Auxiliary online material to Kuhlbrodt and Gregory, "Ocean heat uptake and its consequences for the magnitude of sea level rise and climate change"

Attribution of inter-model variance in TCR and OHU

If a quantity $f(\mathbf{x})$ is a function of several parameters x_i, which have uncertainties quantified by their variances $\mathrm{var}(x_i)$ and covariances $\mathrm{cov}(x_i, x_j)$, these uncertainties propagate to give

$$\mathrm{var}(f) = \sum_i \left(\frac{\partial f}{\partial x_i}\right)^2 \mathrm{var}(x_i) + \sum_i \sum_{j \neq i} \frac{\partial f}{\partial x_i}\frac{\partial f}{\partial x_j}\mathrm{cov}(x_i, x_j).$$

The TCR $T_{2\times} = F_{2\times}/(\kappa+\alpha)$ is a function of the three parameters $(F_{2\times}, \kappa, \alpha)$. After exclusion of two models, as discussed in the text, these parameters have insignificant correlation, so their covariances can be neglected.

In that case, the variance can be apportioned among the parameters. The part of the variance of f due to parameter x_i is $(\partial f/\partial x_i)^2 \mathrm{var}(x_i)$. This can simply be estimated as the variance of f obtained as x_i is varied while all the $x_{j\neq i}$ are held constant at their mean values. Denoting a mean by $\langle\rangle$ and recalling the definition of $\mathrm{var}(x) \equiv \langle (x - \langle x \rangle)^2 \rangle$, this partial variance is

$$\langle (f(x_i, x_{j\neq i} = \langle x_{j\neq i}\rangle) - \langle f \rangle)^2 \rangle = \left\langle \left((x_i - \langle x_i \rangle) \frac{\partial f}{\partial x_i} \right)^2 \right\rangle = \langle (x_i - \langle x_i \rangle)^2 \rangle \left(\frac{\partial f}{\partial x_i}\right)^2,$$

which is the required quantity since the first bracket can be identified as $\mathrm{var}(x_i)$. Thus, we can estimate the uncertainty in TCR due to κ alone by evaluating the variance of $F_{2\times}/(\kappa+\alpha)$ from varying κ over the values it takes in the model ensemble while holding $F_{2\times}$ and α at their ensemble-mean values $\langle F_{2\times} \rangle$ and $\langle \alpha \rangle$.

In the 1% CO_2 experiment, the forcing $F = F_{2\times} t/70$ for t in years, and the rate of ocean heat uptake

$$N = \kappa \Delta T_a = \kappa \frac{F}{\kappa + \alpha} = F_{2\times} \frac{\kappa}{\kappa + \alpha} \frac{t}{70}$$

with time-integrated OHU

$$H = \int N\,dt = \frac{t^2}{140} F_{2\times} \frac{\kappa}{\kappa + \alpha}.$$

At the time of doubling $t = 70$,

$$H = H_{2\times} = 35 F_{2\times} \frac{\kappa}{\kappa + \alpha}.$$

The uncertainty due to κ in $H_{2\times}$ can be estimated as for the TCR by varying κ and keeping $F_{2\times}$ and α constant. Because the sample has a finite size, the sample covariances are not exactly zero, even though we assume that in the theoretical infinitely large population of models the parameters would be independent. Therefore the partial variances do not add up exactly to give the total variance. The decomposition of variance can only be regarded as approximate. It is useful to give an indication of the relative importance of different sources of model uncertainty.

In the case where covariances are not zero, we cannot apportion the variance of f among the x_i. We need to know the covariances in order to estimate $\mathrm{var}(f)$ from $\mathrm{var}(\mathbf{x})$. One application of this is to estimate the variance of $H_{2\times}$ from the variances of κ and TCR, which are correlated parameters. Recalling that $T_{2\times} = F_{2\times}/(\kappa + \alpha)$ and defining $\delta x \equiv x - \langle x \rangle$, we have

