top is formed. The wooden strips are fixed onto the glass sheets with marine glue[6] using a warm iron rod. Thus it is arranged that the tank is waterproofed.

If one installs an electric arc lamp 1 m behind the tank then one can project adequate images onto a white screen that is put 2½ m in front of the tank. These images show the various currents in the tank.

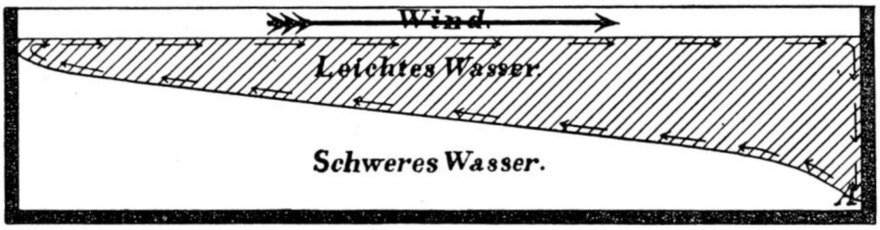
The wind may be generated using an electric wind turbine, a blacksmith's bellows or a pump. It must be distributed as uniformly as possible across the water surface. To this end it is best to use a number of small, slanting tubes that branch off from a large main tube.

When filling the tank one has to pay attention that the different water layers mix as little as possible. Therefore the water has to be poured into the tank with as little velocity as possible, i.e. through openings as large as possible. To this end it is highly advisable to use a rectangular funnel of pyramidal shape with a large rectangular outflow tube that ends about 2 cm above the tank floor. One first pours the least dense water into the tank, and then in turn the denser waters, i.e. the densest last.

The simplest experiment is conducted with two water layers of about the same thickness (10 cm). As long as no forces act on the water, the interface of the two layers is horizontal, but when one blows over the water, it assumes an inclined position as in Fig. 8. Especially remarkable is the depression A at the side wall of the tank against which the wind blows. This depression is a consequence of the circulation in the upper layer. That is, initially the surface water is driven by the wind in the direction in which it blows; as soon as it has reached the end of the tank it submerges and impinges on the interface of the layers; thus the interface is displaced and a depression in it develops. The displaced water settles close by and causes a relative uplift there. The consequence is a slight curvature of the interface, which now takes the approximate shape of a lying integral sign.

[6] Marine glue is a solution of rubber and asphalt in tar oil and is used for caulking.

Fig. 8.



Vom Winde hervorgerufene Zirkulation in geschichtetem Wasser.

Fig. 8. Wind/ Light water/ Heavy water[7]
Wind-driven circulation in stratified water.

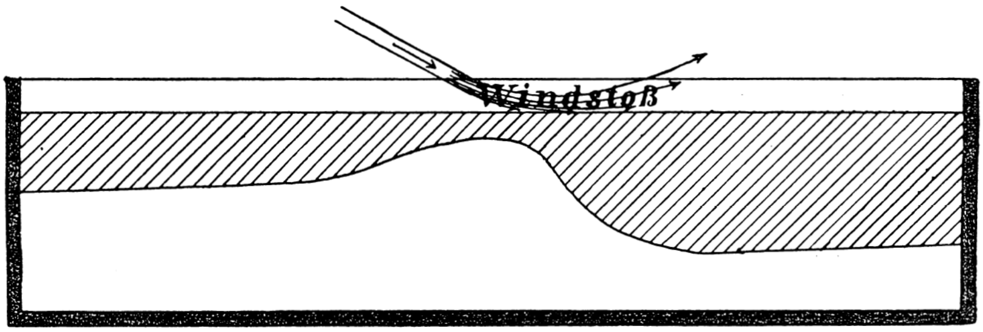
When an image of the experiment is projected, the interface is clearly visible even if the water is not coloured because the projected light is diffracted by the two water layers in different ways. Therefore the interface appears as a sharply defined line in the image. Alternatively one can inject fuchsine solution into the upper layer. Owing to the intense circulation in this layer the colour spreads quickly, and soon the entire layer is coloured red. Hence one sees how completely the two layers are separated from each other. Above the interface the water is completely red, while below it remains completely clear in spite of the vigorous motion in the tank. In Fig. 8 and the subsequent Figures the hatched areas stand for the water that is coloured by the fuchsine solution.

However, a gale at sea cannot be seen as a constant wind that blows over the whole water surface, but rather as a strong gust of vast extent that normally moves from W to E. In order to see the impact of such a gust on the water of the ocean, we let air stream downwards out of a tube slantwise onto the surface of the water of the above experiment after it has settled. We then find that a huge elevation in the shape of a wave develops, as in Fig. 9, and that, if we direct the air tube further forward, a large underwater wave rolls forward that continues even when the air tube is taken away.

Thus, gales at sea might be accompanied by huge underwater waves that could possibly reach heights of 100 m or more. If the gale ceases the underwater wave continues to move, and when it strikes a coast later on, probably a kind of underwater surf develops. Such surfs might be the cause of the following phenomenon that has been sometimes noticed by fishermen on the west coast of Sweden. In calm, fair weather, while the water surface is completely even and calm, it is suddenly set into a most vigorous motion by an invisible underwater force.

[7] If there are labels in the figure panels, their translations appear on the first line in the respective figure legend, and then the translation of the caption in the following lines.

Fig. 9.



Tiefwasserwelle, hervorgerufen durch einen Windstoß.

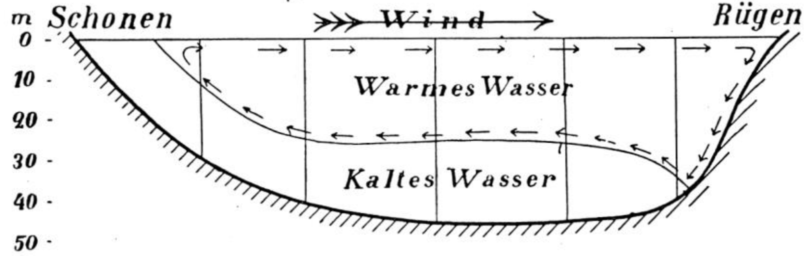
Fig. 9. Gust
Deep water wave caused by a gust.

