Sea ice formulae

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See Fig. 1 for definition of the principal symbols.

Relationship between thickness of the ice and the freeboard

The total mass of the ice and snow: $M = L^2 t_{\rm ice}\rho_{\rm ice} + L^2 h_{\rm snow}\rho_{\rm snow}$.
Volume of ice beneath water: $V_{\rm sub} = L^2(t_{\rm ice} - f)$.
Mass of sea water displaced: $m_{\rm dis} = \rho_{\rm water} V_{\rm sub}$.
Archimedes principle:

$$\begin{aligned} g m_{\rm dis} &= gM \\ \rho_{\rm water} L^2 (t_{\rm ice} - f) &= L^2 t_{\rm ice}\rho_{\rm ice} + L^2 h_{\rm snow}\rho_{\rm snow} \\ \rho_{\rm water}(t_{\rm ice} - f) &= t_{\rm ice}\rho_{\rm ice} + h_{\rm snow}\rho_{\rm snow}. \end{aligned}$$

Relationships between thickness of the ice and the freeboard:

$$t_{\rm ice} = \frac{h_{\rm snow}\rho_{\rm snow} + f\rho_{\rm water}}{\rho_{\rm water} - \rho_{\rm ice}},$$

$$f = \frac{t_{\rm ice}(\rho_{\rm water} - \rho_{\rm ice}) - h_{\rm snow}\rho_{\rm snow}}{\rho_{\rm water}}.$$

Snow correction to radar measurement of freeboard

Let the time taken for the radar signal to leave the satellite and bounce back from the top of the sea ice be T. Let f_1 be the freeboard found when neglecting the change of speed of light in the snow: $T = 2(D - f_1)/c_{\rm v}$, where $c_{\rm v}$ is the speed of light in vacuum.
Now account for the speed of light in ice, $c_{\rm ice}$: $T = 2\left\{(D - f - h_{\rm snow})/c_{\rm v} + h_{\rm snow}/c_{\rm snow}\right\}$, where $c_{\rm snow}$ is the speed of light in snow.
Develop relationship between the corrected freeboard, f, and f_1:

$$\frac{2(D - f_1)}{c_{\rm v}} = \frac{2(D - f - h_{\rm snow})}{c_{\rm v}} + \frac{2h_{\rm snow}}{c_{\rm snow}}$$

$$f = f_1 + h_{\rm snow}\left(\frac{c_{\rm v}}{c_{\rm snow}} - 1\right).$$

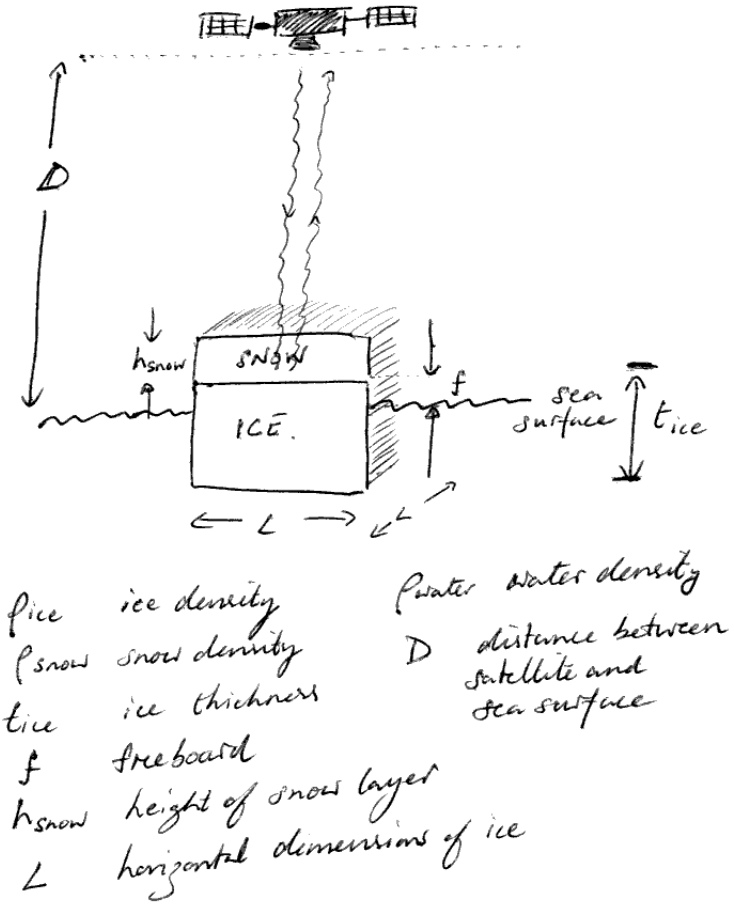

Figure 1: Sea ice schema and definition of the symbols