$$\begin{aligned} \mathrm{cov}(\kappa, T_{2\times}) &\equiv \langle \kappa T_{2\times} \rangle - \langle \kappa \rangle \langle T_{2\times} \rangle = \left\langle \frac{F_{2\times}\kappa}{\kappa+\alpha} \right\rangle - \left\langle \frac{F_{2\times}\langle\kappa\rangle}{\kappa+\alpha} \right\rangle = \left\langle \frac{F_{2\times}(\kappa - \langle\kappa\rangle)}{\kappa+\alpha} \right\rangle \\ &= \left\langle \frac{F_{2\times}}{\kappa+\alpha} \delta\kappa \right\rangle \end{aligned}$$

$$
\begin{aligned}
&= \left\langle \frac{\langle F_{2\times}\rangle + \delta F_{2\times}}{\langle \kappa+\alpha\rangle + \delta\kappa + \delta\alpha}\delta\kappa \right\rangle \\
&= \left\langle \frac{\langle F_{2\times}\rangle + \delta F_{2\times}}{\langle \kappa+\alpha\rangle}\left(1+\frac{\delta\kappa+\delta\alpha}{\langle\kappa+\alpha\rangle}\right)^{-1}\delta\kappa \right\rangle \\
&\simeq \left\langle \left(\left\langle \frac{F_{2\times}}{\kappa+\alpha}\right\rangle + \frac{\delta F_{2\times}}{\langle\kappa+\alpha\rangle}\right)\left(1-\frac{\delta\kappa}{\langle\kappa+\alpha\rangle}-\frac{\delta\alpha}{\langle\kappa+\alpha\rangle}\right)\delta\kappa \right\rangle \\
&= -\left\langle \frac{F_{2\times}}{\kappa+\alpha}\right\rangle \frac{1}{\langle\kappa+\alpha\rangle}\langle(\delta\kappa)^2\rangle
\end{aligned}
$$

to first order in small quantities, because $\langle \delta F_{2\times}\, \delta\kappa\rangle = 0$ and $\langle \delta\alpha\, \delta\kappa\rangle = 0$ by the assumption that these parameters are independent. Thus

$$
\mathrm{cov}(\kappa, T_{2\times}) = -\frac{T_{2\times}}{\kappa+\alpha}\mathrm{var}(\kappa),
$$

using the formula for $T_{2\times}$ and the definition of variance, and dropping the $\langle\rangle$ notation. It is negative because κ and TCR are anticorrelated.

Defining $H_1 = \kappa T_{2\times} (= H_{2\times}/35)$ for convenience and using the general formula for propagation of uncertainty,

$$
\mathrm{var}(H_1) = \left(\frac{\partial H_1}{\partial T_{2\times}}\right)^2 \mathrm{var}(T_{2\times}) + \left(\frac{\partial H_1}{\partial \kappa}\right)^2 \mathrm{var}(\kappa) + 2\frac{\partial H_1}{\partial T_{2\times}}\frac{\partial H_1}{\partial \kappa}\mathrm{cov}(T_{2\times}, \kappa).
$$

Then since $\partial H_1/\partial T_{2\times} = \kappa$ and $\partial H_1/\partial\kappa = T_{2\times}$,

$$
\begin{aligned}
\mathrm{var}(H_1) &= \kappa^2\mathrm{var}(T_{2\times}) + T_{2\times}^2\mathrm{var}(\kappa) - 2T_{2\times}^2\frac{\kappa}{\kappa+\alpha}\mathrm{var}(\kappa) \\
&= \kappa^2\mathrm{var}(T_{2\times}) - T_{2\times}^2\frac{\alpha-\kappa}{\alpha+\kappa}\mathrm{var}(\kappa)
\end{aligned}
$$

The model runs: scenarios and drift

For most of the calculations we used the scenarios with a 1%/year increase of atmospheric CO_2. This has the advantage that differences in forcing across the models are small. However, for the CMIP3 models more data were available for the historical 20C3M and the future SRESA1B scenarios. Therefore we used these data for Fig. 4 in the main text.