During the underwater wave experiment, too (Fig. 9), the upper layer remains completely red and the lower layer remains entirely clear. It is only if the experiment is not conducted carefully enough, such that the wave crests break or surf develops, that a minor mixing of the two layers occurs at the interface. Therefore as a preliminary empirical consequence of the experiments (Fig. 8 and 9) it is justified to make the following statement: *Every wind-driven current is confined to a single layer.*

4.

From hydrographic observations it is likely that phenomena similar to the experiments (Fig. 8 and 9) occur in the ocean as well. On 30 July 1907 the temperature of the sea water was measured in the southern Baltic Sea somewhat north of the island of Rugia. It turned out that the interface between the warm surface water and the underlying deep water was at a depth of 20 m. The depth of the sea floor was 40 m. It was intended to sample some further stations in the direction of Scania, but a north-westerly gale came up, and so the observations had to be postponed until 1 August. On that day, the temperature was firstly measured at the same place as the first time. It emerged that the interface now lay at a depth of 35 m. The north-westerly gale had obviously piled up the warm surface water on the German side of the Baltic. Later that day four more stations were sampled in the direction of Scania. The following depths were found for the interface: 25, 25, 25, and 10 m. If we plot these depths in a section we obtain Fig. 10, which shows the same shape of the interface as the experiment in Fig. 8. The depression off the German coast suggests that the surface layer is circulating.

Fig. 10.



Wind, Temperaturverteilung und wahrscheinliche Wasserbewegung in der südlichen Ostsee am 1. August 1907.

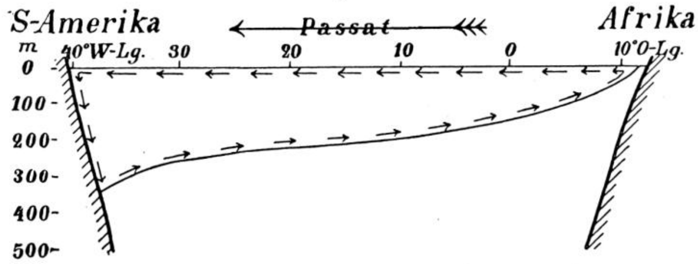
Fig. 10. Scania/ Wind/ Rugia/ Warm water/ Cold water
Wind, temperature distribution and probable water motion in the southern Baltic Sea on 1 August 1907.

Let us imagine a section through the Atlantic Ocean at 20°S. The trade winds blow along this section from Africa towards South America. We can take the depths of the 15°C isotherm from the atlas of the Valdivia expedition and find the following values:

	Depth of the +15°C isotherm at 20°S					
Geographical longitude	10°E	0°	10°W	20°W	30°W	40°W
Depth in metres	40	140	190	220	260	330

If we plot these depths in a section we obtain Fig. 11, which shows that the trade winds pile up the warm surface water against the South American coast. The deepening of the 15°C isotherm by the South American coast suggests that the surface water is set into motion by the trade winds.

Fig. 11.



Wind, Lage der 15° C. Isotherme und wahrscheinliche Wasserbewegung zwischen Afrika und Süd-Amerika auf 20° S-Br.

Fig. 11. S-America/ Trade winds/ Africa
Wind, position of the 15°C isotherm, and probable water motion between Africa and South America at 20°S.

Underwater waves can be observed only by vessels that are moored for a long time at a fixed location, continually taking observations of temperature and salinity of the water in various depths. Such vessels are e.g. the lightships in the Kattegat, and these sometimes observe quite enormous underwater waves that intrude from the Skagerrak into the Kattegat. Professor *O. Petterson*[8] has described such a wave. The isohaline of 34‰ salinity that lay at

[8] *Otto Pettersson*, Über die Wahrscheinlichkeit von periodischen und unperiodischen Schwankungen in dem atlantischen Strom und ihre Beziehungen zu meteorologischen und biologischen Phänomenen. Printed in "Svenska Hydrografisk-Biologiska Kommissionens Skrifter". Gothenburg 1905. [This is a footnote from the original text. The title

Skagens lightship at a depth of 40 m from 16 to 22 October 1898 rose to a depth of only 10 m in the days from 23 to 28 October, and on 26 October the depth was only 5 m. This means that during these days an underwater wave of 35 m depth intruded; and this same wave passed, six days later, the lightship Läsö Rinne that lay a distance of 34’ away. Thus the wave speed was 0.118 m per second. However, if one adds the speed of the countercurrent, which was 0.35 m per second, one obtains a speed of 0.468 m per second. This is the speed that the same wave would have had in stagnant water of the same properties. Seeing that such an enormous underwater wave could appear in a bottom depth of only 40 m, the underwater wave height in the open ocean might amount to hundreds of metres.

The hydrographic phenomena described here show such a large similarity to the experiments described above (Figs. 8 and 9) that we have to assume that in the ocean, too, any circulation caused by the wind is confined to a single layer.

The experiments (Fig. 8 and 9) were conducted in a tank whose shape permitted only vertical circulations. Yet the ocean extends in two horizontal directions as well. Therefore, in the ocean there may also be horizontal circulations that fulfil the requirements mentioned above. All surface currents that form closed circulations may be counted in this class. Such circulating currents exist in the Atlantic, Indian and Pacific Oceans between 5°S and 45°S, and between 10°N and 45°N in the Atlantic and Pacific Oceans. These currents, which all circulate anticyclonically, are obviously driven by the atmospheric anticyclones of the horse latitudes.

5.

We shall now study whether the statement made above, that any wind-driven current is confined to a single layer, is valid for the Gulf Stream too.

It is known that the Gulf Stream forms a layer of about 300 m thickness between the West Indies and Spitsbergen. Underneath this layer the sea water is colder and denser. If the Gulf Stream were wind-driven, then according to what was elaborated above, the entire Gulf Stream circulation would be confined to this Gulf Stream layer, and thus within this layer as much water would flow from Spitsbergen to the West Indies as in the opposite direction. Yet the hydrographic observations show that all the water in the Gulf Stream layer flows in the direction from the West Indies towards Spitsbergen. It follows that the Gulf Stream is not wind-driven.

Indeed, in the Arctic regions all the Gulf Steam water must penetrate downwards through the interface between the Gulf Stream layer and the underlying water; since otherwise the Gulf Stream layer would grow thicker and thicker due to the permanent supply of Gulf Stream Water from the South; but this does not happen. The experiment (Fig. 8) shows that the wind is not able to

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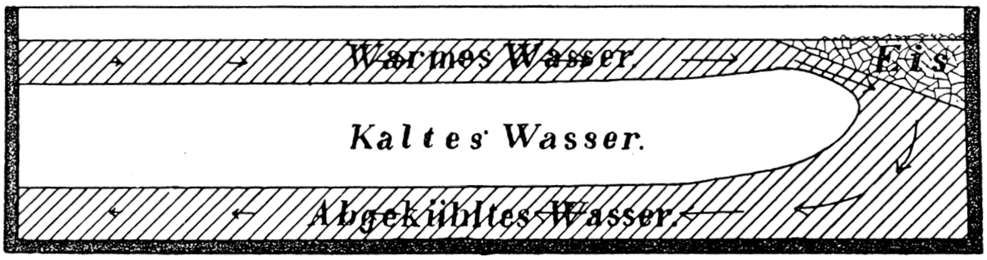
of the cited paper translates as “On the probability of periodic and aperiodic fluctuations in the Atlantic current and their relations to meteorological and biological phenomena“.]

push the light Gulf Stream water into the heavier deeper water. Thus there must be another circumstance that causes this, and that is the cooling of the Gulf Stream water due to ice-melt or radiation.

To prove clearly the difference between currents driven by the wind and due to ice-melt I have conducted the following experiment. The tank was filled with two kinds of water of the same salinity but different temperatures[9]. This operation is achieved most easily by first filling the tank to a height of 20 cm with water of 20‰ salinity, and then installing in the middle of the tank at a water depth of 10 cm a heat source in form of a metal pipe through which water of 40°C flows. Then the water above the level of the metal pipe starts to circulate and becomes warmer than the water under that level. If now fuchsine solution is injected into the water above the pipe, then the upper, warm water is coloured reddish while the lower, cold water stays uncoloured.

Fig. 12.



Abkühlung und Herabsinken des Wassers infolge Eisschmelzung.

Fig. 12. Warm water/ Ice/ Cold water/ Cooled water
Cooling and sinking of the water due to ice-melt.

The tank is now filled with two different kinds of water. The upper water, coloured red, is less dense than the lower, uncoloured water. If one now blows over the water, the phenomenon pictured in Fig. 8 appears, i.e. two circulations, namely one in the red and one in the uncoloured water, and at the same time a sharply defined interface appears between the two water layers that neither of the two kinds of water can penetrate. However, if instead of blowing, one adds a mixture of ice and salt to the water at one end of the tank, then one finds that the red water penetrates the uncoloured water with great ease and comes to rest underneath it, see Fig. 12.

The cause of this phenomenon is obviously the circumstance that the water that comes close to the ice is cooled and thus obtains a larger density than before. Therefore it sinks until it has reached the level it belongs to according to its new density. In this experiment the cooled water becomes heavier than all other waters in the tank and thus sinks to the bottom.

The most substantial difference between the influence of the wind and that of the ice-melt is thus the following: The wind causes only a redistribution of the water and cannot change the density of the individual water-particles. When the wind ceases each water-particle is driven by the Bjerknes forces to that

[9] The literal translation would read "degree of heat", but it seems obvious that "temperature" was meant. Hence "temperature" has been used in all instances.

level where it belongs according to its density, and that is the same level where it was before the wind started blowing. *As soon as the water has attained equilibrium again after a storm it is stratified in exactly the same way as before the storm.* If however ice melts in the sea water, then the individual water-particles are subject to a physical change that results in an increase of their density. For that reason they sink from the surface layer to a deeper layer, i.e. the surface layer gets thinner, and the lower layers thicken. *The ice-melt therefore leads to a permanent change in the stratification of the water.*

The difference between currents caused by the wind and those caused as a consequence of ice-melt can be especially clearly perceived if one considers the matter form the viewpoint of vector theory. All currents created by the wind are of a rotating nature, which is the reason why they are called *eddy currents* in vector theory. By contrast, in case of ice-melt the process may be considered as taking water out of the surface layer and pouring water into a layer that lies deeper. The currents that are created by such processes however are *non-rotational.* Thus, the wind creates rotating currents, while the ice-melt causes non-rotating currents.

The experiment in Fig. 12 demonstrates the necessity of ice-melt for the drift of the Gulf Stream: the ice-melt transforms the warm water that arrives in the Arctic regions into cold water which then sinks because it is denser. At the same time, however, the experiment shows that there must be another process at work in the ocean that is equally important for the drift of the Gulf Stream. Namely, as much warm Gulf Stream water must be produced in the Tropical regions as cold water is produced by ice-melt in the Arctic regions, because otherwise all the Gulf Stream water would soon be transformed into deep water.