Some of the model runs have long-term temperature trends ("drift") that reflect the model's path into statistical equilibrium rather than real-world climate processes. We have excluded drift from our analysis, for all models, by subtracting the respective parts of the control run. For instance, the ocean heat uptake in Fig. 4 (main text) was calculated as

$$
\begin{aligned}
\mathrm{OHU} &= \Delta\mathrm{OHC}_{\mathrm{scenario}} - \Delta\mathrm{OHC}_{\mathrm{control}} \\
&= [\mathrm{OHC}_{\mathrm{scenario}}(21\mathrm{C}) - \mathrm{OHC}_{\mathrm{scenario}}(20\mathrm{C})] - [\mathrm{OHC}_{\mathrm{control}}(21\mathrm{C}) - \mathrm{OHC}_{\mathrm{control}}(20\mathrm{C})]
\end{aligned}
$$

where OHC is the vertical integral of potential temperature multiplied with the heat capacity of sea water and a reference density, and "21C" and "20C" denote 20-year averages over the periods 2080-2099 and 1980-1999, respectively.

For calculations based on the 1%/year CO_2 scenarios the calculation is simpler because they start directly from the control runs. To subtract the drift it is sufficient to subtract the respective part of the control run.

Table 1: Forcing at the time of CO_2 doubling F_{2x} (in W m^{-2}), climate feedback parameter α (α_{4x} from 4xCO_2 and α_{1p} from 1%CO_2/year runs), ocean heat uptake efficiency κ, climate resistance ρ (all in W m^{-2} K^{-1}), transient climate response (in K) and expansion efficiency of heat ϵ (in m YJ^{-1}) for the CMIP3 models (with numbers, data for α_{1p}, κ and ρ from *Gregory and Forster*, 2008) and the CMIP5 models (with letters). For the CMIP3 models, F_{2x} and α_{4x} could not be diagnosed.

	Model	α_{1p}	κ	ρ	TCR	ϵ		Model	F_{2x}	α_{4x}	α_{1p}	κ	ρ	TCR	ϵ
1	bcc_bcm2_0	—	—	—	—	—	A	ACCESS1.0	2.91	0.75	0.73	0.67	1.40	1.98	—
2	cccma_cgcm3_1_t47	1.28	0.55	1.83	1.90	0.139	B	BCC-CSM1.1	3.37	1.20	1.22	0.56	1.78	1.76	—
3	cccma_cgcm3_1_t63	—	—	—	—	—	C	CNRM-CM5	3.68	1.13	1.13	0.46	1.59	2.08	0.107
4	cnrm_cm3	1.60	0.58	2.18	1.60	0.098	D	CSIRO-Mk3.6.0	2.56	0.62	0.72	0.63	1.35	1.78	0.116
5	csiro_mk3_0	1.60	0.83	2.44	1.40	0.112	E	CanESM2	3.81	1.03	1.03	0.49	1.52	2.41	0.120
6	csiro_mk3_5	—	—	—	—	—	F	GFDL-CM3	2.97	0.74	0.74	0.65	1.39	1.95	—
7	gfdl_cm2_0	1.96	0.64	2.60	1.60	0.119	G	GFDL-ESM2G	3.01	1.24	1.73	0.84	2.57	1.05	—
8	gfdl_cm2_1	1.74	0.73	2.48	1.50	0.120	H	GFDL-ESM2M	3.37	1.38	1.69	0.86	2.56	1.34	—
9	giss_aom	—	—	—	—	0.115	I	HadGEM2-CC	—	—	—	—	—	—	0.114
10	giss_model_e_h	1.46	0.77	2.23	1.60	0.124	J	HadGEM2-ES	2.89	0.62	0.61	0.46	1.07	2.50	0.112
11	giss_model_e_r	—	—	—	—	0.101	K	INM-CM4	3.02	1.46	1.38	0.71	2.10	1.29	—
12	inmcm3_0	1.77	0.48	2.24	1.60	0.109	L	IPSL-CM5A-LR	3.15	0.78	0.82	0.62	1.45	2.04	0.092
13	ipsl_cm4	1.03	0.70	1.73	2.10	—	M	IPSL-CM5A-MR	3.27	0.79	0.84	0.61	1.45	2.03	0.098
14	miroc3_2_hires	0.87	0.56	1.43	2.60	0.121	N	MIROC-ESM	4.24	0.91	1.11	0.70	1.81	2.16	0.118
15	miroc3_2_medres	0.97	0.81	1.77	2.10	0.116	O	MIROC-ESM-CHEM	—	—	—	—	—	—	0.117
16	miub_echo_g	1.56	0.27	1.82	1.70	—	P	MIROC5	4.10	1.50	1.78	0.81	2.59	1.51	0.118
17	mpi_echam5	1.01	0.66	1.67	2.20	0.130	Q	MPI-ESM-LR	4.06	1.11	1.23	0.61	1.85	2.06	0.127
18	mri_cgcm2_3_2a	1.23	0.41	1.63	2.20	0.102	R	MPI-ESM-MR	4.04	1.16	1.35	0.50	1.85	2.04	—
19	ncar_ccsm3_0	1.84	0.67	2.51	1.50	0.118	S	MRI-CGCM3	3.10	1.17	1.38	0.54	1.92	1.56	0.122
20	ncar_pcm1	2.08	0.45	2.52	1.30	0.117	T	NorESM1-M	—	—	—	—	—	—	0.118
21	ukmo_hadcm3	1.09	0.53	1.62	2.00	0.114									
22	ukmo_hadgem1	1.27	0.56	1.87	1.90	—									
	mean	1.43	0.60	2.04	1.81	0.116		mean	3.37	1.04	1.15	0.64	1.79	1.83	0.113
	SD	0.37	0.15	0.38	0.35	0.011		SD	0.50	0.28	0.36	0.12	0.45	0.40	0.009