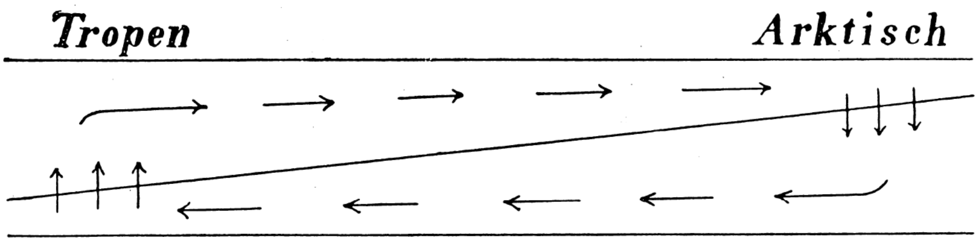
Hence, as much water must flow upwards through the lower interface of the Gulf Stream in the Tropics as flows downwards in the Arctic regions. The upward-streaming water originates from a layer that consists of cold and dense water, and therefore it must undergo a physical change such that it becomes less dense before it can penetrate upwards through the lower interface of the Gulf Stream. This physical change, which is obviously a warming of the water, must happen in the lower cold water layer. Now, since the lower interface of the Gulf Stream in the Tropics lies at a depth of more than 500 m, there must be a heat source at that depth that is large enough to release in any given second an amount of heat that is sufficiently powerful to change the temperature of all the water that is driven forward in one second by the Gulf Stream from the lower temperature of the lower layer to the higher temperature of the upper layer.

Obviously this heat source is the solar radiation in the Tropics. Yet since the solar radiation can penetrate to a depth of only some tens of metres, and since it therefore can warm directly only the water in close proximity to the surface, there must be something else that conducts the heat downwards. This can only be convection, since radiation and conduction from layer to layer are too minor to carry downwards such amounts of heat as are necessary here. According to the common view, convection can only happen in water that is stratified homogeneously or stably[10], but not in water whose density grows with depth. The latter however is just the case in the Tropics, which is why one would expect that convection does not happen there. However, a closer inspection shows that it can happen in stably stratified water too, provided that temperature as well as salinity decreases with depth. This is indeed the case in the Tropics. A water mass at the sea surface becomes more and more saline due to evaporation there until it is so saline that, in spite of its high temperature, it becomes denser than the immediately underlying water. Consequently it sinks to the level where the surrounding water has the same density as the sinking water. The sinking water is then saltier and warmer than, but as dense as, the surrounding water. It subsequently retains its high salinity, but releases some heat into the surrounding water. Owing to this cooling it becomes heavier and sinks further to an even deeper level where the same process is repeated. The water sinks as long as its temperature as well as its salinity decreases with depth. Of course this process does not happen stepwise, as we have described it here for the sake of simplicity, but continuously, such that the sinking water is always saltier, warmer and denser than the surrounding water and therefore always tends to sink. Owing to the persistent cooling the density of the water increases continuously during the sinking.

Once the sinking warm and saline water has entered the cold layer under the Gulf Stream it releases its heat to the surrounding water. Thus this water becomes lighter than its surroundings and starts to rise. In doing so it reaches warmer and saltier layers. It retains its low salinity but continuously takes up heat from its surroundings and is therefore always lighter than the surrounding water. It rises until it reaches the surface, where it partly evaporates, becomes saline and warm, and then sinks again with a new amount of heat. One can imagine that heat is conveyed in this manner from the surface to the cold layer under the Gulf Stream in the Tropics.

Thus we have a warming under the Gulf Stream in the Tropics and a cooling of the surface of the Gulf Stream in the Arctic regions. In the Tropics the water flows upwards through the lower interface of the Gulf Steam while in the Arctic regions it flows downwards through the same interface. Above the interface the warm Gulf Stream water flows northwards, and under the interface the cold Gulf Stream water flows southwards.

Fig. 13.



Schematische Darstellung der Golfstromzirkulation.

Fig. 13. Tropics/ Arctic
Schematic illustration of the Gulf Stream circulation.

[10] It seems that Sandström meant "unstably".

Hence one can imagine the Gulf Stream as a closed circulation in two layers. If we consider the upper layer alone, water is continuously supplied in the Tropics and extracted in the Arctic regions. Thus this layer thickens more in the Tropics than in the Arctic regions. Therefore the lower interface of this layer is slanted and the layer itself wedge-shaped, see Fig. 13. Hence in this layer the Bjerknes forces act northwards, and it is these forces that drive the northward flowing branch of the Gulf Stream. By contrast, in the lower layer water is supplied in the Arctic regions and extracted in the Tropical regions. Thus this layer thickens more in the Arctic regions than in the Tropics, and its lower interface becomes almost horizontal; see Fig. 13. The Bjerknes forces are directed southwards in this layer, and it is they that drive the cold southward branch of the Gulf Stream.

The amount of water that is driven forward in the Gulf Stream depends solely on the amount of heat that is supplied to the lower Gulf Stream layer in the Tropics and that is released from the upper Gulf Stream layer in the Arctic regions.

The Bjerknes forces that drive the Gulf Stream forward always adjust to the circumstances so as to attain exactly the strength that is required to carry away the water that is transformed in the Tropics and in the Arctic regions. The regulation of the Bjerknes forces required for this purpose happens entirely automatically, as will be illustrated by the following examples.

If, for instance, a northerly wind is blowing along the whole Gulf Stream, this initially hinders its motion. However, the warming in the Tropics and the cooling in the Arctic regions are still going on, and thus the same amounts of water are being transformed as before in any amount of time. This results in the Gulf Stream, with its reduced speed, not being able to carry away all the transformed water. This in turn results in warm water accumulating in the Tropics and cold water in the Arctic regions. This accumulation causes the interface between the two Gulf Stream layers (see Fig. 13) to acquire a steeper profile, in other words a strengthening of the Bjerknes forces that drive the Gulf Stream. This strengthening of the Bjerknes forces continues as long as the abovementioned accumulation goes on, i.e. as long as the Gulf Stream has a speed which is too low. Thus the Bjerknes forces increase until they become strong enough to be able to drive the normal amount of water in spite of the headwind.

By contrast, if southerly wind is blowing along the Gulf Stream, its speed is initially accelerated, and it is able to carry away more water than is being transformed in the Tropics and the Arctic regions. The consequence is that the interface between the two Gulf Stream layers that originally lay slanted now becomes almost horizontal; the Bjerknes forces that drive the Gulf Stream decrease, and they continue to decrease as long as the Gulf Stream has a speed which is too large, i.e. until the Bjerknes forces have become small enough that they are not able to drive forward more than the normal amount of water, in spite of the wind.

Thus, the Bjerknes forces that drive the Gulf Stream always adjust to the resistance that the Gulf Stream has to overcome. When the weather is calm this resistance stems only from the internal friction of the water. *Thus one would need just to measure the Bjerknes forces in the Gulf Stream in calm weather to determine the strength of the resistance due to the internal friction of the sea water that the Gulf Stream has to overcome in moving forwards.* When the wind blows over the Gulf Stream, the Bjerknes forces equal the algebraic sum of the resistances that are caused by the internal friction and the wind.

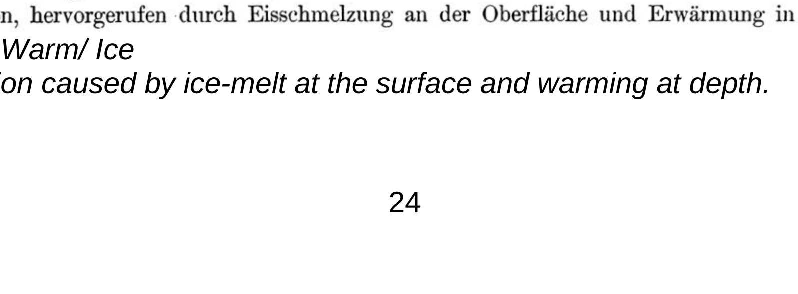
6.

It is quite difficult experimentally to reproduce the slanted interface between the two Gulf Stream layers. The difficulty is that it is the same water that is flowing above and below the interface. Hence, if one were to colour part of the water, very soon all of the water would acquire the same colour. Thus it would be impossible to determine the interface. Since however it is this very interface that reveals the force field that drives the Gulf Stream forward, and since it is therefore of the greatest interest, I conducted experiments until I succeeded in rendering this interface visible.

This is achieved by inserting an intermediate layer between the two layers that are to represent the two Gulf Stream layers in the experiment. Since this layer does not take part in the simulated Gulf Stream circulation in the experiment, it can be coloured in a different way from the circulating water. The latter entrains water-particles from the intermediate layer, which eventually becomes rather thin and gives the impression of an inclined interface.

When conducting the experiment it is advisable to start from the last experiment depicted in Fig. 12. If the experiment has run as far as is shown in Fig. 12, one inserts at one end of the tank at the bottom – as can be seen from Fig. 14 – a heat source in form of a metal pipe through which hot water flows. Then the red-coloured water sinks at the position of the ice – A – and rises at the position where the heat source – B – has been inserted, whereby a closed circulation around the uncoloured water band – C – is created. This water forms, as Fig. 14 shows, a slanted, lens-shaped layer that gets thinner and thinner as time passes and the circulation of the surrounding red water continues, so that it eventually gives the impression of a slanted surface.

Fig. 14.



Zirkulation, hervorgerufen durch Eisschmelzung an der Oberfläche und Erwärmung in der Tiefe.

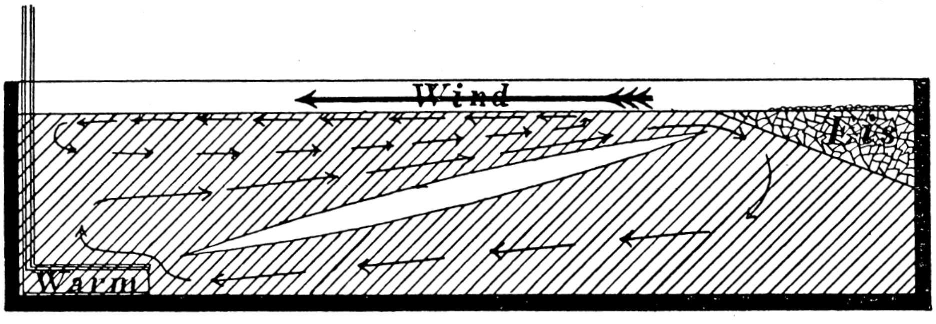
Fig. 14. Warm/ Ice

Circulation caused by ice-melt at the surface and warming at depth.

In this experiment we can represent on a small scale everything that we said above about the Gulf Stream. The ice – A – in the experiment, Fig. 14, corresponds to the polar ice, and the heat source – B – on the tank bottom corresponds to the deep convection in the Tropics. The upper warm water corresponds to the northward-flowing branch of the Gulf Stream, and the lower cool water corresponds to the southward-flowing branch of the Gulf Stream. Between them there is a thin and slanted layer that corresponds to the slanted interface between the two Gulf Stream layers. Owing to the slant of the interface the upper warmer layer and the lower cooler layer take a wedge-shaped form, and in both layers Bjerknes forces appear that are directed from the thicker to the thinner ends of these layers. These forces drive warm upper water towards the location of the ice – A – and they drive the cold lower water towards the location of the heat source – B – exactly as in the case of the Gulf Stream. The forces adjust to the resistance that the current has to overcome and become just strong enough to carry away the water cooled by the ice and the water warmed at the other end of the tank. Yet the resistance against the motion of the water depends only on its inherent friction. Thus the Bjerknes forces in the tank give a measure of the resistance opposing the current due to the inherent friction of the water.

If we now blow over the surface in the direction opposite to the surface current, a separate circulation emerges, in the direction of the wind, in the part of the water closest to the surface, see Fig. 15. But as much water as before is being warmed at one end of the tank and cooled at the other end, and the water masses being transformed in this way must flow along the tank. However, the opening through which these water masses flow has narrowed because the wind-driven circulation also takes up space in the tank. Thus the resistance against the current grows, the slanted interface steepens, and the Bjerknes forces increase until they are able to carry away the warmed and the cooled water in spite of the narrowed opening, until once again the Bjerknes forces equal the sum of the resistances due to wind and friction.

Fig. 15.