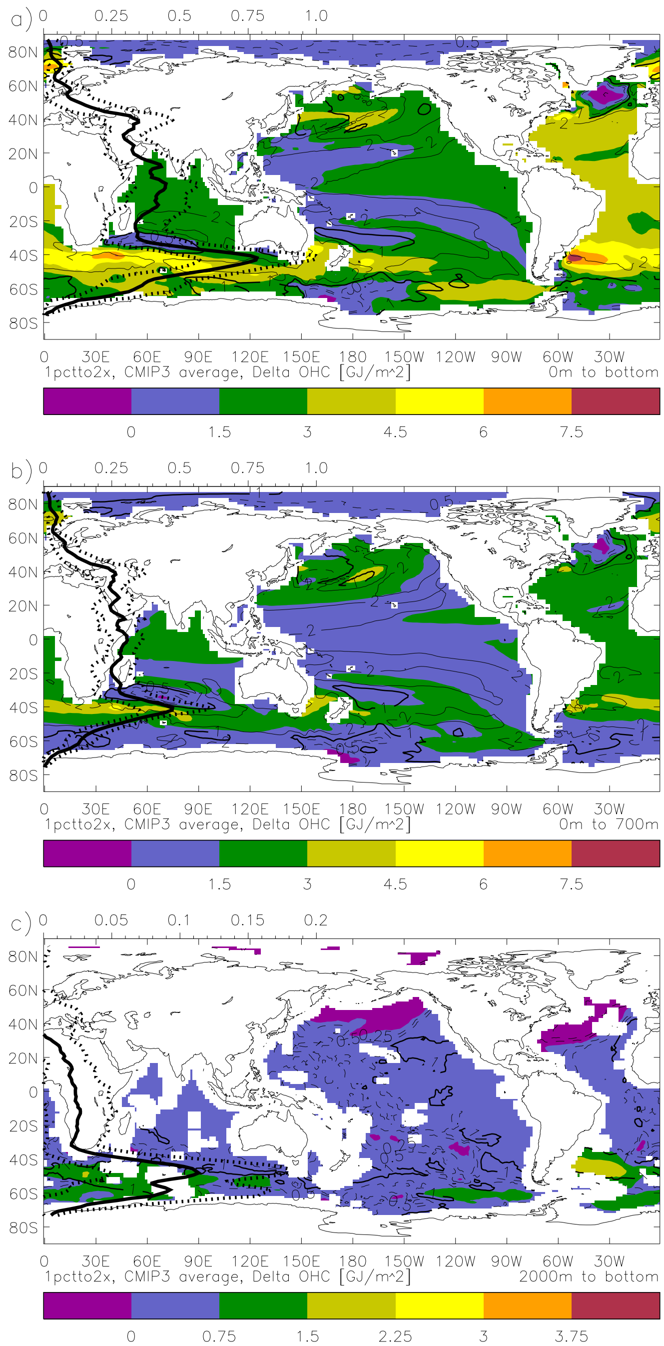

Figure 1: Vertically integrated ocean heat uptake (colour shading; in GJ m^{-2}) in the ensemble mean of the 1%CO_2/year runs of 10 CMIP3 models for (a) the total water column, (b) the upper 700 m and (c) below 2000 m. Thick black line: zonal total in 10^{15} J m^{-1} (scale in the upper left corner), with ± 1 standard deviation (dotted). Note the different scales in (c). Thin black contours show the ratio