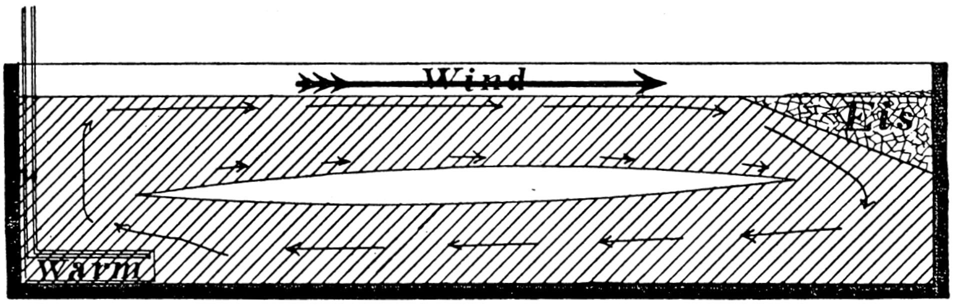


Thermische Zirkulation bei entgegenwirkendem Winde.

Fig. 15. Wind/ Ice/ Warm
Thermal circulation with counteracting wind.

By contrast, if one blows over the surface in the direction of the current, the surface water is driven forward faster than before. Meanwhile just the same amounts of water as before are being warmed at the one end and cooled at the other end, and the water masses being transformed in this way need to flow along the tank. As the surface water is now flowing faster than before, the water underneath does not have to flow as fast as before; the Bjerknes forces driving it forward do not have to be as strong as before. The slanted interface therefore assumes an almost horizontal position, see Fig. 16. Now the Bjerknes forces equal the difference between the resistance due to the inherent friction and the reinforcing force of the wind.

Fig. 16.



Thermische Zirkulation, vom Winde unterstützt.

Fig. 16. Wind/ Ice/ Warm
Thermal circulation reinforced by the wind.

These experiments show that those currents caused by the wind and those caused by thermal influences actually run in different and separated ways. In Fig. 15 the wind-driven rotating current has even developed its own circulation that is entirely separated from the thermal, non-rotating current. In Fig. 16 the two motions are mixed, but a superposition can easily by made whereby one can see immediately how much of the motion may be ascribed to the wind and how much to thermal causes.

7.

The two experiments displayed in Fig. 12 and 14 show that there has to be warming as well as cooling in the sea if a closed circulation is to develop from thermal causes. The following two experiments will show that for this outcome it is necessary in addition that the level at which the water is warmed lies below the level at which it is cooled.

We fill the tank with a single type of water of about 20‰ salinity and install two metal tubes in the tank at water depths of about 6 and 14 cm. Water of about 40°C flows through the former tube, and ice-cold water of about 0°C through the latter. The thin tubes that conduct the cold and warm water to and from [the metal tubes in the tank][11] must be insulated by a rubber coating. The water above the warm water tube and that below the ice-cold water tube soon begin to circulate rather strongly. These circulations later decrease and finally cease completely. The water lying between is always very quiet and finally calms down entirely. Thus at the end of the experiment all the water in the tank is calm, in spite of the constant throughflow of warm and cold water. The motion of the water may be observed at different stages of the experiment by

[11] From the German text it obvious that those tubes are meant (i.e. the thin vertical tubes drawn in Fig. 17), but they are not mentioned explicitly.

sprinkling some pulverized potassium permanganate into the water and observing the coloured vertical lines thus produced.

It is easy to see why the water eventually calms down entirely in this experiment. It is because the water which surrounds the warm metal tube becomes warmer and lighter than its surroundings at the beginning of the experiment and rises to the surface. This produces a warm layer that thickens due to the constant addition of warm water from below. When the layer has become so thick that its lower interface touches the warm metal tube it stops growing because the warm tube is now within the warm layer. Instead of a further growth of the layer a further warming happens such that all the water in the layer attains the same temperature as the metal tube. After that the warm tube releases no more heat into the surrounding water, the warm water layer homogenizes, the Bjerknes forces in this layer vanish, and the water itself calms down.

In very much the same way a cold bottom layer develops in the tank whose upper interface touches the upper edge of the ice-cold water tube. If all the water in this layer attains the same temperature as the cold tube and thus homogenizes, then the Bjerknes forces vanish in the cold bottom layer too, and the water in that layer calms down as well.

Fig. 17.

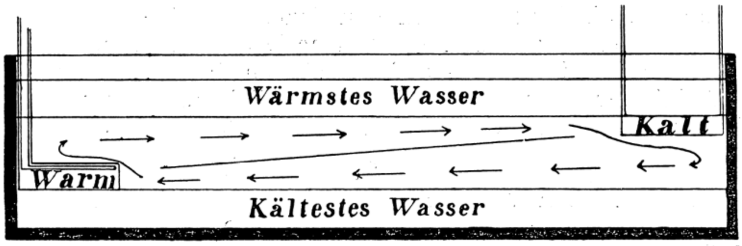
Wärmequelle oberhalb der Kältequelle. Das Wasser in der oberen und unteren homogenen Schicht, ebenso wie in der stabil gelagerten Zwischenschicht bleibt schließlich gänzlich stillstehen.

Fig. 17. Warm/ Warmest water/ Coldest water/ Cold
Heat source above the cold source. The water in the upper and lower homogeneous layers and that in the stably stratified intermediate layer finally calm down entirely.

Water in the intermediate layer is initially entrained by the vigorous motions above and below the layer; but when these motions decrease the intermediate layer calms down, too. However this water does not form a homogeneous layer but is stably stratified because its temperature decreases with depth; thus, strictly speaking, this layer consists of several thin horizontal layers. The horizontal lines in Fig. 17 representing isotherms are meant to demonstrate this.

Now the tank is emptied and filled anew with water of 20‰ salinity. We deploy the metal tube through which warm water flows at a depth of 14 cm and the ice-cold water tube at a depth of 6 cm. The water in the tank immediately starts to move vigorously, and this lasts for a while. Eventually however the water above the cold tube and below the warm tube calms down almost entirely whereas the intermediate water circulates steadily as depicted in Fig. 14. If one measures the temperature in the tank one finds the heat distribution drawn in Fig. 18. The warmest water lies above the level of the cold tube and the coldest water below the level of the warm tube. In the middle of the intermediate layer lies the slanted interface that produces the Bjerknes forces needed to sustain the circulation in this layer. The water is somewhat warmer above this interface than below it.

Fig. 18.



Wärmequelle unterhalb der Kältequelle. Es entsteht eine thermische Zirkulation zwischen zwei stillstehenden Schichten.

Fig. 18. Warmest water/ Warm/ Cold/ Coldest water
Heat source below the cold source. A circulation between to calm layers develops.

The warm surface water and the cold bottom water are both entirely calm. Any minor motion they may have obviously arises from friction against the currents of the intermediate layer. If one injects fuchsine solution into the water of the intermediate layer, this layer will soon be completely coloured red, whereas the surface and the bottom layer remain colourless. Hence the water circulating between the heat source and the cold source is not related to the water that is above the level of the cold source or below the level of the heat source.

It appears that this situation arises as follows. At the beginning of the experiment the water surrounding the warm metal tube warms up. Having become lighter than its surroundings it rises to the surface, where it forms a warm surface layer. By contrast, the water surrounding the cold metal tube cools down and, having thus become heavier, sinks to the bottom, where it forms a cold bottom layer. These two layers grow thicker and thicker due to the continuous addition of warm or cold water. If the cold bottom layer has thickened so much that it reaches the warm metal tube, its upper part warms up, and then the warmed water rises. But this originally cold water is not warmed to the high temperature of the surface layer precisely because it was originally cooler; therefore it forms a layer of its own below the surface layer. In a very similar manner, as time passes the warm surface layer thickens so much that it reaches the cold metal tube. Then the lower part of the layer is cooled by the metal tube, and the cooled water sinks. However, it is too warm to cool down to the low temperature of the bottom layer. Therefore it does not sink down to that layer, but also forms a layer of its own, above the bottom layer. But as soon as this water reaches the other end of the tank it comes into contact with the warm tube, warms up and rises to the lower interface of the warm surface layer, under which it spreads out. At the other end of the tank it comes into contact with the cold tube, cools down and sinks to the upper interface of the cold bottom layer, above which it again spreads out to repeat the same circulation. Thus the intermediate circulation develops.

The warm surface layer and the cold bottom layer are now completely isolated from the heat and cold sources, so there is no further exchange between these water layers in the tank. A further exchange will only happen when one runs the experiment for so long that the two calm layers have received enough heat from the exterior or released enough heat to the exterior that they both approximately reach room temperature.

From these two experiments we draw the conclusion that a circulation can develop from thermal causes only if the level of the heat source lies below the level of the cold source. Such a circulation is confined to the space between these two levels, and thus the water that is above the cold source or below the heat source does not participate in the circulation.

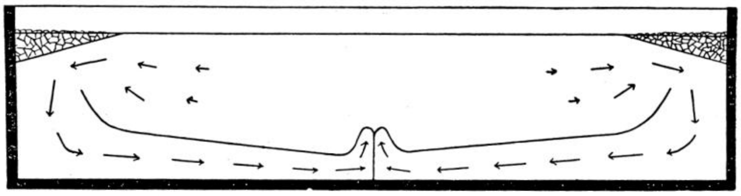
In the ocean, the cold source is at the surface in the Arctic regions, and the heat source in the Tropics will hardly penetrate further than 1000 m. Hence the circulations driven by thermal causes are confined to the upper 1000 m of the sea, and the lower water is completely separated from the upper water provided that the lower water is not warmed in some way.

8.

I will now describe one more experiment to emphasize the facts found above even further.

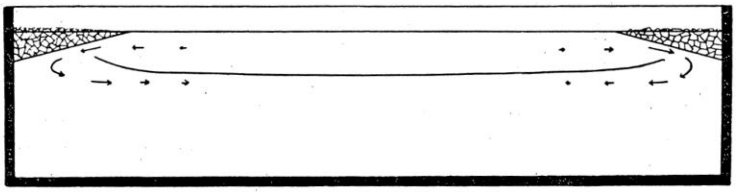
We fill the tank with a single kind of water of about 20‰ salinity, and put a mixture of ice and salt into the tank at both ends at once. At both ends of the tank the water sinks, and two bottom currents develop which meet in the middle of the tank. This is clearly visible if one projects an image of the experiment. Right at the moment where they meet, a bulge of the bottom water rises up, and subsides directly afterwards. Fig. 19 shows the shape of the bottom water at the moment where they meet.

Fig. 19.



Eisschmelzung an beiden Enden des Gefäßes. Es entstehen zwei symmetrische Bodenströme, die in der Mitte des Gefäßes zusammenstoßen.

Fig. 20.



Fortsetzung des in Fig. 19 dargestellten Versuchs. Beinahe alles Wasser im Gefäß ist abgekühlt worden. Die Wasserbewegung hat jetzt beträchtlich abgenommen und wird bald gänzlich aufhören.

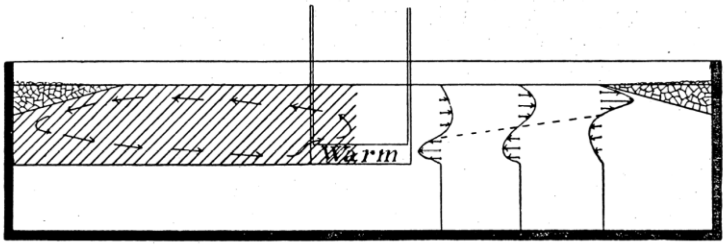
Fig. 19. Ice melting at both ends of the tank. Two symmetrical bottom currents emerge which meet in the middle of the tank.

Fig. 20. Continuation of the experiment depicted in Fig. 19. Nearly all the water in the tank has cooled down. The motion of the water has decreased significantly and will soon cease altogether.