R of ensemble mean and ensemble standard deviation as in Fig. 4 in the main text. The Southern Ocean dominates heat uptake while the deep water formation regions cool down.

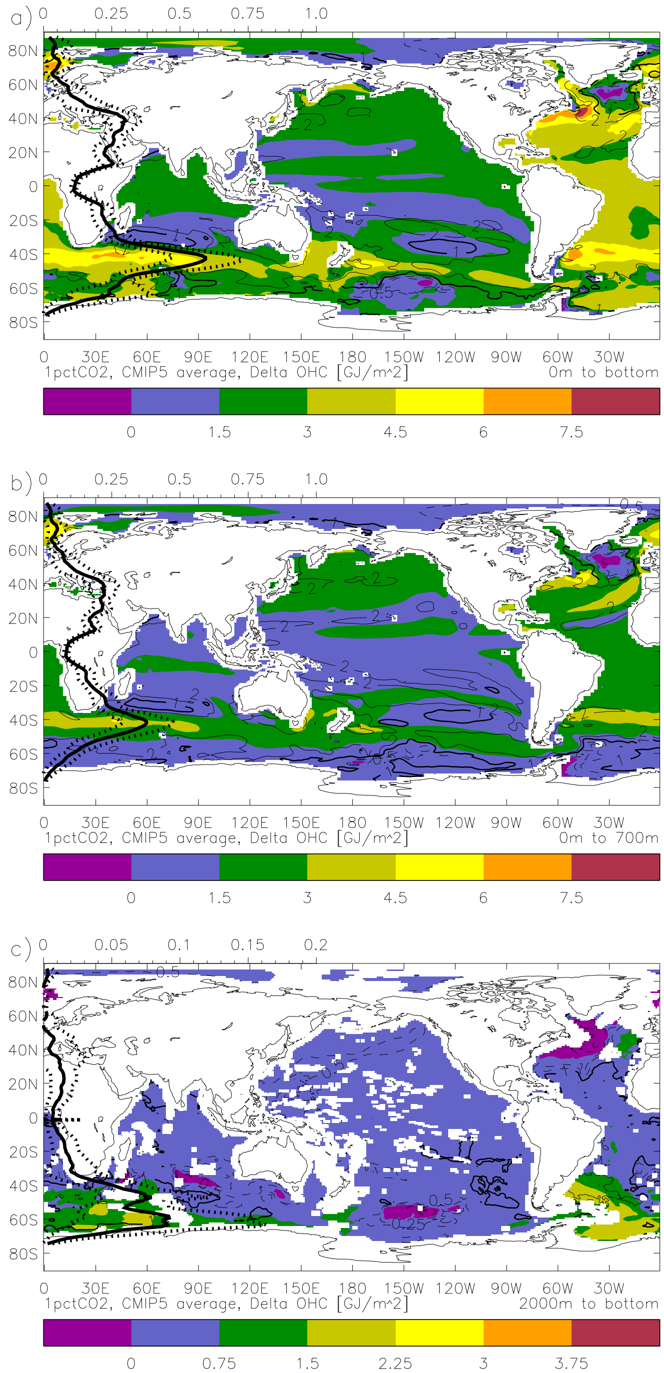

Figure 2: As in Fig. 1, but for the ensemble mean of the 1%CO_2/year runs of 12 CMIP5 models. Note the similarity of the geographical distribution of OHU with the CMIP3 ensemble mean. However, the spread of the zonal total is smaller.