This experiment shows how the water may become stratified through ice-melt. Initially it is homogeneous in the whole tank, but as soon as the ice starts to melt a denser bottom water layer develops such that now the water consists of two layers of different density. During the ice-melt the upper layer gets progressively thinner and the lower layer progressively thicker. Soon the situation depicted in Fig. 20 arises, in which there are only weak currents in the surface layer and in the upper part of the lower layer, while the bottom water is completely calm. We conclude from Fig. 19 and 20 that both the Bjerknes forces and the water speed decrease during the ice-melt. Eventually all the surface water has cooled down; hence the upper layer has vanished, and all the water in the tank forms a single homogeneous layer of cold water. Thus the ice-melt can also turn stratified water into homogeneous water. Now there are no longer any Bjerknes forces acting and the water is entirely calm. The ice lies in the water completely untouched and does not melt any more.

Now we insert a heat source into the middle of the tank, in the form of a metal tube through which warm water flows, see Fig. 21. The water that surrounds this metal tube becomes warmer and lighter than before and starts to rise. Thus all the water above the lower level of this metal tube soon becomes warmer than the water lying below, and hence we again have two layers in the tank. Thus a heat source inserted into the tank can also stratify homogeneous water. If one injects some fuchsine solution into the surface water, one finds that the circulations caused by the heat source in the middle and by the pieces of ice at the two ends of the tank are confined to the layer above the level of the metal tube, since this layer is soon coloured red while the lower layer remains uncoloured. The hashed part in Fig. 21 is meant to depict the coloured water.

Fig. 21.



Fortsetzung des in Fig. 19 und 20 dargestellten Versuchs. Eine Wärmequelle ist mitten im Gefäß angebracht, und infolgedessen sind zwei symmetrische thermische Zirkulationen neu entstanden. Das zirkulierende Wasser ist links mit Fuchsin gefärbt. Rechts sind mittels Kaliumpermanganat drei — gefärbte — Vertikale hergestellt, die vom Strome gebogen worden sind; unten sind sie noch vertikal, weil es hier keinen Strom gibt.

Fig. 21. Warm
Continuation of the experiment depicted in Fig. 19 and 20. A heat source has been installed in the middle of the tank, and consequently two new symmetric thermal circulations have developed. To the left the circulating water has been coloured with fuchsine. To the right three vertical lines are shown – coloured with potassium permanganate – that have been bent by the current; below they are still vertical because there is no current there.

If one sprinkles pulverized potassium permanganate into the tank in such a way that one obtains a couple of coloured vertical lines, then we can see from the motion of these vertical lines that the water in the upper layer is circulating quite vigorously while it is almost calm in the lower layer. Three such lines that are bent in their upper parts are drawn in Fig. 21. We see further that between the heat source and each piece of ice a circulation develops which is of the same kind as depicted in Fig. 14. We can even see the slanted interface of Fig. 14 in this experiment if we draw arrows, with the help of the bent lines in Fig. 21, showing the direction and strength of the motion of the water, and then connect the points where the motion of the water equals zero. (See the dashed slanted line in Fig. 21.)

The state last described illustrates to some extent the circumstances in the ocean. The pieces of ice at the two ends of the tank represent the Arctic and Antarctic ice masses, while the warm metal tube in the middle of the tank represents the heat convection in the Tropics and the equatorial regions. The experiment shows that currents arising from thermal causes extend from the surface to the level reached by thermal convection in the Tropics, and that the water below this level is unaffected by thermal processes.

Thus this deep water is isolated from thermal influences as well as from the winds. It might be of interest to investigate where it can still have some motion.

If the layer were entirely homogeneous, then due to friction against the circulating layer above, two circulations would develop that would be separated by an approximately vertical interface. In that case there would be quite a vigorous motion in the lower layer. However, if the deep water consists of several homogeneous layers lying on top of one another, friction against the layer with the thermal circulation sets into motion the uppermost of these layers, which transmits its motion to the next layer and so forth, somewhat as depicted in

Fig. 5. It is well known that the stratification of the waters in the deep ocean is almost homogeneous, yet marginally stable, and hence it is not easy to say how this water is set into motion by friction against the circulating upper layer.

The wind might well also have a certain influence on the motion of the deep water. If, in the last experiment, we blow over the surface as shown in Fig. 9, the horizontal interface between the thermally circulating and the deep water bulges like a wave. As long as the wind remains constant this bulge does not change; but if the wind changes the bulge changes accordingly. Probably the prevailing winds at sea cause permanent bulges and slants of the corresponding interfaces in an analogous manner, while changing winds must cause corresponding changes in these bulges and slants.

Finally we must stress that the deep water does not remain entirely uninfluenced by thermal processes, because it takes up all the heat that is conducted from Earth's interior to the ocean floor. Although these amounts of heat are tiny, with the passage of time they are able to warm the bottom water enough to make it rise within the nearly homogeneous deep water layer.

The degree to which each of these three causes has an influence on the motion of sea water can perhaps be assessed by a dynamical analysis of hydrographic observations of the deep ocean. Such observations immediately show that the deep water must be in rather vigorous motion, since it is so well ventilated that it can provide the countless animals of those depths with sufficient oxygen. It could hardly be so well ventilated if it were not in touch with the atmosphere from time to time. But this water cannot reach the surface without being heated from below, and hence it appears that the heat coming from the Earth's interior plays at least some role in the motion of these waters.

9.

We can summarise the results of the above study with the following conclusion. The causes of oceanic motion may be divided in two groups. To the first group belong all forces of a mechanical nature that can move the water without changing it physically, such as the wind, the deflecting motion of the Earth's rotation, the resistance due to the inherent friction of the water, and the inertial force that a moving water mass exerts against other water masses due to impact or friction. Each current driven by such forces has the characteristic property that it moves in a circular way within a single layer.

To the second group of the causes of motion belong the processes that result in a change of the density of sea water, such as warming and cooling, increase of salinity through evaporation or freezing and, furthermore, decrease of salinity through influx of fresh water or ice-melt. In each current driven by these causes the Bjerknes forces equal the total resistance against the flow and they have the same direction as the flow. The current itself has the characteristic property that it is non-rotating and runs in several layers.